As Per Latest CBSE Syllabus 2022-23
Issued on 21 April, 2022...

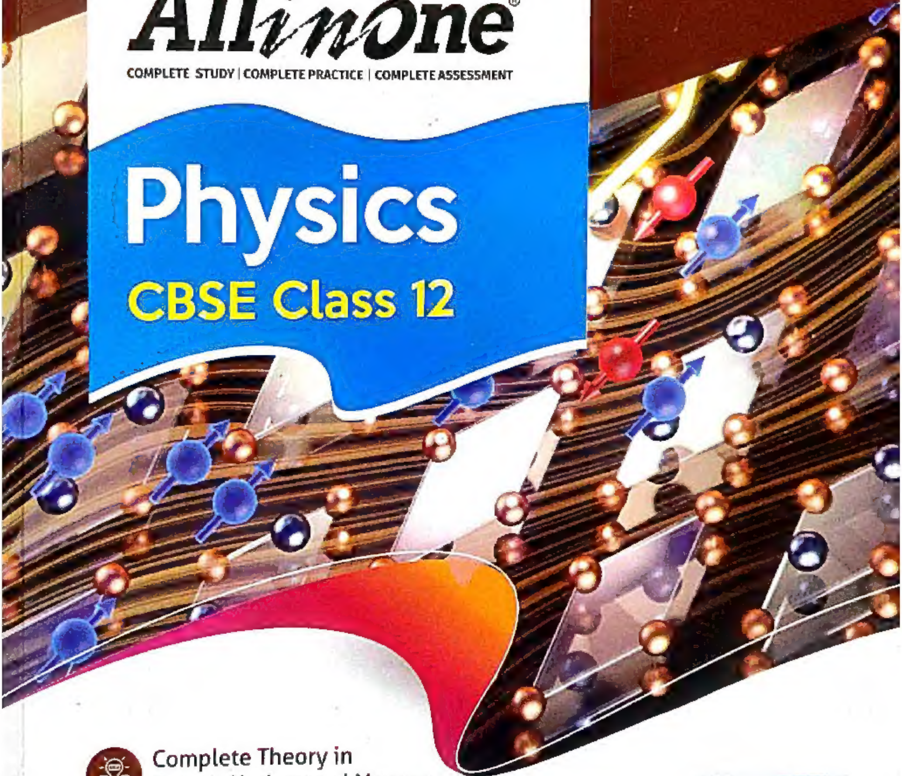
arihant

All in One®

COMPLETE STUDY | COMPLETE PRACTICE | COMPLETE ASSESSMENT

Physics

CBSE Class 12



Complete Theory in Easy to Understand Manner

All Types of Questions ; Including MCQs & Case Based

Included
SAMPLE PAPERS

As Per Latest CBSE Syllabus 2022-23
Issued on 21 April, 2022

All in One®

COMPLETE STUDY | COMPLETE PRACTICE | COMPLETE ASSESSMENT

Physics

CBSE Class 12

Author
Keshav Mohan (M.Sc.)

arihant
ARIHANT PRAKASHAN (School Division Series)

CONTENTS

01

When a glass rod is rubbed with silk, it acquires a power to attract light bodies such as, small pieces of paper. The objects which acquire the attracting power are said to be electrified or charged. Benjamin Franklin demonstrated that lighting was related to static electricity. The branch of Physics which deals with static electricity is called **electrostatics.**

ELECTRIC CHARGES AND FIELDS

All directly experienced forces except the gravitational force are manifestations of electromagnetic force.

Electrostatics deals with study of forces, fields and potentials arising from static charges or charges at rest. In this particular chapter, we will discuss all the above mentioned topics in a detailed form in order to understand them very thoroughly.

CHAPTER CHECKLIST
- Electric Charges
- Coulomb's Law and Electrostatic Field
- Electric Dipole
- Electric Flux

|TOPIC 1|
Electric Charges

The physical property of matter that causes it to experience a force when placed in an electromagnetic field is called electric charge. Electric charge is a characteristic that accompanies fundamental particles, wherever they exist.

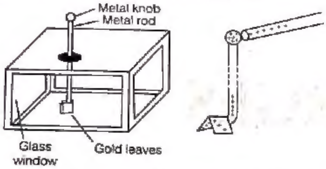
According to William Gilbert, charge is something possessed by material objects that makes it possible for them to exert electrical force and respond to the electrical force.

Electric charge is a scalar quantity.

[redacted]

Gold Leaf Electroscope

It is an instrument which detects the electric charges by means of electrostatic forces. It consists of two gold leaves which are suspended side by side from a conducting rod which is held by an insulated support and placed in a grounded enclosure, such as a glass jar. When a charge is applied to a plate to which the rod is connected, the leaves separate due to their mutual repulsion. One variation involves having one fixed plate along with a single leaf.



There are two kinds of charges such as positive charge and negative charge.

An object can attain positive charge by loosing electrons while other can attain negative charge by gaining electrons. Charges with same sign, i.e. like charges repel each other while charges with opposite sign, i.e. unlike charges attract each other.

Charges always reside on the surface of the charged conducting object. An object can be charged by different methods like friction, conduction and induction.

Charges can be added and subtracted as a number.

Conductors and Insulators

Conductors are those substances which can be used to carry or conduct electric charge/electron from one point to other. They allow electricity to pass through them easily.

e.g. Silver, copper, iron, aluminium, etc.

Insulators are those substances which cannot conduct electricity. They are also called dielectrics. They offer high resistance to the passage of electricity through them,

e.g. Glass, rubber, plastic, ebonite, mica, etc.

Difference between Dielectrics and Conductors

Dielectrics are non-conductors and do not have free electrons at all, while conductors have free electrons in their any volume which makes them able to pass the electricity through them.

Charging by Induction

The process of charging a neutral body by bringing a charged body nearby it without making contact between the two bodies is known as **charging by induction.**

Figures given below are showing the sequential steps of charging a conductor permanently by using the process of charging by induction.

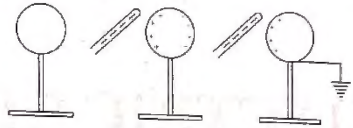


Fig. (a) Fig. (b) Fig. (c)

Using the process of charging by induction, a conductor may be charged permanently.

EXAMPLE |1| A comb run through one's hair attracts small bits of paper. What happens, if the hairs are wet or it is a rainy day?

Sol. If the hairs are wet or it is a rainy day, then the friction between the hair and the comb reduces. The comb does not get charged and it will not attract small bits of paper.

BASIC PROPERTIES OF ELECTRIC CHARGE

Some basic properties of the electric charge are discussed below

Additive Nature of Electric Charge

Electric charge is additive in nature. In general, if a system consists of n charges $q_1, q_2, q_3, \ldots, q_n$, then the total charge of the system will be $q_1 + q_2 + q_3 + \ldots + q_n$.

In order to calculate the net charge on a system, we have to just add algebraically, all the charges present in the system. This is known as the **principle of superposition of charge.**

If the sizes of charged bodies are very small as compared to distance between them, then they can be considered as point charges.

Conservation of Electric Charge

During any process, the net electric charge of an isolated system remains constant (i.e. conserved). In simple words, charge can neither be created nor be destroyed.

[redacted]

In any physical process, the charge may get transferred from one part of the system to another, but the net charge will always remain the same.

Quantisation of Electric Charge

The charge on any body can be expressed as an integral multiple of basic unit of charge, i.e. charge on one electron. This phenomena is called **quantisation of electric charge**.

It can be written as $\boxed{q = \pm ne}$

where, $n = 1, 2, 3, \ldots$ is any integer, positive or negative and e is the basic unit of charge.

The SI unit of charge is called coulomb and denoted by C and its value is $e = 1.602192 \times 10^{-19}$ C or 1.6×10^{-19} C.

EXAMPLE |2| A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.

(i) Estimate the number of electrons transferred from which to which?

(ii) Is there a transfer of mass from wool to polythene?

NCERT

Sol. (i) Here, $q = -3 \times 10^{-7}$ C

∵ Charge on one electron, $e = -1.6 \times 10^{-19}$ C

∴ Number of electrons transferred from wool to polythene piece,

$$n = \frac{q}{e} = \frac{-3 \times 10^{-7}\,\text{C}}{-1.6 \times 10^{-19}\,\text{C}} = 1.875 \times 10^{12}$$

(ii) Yes, there is a transfer of mass from wool to polythene piece.

As, mass of each electron, $m_e = 9 \times 10^{-31}$ kg

∴ Mass transferred from wool to polythene,

$$m = n \times m_e = 1.875 \times 10^{12} \times 9 \times 10^{-31}\,\text{kg}$$
$$= 1.687 \times 10^{-18}\,\text{kg}$$

EXAMPLE |3| A copper slab of mass 2 g contains 2×10^{22} atoms. The charge on the nucleus of each atom is 29 e. What fraction of the electrons must be removed from the sphere to give it a charge of $+2\,\mu$C?

Sol. Total number of electrons in the slab

$$= 29 \times 2 \times 10^{22}$$

Number of electrons removed

$$= \frac{q}{e} = \frac{2 \times 10^{-6}}{1.6 \times 10^{-19}} = 1.25 \times 10^{13}$$

∴ Fraction of electrons removed

$$= \frac{1.25 \times 10^{13}}{29 \times 2 \times 10^{22}} = 2.16 \times 10^{-11}$$

Difference between Charge and Mass

The difference between charge and mass is given in the following table

Charge	Mass
Electric charge on a body may be positive, negative or zero	Mass of a body is a positive quantity
Charge carried by a body does not depend upon velocity of the body.	Mass of a body increases with its velocity as $m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$, where c is velocity of light in vacuum, m is the mass of the body moving with velocity v and m_0 is rest mass of the body.
Charge is quantised	The quantisation of mass is yet to be established
Electric charge is always conserved.	Mass is not conserved as it can be changed into energy and vice-versa
Force between charges can be either attractive or repulsive, as charges are unlike or like charges	The gravitational force between two masses is always attractive

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. One metallic sphere A is given positive charge whereas another identical metallic sphere B of exactly same mass as of A is given equal amount of negative charge. Then,

(a) mass of A and mass of B still remain equal
(b) mass of A increases
(c) mass of B decreases
(d) mass of B increases

2. In general, metallic ropes are suspended from the carriers to the ground which take inflammable material. The reason is

(a) their speed is controlled
(b) to keep the gravity of the carrier nearer to the earth
(c) to keep the body of the carrier in contact with the earth
(d) nothing should be placed under the carrier

[redacted]

3. In charging by induction,
(a) body to be charged must be an insulator
(b) body to be charged must be a semiconductor
(c) body to be charged must be a conductor
(d) any type of body can be charged by induction

4. Charge on a body is q_1 and it is used to charge another body by induction. Charge on second body is found to be q_2 after charging. Then,
(a) $\frac{q_1}{q_2}=1$ (b) $\frac{q_1}{q_2}<1$
(c) $\frac{q_1}{q_2}\leq 1$ (d) $\frac{q_1}{q_2}\geq 1$

5. An object of mass 1 kg contains 4×10^{20} atoms. If one electron is removed from every atom of the solid, the charge gained by the solid of 1 g is
(a) 2.8 C (b) 6.4×10^{-2} C
(c) 3.6×10^{-5} C (d) 9.2×10^{-4} C

6. Number of electrons present in a negative charge of 8 C is
(a) 5×10^{19} (b) 2.5×10^{19}
(c) 12.8×10^{19} (d) 1.6×10^{19}

VERY SHORT ANSWER Type Questions

7. A glass rod when rubbed with silk cloth acquires a charge 1.6×10^{-13} C. What is the charge on the silk cloth?

8. Consider three charged bodies A, B and C. If A and B repel each other and A attracts C, then what is nature of the force between B and C?

9. What does $q_1+q_2=0$ signify in electrostatics?

10. Which property of dielectrics make them different from conductors?

11. Two insulated charged copper spheres A and B of identical size have charges q_A and q_B, respectively. A third sphere C of the same size but uncharged is brought in contact with the first and then in contact with the second and finally removed from both. What are the new charges on A and B?

12. What is the basic cause of quantisation of charge?

13. Can a body has charge 1.5 e, where e is the electronic charge?

14. Which is bigger, a coulomb of charge or a charge on an electron?

15. "An object becomes positively charged through the removal of negatively charged electrons rather than through the addition of positively charged protons". Explain, why?

16. A glass object is charged to +3 nC by rubbing it with a silk cloth. In this rubbing process, have protons been added to the object or have electrons been removed from it?

SHORT ANSWER Type Questions

17. In filling the gasoline tank of an aeroplane, the metal nozzle of hose from the gasoline truck is always carefully connected to the metal body of the aeroplane by a wire, before the nozzle is inserted in the tank. Explain, why?

18. Automobile ignition failure occurs in damp weather. Explain, why?

19. A bird perches on a bare high power line and nothing happens to the bird. A man standing on the ground touches the same line and gets a fatal shock. Why?

20. An ebonite rod held in hand can be charged by rubbing with flannel but a copper rod cannot be charged like this, why?

21. Ordinary rubber is an insulator. But the special rubber tyres of aircrafts are made slightly conducting. Why is this necessary?

22. Why does a charged glass rod attract a piece of paper?

23. Can a charged body attract another uncharged body? Explain.

24. Can two balls having same kind of charge on them attract each other? Explain.

25. Can ever the whole excess charge of a body P be transferred to the other body Q? If yes, how and if not, why?

26. When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain, how this observation is consistent with the law of conservation of charge? NCERT

27. Give any two points of difference between charge and mass.

28. A balloon gets negatively charged by rubbing ceilings of a wall. Does this mean that the wall is positively charged? Why does the balloon eventually fall?

[redacted]

29. A paisa coin is made up of Al-Mg alloys and weighs 0.75 g. It has a square shape and its diagonal measures 17 mm. It is electrically neutral and contains equal amount of positive and negative charges. Treating the paisa coins made up of only Al, find the magnitude of equal number of positive and negative charges. What conclusion do you draw from this magnitude? **NCERT Exemplar**

LONG ANSWER Type I Questions

30. Describe some of the differences between charging by induction and charging by contact.

31. (i) Explain the meaning of the statement "electric charge of a body is quantised".
(ii) Why can one ignore quantisation of electric charge, when dealing with macroscopic, i.e. large scale charges? **NCERT**

32. Two insulated rods A and B are oppositely charged on their ends. They are mounted at the centres, so that they are free to rotate and then held in the position shown in the figure, in a view from above. The rods rotate in the plane of the paper. Will the rods stay in those positions when released? If not, then what position(s) will they move? Will their final configuration(s) be stable?

33. It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, so called 'up' quark (denoted by u) of charge $+\left(\frac{2}{3}\right)e$ and the 'down' quark (denoted by d) of charge $\left(-\frac{1}{3}\right)e$, together with electrons build up ordinary matter. (Other types of quark have also been found which give rise to different unusual varieties of matter). Suggest a possible quark composition of a proton and a neutron. **NCERT**

NUMERICAL PROBLEMS

34. What is the total charge of a system containing five charges $+1, +2, -3, +4$ and -5 in some arbitrary unit?

35. How many electrons are there in one coulomb of negative charge?

36. A metal sphere has a charge of $-6\,\mu C$. When 5×10^{12} electrons are removed from the sphere, what would be net charge on it?

37. A sphere of lead of mass 10 g has net charge -2.5×10^{-9} C.
(i) Find the number of excess electrons on the sphere.
(ii) How many excess electrons are per lead atom? Atomic number of lead is 82 and its atomic mass is 207 g/mol.

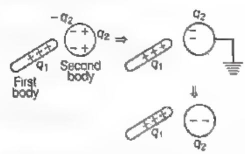
HINTS AND SOLUTIONS

1. (d) When a body is negatively charged more electrons are given to it, so its mass increases

2. (c) During its motion, body of carrier is charged due to rubbing with dry air and dust. If spark occurs near container, then inflammable material may catch fire. So, metallic ropes are suspended so that excess charge flows away from carrier, to ground (for earthing).

3. (c) Induction requires shifting of free charge carrier which are present only in conductors.

4. (d)



Numerically, $q_1 \geq q_2 \Rightarrow \frac{q_1}{q_2} \geq 1$

5. (b) Here, number of electrons removed in 1 g
= number of atoms in 1 g

or $n = \frac{4\times10^{20}}{10^{3}} = 4\times10^{17}$

$\therefore$ Charge, $q = ne = 4\times10^{17}\times1.6\times10^{-19} = 6.4\times10^{-2}$ C

6. (a) Given, charge, $q = 8$ C and
charge of electron, $e = 1.6\times10^{-19}$ C
$\therefore$ Charge, $q = ne$
$\therefore$ Number of electrons, $n = \frac{q}{e} \Rightarrow n = \frac{8}{1.6\times10^{-19}} = 5\times10^{19}$

[redacted]

7. Silk cloth will also acquire a charge 1.6×10^{-13} C. However, it will be negative in nature.
8. It is also attractive in nature
9. The charges q_1 and q_2 are equal and opposite.
10. Dielectrics do not have free electrons at all. They offer high resistance to passage of electricity through them. e.g. Glass, rubber, plastic, etc.
11. When sphere C is brought in contact with A, then charge on sphere C,

$$q_C = \frac{q_A + 0}{2} = \frac{q_A}{2}$$

and new charge on sphere A, $q_A' = \frac{q_A}{2}$

When sphere C is brought in contact with B, then charge on sphere C,

$$q_C' = \frac{q_C + q_B}{2} = \frac{\frac{q_A}{2} + q_B}{2}$$

$$= \frac{q_A + 2q_B}{4}$$

$\therefore$ New charge on sphere B, $q_B' = \frac{q_A + 2q_B}{4}$

12. The basic cause of quantisation of charge is only the integral number of electrons which is transferred from one body to another i.e. $\pm ne$.
13. No, a body cannot have charge $1.5\,e$. It is because the physically existing charge is always an integral multiple of e, i.e. 1.6×10^{-19} C.
14. We know that, $q = ne$

$\Rightarrow \quad 1 = n\times1.6\times10^{-19}$ [given, $q = 1\,\text{C}$]

i.e., $\quad n = \frac{1}{1.6\times10^{-19}} = 6\times10^{18}$

$\therefore$ 1C is the charge of 6×10^{18} electrons.

So, a coulomb of charge is bigger than the charge on an electron.

15. In ordinary matter, a positive charge is much less mobile than a negative charge. For this reason, an object becomes positively charged through the removal of negatively charged electrons rather than through the addition of positively charged protons.
16. Electrons have been removed from the object.
17. Since, the aeroplane and the gasoline truck usually have wheels with rubber tyres, they are insulated from the ground. Further, the service ramps are usually made of concrete and are not necessarily good conductors to the earth. Therefore, inspite of grounding metallic ropes, the aeroplane and the truck could remain charged.
18. The insulating porcelain of the spark plugs accumulates a film of dirt.

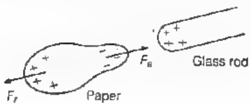
The surface dirt is hygroscopic and picks up moisture from the air. Therefore, in humid weather, the insulating porcelain of the plugs becomes quasi-conductor.

This allows an appreciable proportion of the spark to leak across the surface of the plug instead of discharging across the gap

19. When a bird is perched on a bare high power line, the circuit does not get completed between the bird and the earth, therefore nothing happens to the bird.

When a man standing on ground touches the same line, the circuit between the man and the earth gets completed. As a result, he gets a fatal shock.

20. Both the human body and the copper rod conduct electricity. When it is attempted to charge a copper rod by rubbing, the charge flows from the rod to the earth through the hand. However, when ebonite rod is charged by rubbing, the charges so produced stay on the ebonite rod as it is a bad conductor of electricity.
21. During landing or take off, the tyres of aircrafts get charged due to the friction between tyres and ground. In case the tyres are slightly conducting, the charge developed on the tyres will not stay on them and it finds its way to the earth
22. Paper is a dielectric, so when a positively charged glass rod is brought near it, atoms of paper get polarised, with centre of negative charge of atoms coming closer to the glass rod.



Therefore, force of attraction F_a between glass rod and piece of paper becomes greater than the force of repulsion F_r between the glass rod and the piece of paper. This results in attraction of the piece of paper towards the glass rod.

23. Yes, because when a charged body is brought near to uncharged body, opposite kind of induced charge is produced on an uncharged body. Therefore, the charged body attracts the uncharged body.
24. Yes, two balls having same kind of charge can attract each other. If any one of them has more charge as compared to the other, then due to the induction, they induce opposite kind of charges on the faces of each other when they are brought nearer. Therefore, they behave as oppositely charged balls and hence they attract each other
25. Yes, the whole charge of a body P can be transferred to a conducting body Q, when P is enclosed by Q and is connected to it. This is because the charge always resides on the outer surface of the conductor.

[redacted]

26. When a glass rod is rubbed with a silk cloth, charges appear on both, these charges are equal in magnitude and opposite in sign so that algebraic sum of the charges produced on both is zero. The net charge on the two bodies was zero even before rubbing them. Thus, we find that charges can be created only in equal and unlike pairs. This is consistent with the law of conservation of charge

27. Refer to text on page 3

28. No this does not imply that the wall is positively charged The balloon induces a charge of opposite sign in the ceiling of the wall, causing the balloon and the ceiling to be attracted to each other. The balloon eventually falls because its charge slowly diminishes as it leaks to ground. Some of the charge on the balloon could also be lost due to the presence of positive ions in the surrounding atmosphere, which would tend to neutralise the negative charges on the balloon.

29. Given, mass of a paisa coin, $m = 0.75$ g

Atomic mass of aluminium, $M = 26.9815$ g

Length of the diagonal of square shaped paisa coin = 17 mm

Avogadro's number, $N_A = 6.023 \times 10^{23}$

$$\Rightarrow \quad n = \frac{N_A}{M} \times m = \frac{6.023 \times 10^{23}}{26.9815 \text{ g}} \times 0.75 \text{ g} = 1.6742 \times 10^{22}$$

Since, atomic number (Z) of Al is 13, therefore each atom of Al contains 13 protons and 13 electrons. Now, find out the magnitude of positive and negative charges present in one paisa coin.

$$nZe = 1.6742 \times 10^{22} \times 13 \times 1.6 \times 10^{-19} \text{ C} = 34.8 \text{ kC}$$

Now, write the conclusion drawn from this magnitude of charge.

34.8 kC is a very large amount of charge. This concludes that ordinary neutral matter contains an enormously large amount of positive and negative charges.

30. (i) When an object is charged by induction, there is no physical contact between the object being charged and the object used to do the charging. In contrast, charging by contact, as the name implies, involves the direct physical contact to transfer charge from one object to the another

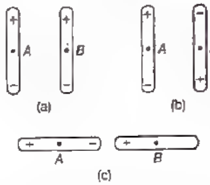
(ii) When an object is charged by induction, the sign of the charge that the object acquires is opposite to that of the object used to do the charging. Charging by contact gives the object being charged the same sign of charge as the original charged object

31. (i) Refer to text on page 3.

(ii) In practice, the charge on a charged body is very large. On the other hand, the charge on an electron is very small. When electrons are added to a body or removed from a body, the change taking place in the total charge on the body is so small that the charge seems to be varying in a continuous manner. Therefore, quantisation of electric charge can be ignored, when dealing with a large scale charged body.

32. Initially, the configuration shown is unstable. The negative charges repel each other. If there is any slight rotation of one of the rods, the repulsion can result in further rotation away from this configuration.

There are three possible final configurations as shown below.



Configuration (A) is stable. If the positive upper ends of both the rods are pushed towards each other, then their mutual repulsion will move the system back to the original configuration. Configuration (B) is an equilibrium configuration. but it is unstable. If the lower ends of both the rods are moved towards each other, then their mutual attraction will be larger than that of the upper ends and thus, the configuration will shift to (c), another possible stable configuration.

33. For the protons, the charge on proton is $+e$.

If the number of up quarks are a, then the number of down quarks are $(3-a)$ as the total number of quarks are 3. So, $a \times$ up quark charge + $(3-a)$ down quark charge) = $+e$

$$a \times \left(\frac{2}{3}e\right) + (3-a)\left(-\frac{e}{3}\right) = e$$

$$\Rightarrow \quad \frac{2ae}{3} - \frac{(3-a)e}{3} = e \Rightarrow 2a - 3 + a = 3$$

$$\Rightarrow \quad 3a = 6$$

$$\Rightarrow \quad a = 2$$

Thus, in the proton, there are two up quarks and one down quark

$\therefore$ Possible quark composition for proton = uud.

For the neutron, the charge on neutron is 0.

Let the number of up quarks be b and the number of down quarks be $3-b$.

So, $b \times$ up quark charge + $(3-b)$ down quark charge = 0

$$\Rightarrow \quad b\left(\frac{2e}{3}\right) + (3-b)\left(-\frac{e}{3}\right) = 0$$

$$\Rightarrow \quad 2b - 3 + b = 0$$

$$\Rightarrow \quad 3b = 3$$

$$\Rightarrow \quad b = 1$$

Thus, in neutron, there is one up quark and two down quarks.

$\therefore$ Possible quark composition for neutron = udd.

[redacted]

34. As charges are additive in nature, i.e. the total charge of a system is the algebraic sum of all the individual charges located at different points inside the system, i.e.

$$q_{net} = q_1 + q_2 + q_3 + q_4 + q_5$$

$\therefore$ Total charge $= +1+2-3+4-5 = -1$ in the same unit.

35. The negative charge is due to the presence of excess electrons. Because an electron has a charge whose magnitude is $e = 1.6\times10^{-19}$ C, the number of electrons is equal to the charge q divided by the charge e on each electron.

Therefore, the number of electrons is

$$n = \frac{q}{e} = \frac{1.0}{1.6\times10^{-19}} = 6.25\times10^{18} \text{ electrons}$$

36. Here, $q_1 = -6\,\mu C$

and $q_2 = ne = 5\times10^{12}\times(1.6\times10^{-19})$

$= 8.0\times10^{-7}$ C

$= 0.8\times10^{-6}$ C $= 0.8\,\mu$C

Since, electrons are removed from the sphere, q_2 is positive.

Therefore, net charge on the sphere,

$q = q_1 + q_2$

$= (-6.0 + 0.8)\,\mu$C

$= -5.2\,\mu$C

37. (i) The charge of an electron $= -1.6\times10^{-19}$C

Net charge on sphere $= -2.5\times10^{-9}$C

So, the number of excess electrons

$$= \frac{-2.5\times10^{-9}\text{ C}}{-1.6\times10^{-19}\text{ C}} = 1.5\times10^{10} \text{ electrons}$$

(ii) Atomic number of lead is 82.

Atomic mass of lead is 207 g/mol.

$\therefore$ 10 g of lead will have

$$\frac{10\text{ g}}{207\text{ g / mol}}\times6.02\times10^{23} \text{ atoms/mol}$$

$= 2.91\times10^{22}$ atoms

$\therefore$ The number of excess electrons per atom

$$= \frac{1.56\times10^{10}}{2.91\times10^{22}}$$

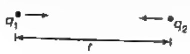
$= 5.36\times10^{-13}$ electrons

|TOPIC 2|

Coulomb's Law and Electrostatic Field

COULOMB'S LAW

The force of interaction (attraction or repulsion) between two stationary point charges in vacuum is directly proportional to the product of the charges and inversely proportional to the square of distance between them. Mathematically, electrostatic force between two stationary charges is given by



$$\boxed{F = \frac{k|q_1q_2|}{r^2}}$$

where, k is a proportionality constant.

In SI unit, k is given by

$$k = \frac{1}{4\pi\varepsilon_0}$$

$$= 9\times10^9 \text{ N-m}^2\text{C}^{-2}$$

where, $\varepsilon_0 = 8.85\times10^{-12}$ C^{2}N^{-1}m^{-2} and is called the permittivity of free space.

i.e. $$F = 9\times10^9\frac{|q_1q_2|}{r^2}$$

The Coulomb force acts along the straight line connecting the points of location of the charges. It is central and spherically symmetric.

If $q_1 = q_2 = 1$C

and $r = 1$ m

Then, $$F = 9\times10^9\frac{1\times1}{(1)^2}$$

$$\boxed{F = 9\times10^9 \text{ N}}$$

i.e. **One coulomb** is the charge, that when placed at distance of 1m from another charge of same magnitude in vacuum, experiences an electric force of repulsion of magnitude 9×10^9 N. Coulomb is a bigger unit, in practice we use smaller units like mC or μC.

[redacted]

Absolute Permittivity of Medium (Dielectric Constant)

The force between two charges q_1 and q_2 located at a distance r apart in a medium may be expressed as,

$$F_{\text{medium}} = \frac{1}{4\pi\varepsilon}\frac{|q_1 q_2|}{r^2}$$

where, ε is absolute permittivity of the medium.

Now, $$\frac{F_{\text{vacuum}}}{F_{\text{medium}}} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}}{\frac{1}{4\pi\varepsilon}\frac{|q_1 q_2|}{r^2}} = \frac{\varepsilon}{\varepsilon_0}$$

The ratio $\frac{\varepsilon}{\varepsilon_0}$ is denoted by ε_r, which is called relative permittivity of the medium with respect to vacuum. It is also denoted by K called dielectric constant of the medium. It has no unit being a ratio.

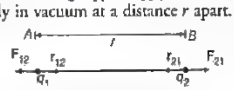
$\therefore$ $$K \text{ (or } \varepsilon_r) = \frac{\varepsilon}{\varepsilon_0} = \frac{F_{\text{vacuum}}}{F_{\text{medium}}}$$

$\Rightarrow$ $$\varepsilon = K\varepsilon_0$$

$\therefore$ $$\boxed{F_{\text{medium}} = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{Kr^2}}$$

Coulomb's Law in Vector Form

Consider two like charges q_1 and q_2 present at points A and B respectively in vacuum at a distance r apart.



Coulomb force between two charges

According to Coulomb's law, the magnitude of force on charge q_1 due to q_2 (or on charge q_2 due to q_1) is given by

$$F_{12} = F_{21} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2} \quad \text{...(i)}$$

Let $\hat{\mathbf{r}}_{12}$ be the unit vector pointing from charge q_1 to q_2.

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{21} \quad \text{...(ii)}$$

[$\because$ $\mathbf{F}_{12}$ is along the direction of unit vector $\mathbf{r}_{21}$]

Also, $\mathbf{r}_{21}$ be the unit vector pointing from charge q_2 to q_1.

$\therefore$ $$\mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{12} \quad \text{...(iii)}$$

[$\because$ $\mathbf{F}_{21}$ is along the direction of unit vector $\mathbf{r}_{12}$]

$\therefore$ $$\hat{\mathbf{r}}_{21} = -\hat{\mathbf{r}}_{12}$$

Eq. (ii) becomes,

$$\mathbf{F}_{12} = \frac{-1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r^2}\hat{\mathbf{r}}_{12} \quad \text{...(iv)}$$

On comparing Eq. (iii) with Eq. (iv), we get

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

i.e. Coulomb's law agrees with Newton's third law.

Comparison of Coulomb's Law with Gravitational Law

Both the Coulomb's and Newton's law follow inverse square law. According to Newton's universal law of gravitation, "every body in the universe attracts every other body with a force which is directly proportional to the product of the masses of two bodies and inversely proportional to the square of distance between them." i.e.

$$F = \frac{Gm_1 m_2}{r^2}$$

As discussed earlier, according to Coulomb's law

$$F = \frac{kq_1 q_2}{r^2}$$

The electric force is much stronger than the gravitational force between two electrons.

i.e. $$F_E = 10^{39} F_G$$

EXAMPLE |1| What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air? **NCERT**

Sol. Given, $q_1 = 2\times10^{-7}$ C

$q_2 = 3\times10^{-7}$ C

$r = 30$ cm

$= 30\times10^{-2}$ m

$= 0.3$ m

$k = 9\times10^9$ N-m^{2}C^{-2}

$F = ?$

We have, $$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2} = \frac{k|q_1 q_2|}{r^2} \quad \left[\because \frac{1}{4\pi\varepsilon_0} = k\right] \text{...(i)}$$

Substituting the given values in Eq. (i), we get

$$F = \frac{(9\times10^9\,\text{N-m}^2\text{C}^{-2})(2\times10^{-7}\,\text{C})\times(3\times10^{-7}\,\text{C})}{(0.3\,\text{m})^2}$$

$\therefore$ $F = 6\times10^{-3}$ N

This force is repulsive, since the spheres have same charges.

[redacted]

EXAMPLE |2| The sum of two point charges is $7\mu C$. They repel each other with a force of 1 N when kept 30 cm apart in free space. Calculate the value of each charge.

Foreign 2009

Sol. Let one of two charges be $x\mu C$. Therefore, other charge will be $(7-x)\mu C$.

By Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2}$$

$$\Rightarrow \quad 1 = 9\times10^9\times\frac{(x\times10^{-6})(7-x)\times10^{-6}}{(0.3)^2}$$

$$\Rightarrow \quad 9\times10^{-2} = 9\times10^{9-12}x(7-x)$$

$$\Rightarrow \quad 10 = x(7-x)$$

$$\Rightarrow \quad x^2-7x+10=0$$

$$\Rightarrow \quad (x-2)(x-5)=0$$

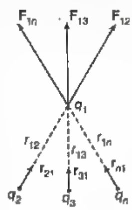
$$\therefore \quad x = 2\mu C \quad \text{or} \quad 5\mu C$$

Therefore, charges are $2\mu C$ and $5\mu C$.

FORCES BETWEEN MULTIPLE CHARGES: SUPERPOSITION PRINCIPLE

According to the superposition principle, forces on any charge due to number of other charges is the vector sum of all the forces on that charge due to other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges.

Consider a system of n point charges $q_1, q_2, q_3, \ldots, q_n$ be distributed in space in a discrete manner. The charges are interacting with each other. Let the charges be $q_2, q_3, \ldots, q_n$ exert forces $\mathbf{F}_{12}, \mathbf{F}_{13}, \ldots, \mathbf{F}_{1n}$, respectively on charge q_1.



Then, according to the principle of superposition, the total force on charge q_1 is given by

$$\mathbf{F}_1 = \mathbf{F}_{12}+\mathbf{F}_{13}+\ldots+\mathbf{F}_{1n} \quad \ldots(i)$$

If the distance between the charges q_1 and q_2 is denoted as r_{12} and $\hat{\mathbf{r}}_{21}$ is unit vector from charge q_2 to q_1, then

$$\mathbf{F}_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r_{21}^2}\hat{\mathbf{r}}_{21}$$

Similarly, the force on charge q_1 due to other charges is given by

$$\mathbf{F}_{13} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_3}{r_{31}^2}\hat{\mathbf{r}}_{31}, \quad \mathbf{F}_{1n} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_n}{r_{n1}^2}\hat{\mathbf{r}}_{n1}$$

Substituting these values in Eq.(i), we get

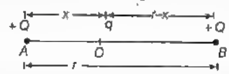
$$\mathbf{F}_1 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1q_2}{r_{21}^2}\hat{\mathbf{r}}_{21}+\frac{q_1q_3}{r_{31}^2}\hat{\mathbf{r}}_{31}+\cdots+\frac{q_1q_n}{r_{n1}^2}\hat{\mathbf{r}}_{n1}\right)$$

$$\Rightarrow \quad \mathbf{F}_{1r} = \frac{q_1}{4\pi\varepsilon_0}\sum_{i=2}^{n}\frac{q_i}{r_{i1}^2}\hat{\mathbf{r}}_{i1}$$

Note This force is on the charge which is to be studied due to other charges

EXAMPLE |3| Two charges each of $+q$ Coulomb are placed along a line. A third charge $-q$ is placed between them. At what position will the system be in equilibrium?

Sol.



For charge $-q$ to be in equilibrium, force on the charge q at point O due to the charge $+Q$ at point A should be equal and opposite to that due to the charge $+Q$ at the point B, i.e. $\frac{1}{4\pi\varepsilon_0}\frac{Qq}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{(r-x)^2}$

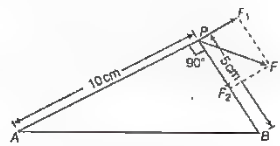
$$\Rightarrow \quad x^2 = (r-x)^2$$

$$\text{or} \quad x = r-x$$

$$\therefore \quad x = \frac{r}{2}$$

Hence, for equilibrium the charge $-q$ should be kept at the middle of the line joining the points A and B.

EXAMPLE |4| Find the magnitude of the resultant force on a charge of $1\mu C$ held at P due to two charges of $+2\times10^{-8}$ C and -10^{-8} C at A and B, respectively. Given, $AP = 10$ cm, $BP = 5$ cm and $\angle APB = 90°$.



Sol. Here, charge at P, $q = 1\mu C = 10^{-6}$ C

Charge at A, $q_1 = 2\times10^{-8}$ C

Charge at B, $q_2 = -10^{-8}$ C

$AP = 10\text{ cm} = 0.1\text{ m}, BP = 5\text{ cm} = 0.05\text{ m}$,

$\angle APB = 90°, F = ?$

Force at P due to q_1 charge at A, $F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q}{AP^2}$, along

AP produced $= \frac{9\times10^9\times2\times10^{-8}\times10^{-6}}{(0.1)^2} = 18\times10^{-3}$ N

[redacted]

Force at P due to q_2 charge at B, $F_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2 q}{BP^2}$, along

PB produced $= \frac{9\times 10^9 \times -10^{-8}\times 10^{-6}}{(0.05)^2} = -36\times 10^{-3}$ N

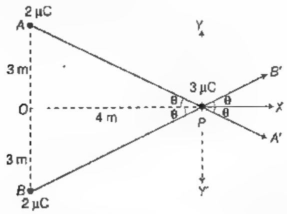
As, angle between F_1 and F_2 is 90°.

$\therefore$ Resultant force,

$$F = \sqrt{F_1^2 + F_2^2} = \sqrt{(18\times 10^{-3})^2 + (-36\times 10^{-3})^2}$$
$$= \sqrt{(324+1296)\times 10^{-6}}$$
$$= \sqrt{1620\times 10^{-6}}$$
$$= 40.2\times 10^{-3}$$
$$= 4.0\times 10^{-2}\text{ N}$$

EXAMPLE |5| Two equal positive charges, each of 2μC interact with a third positive charge of 3μC situated as shown in figure. Calculate the magnitude and direction of the force on the 3μC charge. **NCERT**

Sol. In the figure, $OA = OB = 3$ m, $OP = 4$ m



$\therefore\quad AP = BP = \sqrt{3^2+4^2} = 5$ m

According to Coulomb's law,

force on charge at P due to charge at A,

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{AP^2}$$
$$= \frac{9\times 10^9\times(2\times 10^{-6})\times(3\times 10^{-6})}{5^2} = \frac{54}{25}\times 10^{-3}$$
$$= 2.16\times 10^{-3}\text{ N, along } PA'.$$

It has two rectangular components $F_1\cos\theta$ along PX and $F_1\sin\theta$ along PY'

Similarly, force on charge at P due to charge at B, $F_2 = F_1$ (in magnitude). It is along PB'. It also has two rectangular component $F_2\cos\theta$ along PX and $F_2\sin\theta$ along PY.

The components along PY and PY' cancel. The components along PX add up.

$\therefore$ Total force on 3 μC charge is

$$F = 2F_1\cos\theta$$
$$= 2\times 2.16\times 10^{-3}\times\frac{4}{5}$$
$$= 3.5\times 10^{-3}\text{ N, along } PX.$$

ELECTROSTATIC FORCE DUE TO CONTINUOUS CHARGE DISTRIBUTION

The region in which charges are closely spaced is said to have continuous distribution of charge. Continuous charge distribution is of three types; linear charge distribution (one dimensional), surface charge distribution (two dimensional) and volume charge distribution (three dimensional).

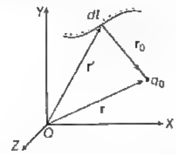
Linear Charge Density

Linear charge density is defined as the charge per unit length of linear charge distribution.

i.e. $\quad\lambda = \frac{dq}{dl}$

Its SI unit is coulomb/metre.

Electric force at a point due to a linear charge distribution is given by $\quad \mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int_l \frac{\lambda\, dl}{r_0^2}\hat{\mathbf{r}}_0$



where, $\mathbf{r}_0 = \mathbf{r}-\mathbf{r}'$, $\mathbf{r}'$ is the position vector of length element dl with respect to origin and $\mathbf{r}$ is the position vector of charge q_0 with respect to origin.

Surface Charge Density

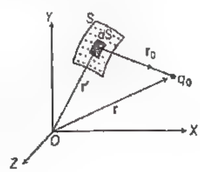
Surface charge density is defined as the charge per unit surface area of surface charge distribution.

i.e. $\quad\sigma = \frac{dq}{dS}$

Its SI unit is coulomb/metre^2.

Electric force at a point due to a surface charge distribution is given by

$$\mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int_S \frac{\sigma\, dS}{r_0^2}\hat{\mathbf{r}}_0$$



where, $\mathbf{r}_0 = \mathbf{r}-\mathbf{r}'$, $\mathbf{r}'$ is the position vector of surface element dS with respect to origin and $\mathbf{r}$ is the position vector of charge q_0 with respect to origin.

[redacted]

Volume Charge Density

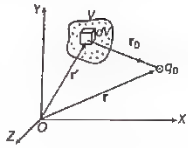
Volume charge density is defined as the charge per unit volume of volume charge distribution.

i.e. $\delta = \dfrac{dq}{dV}$

Its SI unit is coulomb/metre^3.

Electric force at a point due to volume charge distribution is given by

$$\mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int_V \frac{\rho\, dV}{r_0^2}\,\hat{\mathbf{r}}_0$$



where, $\mathbf{r}_0 = \mathbf{r} - \mathbf{r}'$, $\mathbf{r}'$ is the position vector of volume element dV with respect to origin and $\mathbf{r}$ is the position vector of charge q_0 with respect to origin.

EXAMPLE |6| What charge would be required to electrify a sphere of radius 25 cm, so as to get a surface charge density of $\dfrac{3}{\pi}$ Cm^{-2}?

Sol. Here, $r = 25\text{ cm} = 0.25\text{ m}, \sigma = \dfrac{3}{\pi}\text{ Cm}^{-2}$

As, $\sigma = \dfrac{q}{4\pi r^2}$

$\therefore \quad q = 4\pi r^2\sigma = 4\pi \times (0.25)^2 \times \dfrac{3}{\pi}\text{C} = 0.75\text{C}$

EXAMPLE |7| The radius of gold nucleus ($Z = 79$) is about 7.0×10^{-15}m. Assuming that the positive charge is distributed uniformly throughout the nuclear volume, find the volume charge density.

Sol. The total positive charge in the nucleus is given by

$$q = +Ze = 79 \times 1.6 \times 10^{-19}\text{C}$$

$\therefore$ Volume charge density, $\delta = \dfrac{q}{\frac{4}{3}\pi R^3}$

$$= \frac{79 \times 1.6 \times 10^{-19}}{4/3 \times 3.14 \times (7.0 \times 10^{-15})^3}$$

$$= 0.088 \times 10^{26}$$

$$= 8.8 \times 10^{24}\text{ Cm}^{-3}$$

ELECTRIC FIELD

The electric field due to a charge Q at a point in space may be defined as the force that a unit positive charge would experience if placed at that point.

The charge Q which produces the electric field is called source charge and the charge q which experiences the effect of source charge is called test charge.

Electric Field Intensity

The electric field intensity at any point due to source charge is defined as the force experienced per unit positive test charge placed at that point without disturbing the source charge.

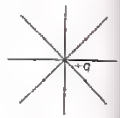
It is expressed as, $\mathbf{E} = \dfrac{\mathbf{F}}{q_0}$

where, E = electric field intensity and

and F = force experienced by the test charge q_0.

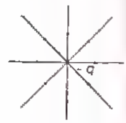
It is a vector quantity and its SI unit is NC^{-1}.

The figure (a) is representing the electric field due to charge $+q$. In this, it can be seen that for a positive charge, the electric field vector is directed radially outwards, i.e. away from positive charge.



(a)

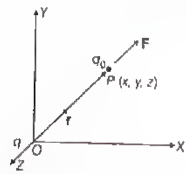
The figure (b) is representing the electric field due to charge $-q$. In this, it can be seen that for a negative charge, the electric field vector is directed radially inwards, i.e. towards negative charge.



(b)

Electric Field due to a Point Charge

We have to find the electric field at a point P due to a point charge $+q$ placed at the origin such that OP = r.



Electric field due to a point charge in coordinate frame

To find the electric field at point P, we have to find the electric force on a test charge q_0 placed at point P, due to source charge q.

According to Coulomb's law, force on the test charge q_0 due to charge q is given by

[redacted]

$$F = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{r}$$

If E is the electric field at a point P, then

$$\mathbf{E} = \lim_{q_0 \to 0}\frac{\mathbf{F}}{q_0} = \lim_{q_0 \to 0}\left(\frac{1}{q_0}\cdot\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{r}\right)$$

$$\Rightarrow \quad \mathbf{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\hat{r} \qquad \text{...(i)}$$

The magnitude of the electric field at a point P is given by

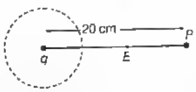
$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}}$$

From the above formula, it is clear that electric field at any point in space due to a charge depends only on the distance. That means, the magnitude of electric field due to point charge is same at all the points of sphere, i.e. it has spherical symmetry.

EXAMPLE |8| A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5×10^3 N/C and points radially inwards, then what is the net charge on the sphere? **NCERT**

Sol. Let the value of unknown charge be q.

Electric field at 20 cm away, $E = 1.5 \times 10^3$ N/C



From the formula, electric field,

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}$$

$$\Rightarrow 1.5 \times 10^3 = \frac{9 \times 10^9 \times q}{(20 \times 10^{-2})^2}$$

$$\therefore \quad q = \frac{1.5 \times 10^3 \times 20 \times 20 \times 10^{-4}}{9 \times 10^9} = 6.67 \times 10^{-9}\,\text{C}$$

As the electric field is radially inwards which shows that the nature of unknown charge q is negative.

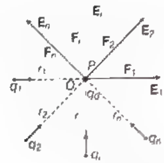
Electric Field due to System of Charges

Consider that n point charges $q_1, q_2, q_3, \ldots, q_n$ exert forces $\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \ldots, \mathbf{F}_n$ on a test charge q_0 placed at origin O.

Let $\mathbf{F}_i$ be the force due to ith charge q_i on q_0, then

$$\mathbf{F}_i = \frac{1}{4\pi\varepsilon_0}\frac{q_i q_0}{r_i^2}\hat{r}_i$$

where, r_i is the distance of the test charge q_0 from q_i.



The electric field at the observation point P is given by

$$\mathbf{E}_i = \lim_{q_0 \to 0}\frac{\mathbf{F}_i}{q_0} = \lim_{q_0 \to 0}\left[\frac{1}{q_0}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i q_0}{r_i^2}\hat{r}_i\right)\right]$$

$$\mathbf{E}_i = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i}{r_i^2}\hat{r}_i \qquad \text{...(i)}$$

If E is electric field at point P due to the system of charges, then by principle of superposition of electric fields,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \cdots + \mathbf{E}_n = \sum_{i=1}^{n}\mathbf{E}_i$$

Using Eq. (i), we get

$$\mathbf{E} = \sum_{i=1}^{n}\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i}{r_i^2}\hat{r}_i$$

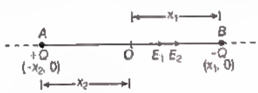
or
$$\boxed{\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i}$$

$\therefore$ E is a vector quantity.

EXAMPLE |9| Two charges $+Q$ and $-Q$ are kept at points $(-x_2, 0)$ and $(x_1, 0)$ respectively, in the XY-plane. Find the magnitude and direction of the net electric field at the origin $(0, 0)$. **All India 2009**

> **Hints:** To find the electric field intensity at a point due to two charges, first of all find the individual electric field due to both charges and then find the resultant field by using vector addition.

Sol.



Electric field intensity at point O due to $+Q$ charge,

$$E_1 = \frac{1}{4\pi\varepsilon_0}\times\frac{Q}{(x_2)^2}\ \text{(towards } B\text{)} \qquad \text{...(i)}$$

Electric field intensity at point O due to $-Q$ charge,

$$E_2 = \frac{1}{4\pi\varepsilon_0}\times\frac{Q}{(x_1)^2}\ \text{(towards } B\text{)} \qquad \text{...(ii)}$$

$\because E_1$ and E_2 act along the same direction.

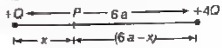
[redacted]

$\therefore$ Net electric field intensity at point O is given by

$$E = E_1 + E_2$$
$$= \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{(x_2)^2} + \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{(x_1)^2} \text{ (towards}B)$$
$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{x_2^2} + \frac{1}{x_1^2}\right]$$

EXAMPLE |10| Two point charges $+Q$ and $+4Q$ are separated by a distance of $6a$. Find the point on the line joining the two charges, where the electric field is zero.

Sol. The electric field is zero at point P only, if the field due to charge $+Q$ balances the field due to charge $+4Q$.



$$\therefore \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{x^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4Q}{(6a - x)^2} \Rightarrow \frac{1}{x^2} = \frac{4}{(6a - x)^2}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{2}{(6a - x)}$$

$$\Rightarrow \quad 2x = 6a - x$$

$$\Rightarrow \quad x = 2a$$

$\therefore$ The required point is at a distance of $2a$ from $+Q$.

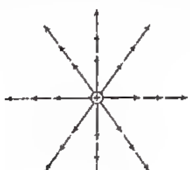
Physical Significance of Electric Field

The physical significance of electric field is that we can readily calculate the magnitude and direction of force experienced by any charge q_0 placed at a point by knowing the electric field intensity at that point.

ELECTRIC FIELD LINES

An electric field line in general is a curve drawn in such a way that the tangent to it at each point is in the direction of the electric field at that point. A field line is a space curve, i.e. a curve in three dimensions.

Electric field lines are thus used to pictorially map the electric field around a charge or a configuration of charges.



Field lines showing electric field of a point charge

The density of field lines is more near the charge. Away from the charge, the field is weak, so the density of field lines is less.

Properties of Electric Field Lines

Electric field lines follow some important properties which are discussed below

(i) Electric field lines start from positive charges and end at negative charges. In the case of a single charge, they may start or end at infinity.

(ii) Tangent to any point on electric field lines shows the direction of electric field at that point.

(iii) Two field lines can never intersect each other because if they intersect, then two tangents drawn at that point will represent two directions of field at that point, which is not possible.

(iv) In a charge free region, electric field lines can be taken to be continuous curves without any breaks.

(v) Electric field lines do not form closed loops (because of conservative nature of electric field).

(vi) Electric field lines are perpendicular to the surface of a charged conductor.

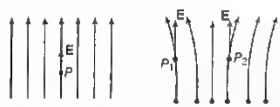
(vii) Electric field lines contract lengthwise to represent attraction between two unlike charges.

(viii) Electric field lines exert sideways pressure to represent repulsion between two like charges.

Note Electric field lines and its properties have been generally asked in the form of questions in previous years All India 2014, 2011, Delhi 2012

Representations of Electric Field

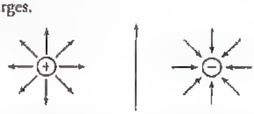
For different types of electric field, lines are represented as shown below



Electric field lines for a uniform field

Electric field lines for a non-uniform field

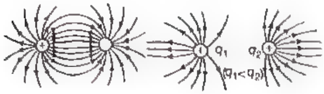
The electric field lines start from positive charges and end at negative charges.



Direction is away from the positive charge

Direction is towards the negative charge

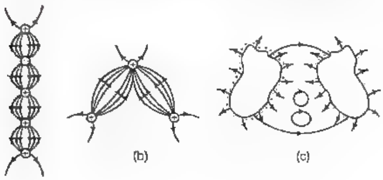
[redacted]



(a) (b)

It is a common misconception that the path traced by a positive charge is a field line. The path traced by a unit positive test charge represents a field line only when it moves along a straight line.

EXAMPLE |11| Explain, why the following curves cannot possibly represent electrostatic field lines? NCERT



(a) (b) (c)

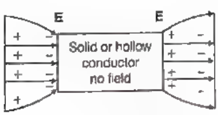
Sol. (a) Electrostatic field lines cannot start from a negative charge.

(b) Electrostatic field lines cannot end at positive charge

(c) Electrostatic field lines cannot form closed loops.

Conductors in an Electrostatic Field

(i) Electric field lines do not pass through a conductor. Hence, the interior of the conductor is free from the influence of the electric field.



Electrostatic shielding

(ii) Total charge of a charged conductor lies at the outer surface of the conductor.

(iii) The magnitude of field strength at any point on the surface of the conductor is proportional to surface charge density at that point.

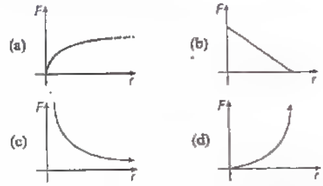
| TOPIC PRACTICE 2 |

OBJECTIVE Type Questions

1. SI unit of electrical permittivity is

(a) $N\text{-}m^2C^{-2}$ (b) Am^{-2} (c) NC^{-1} (d) $C^2N^{-1}m^{-2}$

2. Force between two charges varies with distance between them as



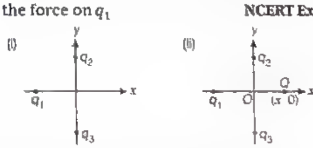
3. Two charges $+1\mu C$ and $+4\mu C$ are situated at a distance in air. The ratio of the forces acting on them is

(a) 1 : 4 (b) 4 : 1 (c) 1 : 1 (d) 1 : 16

4. A charge q is placed at the centre of the line joining two equal charges Q and Q. The system of the three charges will be in equilibrium, if q is equal to

(a) $-Q/2$ (b) $-Q/4$ (c) $+Q/4$ (d) $+Q/2$

5. In figure two positive charges q_2 and q_3 fixed along the y-axis, exert a net electric force in the $+x$-direction on a charge q_1 fixed along the x-axis. If a positive charge Q is added at $(x, 0)$, the force on q_1 NCERT Exemplar



(a) shall increase along the positive x-axis

(b) shall decrease along the positive x-axis

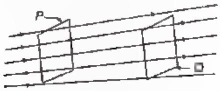
(c) shall point along the negative x-axis

(d) shall increase but the direction changes because of the intersection of Q with q_2 and q_3

[redacted]

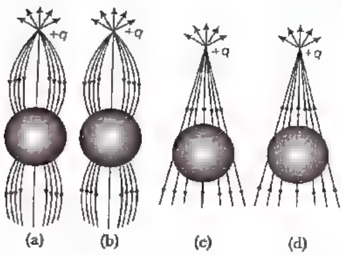
6. A force of 2.25 N acts on a chrage of 15×10^{-4} C. The intensity of electric field at that point is
(a) 150 NC^{-1} (b) 15 NC^{-1}
(c) 1500 NC^{-1} (d) 1.5 NC^{-1}

7. In the diagram shown below,



(a) field strength at P is less than field strength at Q
(b) field strength at P and Q are equal
(c) field is more strong at P and less strong at Q
(d) cannot be tell from the figure

8. A point positive charge is brought near an isolated conducting sphere (figure). The electric field is best given by **NCERT Exemplar**



9. A hemisphere is uniformly charged. The electric field at a point on a diameter away from the centre is directed **NCERT Exemplar**
(a) perpendicular to the diameter
(b) parallel to the diameter
(c) at an angle tilted towards the diameter
(d) at an angle tilted away from the diameter

10. A point charge $+q$ is placed at a distance d from an isolated conducting plane. The field at a point P on the other side of the plane is **NCERT Exemplar**
(a) directed perpendicular to the plane and away from the plane
(b) directed perpendicular to the plane but towards the plane
(c) directed radially away from the point charge
(d) directed radially towards the point charge

VERY SHORT ANSWER Type Questions

11. Does the Coulomb force that one charge exerts on another charge changes, if other charge is brought nearby?

12. In Coulomb's law, $F = \frac{k_e q_1 q_2}{r^2}$, what are the factors on which the proportionality constant k_e depends?

13. If the distance between two equal point charges is doubled and their individual charges are also doubled, then what would happen to the force between them?

14. A metallic spherical shell has an inner radius R_1 and outer radius R_2. A charge Q is placed at the centre of the spherical cavity. What will be surface charge density on (i) the inner surface and (ii) the outer surface? **NCERT Exemplar**

15. The test charge used to measure electric field at a point should be vanishingly small. Why?

16. A point charge q is placed at the origin. How does the electric field due to the charge vary with the distance r from the origin?

17. Force experienced by an electron in an electric field is F newton. What will be the force experienced by a proton in the same field? Take, mass of a proton is 1836 times the mass of an electron.

18. Two point charges of $+3\,\mu C$ each are 100 cm apart. At what point on the line joining the charges will the electric field intensity be zero?

19. A proton is placed in a uniform electric field directed along a positive X-axis. In which direction will it tend to move?

20. Why electrostatic field be normal to the surface at every point of a charged conductor?

21. An electrostatic field line is continuous curve, i.e. a field line cannot have sudden breaks. Why not?

22. Why should electrostatic field be zero inside a conductor?

23. Why do the electric field lines not form closed loops? **Delhi 2015**

24. The dimensions of an atom are of the order of an angstrom. Thus, there must be large electric fields between the protons and electrons. Why, then is the electrostatic field inside a conductor zero? **NCERT Exemplar**

[redacted]

SHORT ANSWER Type Questions

25. In the given statement, point out the correct or incorrect word or phrase with a proper explanation.

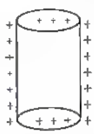
"The mutual forces between two charges do not get affected by the presence of other charges."

26. Plot a graph showing the variation of Coulomb's force (F) *versus* $1/r^2$, where r is the distance between the two charges of each pair of charges ($1\,\mu C$, $2\,\mu C$) and ($1\,\mu C$, $-3\,\mu C$). Interpret the graphs obtained.

27. A charge q is placed at the centre of the line joining two equal charges (Q). Show that the system of three charges will be in equilibrium, if $q = -\frac{Q}{4}$.

28. An uncharged metallic ball is suspended in the region between two vertical metal plates. If the two plates are charged, one positively and one negatively, then describe the motion of the ball after it is brought into contact with one of the plates.

29. Sketch the electric field lines for a uniformly charged hollow cylinder as shown in the figure. **NCERT Exemplar**



LONG ANSWER Type I Questions

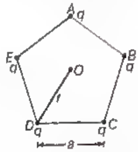
30. Check that the ratio ke^2/Gm_em_p is dimensionless.

Look up a table of physical constants and determine the value of this ratio. What does the ratio signify? **NCERT**

31. Consider three charges Q_1, Q_2, Q_3 each equal to Q at the vertices of an equilateral triangle of side a. What is the force on a charge q (with the same sign as q) placed at the centroid of the triangle? **NCERT**

32. An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55×10^4 N/C in Millikan's oil drop experiment. The density of the oil is $1.26\,g/cm^3$. Estimate the radius of the drop.
(Take, $g = 9.81\,m/s^2$, $e = 1.6 \times 10^{-19}$ C). **NCERT**

33. Five charges, q each are placed at the corners of regular pentagon of side a as shown in the figure.



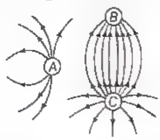
(i) (a) What will be the electric field at O, the centre of the pentagon?

(b) What will be the electric field at O, if the charge from one of the corners (say A) is removed?

(c) What will be the electric field at O, if the charge q at A is replaced by $-q$?

(ii) How would your answer be affected, if pentagon is replaced by n-sided regular polygon with charge q at each of its corners? **NCERT Exemplar**

34. Figure shows the electric field lines around three point charges A, B and C.



(i) Which charges are positive?
(ii) Which charge has the largest magnitude? Why?
(iii) In which region or regions of the picture could the electric field be zero? Justify your answer. **NCERT Exemplar**

(a) Near A (b) Near B
(c) Near C (d) Nowhere

LONG ANSWER Type II Questions

35. Four point charges $q_A = 2\,\mu C$, $q_B = -5\,\mu C$, $q_C = 2\,\mu C$ and $q_D = -5\,\mu C$ are located at the corners of a square $ABCD$ of side 10 cm. What is the force on a charge of $1\,\mu C$ placed at the centre of the square? **NCERT**

[redacted]

36. A free pith-ball of 8 g carries a positive charge of 5×10^{-8}C. What must be the nature and magnitude of charge that should be given to a second pith-ball fixed 5 cm vertically below the former pith-ball, so that the upper pith-ball is stationary? **All India 2011**

NUMERICAL PROBLEMS

37. The dielectric constant of water is 80. What is its permittivity?

38. Two equal balls having equal positive charge q coulombs are suspended by two insulating strings of equal length. What would be the effect on the force when a plastic sheet is inserted between the two? **All India 2014**

39. Two point charges having equal charges separated by 1 m distance experience a force of 8 N. What will be the force experienced by them, if they are held in water, at the same distance? (Given, $K_{water} = 80$) **All India 2011**

40. A charge $q = 1\mu$C is placed at point (1 m, 2 m, 4 m). Find the electric field at point P (0 m, −4 m, 3 m).

41. An infinite number of charges each equal to q are placed along X-axis at $x = 1, x = 2, x = 4, x = 8$ and so on. Find the electric field at the point $x = 0$ due to this set up of charges

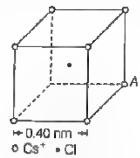
42. The opposite corners of a square carry Q charge each and the other two opposite corners of the same square carry q charge each. If the resultant force on q is zero, how are Q and q related?

43. (i) Two insulated charged copper spheres A and B have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion, if the charge on each is 6.5×10^{-7}C and the radii of A and B are negligible compared to the distance of separation?

(ii) What is the force of repulsion, if each sphere is charged double the above amount and the distance between them is halved? **NCERT**

44. Suppose the spheres A and B in Q. 43 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second and finally removed from both. What is the new force of repulsion between A and B? **NCERT**

45. Figure represents a crystal unit of caesium chloride CsCl. The caesium atoms, represented by open circles are situated at the corners of a cube of side 0.40 nm, whereas a Cl atom is situated at the centre of the cube. The Cs atoms are deficient in one electron while the Cl atom carries an excess electron.



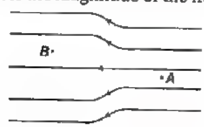
(i) What is the net electric field on the Cl atom due to eight Cs atoms?

(ii) Suppose that the Cs atom at the corner A is missing. What is the net force now on the Cl atom due to seven remaining Cs atoms? **NCERT Exemplar**

46. In the figure below, the electric field lines on the left have twice the separation of those on the right.

(i) If the magnitude of the field of A is 40 N/C, then what force acts on a proton at A?

(ii) What is the magnitude of the field at B?



HINTS AND SOLUTIONS

1. (d) From Coulomb's law, $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$

$\therefore$ Electrical permittivity, $\varepsilon_0 = \frac{q_1q_2}{4\pi\times F\times r^2} = \frac{C\times C}{N\times m^2}$

$\therefore$ Unit of electrical permittivity $= C^2N^{-1}m^{-2}$

2. (c) According to Coulomb's law, force between two point charges, *i.e.*, $F \propto \frac{1}{r^2}$. Therefore, the graph between F and r will be as shown in Fig.(c).

3. (c) According to the Coulomb's law,

Force, $F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1q_2}{r^2}$

[redacted]

where, $q_1 = +1\mu C = 1\times10^{-6}$ C

$q_2 = +4\,\mu C = 4\times10^{-6}$ C

r = distance between the charges.

Force on first charge due to second charge,

$$F_{12} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{10^{-6}\times4\times10^{-6}}{r_{12}^2}$$

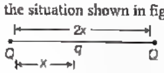
Force on second charge due to first charge,

$$F_{21} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{10^{-6}\times4\times10^{-6}}{r_{21}^2}$$

$\because \quad r_{12} = r_{21} \therefore F_{12} = F_{21}$

$\therefore \quad F_{12} : F_{21} = 1:1$

4. (b) Consider the situation shown in figure.

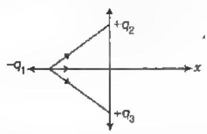


For the system to be in equilibrium,

$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{x^2} = \frac{-1}{4\pi\varepsilon_0}\frac{Q^2}{(2x)^2}$$

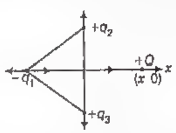
$\Rightarrow \quad q = -Q/4$

5. (a) The net force on q_1 by q_2 and q_3 is along the $+x$-direction, so nature of force between q_1, q_2 and q_1, q_3 is attractive. Thus can be represent by the figure given below



The attractive force between these charges states that q_1 is a negative charge (since, q_2 and q_3 are positive).

Thus, nature of force between q_1 and newly introduced charge Q (positive) is attractive and net force on q_1 by q_2, q_3 and Q are along the same direction as given in the diagram below



The figure given above clearly shows that the force on q_1 shall increase along the positive x-axis due to the positive charge Q.

6. (c) Electric field, $E = \frac{F}{q} = \frac{2.25\text{ N}}{15\times10^{-4}\text{ C}}$

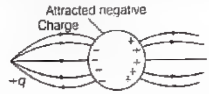
$= 1500\text{ NC}^{-1}$

7. (c) Areas of P-Q are equal but more lines pass through area at P. So, field is stronger at P as compared to Q.

8. (a) The free electrons in the sphere are attracted towards the positive charge. This leaves an excess of positive charge on the rear (right) surface of sphere.

Electric field lines enter or leave perpendicular to the surface of charged conductor.

Thus, the left surface of sphere has an excess of negative charge and the right surface of sphere has an excess of positive charge as given in the figure below



An electric field lines start from positive charge and ends at negative charge (in this case from point positive charge to negative charge created inside the sphere).

Here, all these conditions are fulfilled in Fig. (a).

9. (a) When the point is situated at a point on diameter away from the centre of hemisphere charged uniformly, the electric field is perpendicular to the diameter. The component of electric intensity parallel to the diameter cancel out

10. (a) When a point positive charge brought near an isolated conducting plane, some negative charge developes on the surface of the plane towards the charge and an equal positive charge developes on opposite side of the plane, so field lines are directed perpendicular and away from the plane.

11. Yes, it changes as the distance becomes less.

12. Here, k_e is also called dielectric constant, whose value depends essentially on the type of substance and on the external conditions like temperature, pressure and so on.

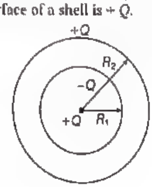
13. We know that by Coulomb's law, $F = k\frac{q_1q_2}{r^2}$

According to question, $q_1' = 2q_1$, $q_2' = 2q_2$, $r' = 2r$

$\therefore \quad F' = \frac{k(2q_1)(2q_2)}{(2r)^2} = \frac{kq_1q_2}{r^2} = F$

Hence, force between these charges remains same.

14. When a charge $+Q$ is placed at the centre of spherical cavity as shown in the figure, then charge induced on the inner surface of a shell is $-Q$ and charge induced on the outer surface of a shell is $+Q$.



[redacted]

Therefore, surface charge density on inner surface of shell is $\frac{-Q}{4\pi R_1^2}$ and on outer surface of shell is $\frac{+Q}{4\pi R_2^2}$.

15. In case, test charge is not vanishingly small, it will produce its own electric field and the measured value of electric field will be different from the actual value of an electric field at that point.

16. The electric field varies inversely as the square of the distance from the point charge.

17. The proton will experience the same force F newton, but in the opposite direction.

18. At the centre, since the electric field due to two charges is equal and opposite at this point.

19. Proton will tend to move along the positive X-axis in the direction of a uniform electric field

20. For the condition of electrostatics, the electric field lines must be normal to the surface of the conductor, otherwise there would be a non-zero component of electric field along the surface of conductor and charges could not be at rest.

21. An electrostatic field line cannot be a discontinuous curve, i.e. it cannot have breaks.If it has breaks, then it will indicate absence of electric field at the break points. But the electric field vanishes only at infinity.

22. Electric field lines do not pass through a conductor. Hence, the interior of the conductor is free from the influence of the electric field.

23. Electric field lines do not form closed loops because they are always directed from positive charge to negative charge.

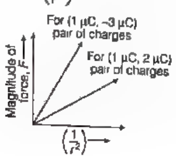
24. The electric fields find the atoms to neutral entity. As it is known that, electrostatic fields are caused by excess charges. However, there is no excess charge on the inner surface of an isolated conductor. Therefore, electrostatic field inside a conductor is zero.

25. Correct, because mutual force acting between two point charges is proportional to the product of magnitude of charges and inversely proportional to the square of the distance between them, i.e. independent of the other charges

26. According to Coulomb's law, the magnitude of force acting between two stationary point charges is given by

$$F = \left(\frac{q_1 q_2}{4\pi\varepsilon_0}\right)\left(\frac{1}{r^2}\right)$$

For given $q_1 q_2$, $F \propto \left(\frac{1}{r^2}\right)$

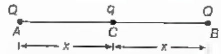


The slope of $F - \frac{1}{r^2}$ graph depends on q_1 and q_2.

Magnitude of $q_1 q_2$ is higher for second pair.

$\therefore$ Slope of $F - \frac{1}{r^2}$ graph, corresponding to second pair (1 μC, −3 μC) is greater. Higher the magnitude of product of charges q_1 and q_2, higher will be the slope.

27. Suppose the three charges be placed as shown in the figure.



As the net force on q is zero, so it is already in equilibrium. For equilibrium of other two charges, the net force on each charge must be zero

Total force on charge Q at B is

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{x^2} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q\cdot Q}{(2x)^2} = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{x^2} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{4x^2}$$

$$\therefore \quad q = -\frac{Q}{4}$$

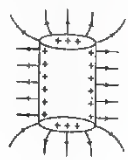
28. The two charged plates create a region with a uniform electric field between them, directed from the positive towards the negative plate.

Once the ball is disturbed so as to touch one plate (say, the negative one), some negative charge will be transferred to the ball and an electric force will act on the ball, that will accelerate it to the positive plate.

Once the ball touches the positive plate, it will release its negative charge, acquire a positive charge and accelerate back to the negative plate. The metallic ball will continue to move back and forth between the plates until it has transferred all their net charges, thereby making both the plates neutral.

29. Here, the hollow cylinder is positively charged.

We know that, the electric field lines appear to come out from the conductor. Thus, the field lines for a uniformly positive charged hollow cylinder is shown in the figure.



[redacted]

30. In the ratio $\frac{ke^2}{Gm_e m_p}$, $k = 4\pi\varepsilon_0$ (constant)

where, G = gravitational constant,

m_e = mass of an electron and m = mass of a proton.

From Coulomb's law, $F = k\frac{q_1 q_2}{r^2}$

$$\Rightarrow \quad k = \frac{Fr^2}{q_1 q_2}$$

$$\text{or} \quad k = \frac{Fr^2}{q^2}$$

The dimension of $k = \left(\frac{1}{4\pi\varepsilon_0}\right) = \frac{[MLT^{-2}][L^2]}{[AT]\,[AT]}$

$= [ML^3T^{-4}A^{-2}]$

The dimension of e (electronic charge) = [AT]

The dimension of G (universal gravitational constant)

$= \frac{[MLT^{-2}]\,[L^2]}{[M^2]} = [M^{-1}L^3T^{-2}]$

The dimension of m_e or m_p (mass of electron or mass of proton) = [M]

The dimension of $\frac{ke^2}{Gm_e m_p} = \frac{[ML^3T^{-4}A^{-2}]\,[A^2T^2]}{[M^{-1}L^3T^{-2}]\,[M^2]}$

$= [M^0L^0T^0A^0]$

Thus, the given ratio is dimensionless.

The value of $k = \left(\frac{1}{4\pi\varepsilon_0}\right) = 9\times10^9$ N-m^2/C^2

The value of e (charge of an electron)

$= 1.6\times10^{-19}$ C

The value of G (universal gravitational constant)

$= 6.67\times10^{-11}$ N-m^2/kg^2

The value of m_e (mass of electron) = 9.1×10^{-31} kg

The value of m_p (mass of proton) = 1.67×10^{-27} kg

The value of $\frac{ke^2}{Gm_e m_p}$

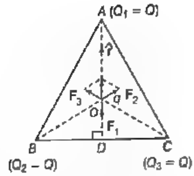
$$= \frac{9\times10^9\times(1.6\times10^{-19})^2}{6.67\times10^{-11}\times9.1\times10^{-31}\times1.67\times10^{-27}}$$

$= 2.29\times10^{39}$

The ratio signifies that the ratio of electrostatic force to the gravitational force is 2.29×10^{39}. This means the electrostatic force between an electron and a proton is 2.29×10^{39} times the gravitational force between an electron and a proton.

31. As shown in the figure, draw $AD \perp BC$.

$\therefore \quad AD = AB\cos30° = \frac{a\sqrt{3}}{2} \qquad \left[\because \cos30° = \frac{\sqrt{3}}{2}\right]$



Distance AO of the centroid O from A

$$= \frac{2}{3}AD = \frac{2}{3}\cdot\frac{a\sqrt{3}}{2} = \frac{a}{\sqrt{3}}$$

$\therefore$ Force F_1 on q at O due to charge $(Q_1 = Q)$ at A,

$$F_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Qq}{\left(\frac{a}{\sqrt{3}}\right)^2} = \frac{3Qq}{4\pi\varepsilon_0 a^2}, \text{ along } AO$$

Similarly, force F_2 on q due to charge $(Q_2 = Q)$ at B,

$$F_2 = \frac{3Qq}{4\pi\varepsilon_0 a^2}, \text{ along } BO$$

Force F_3 on q due to charge $(Q_3 = Q)$ at C,

$$F_3 = \frac{3Qq}{4\pi\varepsilon_0 a^2}, \text{ along } CO$$

The resultant of forces F_2 and F_3 is $\left(\frac{3}{4\pi\varepsilon_0}Q\frac{q}{a^2}\right)$ along OA by the parallelogram law.

Therefore, the total force on

$$q = \frac{3}{4\pi\varepsilon_0}Q\frac{q}{a^2}(\hat{r} - \hat{r}) = 0$$

where, $\hat{r}$ is the unit vector along OA. It is also clear by symmetry that the sum of three forces will zero.

32. **Hints:** Here, oil drop is held stationary under electric field that means the weight of the drop is balanced by the electrostatic force applied on it.

Given, the number of excess electrons, $n = 12$

Electric field, $E = 2.55\times10^4$ N/C

Density of oil, $\rho = 1.26$ g/cm^3

$= 1.26\times10^3$ kg/m^3

Electronic charge, $e = 1.6\times10^{-19}$ C

$\Rightarrow \quad g = 9.81$ m/s^2

Let the radius of drop be r.

The electrostatic force on drop $= qE = neE \qquad [\because q = ne]$

The gravitational force on the drop $= mg$

[where, m = mass of the drop]

$=$ Volume $\times$ Density $\times g \qquad [\because$ mass = volume $\times$ density$]$

$= \frac{4}{3}\pi r^3\times\rho\times g$

[redacted]

As the drop is held stationary. So, the net force on the drop is zero.

$\therefore$ Electrostatic force = Gravitational force

$$\Rightarrow \quad neE = \frac{4}{3}\pi r^3 \rho g$$

$$\Rightarrow \quad r^3 = \frac{3neE}{4\pi\rho g}$$

$$= \frac{3 \times 12 \times 1.6 \times 10^{-19} \times 2.55 \times 10^4}{4 \times 3.14 \times 1.26 \times 10^3 \times 9.81}$$

$$\Rightarrow \quad r^3 = 0.94 \times 10^{-18}$$

$$\Rightarrow \quad r = (0.94 \times 10^{-18})^{1/3} = 9.81 \times 10^{-7}$$

Thus, the radius of the drop is 9.81×10^{-7}m.

33. (i) (a) The point O is equidistant from all the charges at the end points of pentagon. Thus, due to symmetry, the forces due to all the charges are cancelled out. As a result, electric field at O is zero.

(b) When charge q is removed from A, electric field at O would become

$$E = \frac{q \times 1}{4\pi\varepsilon_0 r^2} \text{(along } OA)$$

(c) If charge q at A is replaced by $-q$, then it is equivalent to adding charge $-2q$. Thus, the electric field at O would become

$$E = \frac{2q}{4\pi\varepsilon_0 r^2} \text{(along } OA)$$

(ii) When pentagon is replaced by n-sided regular polygon with charge q at each of its corners, the electric field at O would continue to be zero as symmetricity of the charges is due to the regularity of the polygon. It does not depend on the number of sides or the number of charges.

34. Electric field lines always start from a positive charge and end at a negative charge. In case of a single charge, electric lines of force start from positive charge and end at infinity.

The magnitude of a charge depends on the number of lines of force emanating from a charge, i.e. higher the number of lines of force, higher the magnitude of charge and *vice-versa*.

(i) In the given figure, the electric lines of force emanate from A and C. Therefore, charges A and C must be positive.

(ii) The number of electric lines of force emanating is maximum from charge C here, so C must have the largest magnitude.

(iii) Point between two like charges, where electrostatic force is zero, is called neutral point. So, the neutral point lies between A and C only.

Now, the position of neutral point depends on the strength of the forces of charges. Here, more number of electric lines of force show higher strength of charge C than A. So, neutral point lies near A.

35. **Hints:** Charge placed at the centre is in the influence field of four charges located at the corners of the square. Therefore, we can find force acting on charge placed at the centre using superposition principle. Use the law of vectors to find the net resultant force because force is a vector quantity.

Let the centre of the square be at O. The charge placed on the centre is $1\,\mu$C.

$AB = BC = CD = DA = 10$ cm

$AC = \sqrt{2} \times 10 = 10\sqrt{2}$ cm

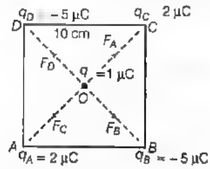
$AC = BD = 10\sqrt{2}$ cm

$AO = BO = CO = DO$

$= \frac{10\sqrt{2}}{2} = 5\sqrt{2}$ cm

Let the force on charge 1μC due to q_A be F_A which is directed away from both charges q_A and q (because both charges are positive in nature, so they will repel each other).

The force on charge 1μC due to q_B is F_B which is towards q_B (because q_B is negatively charged and q is positively charged, so they will attract each other).



The force on charge 1μC due to q_C is F_C which is directed away from both q_C and q (as they both are positive in nature, so will repel each other).

The force on charge 1μC due to q_D is F_D which is towards q_D (because q_D is negatively charged and q is positively charged, so they will attract each other).

Force between q and q_A

$$F_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|qq_A|}{(OA)^2} = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 2 \times 10^{-6}}{(5\sqrt{2} \times 10^{-2})^2}$$

$$= \frac{9 \times 2 \times 10^{-3}}{25 \times 2 \times 10^{-4}} = \frac{90}{25} = \frac{18}{5}$$

$= 3.6$ N [direction towards O to C]

Force between q and q_B

$$F_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|qq_B|}{(OB)^2}$$

$= 9.0$ N [direction towards O to B]

Force between q and q_C

$$F_C = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|qq_C|}{(OC)^2} = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 2 \times 10^{-6}}{(5\sqrt{2} \times 10^{-2})^2}$$

[redacted]

$= \frac{90}{25} = \frac{18}{5} = 3.6\text{N}$ [direction towards O to A]

Here, we observe that F_A and F_C are of same magnitude and opposite in direction. So, the resultant force of F_A and F_C is zero.

Force between q and q_D

$$F_D = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|qq_D|}{(OD)^2} = \frac{9\times 10^9 \times 1\times 10^{-6} \times 5\times 10^{-6}}{(5\sqrt{2}\times 10^{-2})^2}$$

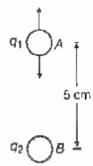
$= 2.25$ N [direction towards O to D]

Here, we observe that F_B and F_D are of same magnitude and opposite in direction. So, the resultant force of F_D and F_B is zero.

Thus, the net resultant force on 1 μC (placed at O) is zero, as all the forces balance each other.

36. Here, charge on the pith-ball A, $q_1 = 5\times 10^{-8}$C

Mass of the pith-ball A, $m_1 = 8\text{ g} = 8\times 10^{-3}$ kg



The weight $m_1 g$ of the pith-ball A acts vertically downwards.

Let q_2 be charge on the pith ball B held 5 cm below the pith ball A, so that the pith-ball A remains stationary.

It can be possible only, if the charges on two pith-balls are of same signs, i.e. if charge on the pith-ball A is positive, the charge on B should also be positive. As such the force on the pith-ball A due to B, i.e. F_{AB} will act vertically upwards.

For charge q_1 to remain stationary,

$$F_{AB} = m_1 g$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{AB^2} = m_1 g$$

Here, $AB = 5\text{ cm} = 0.05$ m

$$\Rightarrow \quad 9\times 10^9 \times \frac{5\times 10^{-8}\times q_2}{(0.05)^2} = 8\times 10^{-3}\times 9.8$$

$$\therefore \quad q_2 = 4.36\times 10^{-7}\text{ C (positive)}$$

37. Given, $K = 80$

We have, $K = \frac{\varepsilon_m}{\varepsilon_0}$

$\therefore \quad \varepsilon_m = K\varepsilon_0 \quad [\because \varepsilon_0 = 8.85\times 10^{-12}\text{C}^2\text{N}^{-1}\text{m}^{-2}]$

$= 80\times 8.85\times 10^{-12}$

$= 708\times 10^{-12}$

$= 7.08\times 10^{-10}\text{C}^2\text{N}^{-1}\text{m}^{-2}$

38. From Coulomb's law, electric force between the two charged bodies, in a medium,

$$F = \frac{1}{4\pi\varepsilon_0 K}\frac{|q_1 q_2|}{r^2}$$

where, K = dielectric constant of the medium.

For vacuum, $K = 1$

For plastic, $K > 1$

Therefore, after insertion of plastic sheet, the force between the two balls will reduce.

39. Two point charges system is taken from air to water keeping other variables (e.g. distance, magnitude of charge) unchanged. So, the only factor which may affect the interacting force is dielectric constant of medium.

Force acting between two point charges,

$$F = \frac{1}{4\pi\varepsilon_0 K}\frac{q_1 q_2}{r^2} \text{ or } F \propto \frac{1}{K} \Rightarrow \frac{F_{\text{air}}}{F_{\text{medium}}} = K$$

$$\Rightarrow \frac{8}{F_{\text{water}}} = 80 \quad \Rightarrow F_{\text{water}} = \frac{8}{80} = \frac{1}{10}\text{N}$$

40. Here, $\mathbf{r}_q = \hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 4\hat{\mathbf{k}}$ and $\mathbf{r}_p = -4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}$

$\therefore \quad \mathbf{r}_p - \mathbf{r}_q = -\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - \hat{\mathbf{k}}$

or $|\mathbf{r}_p - \mathbf{r}_q| = \sqrt{(-1)^2 + (-6)^2 + (-1)^2} = \sqrt{38}$ m

Now, electric field, $\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{|\mathbf{r}_p - \mathbf{r}_q|^3}(\mathbf{r}_p - \mathbf{r}_q)$

Substituting the values, we get

$$\mathbf{E} = \frac{(9.0\times 10^9)(1.0\times 10^{-6})}{(38)^{3/2}}(-\hat{\mathbf{i}} - 6\hat{\mathbf{j}} - \hat{\mathbf{k}})$$

$$= (-38.42\,\hat{\mathbf{i}} - 230.52\,\hat{\mathbf{j}} - 38.42\,\hat{\mathbf{k}})\text{N/C}$$

41. At the point $x = 0$, the electric field due to all the charges are in the same negative x-direction and hence get added up, i.e.

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{1^2} + \frac{q}{2^2} + \frac{q}{4^2} + \frac{q}{8^2} + \ldots\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[1 + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \ldots\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{1}{[1 - 1/4]} = \frac{q}{3\pi\varepsilon_0}$$

This electric field is along negative X-axis.

42. Let each side of square be x.

Diagonal $= \sqrt{x^2 + x^2} = x\sqrt{2}$

$$F_1 = F_2 = \frac{Qq}{4\pi\varepsilon_0 x^2}$$

and $\quad F_3 = \frac{qq}{4\pi\varepsilon_0 (x\sqrt{2})^2} = \frac{q^2}{2\times 4\pi\varepsilon_0 x^2}$

As, F_1 and F_2 are perpendicular to each other, their resultant force,

$$F = \sqrt{F_1^2 + F_2^2}$$

[redacted]

$$= \sqrt{F_1^2 + F_1^2}$$

$$\Rightarrow \quad F = F_1\sqrt{2}$$

As, net force on q is zero, therefore

$$F_1\sqrt{2} = -F_3$$

$$\Rightarrow \quad \frac{Qq\sqrt{2}}{4\pi\varepsilon_0 x^2} = \frac{-q^2}{2 \times 4\pi\varepsilon_0 x^2}$$

$$\Rightarrow \quad q = -2\sqrt{2}\,Q$$

43. (i) Here, $q_1 = q_2 = 6.5 \times 10^{-7}C$, $r = 50$ cm $= 0.5$ m

Electrostatic force of repulsion,

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = \frac{9 \times 10^9 \times (6.5 \times 10^{-7})^2}{(0.5)^2}$$

$$= 1.521 \times 10^{-2}\text{N}$$

(ii) Now, q_1, q_2 both are doubled and r is halved in

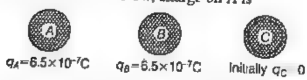
$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}, \text{ then}$$

F becomes 16 times, i.e. $F' = 16F$.

$F' = 16 \times 1.521 \times 10^{-2}$N or $F' = 0.24$ N

44. **Hints:** It is based on the distribution of charges when the two identical bodies come into contact, charge is distributed equally on identical bodies.

Now, the sphere C comes in contact with A, the charges will be divided equally on both spheres as they have same mass and size. Now, charge on A is



$$q'_A = \frac{q_A + q_C}{2} = \frac{6.5 \times 10^{-7} + 0}{2}$$

$$= 3.25 \times 10^{-7}\text{C}$$

Now, the charge on C will also be 3.25×10^{-7}C.

$\therefore \quad q'_C = 3.25 \times 10^{-7}$C

Now, the sphere C comes in contact with B, the charges are shared again.

Then, charge on B is

$$q'_B = \frac{q_B + q'_C}{2}$$

$$= \frac{6.5 \times 10^{-7} + 3.25 \times 10^{-7}}{2}$$

$$= 4.875 \times 10^{-7}\text{C}$$

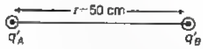
Finally, the charge on C is $q'_C = 4.875 \times 10^{-7}$C

Finally, the charge on A is $q'_A = 3.25 \times 10^{-7}$C

The charge on B is $q'_B = 4.875 \times 10^{-7}$C

From the Coulomb's law, the force between two spheres,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q'_A \cdot q'_B}{r^2}$$



$$= \frac{9 \times 10^9 \times 3.25 \times 10^{-7} \times 4.875 \times 10^{-7}}{(50 \times 10^{-2})^2}$$

$$= \frac{9 \times 3.25 \times 4.875 \times 10^{-5}}{50 \times 50 \times 10^{-4}} = 5.7 \times 10^{-3} \text{ N}$$

This force will be repulsive in nature because both spheres have like charges

45. **Hints:** Net force on a charge due to two equal and opposite charges will be zero. Also, electric field on a charge is given by

$$E = \frac{F}{q}$$

where, E = electric field, F = force on charge q due to electric field and q = magnitude of charge.

If a Cs atom is removed from the corner A, then a singly charged negative Cs ion at A will appear.

(i) From the given figure, we can analyse that the chlorine atom is at the centre of the cube, i.e. at equal distance from all the eight corners of cube, where caesium atoms are placed.

Thus, due to symmetry, the force due to all Cs atoms, on Cl atom will cancel out.

Hence, $\quad E = \frac{F}{q'}$

where, $\quad F = 0$

$\therefore \quad E = 0$

(ii) Thus, net force on Cl atom at A would be given by

$$F = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

where, r = distance between Cl ion and Cs ion.

Applying Pythagoras theorem, we get

$$r = \sqrt{(0.20)^2 + (0.20)^2 + (0.20)^2} \times 10^{-9}\text{m} = 0.346 \times 10^{-9}\text{m}$$

$$\therefore \quad F = \frac{q^2}{4\pi\varepsilon_0 r^2} = \frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{9 \times 10^9(1.6 \times 10^{-19})^2}{(0.346 \times 10^{-9})^2}$$

$$= 1.92 \times 10^{-9} \text{ N}$$

46. (i) Charge of proton, $q = 1.6 \times 10^{-19}$C

Force on proton at A is $F = qE_A$

$= (1.6 \times 10^{-19}\text{C})(40 \text{ N/C}) = 6.4 \times 10^{-18}$N

(ii) Since, electric field,

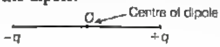
$$E \propto \frac{\text{Number of electric field lines}}{\text{Area}}$$

Hence, $\quad E_B = \frac{1}{2}E_A = \frac{1}{2}(40 \text{ N/C}) = 20\text{N/C}$

[redacted]

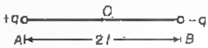
|TOPIC 3|
Electric Dipole

An electric dipole is a pair of point charges with equal magnitude and opposite in sign separated by a very small distance. The mid-point of locations of $-q$ and q is called the centre of the dipole.



Dipole Moment of an Electric Dipole

The strength of an electric dipole is measured by a vector quantity known as **electric dipole moment** (p) which is the product of the charge (q) and separation between the charges ($2l$).



Electric dipole

i.e. $p = q \times 2l$

or $\boxed{|\mathbf{p}| = q(2l)}$

It is a vector quantity and its direction is always from negative charge to positive charge. The SI unit of dipole moment is coulomb-metre (C-m).

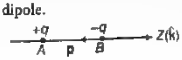
If charge q gets larger and the distance $2l$ gets smaller and smaller, keeping the product $p = q \times 2l =$ constant, we get what is called an **ideal dipole or point dipole**. Thus, an ideal dipole is the smallest dipole having almost no size.

Physical Significance of Dipoles

In most molecules, the centres of positive charges and of negative charges lie at the same place, hence their dipole moment is zero, e.g. CO_2, CH_4. However, they develop a dipole moment when an electric field is applied. But some molecules have permanent dipole moment, e.g. H_2O which are called **polar molecules**. If the centre of mass of positive charges coincides with the centre of mass of negative charges, the molecule behaves as a **non-polar molecule**.

EXAMPLE |1| A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points $A(0, 0, -15 \text{ cm})$ and $B(0, 0, +15 \text{ cm})$, respectively. What are the total charge and electric dipole moment of the system? **NCERT**

Sol. Two charges q_A and q_B are located at points $A(0, 0, -15\text{cm})$ and $B(0, 0, 15\text{cm})$ on Z-axis. They form an electric dipole.



Total charge $q = q_A + q_B = 2.5 \times 10^{-7} - 2.5 \times 10^{-7}$

$\Rightarrow \quad q = 0$

Also, $AB = 15 + 15 = 30$ cm

or $AB = 30 \times 10^{-2}$ m

Electric dipole moment,

$p =$ Either charge $\times$ BA

$= 2.5 \times 10^{-7} \times (30 \times 10^{-2})(-\hat{k})$

$= -7.5 \times 10^{-8} \hat{k}$ C-m

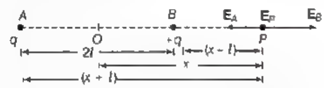
ELECTRIC FIELD INTENSITY DUE TO AN ELECTRIC DIPOLE

Electric field of an electric dipole is the space around the dipole in which the electric effect of the dipole can be experienced.

An electric dipole consists of two charges $+q$ and $-q$, therefore according to the superposition principle, the electric field due to an electric dipole at a point will be equal to the vector sum of the electric fields due to the two individual charges.

At a Point on the Axial Line

We have to calculate the field intensity (E) at a point P on the axial line of the dipole and at a distance $OP = x$ from the centre O of the dipole.



Electric field on axial line of an electric dipole

Resultant electric field intensity at the point P,

$$\mathbf{E}_P = \mathbf{E}_A + \mathbf{E}_B$$

The vectors $\mathbf{E}_A$ and $\mathbf{E}_B$ are collinear and opposite.

$$\therefore \quad E_P = E_B - E_A$$

Here, $E_A = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{(x+l)^2}$ and $E_B = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{q}{(x-l)^2}$

$$\therefore \quad E_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(x-l)^2} - \frac{q}{(x+l)^2}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{4qlx}{(x^2 - l^2)^2}$$

Hence, $E_P = \dfrac{1}{4\pi\varepsilon_0} \cdot \dfrac{2px}{(x^2 - l^2)^2} \qquad [\because p = q \times 2l]$

[redacted]

In vector form, $\mathbf{E}_P = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\mathbf{p}x}{(x^2 - l^2)^2}$

If dipole is short, i.e. $2l << x$, then

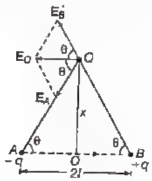
$$\boxed{E_P = \frac{2|\mathbf{p}|}{4\pi\varepsilon_0 x^3}} \quad \text{...(i)}$$

The direction of $\mathbf{E}_P$ is along BP produced.

Clearly, $E_P \propto \frac{1}{x^3}$

At a Point on the Equatorial Line

Consider an electric dipole consisting of two point charges $+q$ and $-q$ separated by a small distance $AB = 2l$ with centre at O and dipole moment, $p = q(2l)$ as shown in the figure.



Resultant electric field intensity at the point Q,

$$\mathbf{E}_Q = \mathbf{E}_A + \mathbf{E}_B$$

The vectors $\mathbf{E}_A$ and $\mathbf{E}_B$ are acting at an angle 2θ.

Here, $E_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(x^2 + l^2)}$ and $E_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(x^2 + l^2)}$

On resolving $\mathbf{E}_A$ and $\mathbf{E}_B$ into two rectangular components, the vectors $E_A \sin\theta$ and $E_B \sin\theta$ are equal in magnitude and opposite to each other and hence cancel out.

The vectors $E_A \cos\theta$ and $E_B \cos\theta$ are acting along the same direction and hence add up.

$$\therefore E_Q = E_A \cos\theta + E_B \cos\theta = 2E_A \cos\theta \qquad [\because E_A = E_B]$$

$$= \frac{2}{4\pi\varepsilon_0} \cdot \frac{q}{(x^2 + l^2)} \cdot \frac{l}{(x^2 + l^2)^{1/2}} \quad \left[\because \cos\theta = \frac{l}{(x^2 + l^2)^{1/2}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{2ql}{(x^2 + l^2)^{3/2}}$$

But $q \times 2l = |\mathbf{p}|$, the dipole moment

$$E_Q = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|\mathbf{p}|}{(x^2 + l^2)^{3/2}}$$

The direction of $\mathbf{E}$ is along $QE \parallel BA$, i.e. opposite to AB. In vector form, we can rewrite as

$$\mathbf{E}_Q = \frac{-\mathbf{p}}{4\pi\varepsilon_0 (x^2 + l^2)^{3/2}}$$

Obviously, $\mathbf{E}_Q$ is in a direction opposite to the direction of $\mathbf{p}$.

If the dipole is short, i.e. $2l << x$, then

$$\therefore \quad \boxed{E_Q = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|\mathbf{p}|}{x^3}} \quad \text{...(ii)}$$

Clearly, $E_Q \propto \frac{1}{x^3}$

From Eqs. (i) and (ii), we get

$$\frac{E_{\text{axial}}}{E_{\text{equatorial}}} = 2$$

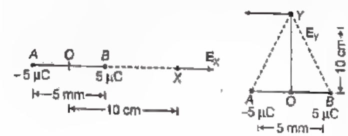
Both the magnitude and the direction of dipole field depend not only on the distance r, but also on the angle between the position vector $\mathbf{r}$ and dipole moment $\mathbf{p}$.

The electric field due to a dipole falls off at large distances, at a much faster rate $\left(\propto \frac{1}{r^3}\right)$ than the electric field due to a single charge $\left(\propto \frac{1}{r^2}\right)$.

EXAMPLE |2| Two charges $\pm 5\,\mu C$ are placed 5 mm apart. Determine the electric field at

(i) a point X on the axis of dipole 10 cm away from its centre O on the side of the positive charge as shown in Fig. (a).

(ii) a point Y, 10 cm away from centre O on a line passing through O and normal to the axis of the dipole as shown in Fig. (b).



(a) (b)

Sol. Given, $q = \pm 5\,\mu C = \pm 5 \times 10^{-6}$ C,

$2l = 5\text{mm} = 5 \times 10^{-3}$ m

$x = OX = OY = 10$ cm

$= 10 \times 10^{-2}$ m

$E_X = ?$

and $E_Y = ?$

Dipole moment, $\mathbf{p} = q \times 2l$

$= 5 \times 10^{-6}\,\text{C} \times 5 \times 10^{-3}\,\text{m}$

$= 25 \times 10^{-9}$ C-m

[redacted]

(i) Now, find out the electric field at point X on the axial line of dipole.

$$E_X = \frac{2px}{4\pi\varepsilon_0(x^2 - l^2)^2}, \text{ along } BX \text{ produced}$$

Since $l << x$, therefore

$$E_X = \frac{2p}{4\pi\varepsilon_0 x^3}$$

$$= \frac{2 \times 25 \times 10^{-9} \times 9 \times 10^9}{(10 \times 10^{-2})^3}$$

$= 4.5 \times 10^5$ NC^{-1}, along BX produced.

(ii) Now, find out the electric field at point Y on equatorial line of dipole.

$$E_Y = \frac{p}{4\pi\varepsilon_0(x^2 + l^2)^{3/2}}, \text{ along a line parallel to } BA$$

Since, $l << x$, therefore $E_Y = \dfrac{p}{4\pi\varepsilon_0 x^3}$

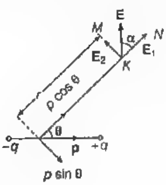
$$\therefore \quad E_Y = \frac{25 \times 10^{-9} \times 9 \times 10^9}{(10 \times 10^{-2})^3}$$

$= 2.25 \times 10^5$ NC^{-1}, along a line parallel to BA.

ELECTRIC FIELD INTENSITY AT ANY POINT DUE TO A SHORT ELECTRIC DIPOLE

Let K be any point which is neither on the axial line nor on the equatorial line.

Let P be the dipole moment of the short electric dipole and O be the mid-point of the dipole. Let the line OK make an angle θ with P. Resolving P along OK and perpendicular to OK, we get $p\cos\theta$ and $p\sin\theta$, respectively.



The electric field at K due to dipole moment $p\cos\theta$ is given by

$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3} \quad [\because K \text{ is on the axial line of the dipole } p\cos\theta]$$

The electric field at K due to dipole moment $p\sin\theta$ is given by

$$E_2 = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3} \text{ [along } KM \text{ which is} \perp p\cos\theta]$$

$\therefore$ Resultant electric field,

$$E^2 = E_1^2 + E_2^2 = \left(\frac{1}{4\pi\varepsilon_0}\frac{2p\cos\theta}{r^3}\right)^2 + \left(\frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3}\right)^2$$

$$\therefore \quad E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}(4\cos^2\theta + \sin^2\theta)^{1/2}$$

i.e. $\boxed{E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{p}{r^3}\sqrt{3\cos^2\theta + 1}}$

If α is the angle between E and E_1, then

$$\tan\alpha = \frac{E_2}{E_1} = \frac{1}{4\pi\varepsilon_0}\frac{p\sin\theta}{r^3} \times \frac{4\pi\varepsilon_0 r^3}{2p\cos\theta}$$

i.e. $\boxed{\tan\alpha = \dfrac{1}{2}\tan\theta}$

Special cases

Case I K lies on the axial line of dipole, then $\theta = 0°$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{3\cos^2 0° + 1} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

$$\Rightarrow \tan\alpha = \frac{\tan 0°}{2} = 0 \Rightarrow \alpha = 0$$

Case II K lies on the equitorial line of dipole, then $\theta = 90°$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}\sqrt{3\cos^2 90° + 1} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^3}$$

$$\Rightarrow \tan\alpha = \frac{\tan 90°}{2} = \infty$$

$$\Rightarrow \quad \alpha = \tan^{-1}\infty \quad \Rightarrow \alpha = 90°$$

EXAMPLE |3| Find the magnitude of electric field intensity due to a dipole of dipole moment 3×10^{-8} C-m at a point distance 1 m from the centre of dipole, when line joining the point to the centre of dipole makes an angle of 60° with the dipole axis.

Sol. Here, $p = 3 \times 10^{-8}$ C-m, $r = 1$ m, $\theta = 60°$ and $E = ?$

$$\text{As, } |E| = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{3\cos^2\theta + 1}$$

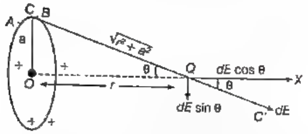
$$\therefore \quad E = \frac{3 \times 10^{-8} \times 9 \times 10^9}{(1)^3}\sqrt{3(\cos 60°)^2 + 1} = 357.17 \text{ N/C}$$

[redacted]

ELECTRIC FIELD INTENSITY AT ANY POINT ON THE AXIS OF UNIFORMLY CHARGED RING

The electric field intensity at any point Q on the axis is given by

$$\boxed{E = \frac{qr}{4\pi\varepsilon_0 (r^2 + a^2)^{3/2}}}$$



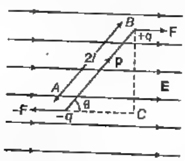
where, q = total charge, a = radius of the ring and r = distance of the point Q from the centre of the ring.

Note The direction of **E** is along QX, the axis of the loop.

DIPOLE IN A UNIFORM EXTERNAL FIELD

Torque on an Electric Dipole in a Uniform Electric Field

Consider an electric dipole consisting of two charges $-q$ and $+q$ placed in a uniform external electric field of intensity **E**.



The length of the electric dipole is $2l$. The dipole moment **p** makes an angle θ with the direction of the electric field. Two forces **F** and $-$**F** which are equal in magnitude and opposite in directions act on the dipole.

$$|\mathbf{F}| = |-\mathbf{F}| = qE$$

The net force is zero. Since, the two forces are equal in magnitude and opposite in direction and act at different points, therefore they constitute a couple. A net torque τ acts on the dipole about an axis passing through the mid-point of the dipole.

Now, τ = Either force × Perpendicular distance BC between the parallel forces $= qE(2l\sin\theta)$

$$\tau = (q \times 2l) E \sin\theta$$

or $\boxed{\tau = pE\sin\theta}$

In vector notation, $\boxed{\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}}$

SI unit of torque is newton-metre (N-m) and its dimensional formula is $[ML^2T^{-2}]$.

Case I If $\theta = 0°$, then $\tau = 0$

The dipole is in stable equilibrium.

Case II If $\theta = 90°$, then $\tau = pE$ (maximum value)

The torque acting on dipole will be maximum.

Case III If $\theta = 180°$, then $\tau = 0$

The dipole is in unstable equilibrium.

EXAMPLE |4| An electric dipole consists of two charges of 0.1 μC separated by a distance of 2.0 cm. The dipole is placed in an external field of 10^5 N/C. What maximum torque does the field exert on the dipole?

Sol. Here, $q = 0.1\ \mu C = 10^{-7}$ C, $2l = 2.0$ cm $= 2 \times 10^{-2}$ m,

$$E = 10^5 \text{ N/C} \Rightarrow \tau = pE\sin\theta = q \times 2l \times E\sin\theta$$

$$\therefore \tau_{max} = 10^{-7} \times 2 \times 10^{-2} \times 10^5 \times 1 \qquad [\because \sin 90° = 1]$$

$$= 2 \times 10^{-4} \text{ N-m}$$

Work Done on a Dipole in a Uniform Electric Field

When an electric dipole is placed in a uniform electric field, it experiences torque and tends to align it in such a way to attain stable equilibrium. Small amount of work done in rotating the dipole through a small angle $d\theta$ against the torque is given by

$$dW = \tau\, d\theta = pE\sin\theta\, d\theta$$

∴ Total work done in rotating the dipole from orientation θ_1 to θ_2, $W = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta = pE(\cos\theta_1 - \cos\theta_2)$

$$\Rightarrow \quad \boxed{W = pE(\cos\theta_1 - \cos\theta_2)}$$

Similarly, potential energy of electric dipole, when it rotates from θ_1 to θ_2, $\boxed{U = W = pE(\cos\theta_1 - \cos\theta_2)}$

Let us assume that the dipole is initially oriented perpendicular to the direction of electric field and brought to the orientation making an angle θ with the field direction, then the work done in rotating the dipole from $\theta_1 = 90°$ to $\theta_2 = \theta$,

$$W = pE(\cos 90° - \cos\theta) = -pE\cos\theta = -\mathbf{p}\cdot\mathbf{E}$$

$$\boxed{W = -\mathbf{p}\cdot\mathbf{E}}$$

[redacted]

EXAMPLE |5| An electric dipole of moment 2×10^{-8} C-m is aligned in a uniform electric field of 2×10^4 N/C. Calculate the work done in rotating the dipole from 30° to 60°.

Sol. Here, $p = 2\times10^{-8}$ C-m, $E = 2\times10^4$ N/C,

$\theta_1 = 30°, \theta_2 = 60°, W = ?$

$\therefore \quad W = pE(\cos\theta_1 - \cos\theta_2)$

$= (2\times10^{-8})(2\times10^4)(\cos 30° - \cos 60°)$

$= (2\times10^{-8})(2\times10^4)(0.366)$

$= 1.464\times10^{-4}$ J

EXAMPLE |6| An electric dipole of length 2 cm, when placed with its axis making an angle of 60° with a uniform electric field, experiences a torque of $8\sqrt{3}$ N-m. Calculate the potential energy of the dipole, if it has a charge of ± 4 nC. **Delhi 2014**

Sol. Here, length, $2a = 2$ cm $= 2\times10^{-2}$ m,

$\theta = 60°, \tau = 8\sqrt{3}$ N-m

Charge, $Q = 4$nC $= 4\times10^{-9}$ C, $U = ?$

As we know that, $\tau = Q(2a)E\sin\theta$

$\Rightarrow$ Electric field,

$$E = \frac{\tau}{Q(2a)\sin\theta} = \frac{8\sqrt{3}}{4\times10^{-9}\times2\times10^{-2}\times\sin 60°} \text{ N/C}$$

$\therefore$ Potential energy, $U = -pE\cos\theta = -Q(2a)E\cos\theta$

$$= -4\times10^{-9}\times2\times10^{-2}\times\frac{8\sqrt{3}\times\cos 60°}{4\times10^{-9}\times2\times10^{-2}\times\sin 60°}$$

$$= -\frac{8\sqrt{3}}{\sqrt{3}} = -8 \text{ J}$$

Note Electric dipole and its properties have been frequently asked in previous years 2014, 2012, 2011, 2010

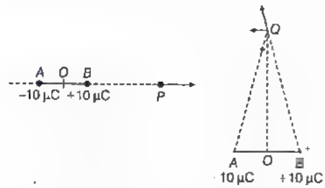
TOPIC PRACTICE 3

OBJECTIVE Type Questions

1. Two equal and opposite charges each of 2C are placed at a distance of 0.04 m. Dipole moment of the system will be
(a) 6×10^{-8} C-m (b) 8×10^{-2} C-m
(c) 1.5×10^{2} C-m (d) 8×10^{-6} C-m

2. What is the angle between the electric dipole moment and the electric field strength due to it on the equatorial line?
(a) 0° (b) 90°
(c) 180° (d) None of these

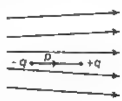
3. Electric charges $q, q, -2q$ are placed at the corners of an equilateral ΔABC of side l. The magnitude of electric dipole moment of the system is
(a) ql (b) $2ql$
(c) $\sqrt{3}ql$ (d) $4ql$

4. 

In given figures, $OP = OQ = 15$ cm, $OA = OB = 2.5$ mm

Magnitudes of electric field at P and Q are respectively
(a) 2.6×10^5 NC^{-1}, 2.6×10^5 NC^{-1}
(b) 1.3×10^5 NC^{-1}, 1.3×10^5 NC^{-1}
(c) 2.6×10^5 NC^{-1}, 1.3×10^5 NC^{-1}
(d) 1.3×10^5 NC^{-1}, 2.6 NC^{-1}

5. Figure shows electric field lines in which an electric dipole P is placed as shown. Which of the following statements is correct? **NCERT Exemplar**



(a) The dipole will not experience any force
(b) The dipole will experience a force towards right
(c) The dipole will experience a force towards left
(d) The dipole will experience a force upwards

6. In an electric field **E**, the torque acting on a dipole moment **p** is
(a) $\mathbf{p}\cdot\mathbf{E}$ (b) $\mathbf{p}\times\mathbf{E}$
(c) zero (d) $\mathbf{E}\times\mathbf{p}$

7. When an electric dipole **p** is placed in a uniform electric field **E**, then at what angle between **p** and **E** the value of torque will be maximum?
(a) 90° (b) 0°
(c) 180° (d) 45°

[redacted]

VERY SHORT ANSWER Type Questions

8. Is it correct to write the unit of electric dipole moment as mC?

9. What do you mean by an "ideal electric dipole"?

10. At what points dipole field intensity is parallel to the line joining the charges?

11. If an electric dipole is placed in a uniform electric field, then state whether it always experiences a torque or not?

12. What happens when an electric dipole is placed in a non-uniform electric field?

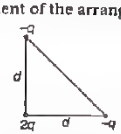
13. A dipole of dipole moment **p** is present in a uniform electric field **E**. Write the value of the angle between **p** and **E** for which the torque, experienced by the dipole is minimum. **All India 2010**

14. A ring of radius R carries a uniformly distributed charge $+Q$. A point charge $-q$ is placed on the axis of the ring at a distance $2R$ from the centre of the ring and released from rest. Will the particle execute simple harmonic motion along the axis of the ring?

SHORT ANSWER Type Questions

15. What is meant by the statement, "the electric field of a point charge has spherical symmetry, whereas that of an electric dipole is cylindrically symmetric"?

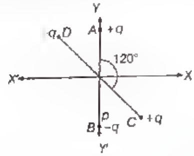
16. Three charges are placed as shown. Find dipole moment of the arrangements.



17. Prove that when an electric dipole is placed in a uniform electric field, potential energy U is given by $U = -\mathbf{p} \cdot \mathbf{E}$.

18. Two small identical dipoles AB and CD, each of dipole moment **p** are kept at an angle of 120° as shown in the figure.

What is the resultant dipole moment of this combination? If this system is subjected to electric field (**E**) directed along positive x-direction, what will be the magnitude and direction of the torque acting on this? **Delhi 2011**



19. A dipole, with a dipole moment of magnitude p, is in stable equilibrium in an electrostatic field of magnitude E. Find the work done in rotating this dipole to its position of unstable equilibrium.

LONG ANSWER Type I Questions

20. An electric dipole of dipole moment **p** consists of point charges $+q$ and $-q$ separated by a distance $2a$ apart. Deduce an expression for the electric field **E** due to the dipole at a distance x from the centre of the dipole on its axial line in terms of the dipole moment **p**. Hence, show that in the limit $x >> a$, $\mathbf{E} \rightarrow 2\mathbf{p}/4\pi\varepsilon_0 x^3$.

21. (i) Derive an expression for electrical field at a point on the equatorial line of an electric dipole.
(ii) Depict the orientation of the dipole in (a) stable, (b) unstable equilibrium in a uniform electric field **Delhi 2017**

22. A charge is distributed uniformly over a ring of radius a. Obtain the expression for the electric field intensity E at a point on the axis of the ring. Hence, show that for points at large distances from the ring, it behaves like a point charge. **Delhi 2016**

23. (i) Obtain the expression for the torque τ experienced by an electric dipole of dipole moment **p** in a uniform electric field **E**.
(ii) What will happen, if the field were not uniform? **Delhi 2017**

24. An electric dipole is held at any angle θ in a uniform electric field E. Will there be any
(i) net translating force and
(ii) torque acting on it?

Explain, what happens to dipole on being released?

[redacted]

LONG ANSWER Type II Questions

25. Define electric dipole moment. Is it a scalar or a vector? Derive the expression for the electric field of a dipole at a point on the equatorial plane of the dipole.

For a short dipole, what is the ratio of electric field intensities at two equidistant points from the centre of the dipole? One along the axial line and other on the equatorial line.

26. Deduce the expression for the torque acting on a dipole of dipole moment p in the presence of a uniform electric field E. **All India 2014**

27. In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is, however not constant but increases uniformly along the positive z-direction, at the rate of 10^5 N C^{-1}m^{-1}. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} C-m in the negative z-direction? **NCERT**

28. (i) Derive the expression for the electric field E due to a dipole of length $2l$ at a point distant r from the centre of the dipole on the axial line.

(ii) Draw a graph of E *versus* r for $r >> a$.

(iii) If this dipole is kept in a uniform external electric field E_0, diagrammatically represent the position of the dipole in stable and unstable equilibrium and write the expressions for the torque acting on the dipole in both the cases. **All India 2017**

29. Derive the expression for the work done in rotating an electric dipole from angle θ_1 to θ_2 in a uniform electric field (E). Hence, find the work done when the dipole is

(i) initially parallel to the field and

(ii) initially perpendicular to the field. **All India 2009**

NUMERICAL PROBLEMS

30. Find the electric dipole moment electron and a proton which distance is 4.3 nm apart.

31. Two charges of $-9\,\mu C$ and $+9\,\mu C$ are placed at the points $P(1, 0, 4)$ and $Q(2, -1, 5)$ located in an electric field $E = 0.20\,\hat{i}$ V/cm. Calculate the torque acting on the dipole.

32. Two charges of $+25 \times 10^{-9}$ C and -25×10^{-9} C are placed 6 m apart. Find the electric field at a point 4 m from the centre of the electric dipole (i) on axial line (ii) on equatorial line. **Delhi 2011**

33. An electric dipole with dipole moment 4×10^{-9} C-m is aligned at 30° with the direction of a uniform electric field of magnitude 5×10^4 N/C. Calculate the magnitude of the torque acting on the dipole.

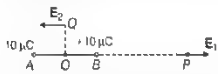
34. A system has two charges $q_A = 3.5 \times 10^{-7}$ C and $q_B = -3.5 \times 10^{-7}$ C located at points A (0, 0, −10 cm) and B (0, 0, +10 cm), respectively. What are the total charge and electric dipole moment of the system?

35. Two charges q_1 and q_2 of 0.1 μμC and − 0.1 μμC respectively are 10Å apart. What is the electric field at a point on the line joining them at a distance of 10 cm from their mid-point?

36. Two charges ±10 μC are placed 5 mm apart. Determine the electric field at

(i) a point P on the axis of dipole 15 cm away from its centre O on the side of the positive charge

(ii) a point Q, 15 cm away from centre O on a line passing through centre O and normal to axis of the dipole as **NCERT**

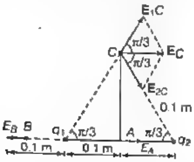


37. An electric dipole consists of two opposite charges each of magnitude 1.0×10^{-6} C separated by 2 cm. The dipole is placed in an external uniform field of 1×10^5 N/C. Find (i) the maximum torque exerted by the field on the dipole, (ii) the work which an external agent will have to do in turning the dipole through 180° starting from the position, $\theta = 0°$.

38. The electric field at a point on the axial line at a distance of 10 cm from the centre of an electric dipole is 3.75×10^5 N/C in air, while at a distance of 20 cm, the electric field is 3×10^4 N/C. Calculate the length of an electric dipole.

[redacted]

39. A two point charges q_1 and q_2 of magnitude 10^{-7} C and -10^{-7} C, respectively are placed 0.2 m apart. Calculate the electric fields at points A, B and C as shown in the figure. **NCERT**



40. (i) Calculate the maximum torque experienced by a water molecule whose electric dipole moment is 6.2×10^{-30} C-m, when it is placed in an electric field of intensity 10^6 N/C.

(ii) Determine the work that must be done to take a water molecule aligned with the above field and set it anti-parallel to the field.

HINTS AND SOLUTIONS

1. (b) Electric dipole moment, $p = q \times d$

Here, q = value of one charge on dipole = 2 C

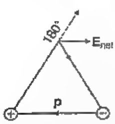
d = distance between the dipoles = 0.04 m

$\therefore$ Electric dipole moment, $p = 2 \times 0.04$

$= 0.08$ C-m

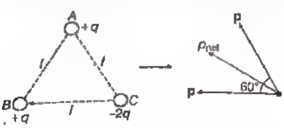
$= 8 \times 10^{-2}$ C-m

2. (c)



Observing $\mathbf{E}_{net}$ and **p** are in opposite directions, so angle between them is 180°.

3. (c)



Net dipole moment, *i.e.*,

$$p_{net} = \sqrt{p^2 + p^2 + 2pp\cos 60^\circ} = \sqrt{3}p$$

$$= \sqrt{3}\, ql \qquad (\because p = ql)$$

4. (c) Here, $a = 2.5$ mm, $r = 15$ cm = 150 mm

As, $r >> a$

$$E_{axis} = \frac{2p}{4\pi\varepsilon_0 r^3} = \frac{2(5\times10^{-3} \times 10 \times 10^{-6}) \times (9 \times 10^9)}{(15 \times 10^{-2})^3}$$

$$= 2.6 \times 10^5 \text{ NC}^{-1}$$

$$E_{\text{equatorial plane}} = \frac{p}{4\pi\varepsilon_0 r^3} = \frac{1}{2}E_{axis} = \frac{1}{2} \times 2.6 \times 10^5$$

$$= 1.3 \times 10^5 \text{ NC}^{-1}$$

5. (c) The space between the electric field lines is increasing, here from left to right and its characteristics states that, strength of electric field decreases with the increase in the space between electric field lines. As a result force on charges also decreases from left to right. Thus, the force on charge $-q$ is greater than force on charge $+q$ in turn dipole will experience a force towards left

6. (b) In electric field (**E**), torque acting on a dipole moment (**p**) is $\tau = pE\sin\theta$

where, θ = angle between **p** and **E**, $\Rightarrow \boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$

7. (a) Torque, $\tau = pE\sin\theta\,\hat{n}$

$$|\tau| = pE\sin\theta$$

$\therefore$ Torque is maximum, when $\theta = 90^\circ$

8. No, it is not correct to write the unit of electric dipole moment as mC. The symbol mC represents milli-coulomb, i.e. unit of electric charge. In SI system, unit symbols are written in alphabetical order

$\therefore$ Unit of dipole moment is C-m

9. If charge q gets larger and distance $2l$ gets smaller; and smaller keeping the product $p = q \times 2l$ = constant. The dipole is called an ideal electric dipole

10. At any point on axial line or equatorial line of dipole.

11. No, it does not experience a torque, when it is placed along the direction of electric field.

12. It experiences some net force and some net torque.

13. $\tau = pE\sin\theta$, τ is minimum when $\theta = 0^\circ$.

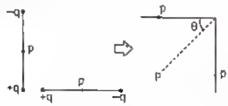
14. Yes, but motion is simple harmonic only when charge $-q$ is not very far from the centre of ring on its axis. Otherwise motion is periodic, but not simple harmonic in nature.

15. The electric field due to a point charge q at a distance r is given by $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$. Clearly, the magnitude of field E will be the at all points on the surface of a sphere of radius r drawn around the point charge and does not depend on the direction **r**. Hence, the field line due to a point charge is spherically symmetric. Electric field at distance r on the equatorial line of a dipole moment p is given by $E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{p}{(r^2 + a^2)^{3/2}}$.

[redacted]

The electric field E is same at all points which lie on a cylinder of radius r with its axis on the dipole axis and the field pattern looks same in all planes passing through the dipole axis. We say that the electric field of an electric dipole is cylindrical symmetric

16. Here, two dipoles are formed. These are shown in diagram below.

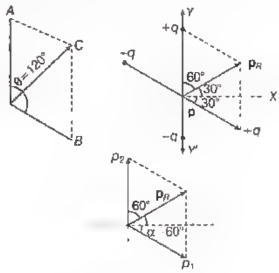


Resultant dipole moment,

$$P = \sqrt{2}p = \sqrt{2}qd,\ \theta = 45^\circ$$

17. Refer to text on page 28.

18. Consider the figure, $|\mathbf{p}_A| = p_C = p$



The magnitude of resultant p_R,

$$p_R = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos\theta}$$
$$= \sqrt{p^2 + p^2 + 2p^2\cos\theta}$$
$$= \sqrt{2p^2(1+\cos\theta)}$$
$$= \sqrt{2p^2 \times 2\cos^2\frac{\theta}{2}} = 2p\cos\frac{\theta}{2}$$

$$\tan\alpha = \frac{p_2\sin\theta}{p_1 + p_2\cos\theta} = \frac{p\sin 120^\circ}{p + p\cos 120^\circ}$$
$$= \frac{p\sqrt{3}/2}{p - \frac{p}{2}} = \sqrt{3}$$

$$\Rightarrow\ |\mathbf{p}_R| = 2p\cos\frac{\theta}{2} = 2p\cos\frac{120^\circ}{2} = 2p \times \frac{1}{2} = p$$

$\mathbf{p}_R$ will subtend an angle of 30° with X-axis.

Now, torque acting on the system,

$$\tau = \mathbf{p}_R \times \mathbf{E} = p_R E\sin\theta = \frac{1}{2}pE$$

Torque will work to align the dipole in the direction of electric field $\mathbf{E}$.

19. The position of stable equilibrium corresponds to $\theta = 0^\circ$

The position of unstable equilibrium corresponds to $\theta = 180^\circ$.

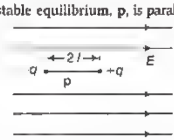
$$\therefore \text{Work done} = \int_{\theta=0^\circ}^{\theta=180^\circ} pE\sin\theta\, d\theta = pE[-\cos\theta]_{0^\circ}^{180^\circ} = 2pE$$

20. Refer to pages 25 and 26 (replacing $2l$ by $2a$)

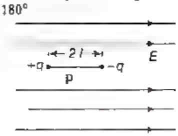
21. (i) Refer to text on page 26.

(ii) The orientation of the dipole

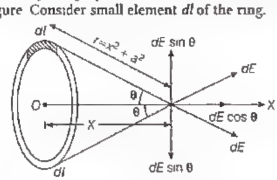
(a) In stable equilibrium, $\mathbf{p}$, is parallel to $\mathbf{E}$, i.e. $\theta = 0^\circ$



(b) In unstable equilibrium, $\mathbf{p}$ is anti-parallel to $\mathbf{E}$, i.e. $\theta = 180^\circ$



22. According to question, suppose that the ring is placed with its plane perpendicular to the X-axis as shown in figure. Consider small element dl of the ring.



As the total charge q is uniformly distributed, so the charge dq on element dl is $dq = \frac{q}{2\pi a}\cdot dl$

$$\Rightarrow dq = \frac{q}{2\pi a}\frac{dl}{r^2}\cos\theta = dE\cos\theta \qquad \left[\text{where } \cos\theta = \frac{x}{r}\right]$$

Since, only the axial component gives the net $\mathbf{E}$ at point P due to charge on ring.

So, $\int_0^E dE = \int_0^{2\pi a} dE\cos\theta = \int_0^{2\pi a}\frac{kq}{2\pi a}\cdot\frac{dl}{r^2}\times\frac{x}{r}$

$$= \frac{kqx}{2\pi a}\frac{1}{r^3}\int_0^{2\pi a} dl = \frac{kq}{2\pi a}\cdot\frac{1}{r^3}\int_0^{2\pi a}[l]_0^{2\pi a}$$

$$= \frac{kqx}{2\pi a}\frac{1}{(x^2+a^2)^{3/2}}\cdot 2\pi a \qquad [\because r^2 = x^2 + a^2]$$

$$\therefore \qquad E = \frac{kqx}{(x^2+a^2)^{3/2}}$$

Now, for points at large distances from the ring $x >> a$.

[redacted]

$$\therefore \quad E = \frac{kq}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}$$

This is same as the field due to a point charge indicating that for far-off axial point, the charged ring behaves as a point charge.

23. (i) Refer to text on page 28.
(ii) If the field is non-uniform, the net force will be non-zero.

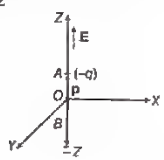
24. Refer to text on page 28.

25. Refer to text on pages 25 and 26.

26. Refer to text on page 28

27. Consider an electric dipole with charge $-q$ at A and charge $+q$ at B, placed along Z-axis, such that its dipole moment is in negative z-direction, i.e. $p_Z = -10^{-7}$ C-m, as shown in the figure.

The electric field is along positive direction of Z-axis, such that $\frac{dE}{dZ} = 10^5 \text{N C}^{-1}\text{m}^{-1}$.



Using, $\quad F = qdE = q \times \frac{dE}{dZ} \times dZ$

$$= (q \times dZ) \times \frac{dE}{dZ} = p\frac{dE}{dZ}$$

$$= 10^{-7} \times 10^5 = -10^{-2} \text{ N}$$

Thus, the force on the dipole is along negative direction of Z-axis.

As, $\quad \theta = 180°$

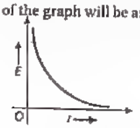
$\therefore$ Torque on dipole,

$$\tau = pE \sin 180° = 0$$

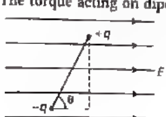
28. (i) Refer to text on pages 25 and 26.

(ii) $E \propto \frac{1}{r^3}$. As r will increase, E will sharply decreases.

The shape of the graph will be as given in the figure.



(iii) When the dipole were kept in a uniform electric field E_0. The torque acting on dipole, $\tau = p \times E$



(a) If $\theta = 0°$, then $\tau = 0$, $\mathbf{p} \parallel \mathbf{E}$

The dipole is in stable equilibrium.

(b) If $\theta = 180°$, then $\tau = 0$, $\mathbf{p} \parallel -\mathbf{E}$

The dipole is in unstable equilibrium.

29. Refer to text on page 28

(i) If the dipole is initially parallel to the field, $\theta_1 = 0°$

$$W = pE(1 - \cos\theta_2)$$

(ii) If the dipole is initially perpendicular to the field, $\theta_1 = 90°$

$$W = -pE\cos\theta_2$$

30. Dipole moment,

$$p = q \times r = 1.6 \times 10^{-19} \times 4.3 \times 10^{-9} = 6.8 \times 10^{-28} \text{ C-m}$$

31. As, $P(1, 0, 4)$ and $Q(2, -1, 5)$

$\therefore \quad 2l = \text{PQ} = [(2-1)\hat{i} + (-1-0)\hat{j} + (5-4)\hat{k}] = (\hat{i} - \hat{j} + \hat{k})$

and $\quad q = \pm 9 \times 10^{-6}$ C, $E = 0.20\,\hat{i}$ V/cm, $\tau = ?$

Since, $\tau = p \times E = q(2l) \times E$

$\therefore \quad \tau = 9 \times 10^{-6}(\hat{i} - \hat{j} + \hat{k}) \times 0.20\hat{i} = 18 \times 10^{-7}(\hat{k} - \hat{j})$

$\therefore$ Magnitude of torque,

$$\tau = 18 \times 10^{-7}\left[\sqrt{(1)^2 + (1)^2}\right] = 25.45 \times 10^{-7} \text{N-m}$$

32. Here, $q = 25 \times 10^{-9}$ C, $2a = 6$ m, $r = 4$ m,

$$p = q(2a) = 25 \times 10^{-9} \times 6 = 1.5 \times 10^{-7} \text{ C-m}$$

Now, $\quad E_{\text{axial}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2pr}{(r^2 - a^2)^2}$

$$= \frac{9 \times 10^9 \times 2 \times 1.5 \times 10^{-7} \times 4}{(4^2 - 3^2)^2} = \frac{2700 \times 4}{49}$$

$$= 220.4 \text{ NC}^{-1}$$

$\therefore \quad E_{\text{equatorial}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{(r^2 + a^2)^{3/2}}$

$$= \frac{9 \times 10^9 \times 1.5 \times 10^{-7}}{(4^2 + 3^2)^{3/2}} = \frac{1350}{125} = 10.8 \text{ N C}^{-1}$$

33. Given, $p = 4 \times 10^{-9}$ C-m, $E = 5 \times 10^4$, $\theta = 30°$

$\therefore \quad \tau = pE\sin\theta$

$$= 4 \times 10^{-9} \times 5 \times 10^4 \times \sin 30°$$

$$= 4 \times 10^{-9} \times 5 \times 10^4 \times \frac{1}{2} \qquad \left[\because \sin 30° = \frac{1}{2}\right]$$

$$= 10 \times 10^{-5} = 10^{-4} \text{ N-m}$$

34. Refer to Example 1 on page 25.

[redacted]

35. Here, $q_1 = q_2 = q = 0.1\mu\mu C = 10^{-13}$ C [in magnitude]

Length of the electric dipole formed by these charges,

$$2a = 10Å = 10^{-9} \text{ m}$$

Thus, electric dipole moment,

$$p = 2aq = 10^{-13} \times 10^{-9} = 10^{-22} \text{ C-m}$$

Distance of the point under consideration on the axial line from the mid-point, $r = 10$ cm $= 0.1$ m

Since, $a << r$, electric field at a point on the axial line,

$$E = k_e \frac{2p}{r^3}$$

$$= (9\times10^9)\frac{2\times10^{-22}}{(0.1)^3}$$

$$= 18\times10^{-10} \text{ N/C}$$

36. Refer to Example 2 on pages 26 and 27.

37. Here, $q = 1\times10^{-6}$ C, $2a = 2$ cm $= 0.02$ m

$\therefore p = q \times 2a = (1\times10^{-6})\times0.02 = 2\times10^{-8}$cm

Intensity of the external electric field, $E = 1.0\times10^5$ N/C

(i) $\tau_{max} = pE = (2\times10^{-8})(1.0\times10^5) = 2\times10^{-3}$N-m

(ii) Net work done in turning the dipole from 0° to 180°,

i.e. $W = \int_{0°}^{180°} \tau d\theta = \int_{0°}^{180°} pE\sin\theta d\theta = pE[-\cos\theta]_{0°}^{180°}$

$= -pE(\cos 180° - \cos 0°) = 2pE$

$= 2\times(2\times10^{-8})(1\times10^5)$ J $= 4\times10^{-3}$ J

38. We know that,

$$E_{axial} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2pr}{(r^2-a^2)^2}$$

Case I When $r = 10$ cm $= 0.1$ m and

$E_{axial} = 3.75\times10^5$ N/C

$$\Rightarrow \quad 3.75\times10^5 = 9\times10^9 \times \frac{2p\times0.1}{[(0.1)^2 - a^2]^2} \quad \text{...(i)}$$

Case II When $r = 20$ cm $= 0.2$ m and

$E_{axial} = 3\times10^4$ N/C

$$\Rightarrow \quad 3\times10^4 = 9\times10^9 \times \frac{2p\times0.2}{[(0.2)^2 - a^2]^2} \quad \text{... (ii)}$$

Solving the Eqs. (i) and (ii), we get

$a = 0.05$ m

Therefore, length of the dipole is $2a$.

So, $2a = 2\times0.05 = 0.1$ m

39. The electric field vector E_{1A} at A due to the positive charge q_1 points towards the right. Its magnitude,

$$E_{1A} = \frac{(9\times10^9 \text{ N-m}^2\text{C}^{-2})(10^{-7}\text{ C})}{(0.1 \text{ m})^2} = 9\times10^4 \text{ N C}^{-1}$$

The electric field vector E_{2A} due to q_2 points to the right and has the same magnitude.

Hence, the magnitude of total electric field E_A at A,

$$E_A = E_{1A} + E_{2A} = 18\times10^4 \text{ NC}^{-1}$$

i.e. E_A is directed towards right.

The electric field E_{1B} at B due to q_1 points towards the left and has a magnitude,

$$E_{1B} = \frac{(9\times10^9 \text{ N-m}^2\text{C}^{-2})(10^{-7}\text{C})}{(0.1 \text{ m})^2} = 9\times10^4 \text{ NC}^{-1}$$

The electric field E_{2B} at B due to the negative charge q_2 points towards the right and has a magnitude

$$E_{2B} = \frac{(9\times10^9 \text{ N-m}^2\text{C}^{-2})(10^{-7}\text{C})}{(0.3 \text{ m})^2} = 1\times10^4 \text{ NC}^{-1}$$

The magnitude of the total electric field at B,

$$E_B = E_{1B} - E_{2B}$$

$$= 8\times10^4 \text{ NC}^{-1}$$

i.e. E_B is directed towards the left.

The magnitude of each electric field vector at point C due to charge q_1 and q_2,

$$E_{1C} = E_{2C}$$

$$= \frac{(9\times10^9 \text{ N m}^2\text{C}^{-2})(10^{-7}\text{ C})}{(0.2 \text{ m})^2}$$

$$= 2.25\times10^4 \text{ NC}^{-1}$$

The resultant of these two vectors,

$$E_C = E_{1C}\cos\frac{\pi}{3} + E_{2C}\cos\frac{\pi}{3}$$

$$= 2.25\times10^4 \text{ NC}^{-1}$$

i.e. E_C points towards the right.

40. (i) Here, $p = 6.2\times10^{-30}$ C-m and $E = 10^6$ N/C

$\therefore \tau = pE\sin\theta$ [for maximum value $\theta = 90°$]

$= pE\sin 90° = 6.2\times10^{-30}\times10^6\times1$

$= 6.2\times10^{-24}$ N-m

(ii) When dipole is aligned anti-parallel to the field, $\theta = 180°$

$\therefore W = pE(1-\cos\theta)$

$= 6.2\times10^{-30}\times10^6(1-\cos180°)$ [$\because \cos 180° = -1$]

$= 6.2\times10^{-30}\times10^6(1-(-1))$

$= 6.2\times10^{-30}\times10^6\times2$

$= 1.24\times10^{-23}$ J

[redacted]

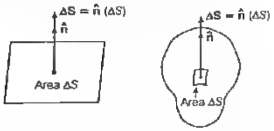
|TOPIC 4|

Electric Flux

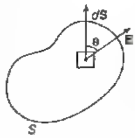
AREA VECTOR

The vector associated with every area element of a closed surface is taken to be in the direction of the outward normal. Thus, the area element vector ΔS at a point on a closed surface is equal to $\Delta S\,\hat{n}$, where ΔS is the magnitude of the area element and $\hat{n}$ is a unit vector in the direction of outward normal at the point.

$$\Delta S = \hat{n}(\Delta S)$$



ELECTRIC FLUX



Electric flux linked with any surface is defined as the total number of electric field lines that normally pass through that surface.

Electric flux $d\phi$ through a small area element dS due to an electric field $\mathbf{E}$ at an angle θ with dS is

$$d\phi = \mathbf{E}\cdot d\mathbf{S} = E\, dS \cos\theta$$

which is proportional to the number of field lines cutting the area element. Total electric flux ϕ over the whole surface S due to an electric field $\mathbf{E}$,

$$\phi = \oint_S \mathbf{E}\cdot d\mathbf{S} = \oint_S E\, dS\cos\theta$$

Electric flux is a scalar quantity. But it is a property of vector field.

SI unit of electric flux is N-m^{2}C^{-1}

and dimensional formula of electric flux is expressed as

$$\phi = [MLT^{-2}][L^2][AT]^{-1}$$
$$= [ML^3T^{-3}A^{-1}]$$

If $\oint \mathbf{E}\cdot d\mathbf{S}$ over a closed surface is negative, then the surface encloses a net negative charge.

Special cases

(i) For $0° < \theta < 90°$, ϕ is positive.

(ii) For $\theta = 90°$, ϕ is zero.

(iii) For $90° < \theta < 180°$, ϕ is negative.

> ***Analogy Between Electric Flux And Liquid Flux***
>
> It should be known that electric flux is analogous to flux of a liquid flowing across a plane which is equal to $\mathbf{v}\cdot\Delta\mathbf{S}$, where $\mathbf{v}$ is the velocity of flow of liquid. In electric flux, there is no flow of a physically observable quantity like liquid

EXAMPLE |1| A box encloses an electrical dipole consisting of charge $5\,\mu C$ and $-5\,\mu C$ and of length 10 cm. What is the total electric flux through the box?

All India 2011

Sol. Since, an electric dipole consists of two equal and opposite charges, the net charge on the dipole is zero.

Hence, the net electric flux coming out of the closed surface of the box or through the box is zero

GAUSS' THEOREM

Statement

The surface integral of the electric field intensity over any closed surface (called Gaussian surface) in free space is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed within the surface.

$$\phi_E = \oint_S \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_0}\sum_{i=1}^{n} q_i = \frac{q}{\varepsilon_0}$$

where, $q = \sum_{i=1}^{n} q_i$ is the algebraic sum of all the charges inside the closed surface.

Hence, total electric flux over a closed surface in vacuum is $\frac{1}{\varepsilon_0}$ times the total charge within the surface, regardless of how the charges may be distributed.

Proof of Gauss' Theorem for Spherically Symmetric Surface

Electric flux through a surface element dS is given by

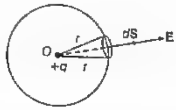
$$d\phi_E = \mathbf{E}\cdot d\mathbf{S} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\hat{\mathbf{r}}\cdot(dS\,\hat{\mathbf{n}})$$

$$\Rightarrow \quad d\phi_E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\,dS}{r^2}\hat{\mathbf{r}}\cdot\hat{\mathbf{n}}$$

[redacted]

Here, $\hat{\mathbf{r}} \cdot \hat{\mathbf{n}} = 1 \cdot 1 \cos 0° = 1$

$$\therefore \quad d\phi_E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q\,dS}{r^2}$$



Total electric flux through the spherical surface,

$$\phi_E = \oint_S d\phi_E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \oint_S dS$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow \quad \boxed{\phi_E = \frac{q}{\varepsilon_0}}$$

If the medium surrounding the charge has a dielectric constant K, then

$$\phi_E = \frac{q}{K\varepsilon_0} = \frac{q\varepsilon_0}{\varepsilon\varepsilon_0} = \frac{q}{\varepsilon}, \text{ where } K = \varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

$$\therefore \quad \phi_E = \frac{q}{\varepsilon}$$

If there is no net charge within the closed surface, i.e. when $q = 0$, then $\phi_E = 0$

∴ The total electric flux through a closed surface is zero, if no charge is enclosed by the surface.

> ***Some Features of Gauss' Law***
>
> (i) Gauss' law is true for any closed surface, no matter what its shape or size be.
>
> (ii) In the situation, when the surface is so chosen that there are some charges inside and some outside, the electric field is due to all the charges, both inside and outside the closed surface.
>
> (iii) Gauss' law is often useful when the system has some symmetry. This is facilitated by the choice of a suitable Gaussian surface.

EXAMPLE |2| A charge q is placed at the centre of a cube of side l. What is the electric flux passing through each face of the cube? **All India 2012; Foreign 2010**

Sol. By Gauss' theorem, total electric flux linked with a closed surface is given by

$$\phi = \frac{q}{\varepsilon_0}$$

where, q is the total charge enclosed by the closed surface.

∴ Total electric flux linked with cube, $\phi = \frac{q}{\varepsilon_0}$

As charge is at centre, therefore electric flux is symmetrically distributed through all 6 faces.

∴ Flux linked with each face $= \frac{1}{6}\phi = \frac{1}{6} \times \frac{q}{\varepsilon_0} = \frac{q}{6\varepsilon_0}$

EXAMPLE |3| Figure shows three point charges, $+2q$, $-q$ and $+3q$. Two charges $+2q$ and $-q$ are enclosed within a surface S. What is the electric flux due to this configuration through the surface S? **Delhi 2010**

Sol. Electric flux through the closed surface S is

$$\phi_S = \frac{\Sigma q}{\varepsilon_0} = \frac{+2q - q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

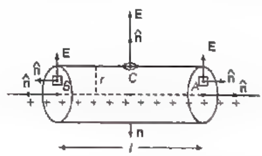
Charge $+3q$ is outside the closed surface S, therefore it would not be taken into consideration in applying Gauss' theorem.

Applications of Gauss' Theorem

The electric field due to some symmetric charge configurations can be obtained using Gauss' law.

Field due to an Infinitely Long Thin Straight Charged Wire

Consider an infinitely long thin straight wire with uniform linear charge density (λ).



From symmetry, the electric field is everywhere radial in the plane cutting the wire normally and its magnitude only depends on the radial distance (r).

From Gauss' law, $\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$

Now, $\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \oint_S \mathbf{E} \cdot \hat{\mathbf{n}}\, dS$

$$= \oint_A \mathbf{E} \cdot \hat{\mathbf{n}}\, dS + \oint_B \mathbf{E} \cdot \hat{\mathbf{n}}\, dS + \oint_C \mathbf{E} \cdot \hat{\mathbf{n}}\, dS$$

[redacted]

$$\therefore \oint_S \mathbf{E}\cdot d\mathbf{S}=\oint_A \mathbf{E}\; dS\cos 90^\circ+\oint_B \mathbf{E}\; dS\cos 90^\circ+\oint_C \mathbf{E}\; dS\cos 0^\circ$$

$$=\oint_C E\, dS=E(2\pi r l)$$

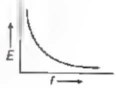
Charge enclosed in the cylinder, $q=\lambda l$

$$\therefore \quad E(2\pi r l)=\frac{\lambda l}{\varepsilon_0} \quad \text{or} \quad \boxed{E=\frac{\lambda}{2\pi\varepsilon_0 r}}$$

Vectorially, $\mathbf{E}=\dfrac{\lambda}{2\pi\varepsilon_0 r}\,\hat{\mathbf{r}}$

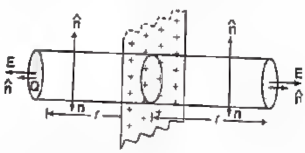
The direction of the electric field is radially outward from the positive line charge. For negative line charge, it will be radially inward.

Thus, electric field (E) due to the linear charge is inversely proportional to the distance (r) from the linear charge. The variation of electric field (E) with distance (r) is shown in figure.



Field due to a Thin Infinite Plane Sheet of Charge

Let σ be the surface charge density of the sheet. From symmetry, $\mathbf{E}$ on either side of the sheet must be perpendicular to the plane of the sheet, having same magnitude at all points equidistant from the sheet.



We take a cylindrical cross-sectional area A and length $2r$ as the Gaussian surface.

On the curved surface of the cylinder, $\mathbf{E}$ and $\hat{\mathbf{n}}$ are perpendicular to each other. Therefore, flux through curved surface = 0.

Flux through the flat surfaces $= EA+EA=2EA$

$\therefore$ Total electric flux over the entire surface of cylinder,

$$\phi_E=2EA$$

Total charge enclosed by the cylinder, $q=\sigma A$

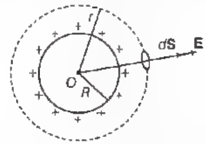
According to Gauss' law, $\phi_E=\dfrac{q}{\varepsilon_0}$

$$\therefore \quad 2EA=\frac{\sigma A}{\varepsilon_0} \quad \text{or} \quad \boxed{E=\frac{\sigma}{2\varepsilon_0}}$$

E is independent of r, the distance of the point from the plane charged sheet.

Field due to a Uniformly Charged Thin Spherical Shell

Let σ be the uniform surface charge density of a thin spherical shell of radius (R). The Gaussian surface will be a spherical surface centered at the centre of shell.



(i) At a point outside the shell ($r>R$)

Since, $\mathbf{E}$ and $d\mathbf{S}$ are in the same direction.

$$\therefore \quad \phi_E=\oint_S \mathbf{E}\cdot d\mathbf{S}=\frac{q}{\varepsilon_0} \quad \text{or} \quad E(4\pi r^2)=\frac{q}{\varepsilon_0}$$

$$\therefore \quad \boxed{E=\frac{q}{4\pi\varepsilon_0 r^2}}$$

Since, $\quad q=\sigma\times 4\pi R^2$

$$\therefore \quad E=\frac{\sigma R^2}{\varepsilon_0 r^2}$$

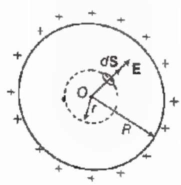
Vectorially, $\quad \mathbf{E}=\dfrac{\sigma R^2}{\varepsilon_0 r^2}\,\hat{\mathbf{r}}$

(ii) At a point on the surface of the shell ($r=R$)

$$\boxed{E=\frac{q}{4\pi\varepsilon_0 R^2}}$$

and $\quad E=\dfrac{\sigma}{\varepsilon_0}$

(iii) At a point inside the shell ($r<R$)



Here, the charge inside the Gaussian surface shell.

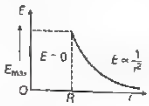
$$q=0$$

$$\therefore \quad E(4\pi r^2)=0$$

[redacted]

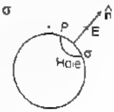
$\therefore \qquad E = 0$

This important result is a direct consequence of Gauss' law which follows from Coulomb's law. The experimental verification of this result confirms $1/r^2$ dependence in Coulomb's law. The variation of electric field intensity (E) with distance from the centre of a uniformly charged spherical shell is shown in figure.



EXAMPLE |4| A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is $\left(\dfrac{\sigma}{2\varepsilon_0}\right)\hat{n}$, where $\hat{n}$ is the unit vector in the outward normal direction and σ is the surface charge density near the hole. **NCERT**

Sol. Surface charge density near the hole $= \sigma$



Unit vector $= \hat{n}$ (normal directed outwards)

Let P be the point on the hole. The electric field at point P closed to the surface to conductor, according to Gauss' theorem,

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

where, q is the charge near the hole.

$$EdS \cos\theta = \frac{\sigma dS}{\varepsilon_0}$$

$[\because \sigma = q/dS \Rightarrow q = \sigma dS$, where dS = area$]$

$\therefore$ Angle between electric field and area vector is $0°$.

$$\therefore \quad E\,dS = \frac{\sigma\,dS}{\varepsilon_0} \qquad [\because \cos 0° = 1]$$

$$\Rightarrow \quad E = \frac{\sigma}{\varepsilon_0} \Rightarrow \mathbf{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$$

This electric field is due to the filled up hole and the field due to the rest of the charged conductor. The two fields inside the conductor are equal and opposite.

So, there is no electric field inside the conductor. Outside the conductor, the electric fields are equal in the same direction.

So, the electric field at point P due to each part

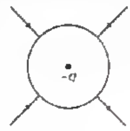
$$= \frac{1}{2}\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\hat{n}$$

EXAMPLE |5| A point charge causes an electric flux $-3\times10^{-14}\ \text{N-m}^2/\text{C}$ to pass through a spherical Gaussian surface.

(i) Calculate the value of the point charge.

(ii) If the radius of the Gaussian surface is doubled, how much flux would pass through the surface? **Foreign 2008**

Sol. (i) By Gauss theorem, total electric flux through closed Gaussian surface is given by



$$\phi = \frac{q}{\varepsilon_0}$$

$$\therefore \quad q = \phi\varepsilon_0$$

But electric flux passing through the surface,

$$\phi = -3\times10^{-14}\ \text{N-m}^2/\text{C}$$

$$\therefore \quad q = (-3\times10^{-14})\times8.85\times10^{-12} = -26.55\times10^{-26}\ \text{C}$$
$$= -2.655\times10^{-25}\ \text{C}$$

(ii) Electric flux passing through the surface remains unchanged because it depends only on charge enclosed by the surface and is independent of its size.

Note Electric flux, Gauss's law and numericals based on them have been frequently asked in previous years 2015, 2014, 2013, 2012, 2011, 2010

TOPIC PRACTICE 4

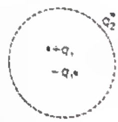
OBJECTIVE Type Questions

1. The SI unit of electric flux is

(a) $\dfrac{\text{volt}}{\text{metre}}$ (b) $\dfrac{\text{newton}}{\text{coulomb}}$

(c) $\dfrac{\text{newton} \times \text{metre}^2}{\text{coulomb}}$ (d) volt $\times$ metre^2

2. Consider the charge configuration and spherical Gaussian surface as shown in the figure. When calculating the flux of the electric field over the spherical surface, the electric field will be due to



(a) q_2 (b) only the positive charges
(c) all the charges (d) $+q_1$ and $-q_1$

3. Total electric flux coming out of a unit positive charge put in air is

(a) ε_0 (b) ε_0^{-1}
(c) $(4\pi\varepsilon_0)^{-1}$ (d) $4\pi\varepsilon_0$

4. In a system, 'n' electric dipole are placed in a closed surface. The value of emergent electric flux from enclosed surface is

(a) $\dfrac{q}{\varepsilon_0}$ (b) $\dfrac{2q}{\varepsilon_0}$ (c) $-\dfrac{2q}{\varepsilon_0}$ (d) zero

[redacted]

5. The intensity of electric field at the surface of conducting hollow sphere is $10\,NC^{-1}$ and its radius is 10 cm. The value of electric field at the centre of sphere is

(a) zero (b) $10\,NC^{-1}$ (c) $1\,NC^{-1}$ (d) $100\,NC^{-1}$

6. The surface densities on the surfaces of two charged spherical conductors of radii R_1 and R_2 are equal. The ratio of electric intensities on the surfaces are

(a) R_1^2/R_2^2 (b) R_2^2/R_1^2 (c) R_1/R_2 (d) 1 : 1

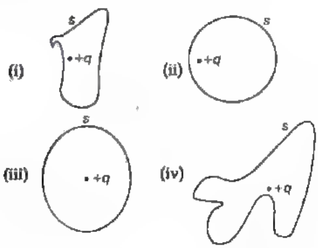
7. The electric flux in a charged spherical conductor is

(a) zero inside and outside the sphere
(b) maximum inside the sphere and zero outside the sphere
(c) zero inside the sphere and decreases outside the sphere with increase of square of distance
(d) maximum inside the sphere and decreases outside the sphere with increase of distance.

8. Radius of a hollow sphere is R and a charge q is placed at the centre of hollow sphere. If the radius of sphere becomes half and charge also becomes half, then the value of emergent total flux from the surface of sphere is

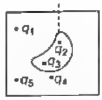
(a) $4q/\varepsilon_0$ (b) $2q/\varepsilon_0$ (c) $q/2\varepsilon_0$ (d) q/ε_0

9. The electric flux through the surface



(a) in Fig. (iv) is the largest
(b) in Fig. (iii) is the least
(c) in Fig. (ii) is same as Fig. (iii) but is smaller than Fig. (iv)
(d) is the same for all the figures

10. Five charges q_1, q_2, q_3, q_4, and q_5 are fixed at their positions as shown in Figure, S is a Gaussian surface. The Gauss' law is given by $\int_s \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$. Which of the following statements is correct?



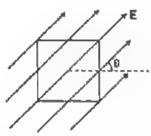
(a) E on the LHS of the above equation will have a contribution from q_1, q_5 and q_1, q_5 and q_3 while q on the RHS will have a contribution from q_2 and q_4 only
(b) E on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_3 only
(c) E on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_1, q_3 and q_5 only
(d) Both E on the LHS and q on the RHS will have contributions from q_2 and q_4 only

VERY SHORT ANSWER Type Questions

11. Can Gauss' law in electrostatics tell us exactly, where the charge is located within the Gaussian surface?

12. An arbitrary surface encloses a dipole. What is the electric flux through this surface? **NCERT Exemplar**

13. A square surface of side l metres in the plane of paper is placed in a uniform electric field E acting along the same plane at an angle θ with the horizontal side of square as shown in the figure. What is the electric flux linked to the surface?



14. What is the net flux of the uniform electric field through a cube of side 20 cm oriented, so that its faces are parallel to the coordinate planes? **NCERT**

15. What is the number of electric field lines that radiate outward from one coulomb of charge in vacuum?

[redacted]

16. Does the strength of electric field due to an infinite long line charge depend upon the distance of the observation point from the line charge?

17. How does electric field at a point charge vary with distance r from an infinitely long charged wire?

18. Does the strength of electric field due to an infinite plane sheet of charge depend upon the distance of the observation point from the sheet of charge? **Delhi 2010**

19. How does the electric flux due to a point charge enclosed by a spherical Gaussian surface get affected when its radius is increased? **Delhi 2016**

20. Two charges of magnitudes $-2Q$ and $+Q$ are located at points $(a, 0)$ and $(4a, 0)$, respectively. What is the electric flux due to these charges through a sphere of radius $3a$ with its centre at the origin? **All India 2013**

21. What is the electric flux through a cube of side 1 cm which encloses an electric dipole? **All India 2015**

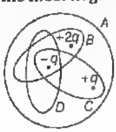
22. (i) A charge q is placed at the centre of a cube. What is the electric flux passing through each face of cube?
   (ii) If radius of Gaussian surface enclosing some charge q is halved, then how does electric flux through Gaussian surface change?

SHORT ANSWER Type Questions

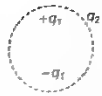
23. If the total charge enclosed by a surface is zero, does it imply that the electric field everywhere on the surface is zero, conversely, if the electric field everywhere on the surface is zero? Does it imply the net charge inside is zero? **NCERT Exemplar**

24. A charge q is enclosed by a spherical surface of radius R. If the radius is reduced to half, how would the electric flux through the surface change?

25. Rank the Gaussian surfaces as shown in the figure. In order of increasing electric flux, starting with the most negative.



26. Consider the charge configuration and a spherical Gaussian surface as shown in the figure.



Which one of the three charges will be the cause of electric field while calculating the flux of the field over the spherical surface?

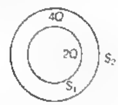
27. Deduce Coulomb's law from Gauss' law.

28. What will be the electric field intensity at the centre of a uniformly charged circular wire of linear charge density?

29. A thin straight infinitely long conducting wire having charge density λ is enclosed by a cylindrical surface of radius r and length l, its axis coinciding with the length of the wire. Find the expression for the electric flux through the surface of the cylinder. **All India 2011**

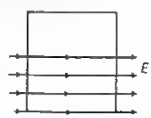
30. A hemispherical body is placed in a uniform electric field E. What is the flux associated with the curved surface, if field is
   (i) parallel to base?
   (ii) perpendicular to base?

31. Consider two hollow concentric spheres S_1 and S_2 enclosing charges $2Q$ and $4Q$ respectively, as shown in the figure.



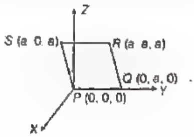
   (i) Find out the ratio of the electric flux through them.
   (ii) How will the electric flux through the sphere S_1 change, if a medium of dielectric constant ε_r is introduced in the space inside S_1 in place of air? Deduce the necessary expression.

32. A square surface of side l metre is in the plane of paper. A uniform electric field E (volt/metre), also in the plane of the paper, is limited only to the lower half of the square surface, (see figure) What is the electric flux associated with this surface?

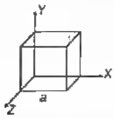


[redacted]

33. Consider an electric field $\mathbf{E} = E_0\hat{x}$, where E_0 is a constant. What is the flux through the shaded area (as shown in figure) due to this field?

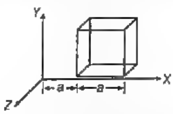


34. Given the electric field in the region $\mathbf{E} = 2x\hat{\mathbf{i}}$, find the net electric flux through the cube and the charge enclosed by it. **All India 2015**



LONG ANSWER Type I Questions

35. State Gauss' law in electrostatics. A cube with each side a is kept in an electric field given by $\mathbf{E} = Cx\hat{\mathbf{i}}$ as shown in the figure, where C is a positive dimensionless constant.



Find out

(i) the electric flux through the cube and

(ii) the net charge inside the cube. **Foreign 2012**

36. Use Gauss' law to derive the expression for the electric field between two uniformly charge parallel, sheets with surface charge densities σ and $-\sigma$, respectively. **All India 2009**

37. Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 8.0×10^3 N-m^{2}C^{-1}.

(i) What is the net charge inside the box?

(ii) If the net outward flux through the surface of the box was zero, could you conclude that there were no charges inside the box. Why or why not? **NCERT**

LONG ANSWER Type II Questions

38. (i) Define electric flux. Write its SI unit. Gauss' law in electrostatics is true for any closed surface, no matter what its shape or size is. Justify this statement with the help of a suitable example.

(ii) Use Gauss' law to prove that the electric field inside a uniformly charged spherical shell is zero. **Delhi 2015**

39. (i) State Gauss' theorem.

(ii) Using Gauss' law, prove that the electric field at a point due to a uniformly charged infinite plane sheet is independent of the distance from it.

(iii) How is the field directed, if

(a) the sheet is positively charged,

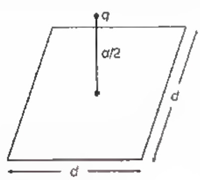
(b) negatively charged? **Delhi 2012**

40. (i) Use Gauss' theorem to find the electric field due to a uniformly charged infinitely large plane thin sheet with surface charge density σ.

(ii) An infinitely large thin plane sheet has a uniform surface charge density $+\sigma$. Obtain the expression for the amount of work done in bringing a point charge q from infinity to a point, distant r, in front of the charged plane sheet. **All India 2017**

41. (a) Define electric flux. Is it a scalar or a vector quantity?

A point charge q is at a distance of $d/2$ directly above the centre of a square of side d, as shown in the figure. Use Gauss' law to obtain the expression for the electric flux through the square. **CBSE 2018**



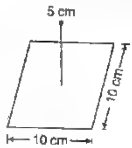
(b) If the point charge is now moved to a distance d from the centre of the square and the side of the square is doubled, explain how the electric flux will be affected.

[redacted]

NUMERICAL PROBLEMS

42. An infinite line charge produces a field of 9×10^4 N/C at a distance of 2 cm. Calculate the linear charge density. **NCERT**

43. A point charge $+10\,\mu$C is at a distance 5 cm directly above the centre of a square of side 10 cm, as shown in figure. What is the magnitude of the electric flux through the square? **NCERT**



44. A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface? **NCERT**

45. Consider a uniform electric field $\mathbf{E} = 3 \times 10^3 \hat{\mathbf{i}}$ N/C.

(i) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the YZ-plane?

(ii) What is the flux through the same square, if the normal to its plane makes an angle 60° with the X-axis? **NCERT**

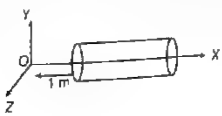
46. Given a uniform electric field $\mathbf{E} = 5 \times 10^3 \hat{\mathbf{i}}$ N/C, find the flux of this field through a square of 10 cm on a side whose plane is parallel to the YZ-plane. What would be the flux through the same square, if the plane makes an angle of 30° with the X-axis? **Delhi 2014**

47. A uniformly charged conducting sphere of diameter 2.4 m has a surface charge density of $80.0\,\mu\text{C/m}^2$.

(i) Find the charge on the sphere.

(ii) What is the total electric flux leaving the surface of the sphere? **NCERT**

48. A hollow cylindrical box of length 1 m and area of cross-section 25 cm^2 is placed in a three dimensional coordinate system as shown in the figure. The electric field in the region is given by $\mathbf{E} = 50\,x\,\hat{\mathbf{i}}$, where E is in NC^{-1}and x is in metre.



Find

(i) net flux through the cylinder and

(ii) charge enclosed by the cylinder.

Delhi 2014

49. A uniform electric field is given as $\mathbf{E} = 100\hat{\mathbf{i}}$ N/C for $x > 0$ and $\mathbf{E} = -100\hat{\mathbf{i}}$ N/C for $x < 0$. A right circular cylinder of length 20 cm and radius 5 cm has its centre at the origin and its axis along the X-axis, so that one face is at $x = +10$ cm and other is at $x = -10$ cm.

(i) What is the net outward flux through each flat face?

(ii) What is the flux through the side of cylinder?

(iii) What is the net outward flux through the cylinder?

(iv) What is the net charge inside the cylinder?

50. Two large thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude $17.0 \times 10^{-22}\,\text{Cm}^{-2}$. What is E

(i) to the left of the plates,

(ii) to the right of the plates and

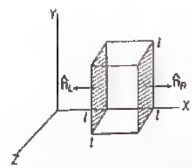
(iii) in between the plates? **NCERT**

51. A point charge causes an electric flux of -1.0×10^3 N-m^2/C to pass through a spherical Gaussian surface of 10.0 cm, radius centred on the charge.

(i) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface?

(ii) What is the value of point charge? **NCERT**

52. The electric field components in given figure are $E_X = \alpha x^{1/2}$, $E_Y = E_Z = 0$, in which $\alpha = 600$ N/ C-m$^{1/2}$.



Calculate (i) the flux through the cube and (ii) the charge within the cube. Assume that $l = 0.1$ m.

[redacted]

HINTS AND SOLUTIONS

1. (c) The SI unit of electric flux is $N\text{-}m^2\,C^{-1}$.

2. (c) As flux $= \frac{q_{enclosed}}{\varepsilon_0}$. So, flux is due to only charges $+q_1$ and $-q_1$ that makes a sum zero. But q_2 produces its own flux and net flux linked with sphere is zero. Electric field will be due to all the charges.

3. (b) By Gauss' law, ϕ = Electric flux through closed surface area

$$= \frac{q_{enclosed}}{\varepsilon_0}, \text{ if } q_{enclosed} = 1 \text{ unit}$$

$$\Rightarrow \quad \phi = \frac{1}{\varepsilon_0} = \varepsilon_0^{-1}$$

4. (d) According to the definition of an electric dipole, net charge in enclosed surface $= +q - q = 0$

Hence, electric flux,

$$\phi_E = \frac{q}{\varepsilon_0} = 0$$

5. (a) Inside the conducting sphere, electric field at every point is zero.

6. (d) Intensity of electric field on the surface of conducting sphere, $(E = \sigma/\varepsilon_0)$.

Since, both charged spheres have same surface charge density, so according to Gauss' theorem, these have same electric intensity i.e., the ratio is $1:1$.

7. (c) Electric flux is zero inside of spherical conductor and outside is $E \propto (1/r^2)$ and decreases outside the sphere with increase of distance.

8. (c) According to Gauss' law, emergent flux, $\phi = \frac{q}{\varepsilon_0}$.

If charge becomes half, then the value of charge in the surface is $q' = \frac{q}{2}$, so $\phi = \frac{q}{2\varepsilon_0}$.

9. (d) Gauss' law of electrostatics state that the total of the electric flux out of a closed surface is equal to the charge enclosed decided by the permittivity i.e., Q electric $= \frac{Q}{\varepsilon_0}$.

Thus, electric flux through a surface doesn't depend on the shape, size or area of a surface but it depends on the number of charges enclosed by the surface.

So, here in this question, all the figures have same electric flux as all of them has single positive charge.

10. (b) According to Gauss' law, the term q on the right side of the equation $\oint_s \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$ includes the sum of all charges enclosed by the surface.

The charges may be located anywhere inside the surface, if the surface is so chosen that there are some charges inside and some outside, the electric field on the left side of equation is due to all the charges, both inside and outside S. So, E on LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_3 only.

11. No, it tells us only about the magnitude of charge enclosed by the Gaussian surface.

12. If any arbitrary surface encloses a dipole, then the net charge is zero because the total charge on the dipole is zero (dipole consists of two equal and opposite charges). According to Gauss' law,

total flux $= \frac{1}{\varepsilon_0} \times$ charge enclosed $= \frac{1}{\varepsilon_0} \times (0) = 0 \Rightarrow \phi = 0$

13. Since, E is acting along the same plane at an angle θ as shown in the figure, therefore electric flux,

$\phi = EA\cos\theta$, where θ is angle between E and normal to surface, i.e.

$$\theta = 90^\circ \Rightarrow \phi = EA\cos 90^\circ = 0$$

14. As we know that, the number of field lines entering in the cube is the same as that the number of field lines leaving the cube. So, no flux is remained on the cube and hence, the net flux over the cube is zero.

15. Number of electric field lines $= \frac{1}{\varepsilon_0} = \frac{1}{8.85 \times 10^{-12}}$

$= 1.13 \times 10^{11} \quad [\because q = 1\,C]$

16. Yes, the electric field due to an infinitely long line charge depends upon the distance of the observation point from the line charge.

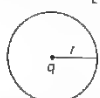
17. The electric field due to a line charge falls of with distance as $1/r$.

18. No, the electric field due to an infinite plane sheet of charge does not depend upon the distance of the observation point from the plane sheet of charge.

19. According to question, electric flux (ϕ) due to a point charge enclosed by a spherical Gaussian surface is given by

$$\phi = E \cdot A$$

$$= \frac{kq}{r^2}\,4\pi r^2 = kq\cdot 4\pi \qquad \left[\because E = \frac{kq}{r^2} \text{ and } A = 4\pi r^2\right]$$



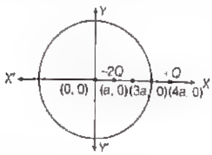
So, there is no effect of change in radius on the electric flux.

20. Gauss' theorem states that the total electric flux linked with closed surface S,

$$\phi_E = \mathbf{E}\cdot d\mathbf{S} = \frac{q}{E_0}$$

[redacted]

where, q is the total charge enclosed by the closed Gaussian (imaginary) surface.



The sphere enclosed charge $= -2Q$

Therefore, $\phi = \dfrac{2Q}{\varepsilon_0}$ (inwards)

21. Since, according to the Gauss' law of electrostatics, electric flux through any closed surface is given by

$$\phi_E = \oint E \cdot dS = \frac{q}{\varepsilon_0} \quad \text{...(i)}$$

where, E = electrostatic field,

q = total charge enclosed by the surface

and ε_0 = absolute electric permittivity of free space.

So, in the given case, cube encloses an electric dipole.

Therefore, the total charge enclosed by the cube is zero.

i.e. $q = 0$

Therefore, from Eq. (i), we get

$$\phi_E = \frac{q}{\varepsilon_0} = 0$$

i.e. Electric flux is zero.

22. (i) Flux through each face

$$= \frac{1}{6}\left(\frac{q}{\varepsilon_0}\right) = \frac{q}{6\varepsilon_0}$$

(ii) Electric flux, $\phi = \dfrac{q}{6\varepsilon_0}$, since q does not change, ϕ will remain same.

23. No, since $\oint E \cdot dS = \dfrac{q}{\varepsilon_0} = 0$, therefore the field may be normal to the surface.

However, the reverse is true, i.e. when $E = 0$ everywhere on the surface, the net charge inside is zero.

24. We know that, $\phi_E = \dfrac{Q_{\text{enclosed}}}{\varepsilon_0}$

Here, Q_{enclosed} remains unchanged. Hence, electric flux through the surface remains same.

25. Since, surface D enclosed negative charge, hence it has least flux negative.

In parts C and A, there is zero net charge, hence flux is zero, surface B has most flux, which is positive in nature, since it consist positive charge, i.e. $+2q$.

26. Refer to Example 3 on page 37.

27. According to Gauss' theorem,

$$\int_S E \cdot dS = \frac{q}{\varepsilon_0} \Rightarrow E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\therefore \quad E = \frac{q}{4\pi\varepsilon_0 r^2}$$

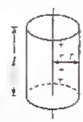
If a charge q_0 is kept on the surface, then

$F = E \times q_0 = \dfrac{qq_0}{4\pi\varepsilon_0 r^2}$, which is Coulomb's law.

28. A uniformly charged circular wire can be considered to be subdivided into pairs of diametrically opposite elements. The electric field intensity at the centre of wire due to each of the pairs is zero, therefore the electric field intensity due to the entire circular wire will be zero.

29. **Hints:** A thin straight conducting wire will have a uniform linear charge distribution.

Let q charge be enclosed by the cylindrical surface.



$\therefore$ Linear charge density, $\lambda = \dfrac{q}{l} \Rightarrow q = \lambda l$ ...(i)

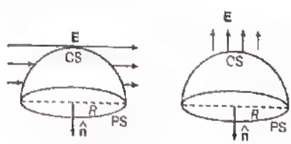
By Gauss' theorem,

total electric flux through the surface of cylinder,

$$\phi = \frac{q}{\varepsilon_0}$$

$$\therefore \quad \phi = \frac{\lambda l}{\varepsilon_0} \quad \text{[from Eq. (i)]}$$

30. Considering the hemispherical body as a closed body with a Curved Surface (CS) and a Plane Surface (PS), the total flux (ϕ) linked with the body will be zero, as no charge is enclosed by the body



$$\therefore \quad \phi = \phi_{CS} + \phi_{PS} = 0 \quad \text{...(i)}$$

[redacted]

(i) When field is parallel to the base,

$$\phi_{PS} = E \times \pi R^2 \cos 90^\circ = 0$$

From Eq. (i), we get

$$\phi = \phi_{CS} = 0$$

(ii) When field is perpendicular to the base,

$$\phi_{PS} = E \times \pi R^2 \cos 180^\circ$$
$$= -E\pi R^2$$

From Eq. (i), we get

$$\phi_{CS} - E\pi R^2 = 0$$

$$\Rightarrow \quad \phi_{CS} = E\pi R^2$$

31. (i) According to Gauss' theorem

$$\phi = \frac{\Sigma q}{\varepsilon_0 \varepsilon_r} \propto \Sigma q$$

$$\therefore \quad \frac{\phi_{S_1}}{\phi_{S_2}} = \frac{2Q}{2Q + 4Q} = \frac{2Q}{6Q} = \frac{1}{3}$$

(ii) If a medium of dielectric constant ε_r is introduced in the space inside S_1 in place of air, then

$$\phi_{S_1} = \frac{\Sigma q}{\varepsilon_0 \varepsilon_r} = \frac{2Q}{\varepsilon_0 \varepsilon_r}$$

32. Electric flux ϕ is a measure of number of field lines crossing a surface. The number of field lines passing through unit area (N/S) will be proportional to the electric field, i.e. $N/S \propto E$

$$\Rightarrow \quad N \propto ES$$

The quantity ES is the electric flux through surface S. As in the given question, the field lines that enter the closed surface leave the surface immediately, so the net electric flux is bound to the system. Thus, electric flux is zero.

33. We have, $\mathbf{E} = E_0 \hat{x}$

Consider $\hat{x}, \hat{y}$ and $\hat{z}$ be the unit vectors along X, Y and Z-axes, respectively.

In the figure, shaded area, $\mathbf{A} = \mathbf{PQ} \times \mathbf{PS}$

$$\therefore \quad \mathbf{A} = (0\hat{x} + a\hat{y} + 0\hat{z}) \times (a\hat{x} + 0\hat{y} + a\hat{z}) = a^2\hat{x} - a^2\hat{z}$$

$\therefore$ Electric flux through the shaded area is given by

$$\phi = \mathbf{E} \cdot \mathbf{A} = (E_0\hat{x}) \cdot (a^2\hat{x} - a^2\hat{z}) = E_0 a^2$$

34. Since, the electric field has only x-component, for faces normal to x-direction, the angle between E and ΔS is $\pm \frac{\pi}{2}$. Therefore, the flux is separately zero for each of the cube except the shaded ones.

The magnitude of the electric field at the left face is

$E_L = 0$ [as $x = 0$ at the left face]

The magnitude of the electric field at the right face is

$E_R = 3a$ [as, $x = a$ at the right face]

The corresponding fluxes are

$$\phi_L = \mathbf{E}_L \cdot \Delta \mathbf{S} = 0$$

and $\phi_R = \mathbf{E}_L \cdot \Delta \mathbf{S}$

$= E_R \Delta S \cos\theta = E_R \Delta S$ $\quad [\because \theta = 0^\circ]$

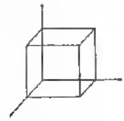
$$\Rightarrow \quad \phi_R = E_R a^2$$

Net flux (ϕ) through the cube

$$= \phi_L + \phi_R = 0 + E_R a^2 = E_R a^2$$

$$\Rightarrow \quad q = 2a(a)^2 = 2a^3$$

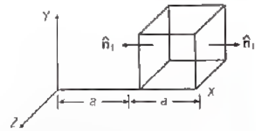
We can use Gauss' law to find the total charge q inside the cube.



$$\phi = \frac{q}{\varepsilon_0} \Rightarrow \phi = \frac{2a^3}{\varepsilon_0}$$

35. Refer to text on page 36

(i) Now, the electric field, $\mathbf{E} = Cx\hat{i}$ is in x-direction only. So, face with surface normal vector perpendicular to this field would give zero electric flux, i.e. $\phi = E\,dS \cos 90^\circ = 0$, through it.



So, flux would be across only two surfaces.

Magnitude of E at left face,

$E_L = Cx = Ca$ $\quad [x = a$ at left face$]$

Magnitude of E at right face,

$E_R = Cx = C2a = 2aC$ $\quad [x = 2a$ at right face$]$

Thus, corresponding fluxes are

$\phi_L = \mathbf{E}_L \cdot d\mathbf{S} = E_L dS \cos\theta$

$= -aC \times a^2 = -a^3C$ $\quad [\because \theta = 180^\circ]$

and $\phi_R = \mathbf{E}_R \cdot d\mathbf{S}$

$= 2aC\,dS \cos\theta$ $\quad [\because \theta = 0^\circ]$

$= 2aCa^2 = 2a^3C$

Now, net flux through cube

$= \phi_L + \phi_R$

$= -a^3C + 2a^3C$

$= a^3C$ N-m^{2}C^{-1}

(ii) Net charge inside the cube, again we can use Gauss' law to find total charge q inside the cube.

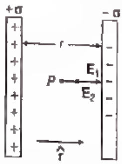
We have, $\phi = \frac{q}{\varepsilon_0}$ or $q = \phi\varepsilon_0$

$$\Rightarrow \quad q = a^3 C \varepsilon_0$$

36. Let us consider two uniformly charge parallel sheets carrying surface charge densities $+\sigma$ and $-\sigma$ respectively and are separated by a small distance from each other

[redacted]

By Gauss' law, it can be proved that, electric field intensity due to a uniformly charged infinite plane sheet as nearby is given by, $E = \frac{\sigma}{2\varepsilon_0}$



The electric field is directed normally outward from the plane sheet, if nature of charge on sheet is positive and normally inward, if charge is of negative nature. Let $\hat{r}$ represents unit vector directed from positive plate to negative plate.

Now, electric field intensity (EFI) at any point P between the two plates is given by

(i) $\mathbf{E}_1 = +\frac{\sigma}{2\varepsilon_0}\hat{r}$ [due to positive plate]

(ii) $\mathbf{E}_2 = +\frac{\sigma}{2\varepsilon_0}\hat{r}$ [due to negative plate]

∴ New EFI at point P,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \frac{\sigma}{2\varepsilon_0}\hat{r} + \frac{\sigma}{2\varepsilon_0}\hat{r} = \frac{\sigma}{\varepsilon_0}\hat{r}$$

Thus, uniform electric field is produced between the two infinite parallel plane sheet of charge which is directed from positive plate to negative plate.

37. Here, $\phi = 8.0 \times 10^3$ N-m^{2}C^{-1}

(i) Suppose that the net charge inside the box is q, then according to Gauss' theorem,

$$\phi = \frac{q}{\varepsilon_0} \text{ or } q = \varepsilon_0 \phi$$

$\because$ $\varepsilon_0 = 8.854 \times 10^{-12}$ C^{2}N^{-1}m^{-2}

$\therefore$ $q = 8.854 \times 10^{-12} \times 8.0 \times 10^3$

$= 70.832 \times 10^{-9}$ C

(ii) If the net outward flux through the surface of the box is zero, then it cannot be concluded that there is no charge inside the box. There may be equal amount of positive and negative charges inside the box. Therefore, if the net outward flux is zero, we cannot conclude that the charge inside the box is zero.

One can only say that the net charge inside the box is zero.

38. (i) Electric flux over an area in an electric field represents the total number of electric field lines crossing the area. The SI unit of electric flux is N - m^{2}C^{-1}.

According to Gauss' law in electrostatics, the surface integral of electrostatic field **E** produced by any source over any closed surface S enclosing a volume V in vacuum, i.e. total electric flux over the closed surface S in vacuum, is $1/\varepsilon_0$ times the total charge (q) contained inside S, i.e.

$$\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

Gauss' law in electrostatics is true for a closed surface, no matter what its shape or size is.

So, in order to justify the above statement, suppose in isolated positive charge q is situated at the centre O of a sphere of radius r.

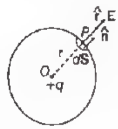
According to Coulomb's law, electric field intensity at any point P on the surface of the sphere is

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0} \cdot \frac{\hat{r}}{r^2}$$

where, $\hat{r}$ is unit vector directed from O to P.

Consider a small area element dS of the sphere around P. Let it be represented by the vector $d\mathbf{S} = \hat{n} \cdot dS$.

where, $\hat{n}$ is unit vector along out drawn normal to the area element.



∴ Electric flux over the area element,

$$d\phi_E = \mathbf{E} \cdot d\mathbf{S}$$

$$= \left(\frac{q}{4\pi\varepsilon_0} \cdot \frac{\hat{r}}{r^2}\right) \cdot (\hat{n} \cdot dS)$$

$$\mathbf{E} \cdot d\mathbf{S} = \frac{q}{4\pi\varepsilon_0} \frac{dS}{r^2} \hat{r} \cdot \hat{n}$$

As normal to a surface of every point is along the radius vector at that point, therefore $\hat{r} \cdot \hat{n} = 1$

$$\mathbf{E} \cdot d\mathbf{S} = \frac{q}{4\pi\varepsilon_0} \cdot \frac{dS}{r^2}$$

Integrating over the closed surface area of the sphere, we get total normal electric flux over the entire sphere,

$$\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{4\pi\varepsilon_0 r^2} \oint_S dS$$

$$= \frac{q}{4\pi\varepsilon_0 r^2} \times \text{total area of surface of sphere.}$$

$$= \frac{q}{4\pi\varepsilon_0 r^2}(4\pi r^2) = \frac{q}{\varepsilon_0}$$

Hence, $\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$, which proves Gauss' theorem.

(ii) **Electric field inside a uniformly charged spherical shell**

Refer to text on pages 38 and 39.

39. (i) Refer to text on page 36.

(ii) and (iii) Refer to text on page 38.

[redacted]

40. (i) Refer to text on page 38.

(ii) Surface charge density of the uniform plane sheet which is infinitely large $+\sigma$. The electric potential (V) due to infinite sheet of a uniform charge density $+\sigma$,

$$V=\frac{-\sigma r}{2\varepsilon_0}$$

The amount of work done in bringing a point charge q from infinite to point, at distance r in front of the charged plane sheet, is

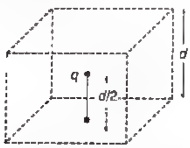
$$W=q'\times V=q'\cdot\frac{\sigma r}{2\varepsilon_0}=\frac{\sigma r\cdot q'}{2\varepsilon_0}\text{ joule}$$

41. (a) **Electric flux** It is defined as the total number of electric field lines that are normally pass through that surface.

Total electric flux ϕ over the whole surface S due to an electric field E is given as

$$\phi=\oint_S \mathbf{E}\cdot d\mathbf{S}=\oint E\,dS\cos\theta$$

It is a scalar quantity.



From the given problem, q is the point charge at a distance of $\frac{d}{2}$ directly above the centre of the square side.

Now, construct a Gaussian surface in form of a cube of side d to evaluate the amount of electric flux.

∴ We can calculate the amount of electric flux for six surfaces by using Gauss'S law,

$$\phi_E=\int_s \mathbf{E}\cdot d\mathbf{S}=\frac{q}{\varepsilon_0}$$

∴ For one surface of the cube, amount of electric flux is given as $\phi_E'=\frac{q}{6\varepsilon_0}$

(b) Even if the point charge is moved to a distance d from the centre of the square and side of the square is doubled, but amount of charge enclosed into the Gaussian surface does not changes.

∴ The amount of electric flux remains same.

42. Here, $E=9\times10^4$ N/C, $r=2\text{ cm}=2\times10^{-2}$ m and $\lambda=?$

As, $E=\frac{\lambda}{2\pi\varepsilon_0 r}\Rightarrow\lambda=2\pi\varepsilon_0 rE$

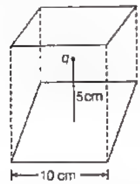
$$=\frac{1}{2\times9\times10^9}\times2\times10^{-2}\times9\times10^4=10^{-7}\text{Cm}^{-1}$$

43. **Hints:** Think of the square as one face of a cube with edge 10 cm.

Electric flux linked with a surface can be calculated using Gauss' theorem, according to which total electric flux linked with a closed surface is $\frac{q}{\varepsilon_0}$.

Now, we imagine an enclosed cubical surface and the given square be one side of this cubical surface. Let a charge q be placed at the centre of cube.

Now, the figure looks like as shown below.



The total flux enclosed through the centre of cube is given by

$$\phi=\frac{q}{\varepsilon_0}\qquad\text{...(i)}$$

[according to Gauss' theorem]

Here, $q=10\,\mu$C

The flux enclosed by one face, i.e. square is 1/6 of total flux (because the cube has six square shaped faces), so the flux linked with each square,

$$\phi'=\frac{\phi}{6}=\frac{1}{6}\cdot\frac{q}{\varepsilon_0}\qquad\text{[from Eq. (i)]}$$

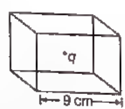
$$\Rightarrow\quad\phi'=\frac{1}{6}\times\frac{10\times10^{-6}}{8.854\times10^{-12}}=1.88\times10^5\text{ N-m}^2/\text{C}$$

Thus, the flux linked with the square is 1.88×10^5 N m^2/C.

44. Let us consider a charge q is placed at the centre of a cubic Gaussian surface. As per the question,

$$q=2\,\mu\text{C}=2\times10^{-6}\text{ C}$$

Length of edge = 9 cm



According to Gauss' theorem, the net electric flux (ϕ) through the surface is given by

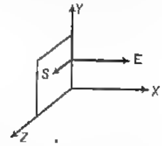
$$\phi=\frac{q}{\varepsilon_0}=\frac{2\times10^{-6}}{8.854\times10^{-12}}=2.26\times10^5\text{ N-m}^2/\text{C}$$

Thus, the net electric flux through the surface is 2.26×10^5 N-m^2/C.

[redacted]

45. Electric field, $\mathbf{E} = 3\times10^3\,\hat{\mathbf{i}}$ N/C, i.e. electric field is directed towards X-axis (due to involvement of $\hat{\mathbf{i}}$).

(i) As the surface is in YZ-plane, so the area vector (normal to the square) is along X-axis.



Area, $S = 10\times10 = 100\ \text{cm}^2 = 10^{-2}\ \text{m}^2$

Area vector, $\mathbf{S} = 10^{-2}\,\hat{\mathbf{i}}\ \text{m}^2$

Using the formula of electric flux,

$$\phi = \mathbf{E}\cdot\mathbf{S} = ES\cos\theta$$

$$= ES \qquad [\because \text{angle between } \mathbf{E} \text{ and } \mathbf{S} \text{ is } 0^\circ]$$

$$= 3\times10^3\times10^{-2} = 30\ \text{N-m}^2/\text{C}$$

(ii) Now, the area vector makes an angle of 60° with X-axis.

$$\mathbf{E} = 3\times10^3\,\hat{\mathbf{i}}\ \text{N/C}$$

$$S = 100\ \text{cm}^2 = 10^{-2}\ \text{m}^2,\ \theta = 60^\circ$$

Using the formula of electric flux, $\phi = \mathbf{E}\cdot\mathbf{S}$

$$\Rightarrow \quad \phi = ES\cos\theta = 3\times10^3\times10^{-2}\cos60^\circ$$

$$= 3\times10\times\frac{1}{2}$$

$$= 15\ \text{N-m}^2/\text{C}$$

46. Given, electric field intensity,

$$\mathbf{E} = 5\times10^3\,\hat{\mathbf{i}}\ \text{N/C}$$

Magnitude of electric field intensity,

$$|\mathbf{E}| = 5\times10^3\ \text{N/C}$$

Side of square, $S = 10\ \text{cm} = 0.1\ \text{m}$

Area of square, $A = (0.1)^2 = 0.01\ \text{m}^2$

The plane of the square is parallel to the yz-plane.

Hence, the angle between the unit vector normal to the plane and electric field is zero, i.e. $\theta = 0^\circ$.

$\therefore$ Flux through the plane,

$$\phi = |\mathbf{E}|\times A\cos\theta$$

$$= 5\times10^3\times0.01\cos0^\circ$$

$$= 50\ \text{N-m}^2/\text{C}$$

If the plane makes an angle of 30° with the X-axis, then $\theta = 60^\circ$.

$\therefore$ Flux through the plane,

$$\phi = |\mathbf{E}|\times A\times\cos60^\circ = 5\times10^3\times0.01\times\cos60^\circ$$

$$= 25\ \text{N-m}^2/\text{C}$$

47. Given, $R = \frac{D}{2} = \frac{2.4}{2} = 1.2$ m

and $\sigma = 80.0\ \mu\text{C/m}^2$

(i) Charge on sphere, $q = 4\pi R^2\cdot\sigma$

$$= 4\times3.14\times(1.2)^2\times80$$

$$= 1446.912\ \mu\text{C}$$

$$= 1.45\times10^{-3}\ \text{C}$$

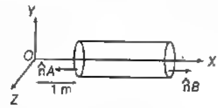
(ii) Electric flux, $\phi = \frac{q}{\varepsilon_0}$

$$= \frac{1.45\times10^{-3}}{8.854\times10^{-12}}$$

$$= 0.1637\times10^9$$

$$= 1.637\times10^8\ \text{N-m}^2/\text{C}$$

48. (i)



Given, $\quad \mathbf{E} = 50x\,\hat{\mathbf{i}}$

and $\quad \Delta S = 25\ \text{cm}^2$

$$= 25\times10^{-4}\ \text{m}^2$$

As the electric field is only along the X-axis, so flux will pass only through the cross-section of cylinder.

Magnitude of electric field at cross-section A,

$$E_A = 50\times1 = 50\ \text{N C}^{-1}$$

Magnitude of electric field at cross-section B,

$$E_B = 50\times2 = 100\ \text{NC}^{-1}$$

The corresponding electric fluxes are

$$\phi_A = \mathbf{E}_A\cdot\Delta\mathbf{S}$$

$$= 50\times25\times10^{-4}\times\cos180^\circ$$

$$= -0.125\ \text{N-m}^2\ \text{C}^{-1}$$

and $$\phi_B = \mathbf{E}_B\cdot\Delta\mathbf{S}$$

$$= 100\times25\times10^{-4}\times\cos0^\circ$$

$$= 0.25\ \text{N-m}^2\text{C}^{-1}$$

So, the net flux through the cylinder,

$$\phi = \phi_A + \phi_B = -0.125 + 0.25$$

$$= 0.125\ \text{N-m}^2\ \text{C}^{-1}$$

(ii) Using Gauss' law,

$$\oint \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0} \qquad \left[\because \oint \mathbf{E}\cdot d\mathbf{S} = \phi\right]$$

$$\Rightarrow \quad 0.125 = \frac{q}{8.85\times10^{-12}}$$

$$\Rightarrow \quad q = 8.85\times0.125\times10^{-12} = 1.1\times10^{-12}\ \text{C}$$

49. (i) On the left face, the outward flux,

$$\phi_L = \mathbf{E}\cdot\Delta\mathbf{S} = -100\hat{\mathbf{i}}\cdot\Delta\mathbf{S} = 100\Delta S,$$

since $\quad \hat{\mathbf{i}}\cdot\Delta\mathbf{S} = -\Delta S$

$$= 100\times\pi(0.05)^2 = 0.785\ \text{N-m}^2\text{C}^{-1}$$

On the right face, $\mathbf{E}$ and $\Delta\mathbf{S}$ are parallel and therefore,

$$\phi_R = \mathbf{E}\cdot\Delta\mathbf{S} = 0.785\ \text{N-m}^2\text{C}^{-1}$$

(ii) For any point on the side of the cylinder, $\mathbf{E}\perp\Delta\mathbf{S}$ and hence $\mathbf{E}\cdot\Delta\mathbf{S} = 0$

$\therefore$ Flux out of the side of the cylinder = 0

(iii) Net outward flux through the cylinder,

$$\phi = 0.785 + 0.785 + 0$$
$$= 1.57\ \text{N-m}^2\text{C}^{-1}$$

(iv) From Gauss' law, the net charge within the cylinder

$$= 1.57 \times 8.854 \times 10^{-12}\text{C}$$
$$- 1.39 \times 10^{-11}\text{C}$$

50. We know that,

$$\sigma = 17.0 \times 10^{-22}\ \text{Cm}^{-2}$$

(i) E on the left of the plates is zero.

(ii) E on the right of the plates is zero.

(iii) In between the plates, $E = \dfrac{\sigma}{\varepsilon_0}$

$$E = \frac{17.0 \times 10^{-22}}{8.85 \times 10^{-12}}$$
$$- 1.9 \times 10^{-10}\ \text{NC}^{-1}$$

51. Refer to Example 5 on page 39

52. Given, $E_X = \alpha x^{1/2}$, $E_Y = 0$, $E_Z = 0$

$$\alpha = 600\ \text{N/C}\ \ \text{m}^{1/2}, l = 0.1\ \text{m}, \phi = ?, q = ?$$

As the electric field has only x-component, therefore $\mathbf{E} \cdot \Delta \mathbf{S} - \phi_E = 0$ for each of four faces of cube perpendicular to Y-axis and Z-axis.

Flux is there only for left face L and right face R of the cube.

At the left face, $x = l$

$$\therefore \quad E_L \quad \alpha l^{1/2}$$

The flux for left face of cube,

$$\phi_L - \mathbf{E}_L \cdot \Delta\mathbf{S}\ \cdot \alpha l^{1/2}(l^2)\cos 180° = -\alpha l^{5/2}$$

Similarly at right face, $x = l + l = 2l$

The flux for right face of cube,

$$E_R = \alpha(2l)^{1/2}$$
$$\phi_R = \mathbf{E}_R \cdot \Delta\mathbf{S}$$
$$= \alpha(2l)^{1/2}(l^2)\cos 0°$$
$$= \alpha l^{5/2}\sqrt{2}$$

The net flux through the cube,

$$\phi - \phi_R + \phi_L$$
$$= \alpha l^{5/2}\sqrt{2} - \alpha l^{5/2}$$
$$= \alpha l^{5/2}(\sqrt{2} - 1)$$
$$= 600\,(0.1)^{5/2}(\sqrt{2} - 1)$$
$$= 0.785\ \text{N-m}^2\ \text{C}^{-1}$$

Apply Gauss' theorem and the charge within the cube,

$$q = \varepsilon_0\phi = 8.85 \times 10^{-12} \times 0.785$$
$$= 6.95 \times 10^{-12}\ \text{C}$$

[redacted]

SUMMARY

- **Electric Charge** It is the intrinsic property of the material which is responsible to exert the electric force.
- **Conductors and Insulators** Conductors are those substances which conduct the electricity, whereas insulators are those substance which cannot conductor electricity.
- **Charging by Induction** The process of charging a neutral body by bringing a charged body nearby it without making contact between the two bodies is called charging by induction.
- **Quantisation of Electric Charge** The charge on any body can be expressed as an integral multiple of basic unit of charge, i.e. charge on one electron. It can be expressed as

  $q = \pm\, ne$, where $n = 1, 2, \ldots$
- **Coulomb's Law** The force of interaction (attraction or repulsion) between two stationary point charges in vacuum is directly proportional to the product of the charges and inversely proportional to the square of distances between them

  i.e. $F = \dfrac{kq_1q_2}{r^2}$
- **Superposition Principle** Force on any charge due to number of charges is the vector sum of all the forces on that charge due to other charges taken one at a time.
- **Electrostatic Force due to Continuous Charge Distribution**

  $\lambda = \dfrac{q}{l}$, where λ is a linear charge density.

  $\sigma = \dfrac{q}{A}$, where σ is a surface charge density.

  $\delta = \dfrac{q}{V}$, where δ is a volume charge density
- **Electric Field** It is the space around the given charge in which another charge experiences an electrostatic force of repulsion or attraction
- **Electric Field Lines** It is a path traversed by a test charge around the given charge.
- **Properties of Electric Field Lines**

  Electric lines of force start from positive charges and end at negative charges.

  Two field lines never intersect each other.

  These are perpendicular to the surface of charged conductor.

  These do not pass through a conductor.

- **Electric Field Intensity** At a point is the force experienced per unit positive test charge placed at that point without disturbing the source charge, i.e. $E = \left(\dfrac{F}{q_0}\right)$
- **Electric Dipole** It is a pair of point charges with equal magnitude and opposite in sign separated by a very small distance
- **Dipole Moment** It's the product of the charge and separation between the charges i.e. $p = 2a \cdot q$
- **Electric Field Intensity due to an Electric Dipole**

  At a point on axial line, $E = \dfrac{2kpx}{(x^2 - l^2)^2}$

  At a point on equatorial line $E = \dfrac{2kp}{(x^2 + l^2)^{3/2}}$

  At any point, $|E| = \dfrac{p\sqrt{3\cos^2\theta + 1}}{4\pi\varepsilon_0 r^3}$
- **Torque on an electric Dipole in a Uniform Electric Field** It is given by, $\tau = pE\sin\theta$
- **Work done on Dipole in a Uniform Electric Field** Total work done in rotating the dipole from orientation θ_1 to θ_2 is

  $W = pE(\cos\theta_1 - \cos\theta_2)$
- **Electric Flux** It is defined as the total number of electric lines of force passing normally through the surface
- **Gauss' Theorem** The surface integral of the electric field intensity over any closed surface in free space is equal to $\dfrac{1}{\varepsilon_0}$ times the net charge enclosed with in the surface

  $$\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$
- **Applications of Gauss' Theorem**

  Field due to an infinitely long straight charged wire, $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$

  Field due to thin infinite sheet of charge, $E = \dfrac{\sigma}{2\varepsilon_0}$

  Field due to a uniformly charged thin spherical shell

  Outside the shell, $E = \dfrac{\sigma R^2}{\varepsilon_0 r^2}$

  On the surface of shell, $E = \dfrac{\sigma}{\varepsilon_0}$

  Inside the shell, $E = 0$

[redacted]

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. A negatively charged object X is repelled by another charged object Y. However, an object Z is attracted to object Y. Which of the following is the most possibility for the object Z? **CBSE SQP (Term-I)**

(a) Positively charged only
(b) Negatively charged only
(c) Neutral or positively charged
(d) Neutral or negatively charged

2. In an experiment, three microscopic latex spheres are sprayed into a chamber and became charged with charges $+3e$, $+5e$ and $-3e$, respectively. All the three spheres came in contact simultaneously for a moment and got separated. Which one of the following are possible values for the final charge on the spheres? **CBSE SQP (Term-I)**

(a) $+5e, -4e, +5e$
(b) $+6e, +6e, -7e$
(c) $+4e, +3.5e, +5.5e$
(d) $+5e, -8e, +7e$

3. An object has charge of 1 C and gains 5.0×10^{18} electrons. The net charge on the object becomes **CBSE SQP (Term-I)**

(a) -0.80 C (b) $+0.80$ C
(c) $+1.80$ C (d) $+0.20$ C

4. The number of electrons that must be removed from an electrically neutral silver dollar to give it a charge of $+2.4$ C is

(a) 2.5×10^{19} (b) 1.5×10^{19}
(c) 1.5×10^{-19} (d) 2.5×10^{-19}

5. Two identical metallic spheres having charges $+4q$ and $-2q$ are placed with their centres r distance apart. Force of attraction between the spheres is F. If the two spheres are brought in contact and then placed at the same distance r apart, the force between them

(a) F (b) $F/2$
(c) $F/4$ (d) $F/8$

6. In the following configuration of charges, force on charge q_2 by q_1 is given by

(a) $\mathbf{F}_{21} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q_1 q_2}{r^2} \cdot \hat{\mathbf{r}}_{21}$

(b) $\mathbf{F}_{21} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q_1 q_2}{r^2} (-\hat{\mathbf{r}}_{21})$

(c) $\mathbf{F}_{21} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q_1 q_2}{r^3} \cdot \hat{\mathbf{r}}_{21}$

(d) $\mathbf{F}_{21} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{q_1 q_2}{r^3} \cdot (-\hat{\mathbf{r}}_{21})$

(here, $r = r_{21} = |\mathbf{r}_2 - \mathbf{r}_1|$)

7. If charges q, q and $-q$ are placed at vertices of an equilateral triangle of side l. If $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ are the forces on the charges respectively, then

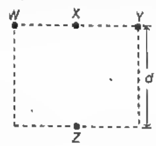
(a) $|\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3| = \sqrt{3}\dfrac{kq^2}{l^2}$

(b) $|\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3| = 0$

(c) $|\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3| = 3\sqrt{2}\dfrac{kq^2}{l^2}$

(d) $|\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3| = \sqrt{2}\dfrac{kq^2}{l^2}$

8. Four objects W, X, Y and Z each with charge $+q$ are held fixed at four points of a square of side d as shown in the figure. Objects X and Z are on the mid-points of the sides of the square. The electrostatic force exerted by object W on object X is F, then the magnitude of the force exerted by object W on Z is **CBSE SQP (Term-I)**

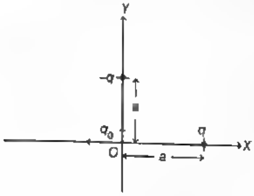


(a) $\dfrac{F}{7}$ (b) $\dfrac{F}{5}$
(c) $\dfrac{F}{3}$ (d) $\dfrac{F}{2}$

[redacted]

9. Three charges $q, -q$ and q_0 are placed as shown in figure. The magnitude of the net force on the charge q_0 at point O is $\left(\text{Take, } K = \frac{1}{4\pi\varepsilon_0}\right)$

CBSE 2021 (Term-I)



(a) 0 (b) $\frac{2Kqq_0}{a^2}$

(c) $\frac{\sqrt{2}Kqq_0}{a^2}$ (d) $\frac{1}{\sqrt{2}} \cdot \frac{Kqq_0}{a^2}$

10. Two point charges placed in a medium of dielectric constant 5 are at a distance r between them, experience an electrostatic force F. The electrostatic force between them in vacuum at the same distance r will be

(a) $5F$ (b) F (c) $F/2$ (d) $F/5$

11. The magnitude of electric field due to a point charge $2q$, at distance r is E. Then, the magnitude of electric field due to a uniformly charged thin sphere shell of radius R with total charge q at a distance $\frac{r}{2}$ $(r >> R)$ will be

CBSE SQP (Term-I)

(a) $\frac{E}{4}$ (b) 0

(c) $2E$ (d) $4E$

12. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$, respectively.

The point on X-axis at which net electric field is zero due to these charges, is

(a) $8L$ (b) $4L$ (c) $2L$ (d) L

13. Two parallel large thin metal sheets have equal surface densities 26.4×10^{-12} C/m^2 of opposite signs. The electric field between these sheets is

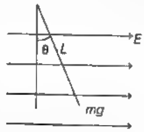
CBSE SQP (Term-I)

(a) 1.5 N/C (b) 1.5×10^{-16} N/C

(c) 3×10^{-10} N/C (d) 3 N/C

14. A small object with charge q and weight mg is attached to one end of a string of length L attached to a stationary support. The system is placed in a uniform horizontal electric field E, as shown in the given figure. In the presence of the field, the string makes a constant angle θ with the vertical. The sign and magnitude of q

CBSE SQP (Term-I)



(a) positive with magnitude mg/E

(b) positive with magnitude $mg/E \tan\theta$

(c) negative with magnitude $mg/E \tan\theta$

(d) positive with magnitude $E\tan\theta/mg$

15. Unit of electric field is

Or Unit of electric field intensity is

Or The unit of intensity of electric field is

(a) N/m (b) C/N (c) N/C (d) J/N

16. Electric field of a system of charges does not depend on

(a) position of charges forming the system

(b) distance of point (at which field is being observed) from the charges forming system

(c) value of test charge used to find out the field

(d) separation of charges forming the system

17. For the dipole shown,

Dipole moment is given by

(a) $\mathbf{p} = q \times 2a\hat{\mathbf{p}}$ (b) $\mathbf{p} = \frac{1}{2} q \times 2a\hat{\mathbf{p}}$

(c) $\mathbf{p} = -q \times 2a\hat{\mathbf{p}}$ (d) $\mathbf{p} = 4q \times 2a\hat{\mathbf{p}}$

18. An electric dipole of moment p is placed parallel to the uniform electric field. The amount of work done in rotating the dipole by 90° is

CBSE SQP (Term-I)

(a) $2pE$ (b) pE

(c) $pE/2$ (d) zero

[redacted]

19. Which statement is true for Gauss' law?
(a) All the charges whether inside or outside the gaussian surface contribute to the electric flux.
(b) Electric flux depends upon the geometry of the gaussian surface.
(c) Gauss' theorem can be applied to non-uniform electric field.
(d) The electric field over the gaussian surface remains continuous and uniform at every point.

20. A square sheet of side a is lying parallel to XY-plane at $z = a$. The electric field in the region is $\mathbf{E} = cz^2\hat{\mathbf{k}}$. The electric flux through the sheet is
CBSE SQP (Term-I)
(a) a^4c (b) $\frac{1}{3}a^3c$ (c) $\frac{1}{3}a^4c$ (d) 0

21. Gauss' law is true only if force due to charges varies as
(a) r^{-1} (b) r^{-2} (c) r^{-3} (d) r^{-4}

22. For a given surface, the $\oint \mathbf{E}\cdot d\mathbf{S} = 0$. From this, we can conclude that
(a) E is necessarily zero on the surface
(b) E is perpendicular to the surface at every point
(c) the total flux through the surface is zero
(d) the flux is only going out of the surface

23. A charge on a sphere of radius 2 cm is 2 μC while charge on sphere of radius 5 cm is 5μC. Find the ratio of an electric field on distance of 10 cm from centre of the sphere.
(a) 1 : 1 (b) 2 : 5 (c) 5 : 2 (d) 4 : 25

ASSERTION AND REASON

Directions (Q. Nos. 24-31) *In the following questions, two statements are given- one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*
(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

24. **Assertion** If a point charge be revolved in a circle around another charge as the centre of circle, then work done by electric field will be zero.
Reason Work done is equal to dot product of force and displacement.

25. **Assertion** A positive point charge initially at rest in a uniform electric field starts moving along electric lines of force. (Neglect all other forces except electric forces).
Reason A point charge released from rest in an electric field always moves along the line of force.

26. **Assertion** Electric force between two charges always acts along the line joining the two charges
Reason Electric force is a conservative force.

27. **Assertion** When a neutral body acquires positive charge, its mass decreases.
Reason A body acquires positive charge when it loses electrons.

28. **Assertion** In a non-uniform electric field, a dipole will have translatory as well as rotatory motion.
Reason In a non-uniform electric field, a dipole experiences a force as well as torque.

29. **Assertion** E outside vicinity of a conductor depends only on the local charge density σ and it is independent of the other charges present anywhere on the conductor.
Reason E in outside vicinity of a conductor is given by $\frac{\sigma}{\varepsilon_0}$.

30. **Assertion** Upon displacement of charges within a closed surface, E at any point on the surface does not change.
Reason The flux crossing through a closed surface is independent of the location of charge within the surface.

31. **Assertion** If Gaussian surface does not enclose any charge, then E at any point on the Gaussian surface must be zero.
Reason No net charge is enclosed by Gaussian surface, so net flux passing through the surface is zero.

CASE BASED QUESTIONS

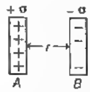
Directions (Q.Nos. 32-34) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

32. **Parallel Sheet of Charge**
Surface charge density is defined as the charge per unit surface area of surface charge distribution.

[redacted]

i.e. $\sigma = \frac{dq}{dS}$.

Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs having magnitude of 17.0×10^{-22} Cm^{-2} as shown below.



The intensity of electric field at a point is $E = \frac{\sigma}{\varepsilon_0}$

where, ε_0 = permittivity of free space.

(i) E in the outer region of the first plate is
(a) 17×10^{-22} N/C (b) 1.5×10^{-15} N/C
(c) 1.9×10^{-10} N/C (d) zero

(ii) E in the outer region of the second plate is
(a) 17×10^{-22} N/C (b) 1.5×10^{-15} N/C
(c) 1.9×10^{-10} N/C (d) zero

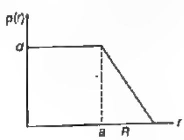
(iii) E between the plates is
(a) 17×10^{-22} N/C (b) 1.5×10^{-15} N/C
(c) 1.9×10^{-10} N/C (d) zero

(iv) The ratio of E from right side of B at distances
2 cm and 4 cm, respectively is
(a) 1 : 2 (b) 2 : 1 (c) 1 : 1 (d) $1 : \sqrt{2}$

(v) In order to estimate the electric field due to a thin finite plane metal plate, the gaussian surface considered is
(a) spherical (b) cylindrical
(c) straight line (d) None of these

33. Electric Field due to Nuclear Charge

The nuclear charge (Ze) is non-uniformly distributed within a nucleus of radius R. The charge density $\rho(r)$ [charge per unit volume is dependent only on the radial distance r from the centre of the nucleus as shown in figure. The electric field is only along the radial direction.

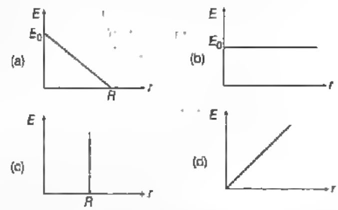


(i) The electric field at $r = R$ is
(a) independent of a
(b) directly proportional to a
(c) directly proportional to a^2
(d) inversely proportional to a

(ii) Net charge on given system is
(a) $Q = \int \rho_r (4\pi r^2) dr$ (b) $Q = \int \rho_r (\pi r^2) dr$
(c) $Q = \int \rho_r \frac{r^2}{2} dr$ (d) $Q = \int \rho_r (4\pi r^2)$

(iii) For $a = 0$, the value d (maximum value of ρ as shown in the figure) is
(a) $\frac{3Ze^2}{4\pi R^3}$ (b) $\frac{3Ze}{\pi R^3}$ (c) $\frac{4Ze}{3\pi R^3}$ (d) $\frac{Ze}{3\pi R^3}$

(iv) The correct graph representing the variation of E with r is



(v) The electric field within the nucleus is generally observed to be linearly dependent on r. This implies
(a) $a = 0$ (b) $a = \frac{R}{2}$
(c) $a = R$ (d) $a = \frac{2R}{3}$

34. Faraday cage

A Faraday cage or Faraday shield is an enclosure made of a conducting material. The fields within a conductor cancel out with any external fields, so the electric field within the enclosure is zero. These Faraday cages act as big hollow conductors. You can put things to shield them from electrical fields. Any electrical shocks the cage receives, pass harmlessly around the outside of the cage.

[redacted]

(i) Which of the following material can be used to make a Faraday cage?
(a) Plastic (b) Glass
(c) Copper (d) Wood

(ii) Example of a real-world Faraday cage is
(a) car (b) plastic box
(c) lightning rod (d) metal rod

(iii) What is the electrical force inside a Faraday cage, when it is struck by lightning?
(a) The same as the lightning
(b) Half that of the lightning
(c) Zero
(d) A quarter of the lightning

(iv) An isolated point charge $+q$ is placed inside the Faraday cage. Its surface must have charge equal to
(a) zero (b) $+q$
(c) $-q$ (d) $+2q$

(v) A point charge of 2C is placed at centre of Faraday cage in the shape of cube with surface of 9 cm edge. The number of electric field lines passing through the cube normally will be
(a) 1.9×10^5 N-m^2/C, entering the surface
(b) 1.9×10^5 N-m^2/C, leaving the surface
(c) 2.01×10^{11} N-m^2/C, leaving the surface
(d) 2.01×10^5 N-m^2/C, entering the surface

VERY SHORT ANSWER Type Questions

35. Electric charge is additive in nature. Explain.

36. Give one difference between the conductors and insulators.

37. "Electrostatic forces are much stronger than the gravitational forces". Give one example to justify this statement.

38. A point charge is placed at the centre of a hollow conducting sphere of internal radius r and outer radius $2r$. The ratio of the surface charge density of the inner surface to that of the outer surface will be .... CBSE 2020

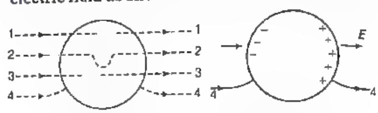
39. The work done in moving a charge particle between two points in a uniform electric field, does not depend on the path followed by the particle. Why? CBSE 2020

40. Draw the pattern of electric field lines, when a point charge $-Q$ is kept near an uncharged conducting plate. CBSE 2019

41. Draw the pattern of electric field lines due to an electric dipole. CBSE 2019

42. Draw a pattern of electric field lines due to two positive charges placed a distance d apart. CBSE 2019

43. A metallic solid sphere is placed in a uniform electric field as shown below.



(a) (b)

Which path is followed by electric field lines?

44. A point charge $+Q$ is placed in the vicinity of a conducting surface. Draw the electric field lines between the surface and the charge. All India 2017 C

45. The electric field induced in a dielectric when placed in an external field is 1/10 times the electric field. Calculate relative permittivity of the dielectric.

SHORT ANSWER Type Questions

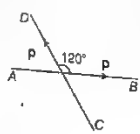
46. Two identical metallic spherical shells A and B having charges $+4Q$ and $-10Q$ are kept a certain distance apart.

A third identical uncharged sphere C is first placed in contact with sphere A and then with sphere B, then spheres A and B are brought in contact and then separated.

Find the charge on the spheres A and B.

47. Deduce the expression for the electric field E due to a system of two charges q_1 and q_2 with position vectors r_1 and r_2 at a point r with respect to a common origin.

48. Two small identical dipoles AB and CD, each of dipole moment p are kept at an angle of 120° as shown in the figure.



[redacted]

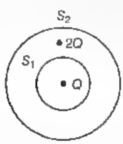
What is the resultant dipole moment of this combination? If this system is subjected to electric field (E) directed along positive x-direction, what will be the magnitude and direction of the torque acting on this?

49. Derive the expression for the torque acting on an electric dipole, when it is held in a uniform electric field. Identify the orientation of the dipole in the electric field, in which it attains a stable equilibrium. **CBSE 2020**

50. What is the use of Gaussian surface? Also, mention the importance of Gauss' theorem.

51. A point charge causes an electric flux of -3.1×10^4 N-m^2/C to pass through a spherical Gaussian surface.
   (i) Find the value of the point charge.
   (ii) If the radius of the Gaussian surface is doubled, how much flux would pass through the surface?

52. S_1 and S_2 are two parallel concentric spheres enclosing charges Q and $2Q$ as shown in the figure.



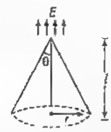
   (i) What is the ratio of the electric flux through S_1 and S_2?
   (ii) How will the electric flux through the sphere S_1 change, if a medium of dielectric constant $5\varepsilon_0$ is introduced in the space inside S_1 in place of air?

LONG ANSWER Type I Questions

53. A spherical conducting shell of inner radius R_1 and outer radius R_2 has a charge Q. A charge q is placed at the centre of the shell.
   (i) What is the surface charge density on the
      (a) inner surface?
      (b) outer surface of the shell?
   (ii) Write the expression for the electric field at a point $X > R_2$ from the centre of the shell.

54. A large plane sheet of charge having surface charge density 5×10^{-6} C/m^2 lies in XY-plane. Find the electric flux through a circular area of radius 0.1 m, if the normal to the circular area makes an angle of 60° with Z-axis.
   [Take, $\varepsilon_0 = 8.85\times10^{-12}$ C^2/N-m^2]

55. Consider a region bounded by conical surface as shown in the figure.



In this region, **E** is in vertical upward direction, then find the electric field when electric flux is passing through curved surface.

56. A hollow conducting sphere of inner radius r_1 and outer radius r_2 has a charge Q on its surface. A point charge $-q$ is also placed at the centre of the sphere.
   (a) What is the surface charge density on the (i) inner and (ii) outer surface of the sphere?
   (b) Use Gauss' law of electrostatics to obtain the expression for the electric field at a point lying outside the sphere.

Or

   (a) An infinitely long thin straight wire has a uniform linear charge density λ. Obtain the expression for the electric field (E) at a point lying at a distance x from the wire, using Gauss' law.
   (b) Show graphically the variation of this electric field E as a function of distance x from the wire.

57. Two large charged plane sheets of charge densities σ and -2σ C/m^2 are arranged vertically with a separation of d between them. Deduce expressions for the electric field at points (i) to the left of the first sheet, (ii) to the right of the second sheet and (iii) between the two sheets. **CBSE 2019**

Or

A spherical conducting shell of inner radius r_1 and outer radius r_2 has a charge Q.

[redacted]

(a) A charge q is placed at the centre of the shell. Find out the surface charge density on the inner and outer surfaces of the shell.

(b) Is the electric field inside a cavity (with no charge) zero; independent of the fact whether the shell is spherical or not? Explain.

58. Total charge $-Q$ is uniformly spread along length of a ring of radius R. A small test charge $+q$ of mass m is kept at the centre of the ring and is given a gentle push along the axis of the ring.

(i) Show that the particle executes a simple harmonic oscillation.

(ii) Obtain its time period.

LONG ANSWER Type II Questions

59. (i) Derive the expression for the electric intensity at any point P, at distance r from the centre of an electric dipole, making angle α, with its axis.

(ii) Two point charges $+4\,\mu C$ and $+1\mu C$ are separated by a distance of 2 m in air. Find the point on the line joining charges at which the net electric field of the system is zero.

All India 2017C

60. (a) Derive an expression for the electric field at any point on the equatorial line of an electric dipole.

(b) Two identical point charges, q each are kept 2 m apart in air. A third point charge Q of unknown magnitude and sign is placed on the line joining the charges such that the system remains in equilibrium. Find the position and nature of Q.

61. (a) Define an ideal electric dipole. Give an example.

(b) Derive an expression for the torque experienced by an electric dipole in a uniform electric field. What is net force acting on this dipole?

(c) An electric dipole of length 2 cm is placed with its axis making an angle of 60° with respect to uniform electric field of 10^5 N/C.

If it experiences a torque of $8\sqrt{3}$ N-m, calculate the

(i) magnitude of charge on the dipole and (ii) its potential energy.

62. (i) State Gauss' law. Using this law, obtain the expression for the electric field due to an infinitely long straight conductor of linear charge density λ.

(ii) A wire AB of length L has linear charge density $\lambda = kx$, where x is measured from the end A of the wire. This wire is enclosed by a Gaussian hollow surface. Find an expression for the electric flux through this surface. All India 2017C

ANSWERS

1. (c)	2. (b)	3. (d)	4. (b)	5. (d)
6. (a)	7. (b)	8. (b)	9. (c)	10. (a)
11. (c)	12. (c)	13. (d)	14. (b)	15. (c)
16. (c)	17. (a)	18. (b)	19. (d)	20. (c)
21. (b)	22. (c)	23. (b)		

24. (a) Force by electric field will be perpendicular to the displacement.

25 (c) If the field lines are curved, then the charged particle follows the straight line path along the direction of tangent drawn to electric field lines at its starting point

26. (b)

27. (a)

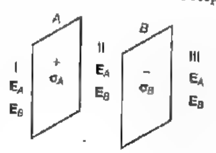
28. (a)

29. (d) E in outside vicinity of conductor's surface depends on all the charges present in the space, but expression $E = \frac{\sigma}{\varepsilon_0}$.

30. (d) Due to displacement of charge within closed surface E at any point may change. But net flux crossing the surface will not change.

31. (d) E at any point on Gaussian surface may be due to outside charges also.

32. (i) (d) There are two plates A and B having surface charge densities $\sigma_A = 17.0 \times 10^{-22}$ C/m^2 on A and $\sigma_B = -17.0 \times 10^{-22}$ C/m^2 on B, respectively.



[redacted]

According to Gauss' theorem, if the plates have same surface charge density but having opposite signs, then the electric field in region I is zero.

$$\mathbf{E}_I = \mathbf{E}_A + \mathbf{E}_B = \frac{\sigma}{2\varepsilon_0} + \left(-\frac{\sigma}{2\varepsilon_0}\right) = 0$$

(ii) (*d*) The electric field in region III is also zero.

$$\mathbf{E}_{III} = \mathbf{E}_A + \mathbf{E}_B$$
$$= \frac{\sigma}{2\varepsilon_0} + \left(-\frac{\sigma}{2\varepsilon_0}\right) = 0$$

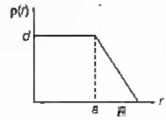
(iii) (*c*) In region II, the electric field

$$\mathbf{E}_{II} = \mathbf{E}_A - \mathbf{E}_B = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$
$$= \frac{\sigma}{\varepsilon_0} = \frac{\sigma_A \text{ or } \sigma_B}{\varepsilon_0}$$
$$= \frac{17.0 \times 10^{-22}}{8.85 \times 10^{-12}}$$
$$E = 1.92 \times 10^{-10}\ \text{NC}^{-1}$$

(iv) (*c*) Since, electric field due to an infinite-plane sheet of charge does not depend on the distance of observation point from the plane sheet of charge. So, for the given distances, the ratio of E will be 1 : 1.

(v) (*b*) In order to estimate the electric field due to a thin finite plane metal plate, we take a cylindrical cross-sectional area *A* and length $2r$ as the gaussian surface.

33. (i) (*a*) Electric field at $r = R$



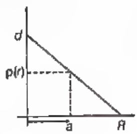
$$E = \frac{KQ}{R^2}; Q = \text{Total charge within the nucleus} = Ze$$

So, $E = \dfrac{KZe}{R^2}$

So, electric field is independent of *a*.

(ii) (*a*) Net charge, $Q = \int \rho_r 4\pi r^2\, dr$

(iii) (*b*) As, $\dfrac{d}{R} = \dfrac{\rho_r}{R-r}$



$$Q = \int_0^R \frac{d}{R}(R-r)\,4\pi r^2 dr$$

$$= \frac{4\pi d}{R}\left(R\int_0^R r^2 dr - \int_0^R r^3 dr\right)$$

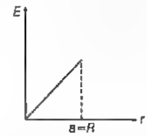
$$= \frac{4\pi d}{R}\left(\frac{R^4}{3} - \frac{R^4}{4}\right) = \frac{\pi d R^3}{3}$$

$$Q = Ze = \frac{\pi d R^3}{3}$$

$$\Rightarrow d = \frac{3Ze}{\pi R^3}$$

(iv) (*d*) From the formula of uniformly (volume) charged sphere $E = \dfrac{\rho r}{3\varepsilon_0}$

$\Rightarrow E \propto r$, so correct graph is as shown,



(v) (*c*) For $E \propto r$, ρ should be constant throughout that of nucleus. This will be possible only when $a = R$.

34. (i) (c), (ii) (a), (iii) (c), (iv) (b)

(v) (c) According to Gauss' law, electric flux,

$$\phi = \frac{q}{\varepsilon_0} = \frac{2}{8.86 \times 10^{-12}} = 2.01 \times 10^{11}\ \text{N-m}^2/\text{C}$$

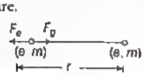
(leaving the surface)

35. Refer to text on page 2.

36. Conductors are those substances which can be used to carry or conduct electric charge/electron from one point to other. Insulators are those substances which cannot conduct electricity.

37. This statement can be proved by taking comparison of electric and magnetic forces between two protons.

Let the distance between two protons having charge $+e$ and mass *m* is placed at a distance *r* from each other as shown in figure.



$$\therefore \quad F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} \qquad \text{[Coulomb's law]}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{e \times e}{r^2}$$

$$\Rightarrow \quad F_g = G\frac{m_1 m_2}{r^2}\ \text{(gravitation law)} = G\frac{m \times m}{r^2}$$

where, F_e and F_g are electric and gravitational force respectively, on putting the values, we get

[redacted]

$$\frac{F_e}{F_g} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}}{\frac{Gm^2}{r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{m^2}\frac{1}{G}$$

$$= \frac{(9\times10^9)(1.6\times10^{-19})^2}{(1.67\times10^{-27})^2}\times\frac{1}{(6.67\times10^{-11})}$$

$$= 1.2\times10^{36} \approx 10^{36}$$

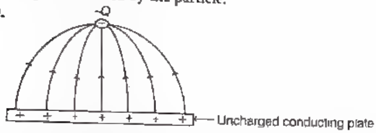
$\Rightarrow \quad F_e >> F_g$

Hence, the electric force is much stronger than gravitational force.

38. 4 : 1, for detail refer to sol 14 given on page 19 and 20.

39. It is because force acting on charge particle in an electric field is conservative which does not depend on path. Hence, work done by electric force is independent of path followed by the particle.

40.

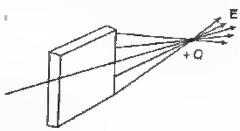


41. Refer to figure on pages 14 and 15.

42. Refer to diagram on pages 14 and 15.

43. Path 4 is followed by electric field lines. Since, there are no electric field lines within the metallic sphere and field lines are normal at each point on the surface

44.



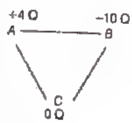
45. We know that, $\quad \varepsilon_r = K = \frac{\varepsilon_0}{\varepsilon}$

Here, $\quad \varepsilon = \frac{1}{10}\varepsilon_0$

$\therefore \quad \varepsilon_r = \frac{\varepsilon_0}{\frac{1}{10}\varepsilon_0} = 10$

Hence, relative permittivity of the dielectric becomes 10 times of the original value

46. When C is in contact with A, the charge developed on C is $-4Q$. When this is brought in contact with B, the charges are distributed and becomes $(-10-4) = 14Q$ on B and C each.



A. last when A and B are brought in contact again charges are re-distributed and become $(-14+4) = 10\,Q$ on each.

47. Refer to text on page 13

48. Refer to Q. 18 on page 30.

49. Refer to text on page 28.

50. Refer to text on pages 36 and 37.

51. Refer to Example 5 on page 39.

52. Refer to Q. 31 on page 41.

53. Refer to text on pages 38 and 39.

54. According to Gauss' law, the electric field due to a plane sheet of charge is

$$E = \frac{\sigma}{2\varepsilon_0}$$

Given that, $\sigma = 5\times10^{-6}$ C/m^2

So, $\quad E = \frac{5\times10^{-6}}{2\times8.85\times10^{-12}} = \frac{5\times10^{6}}{17.7}$

$= 282.5\times10^3$ V/m

$\therefore \quad d\phi = \mathbf{E}\cdot d\mathbf{A}$

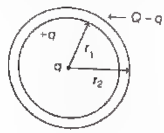
$\Rightarrow \quad \phi = EA\cos\theta$

$= 282.5\times10^3\times3.14\times10^{-2}\times\cos60^\circ$

$= 4.44$ kVm $= 4.44\times10^3$ N-m^2/C

55. Use Gauss' theorem, $\int \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$

56. (a) When a charge $-q$ is placed at the centre of the hollow conducting sphere, then charge induced on the inner surface is $+q$ and on outer surface is $-q$. But charge Q is already present on its outer surface. So, net charge on outer surface is $(Q-q)$ as shown.



Therefore, surface charge density on

(i) inner surface is $\frac{q}{4\pi r_1^2}$

(ii) and on outer surface is $\frac{Q-q}{4\pi r_2^2}$.

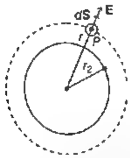
(b) At a point lying outside the sphere by Gauss' law, the electric flux is given by,

$$\phi_E = \oint \mathbf{E}\cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

where, q is the net charge enclosed.

[redacted]

As E and dS are in same direction as shown.



$$\therefore \qquad E\,(4\pi r^2) = \frac{Q-q}{\varepsilon_0}$$

$\Rightarrow$ Electric field, $E = \dfrac{Q-q}{4\pi\varepsilon_0 r^2}$

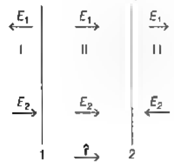
Or

(a) Refer to text on pages 37 and 38 (Replacing r by x).

(b) Refer to text and graph given on page 38 (Replacing r by x).

57. (i) Electric field to the left of plate 1 (region I)

$$E_I = E_1 + E_2 = \frac{\sigma}{2\varepsilon_0}\hat{r} - \frac{2\sigma}{2\varepsilon_0}\hat{r}$$



(ii) Electric field to the right of plate 2 (region III)

$$E_{III} = \frac{\sigma}{2\varepsilon_0}\hat{r} - \frac{2\sigma}{2\varepsilon_0}\hat{r}$$

(iii) Electric field between two plates (region II)

$$E_{II} = \frac{\sigma}{2\varepsilon_0}\hat{r} + \frac{2\sigma}{2\varepsilon_0}\hat{r}.$$

Or

(a) Refer to Sol. 14 on pages 19 and 20.

(b) Yes, the electric field inside a cavity is zero irrespective of shape because the cavity has zero net charge.

58. Refer to Q. 14 on page 30.

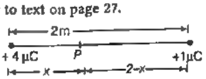
(i) Use relation, $E = \dfrac{qr}{4\pi\varepsilon_0\,(r^2+a^2)^{3/2}}$ to find

$F = qE = -kz$

(ii) Use relation, $T = 2\pi\sqrt{\dfrac{m}{k}}$

59. (i) Refer to text on page 27.

(ii)



Let the net electric field be zero at point P at a distance x from charge $+4\mu C$, then

$$\frac{1}{4\pi\varepsilon_0}\frac{4\times10^{-6}}{x^2} = \frac{1\times10^{-6}\times1}{4\pi\varepsilon_0\times(2-x)^2} = 0$$

$$\Rightarrow \qquad \frac{4}{x^2} = \frac{1}{(2-x)^2},$$

$$\Rightarrow \qquad \frac{2}{x} = \frac{1}{2-x}$$

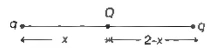
$$\Rightarrow \qquad x = 4 - 2x$$

$$\Rightarrow \qquad x = \frac{4}{3}\text{ m}$$

60. (a) Refer to text on page 26.

(b) Let P be the point at which the system of charges is in equilibrium, then

$$F(x) = F(2-x)$$



$$\frac{1}{4\pi\varepsilon_0}\frac{qQ}{x^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qQ}{(2-x)^2}$$

$$\Rightarrow \qquad \frac{1}{x^2} = \frac{1}{(2-x)^2}$$

$$\Rightarrow \qquad x = (2-x) \Rightarrow x = 1$$

Thus the charge Q should be placed at the centre of line joining two given charges. Also the two given charges are identical, i.e. having same nature, so the third charge could be of any nature (positive or negative). As the forces on it at the centre are equal and opposite.

61. (a) Refer to text on page 25.

(b) Refer to text on page 28.

(c) Given, length of electric dipole, $2l = 2\text{ cm} = 2\times10^{-2}$ m

$\theta = 60°$

$E = 10^5$ N/C

(i) $\because \quad \tau = pE\sin\theta$

$$p = \frac{\tau}{E\sin\theta}$$

$$\Rightarrow \quad 2lq = \frac{\tau}{E\sin\theta}$$

$$\Rightarrow \quad q = \frac{\tau}{(2l)E\sin\theta}$$

$$= \frac{8\sqrt{3}}{2\times10^{-2}\times10^5\times\sin 60°} = 8\times10^{-3}\text{ C}$$

(ii) Potential energy $= -pE\cos\theta = -(2l\times q)E\cos 60°$

$$= -2\times10^{-2}\times8\times10^{-3}\times10^5\times\frac{1}{2}$$

$$= -8\text{ joule}$$

62. (i) Refer to text on pages 36, 37 and 38.

(ii) Use $dQ = \lambda\cdot dl$ and Gauss' theorem $\phi = \dfrac{Q}{\varepsilon_0}$

[redacted]

The degree of electrification of a body is represented by the electrostatic potential of the charged body. The direction of flow of charge between two charged bodies placed in contact with each other is determined by electrostatic potential.

ELECTROSTATIC POTENTIAL AND CAPACITANCE

|TOPIC 1|

Electrostatic Potential, Electrostatic Potential Difference and Electrostatic Potential Energy

CHAPTER CHECKLIST

- Electrostatic Potential, Electrostatic Potential Difference and Electrostatic Potential Energy
- Dielectric and Capacitance

The electrostatic potential at any point in the region of electric field is equal to the amount of work done in bringing a unit positive test charge (without acceleration) from infinity to that point.

$$\therefore \quad \text{Electrostatic potential } (V) = \frac{\text{Work done } (W)}{\text{Charge } (q_0)}$$

It is a scalar quantity. Its SI unit is volt (V) and 1V = 1 J/C and its dimensional formula is $[ML^2T^{-3}A^{-1}]$.

Note

Electrostatic potential (V) at a point is said to be **one volt**, when one joule of work is done in moving one coulomb of positive charge (without acceleration) from infinity to that point.

Work done ($[W_\infty]_{ext}$) by an external force in bringing (without acceleration) a unit positive charge from infinity to a point is equal to the potential (V) at that point,

i.e. $$V = \frac{[W_\infty]_{ext}}{q_0} = \frac{-[W_\infty]_{elec}}{q_0} \qquad [\because [W_\infty]_{ext} = -[W_\infty]_{elec}]$$

[redacted]

where, $(W_\infty)_{elec}$ is the work done by the electric field on a charged particle as that particle moves from infinity to a point. A potential (V) can be positive, negative or zero depending on the signs and magnitudes of q and W_∞.

Note Electric potential is state dependent function as electrostatic forces are conservative forces. No work is done in moving a unit positive test charge over a closed path in an electric field.

EXAMPLE |1| Potential at a point P in space is given as 3×10^5 V. Find the work done in bringing a charge of 2×10^{-6} C from infinity to the point P. Does the answer depend on the path along which the charge is brought?

Sol. Given,

Potential at the point P,

$V = 3\times10^5$ V, charge, $q_0 = 2\times10^{-6}$ C

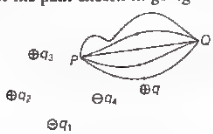
Work done in bringing the charge from infinity to the point P is

$$W_\infty = q_0 V = 2\times10^{-6}\times3\times10^5$$
$$= 6\times10^{-1} = 0.6 \text{ J}$$

No, the work done will be path independent.

ELECTROSTATIC POTENTIAL DIFFERENCE

Electrostatic potential difference between two points P and Q of a charge configuration consisting of charges q_1, q_2, q_3, q_4, and q is equal to the work done by an external force in moving a unit positive test charge against the electrostatic force from point Q to P along any path between these two points. Figure shows that work done on a test charge q_0 by the electrostatic field due to any given charge configuration depends only on the position of initial point Q and position of final point P. Work done is independent of the path chosen in going from Q to P.



Electrostatic potential difference between two points P and Q

If V_Q and V_P are the electrostatic potentials at Q and P respectively, then electrostatic potential difference between points Q and P is

$$\Delta V = V_P - V_Q$$

Thus,
$$\Delta V = \frac{W_{QP}}{q_0}$$

The dimensional formula for electrostatic potential difference is given by

$$\Delta V = \frac{W_{QP}}{q_0} = \frac{[ML^2T^{-2}]}{[AT]} = [ML^2T^{-3}A^{-1}]$$

The SI unit of electrostatic potential difference is volt.

$$1 \text{ V} = 1 \text{ JC}^{-1} = 1 \text{ N-m C}^{-1}$$

Thus, electrostatic potential difference between any two points in an electrostatic field is said to be one volt, when one joule of work is done by an external force in moving a positive charge of one coulomb from one point to the other against the electrostatic force of field without any acceleration.

Note One electron-volt (1 eV) is the energy equal to the work required to move a single elementary charge e such as an electron or the proton through a potential difference of exactly one volt (1 V).

$$1 \text{ eV} = e\,(1\text{V}) = (1.60\times10^{-19}\text{C})\,(1\text{ J/C}) = 1.60\times10^{-19}\text{ J}$$

EXAMPLE |2| The potential difference between two points is 20 V. How much work will be done in carrying a charge of 400 μC from one point to the another?

Sol. Given, $\Delta V = 20$ V and $q = 400\,\mu\text{C} = 400\times10^{-6}$ C

We know that

$$\text{Electrostatic potential difference} = \frac{\text{Work done}}{\text{Charge}}$$

$$\Rightarrow \quad \Delta V = \frac{W}{q}$$

$$\Rightarrow \quad 20 = \frac{W}{400\times10^{-6}}$$

$$W = 20\times400\times10^{-6} = 8\times10^{-3} \text{ J}$$

EXAMPLE |3| If 100 J of work must be done to move an electric charge of magnitude 4 C from a place A, where potential is −10 V to another place B where potential is V volt. Find the value of V.

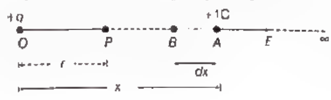
Sol. Given, $W_{AB} = 100$ J, $q = 4$ C, $V_A = -10$ V, $V_B = V = ?$

Since, $W_{AB} = q(V_B - V_A)$

$\Rightarrow \quad 100 = 4(V + 10) \Rightarrow V = 15$ V

ELECTROSTATIC POTENTIAL DUE TO A POINT CHARGE

Let P be the point at a distance r from the origin O at which the electric potential due to charge $+q$ is required.



[redacted]

The electric potential at a point P is the amount of work done in carrying a unit positive charge from ∞ to P. As, work done is independent of the path, we choose a convenient path along the radial direction from infinity to the point P without acceleration. Let A be an intermediate point on this path where $OA = x$. The electrostatic force on a unit positive charge at A is given by

$$F = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q\times 1}{x^2} \quad \text{[along } OA] \quad \ldots(i)$$

Small work done in moving the charge through a distance dx from A to B is given by

$$dW = \mathbf{F}\cdot d\mathbf{x}$$

$$= F dx \cos 180^\circ = -F dx \quad [\because \cos 180^\circ = -1]$$

$$\Rightarrow \quad dW = -F dx \quad \ldots(ii)$$

Total work done in moving a unit positive charge from ∞ to the point P is given by

$$W = \int_\infty^r -F\,dx$$

$$= \int_\infty^r -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x^2}\,dx$$

$$= -\frac{q}{4\pi\varepsilon_0}\int_\infty^r x^{-2}dx$$

$$= -\frac{q}{4\pi\varepsilon_0}\left[\frac{-1}{x}\right]_\infty^r \quad \left[\because \int x^{-2}\,dx = -\frac{1}{x}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

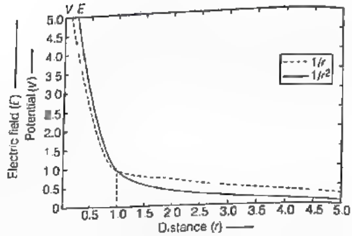
$$\Rightarrow \quad W = \frac{q}{4\pi\varepsilon_0 r} \quad \ldots(iii)$$

From the definition of electric potential, this work is equal to the potential at point P.

$$\boxed{V = \frac{q}{4\pi\varepsilon_0 r}} \quad \ldots(iv)$$

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential. Here, we assume that electrostatic potential is zero at infinity. Eq.(iv) shows that at equal distances from a point charge q, value of V is same.

Hence, electrostatic potential due to a single charge is spherically symmetric. Figure given below shows the variation of electrostatic potential with distance, i.e. $V \propto \frac{1}{r}$ and also the variation of electrostatic field with distance, i.e. $E \propto \frac{1}{r^2}$.



Variation of electrostatic potential V and electric field E with distance r

Due to a single charge, $F \propto \frac{1}{r^2}$, $E \propto \frac{1}{r^2}$ but $V \propto \frac{1}{r}$, where r is the distance from the charge.

EXAMPLE |4| What is the electrostatic potential at the surface of a silver nucleus of diameter 12.4 fermi? Atomic number (Z) for silver is 47.

Sol. Given, $r = \frac{12.4}{2} = 6.2 \text{ fermi} = 6.2\times 10^{-15}$ m and $Z = 47$

$\therefore$ Charge of the nucleus, $q = Ze = 47\times 1.6\times 10^{-19}$ C

$[\because e = 1.6\times 10^{-19}\text{C}]$

$\therefore$ Electrostatic potential at the surface,

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{9\times 10^9\times 47\times 1.6\times 10^{-19}}{6.2\times 10^{-15}} = 1.09\times 10^7\text{V}$$

EXAMPLE |5| A point charge Q is placed at point O as shown in the figure. Is the potential difference $(V_A - V_B)$ positive, negative or zero, if Q is

(i) positive? (ii) negative?

$\overset{Q}{O}$ - - - - - - - - A - - - - - - - - B **All India 2011**

Sol. Let the distance of points A and B from charge Q be r_A and r_B, respectively.

$\therefore$ Potential difference between points A and B,

$$V_A - V_B = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

As, $r_A = OA$, $r_B = OB$ and $r_A < r_B$

$\Rightarrow \quad \frac{1}{r_A} > \frac{1}{r_B}$

Therefore, $\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$ has positive value.

$(V_A - V_B)$ depends on the nature of charge Q.

(i) $(V_A - V_B)$ is positive when $Q > 0$, then

(ii) $(V_A - V_B)$ is negative when $Q < 0$.

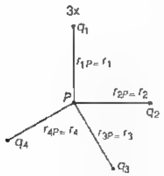
[redacted]

ELECTROSTATIC POTENTIAL DUE TO A SYSTEM OF CHARGES

Let there be a number of point charges $q_1, q_2, q_3, \ldots, q_n$ at distances $r_1, r_2, r_3, \ldots, r_n$ respectively from the point P, where electric potential is to be calculated.

Potential at P due to charge q_1,

$$V_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_{1P}}$$



A system of charges

Similarly, $V_2 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_{2P}}$, $V_3 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_3}{r_{3P}}, \ldots$

$$V_n = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_n}{r_{nP}}$$

Using superposition principle, we obtain resultant potential at point P due to total charge configuration as the algebraic sum of the potentials due to individual charges.

$$\therefore \quad V = V_1 + V_2 + V_3 + \cdots + V_n$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_{1P}} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_{2P}} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_3}{r_{3P}} + \cdots + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_n}{r_{nP}}$$

$$\Rightarrow \quad V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + \frac{q_3}{r_{3P}} + \cdots + \frac{q_n}{r_{nP}}\right)$$

$$\Rightarrow \quad \boxed{V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_{iP}}}$$

The net electrostatic potential at a point due to multiple charges is equal to the algebraic sum of the potentials due to individual charges at that particular point.

Mathematically, it is expressed as

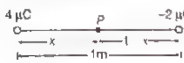
$$V_{\text{net}} = \sum_{i=1}^{n} V_i$$

Important Results

- If $r_1, r_2, r_3, \ldots, r_n$ are position vectors of the charges $q_1, q_2, q_3, \ldots, q_n$ respectively, then electrostatic potential at point P whose position vector is $\mathbf{r}_0$, would be
$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\mathbf{r}_0 - \mathbf{r}_i|}$$
- If we have to calculate electric potential due to a continuous charge distribution characterised by volume charge density $\rho(r)$, we divide the entire volume into a large number of small volume elements each of volume ΔV.
Charge on each element $= \rho \Delta V$. $\left[\because \rho = \frac{q}{\Delta V}\right]$
- For a uniformly charged conducting spherical shell, the electric field outside the shell is as, if the entire charge is concentrated at the centre. Thus, the potential outside the shell is given by
$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r} \qquad [\because r \geq R]$$
where, q is the total charge on the shell and R is its radius. The electric field inside the shell is zero. This implies that potential is constant inside the shell (as no work is done in moving a charge inside the shell) and therefore equal to its value at the surface, which is
$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$$

EXAMPLE |6| Two point charges of 4 μC and −2 μC are separated by a distance of 1 m in air. Find the location of a point on the line joining the two charges, where the electric potential is zero.

Sol. Let the electrostatic potential be zero at point P between the two charges separated by a distance x metre.



At point P, $\quad V_P = V_1 + V_2 = 0$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_2} = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0} \cdot \frac{4\times10^{-6}}{x} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{(-2\times10^{-6})}{(1-x)} = 0$$

$$\Rightarrow \quad \frac{4\times10^{-6}}{x} = \frac{2\times10^{-6}}{(1-x)}$$

$$\Rightarrow \quad \frac{4}{x} = \frac{2}{(1-x)}$$

$$\Rightarrow \quad 2(1-x) = x$$

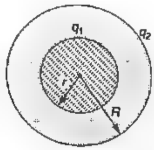
$$\Rightarrow \quad 2 = 3x \quad \text{or} \quad x = \frac{2}{3}$$

$\therefore$ Electrostatic potential is zero at a distance 2/3 m from charge 4μC between the two charges.

[redacted]

EXAMPLE |7| A charge Q is distributed over two concentric hollow spheres of radii r and R ($>r$) such that the surface densities are equal. Find the potential at the common centre.

Sol. Let q_1 and q_2 be the charges on them.



$$\sigma_1 = \sigma_2$$

$$\therefore \quad \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$$

$$\therefore \quad \frac{q_1}{q_2} = \frac{r^2}{R^2}$$

i.e. charge on them is distributed in above ratio

or $q_1 = \dfrac{r^2}{r^2+R^2}Q$ and $q_2 = \dfrac{R^2}{r^2+R^2}Q$

$\therefore$ Potential at centre

$V =$ Potential due to q_1 + Potential due to q_2

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{R} = \frac{Q(R+r)}{4\pi\varepsilon_0(r^2+R^2)}$$

EXAMPLE |8| Two spherical metal shells with different radii r and R are far apart and connected by a thin conducting wire. A charge Q is placed on one of them. The charge redistributes so that same is on each sphere. How much charge is on the sphere with radius r?

Sol. The electrical potential of a spherical shell with charge q and radius r is kq/r, where $k = 1/(4\pi\varepsilon_0)$.

Since, the shells are joined by a conductor the charge will distribute between them so that they attain the same electrical potential.

Let the charge on the sphere with radius r be q_r and that on the another sphere q_R. Then, equating the potentials gives $q_r/r = q_R/R$.

$$\Rightarrow \quad q_r = q_R(r/R) \quad \text{...(i)}$$

$\because$ The total charge equals the original charge.

$$\therefore \quad Q = q_r + q_R \Rightarrow q_R = Q - q_r$$

By Eq. (i), $\quad q_r = (Q - q_r)(r/R)$

Solving for q_r gives $\quad q_r(1 + r/R) = Q(r/R)$

$$\Rightarrow \quad q_r = Qr/(R+r)$$

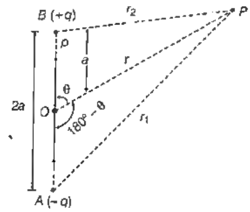
which is the required charge.

Note The value of k is not needed, stating proportionally is sufficient

ELECTROSTATIC POTENTIAL DUE TO AN ELECTRIC DIPOLE

Let us consider an electric dipole consisting of charges $+q$ and $-q$ separated by a distance $2a$.

The dipole moment $|\mathbf{p}| = q \times 2a$.



Electric potential at point P due to electric dipole

Let O be the centre of the dipole, P be any point near the electric dipole inclined at an angle θ as shown in the figure.

Let P be the point at which electric potential is required.

Potential at P due to $-q$ charge, $V_1 = \dfrac{-q}{4\pi\varepsilon_0 r_1}$

Potential at P due to $+q$ charge, $V_2 = \dfrac{q}{4\pi\varepsilon_0 r_2}$

As, potential is related to work done by the field, electrostatic potential also follows the superposition principle. Therefore, potential at P due to the dipole,

$$V_P = V_1 + V_2 = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \quad \text{...(i)}$$

Now, by geometry,

$$r_1^2 = r^2 + a^2 + 2ar\cos\theta$$

Similarly, $\quad r_2^2 = r^2 + a^2 + 2ar\cos(180° - \theta)$

or $\quad r_2^2 = r^2 + a^2 - 2ar\cos\theta \quad [\because \cos(180° - \theta) = -\cos\theta]$

and $\quad r_1^2 = r^2\left(1 + \dfrac{a^2}{r^2} + \dfrac{2a}{r}\cos\theta\right)$

[redacted]

If $r >> a$, $\frac{a}{r}$ is small.

Therefore, $\frac{a^2}{r^2}$ can be neglected.

$$r_1^2 = r^2\left(1+\frac{2a}{r}\cos\theta\right)$$

$$\Rightarrow \qquad r_1 = r\left(1+\frac{2a}{r}\cos\theta\right)^{1/2}$$

or $$\frac{1}{r_1} = \frac{1}{r}\left(1+\frac{2a}{r}\cos\theta\right)^{1/2}$$

Similarly, $$\frac{1}{r_2} = \frac{1}{r}\left(1-\frac{2a}{r}\cos\theta\right)^{1/2}$$

Putting these values in Eq. (i), we obtain

$$V_P = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\left(1-\frac{2a}{r}\cos\theta\right)^{-\frac{1}{2}} - \frac{1}{r}\left(1+\frac{2a}{r}\cos\theta\right)^{-\frac{1}{2}}\right]$$

Using Binomial theorem, $[(1+x)^n = 1+nx, x<<1]$ and retaining terms upto the first order in $\frac{a}{r}$, we set

$$V_P = \frac{q}{4\pi\varepsilon_0 r}\left[\left(1+\frac{a}{r}\cos\theta\right)-\left(1-\frac{a}{r}\cos\theta\right)\right]$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left[1+\frac{a}{r}\cos\theta - 1 + \frac{a}{r}\cos\theta\right]$$

$$= \frac{q}{4\pi\varepsilon_0 r}\left(\frac{2a\cos\theta}{r}\right)$$

$$= \frac{q\times 2a\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$\Rightarrow \qquad \boxed{V_P = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}} \qquad [\because p = q\times 2a]$$

As, $\qquad p\cos\theta = \mathbf{p}\cdot\hat{\mathbf{r}}$

where, $\hat{\mathbf{r}}$ is a unit vector along the position vector $\mathbf{OP} = \mathbf{r}$.

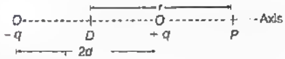
$\therefore$ Electrostatic potential at point P due to a short dipole ($a<<r$) is given by $\boxed{V = \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{4\pi\varepsilon_0 r^2}}$

The potential depends just not only on the position vector $\mathbf{r}$, but also on the angle between the position vector $\mathbf{r}$ and the dipole moment $\mathbf{P}$. The electric potential due to an electric dipole at point P varies inversely with square of r, i.e. the distance of point P from the centre of the dipole.

Electrostatic Potential due to Dipole on its Axis and Equatorial Plane

On the dipole axis, $\qquad \theta = 0°$ or π

$$\therefore \qquad V = \pm\frac{p}{4\pi\varepsilon_0 r^2}$$

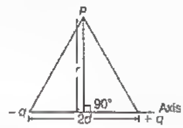


Positive sign for $\theta = 0°$ and negative sign for $\theta = \pi$.

In the equatorial plane, $\theta = \frac{\pi}{2}$

$$\cos\theta = \cos\frac{\pi}{2} = 0$$

$$\therefore \qquad V = 0$$



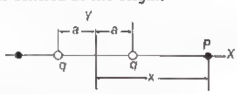
Thus, electrostatic potential at any point in the equatorial plane of dipole is zero

Differences between electric potential due to an electric dipole and due to a single charge are given as below:

(i) The potential due to a dipole depends not just on r but also on the angle between the position vector $\mathbf{r}$ and dipole moment vector $\mathbf{p}$

(ii) The electric potential due to dipole falls off at large distance as $1/r^2$ not as $1/r$, which is a characteristic of the potential due to single charge

EXAMPLE |9| An electric dipole consists of two charges of equal magnitude and opposite signs separated by a distance $2a$ as shown in figure. The dipole is along the X-axis and is centred at the origin.



(i) Calculate the electric potential at point P.

(ii) Calculate V at a point far from the dipole.

Sol. (i) For the point P in figure,

$$V = k_e\,\Sigma\frac{q_i}{r_i} = k_e\left(\frac{q}{x-a} - \frac{q}{x+a}\right) = \frac{2k_e qa}{x^2-a^2}$$

(ii) If point P is far from the dipole, such that $x >> a$, then a^2 can be neglected in the terms, $x^2 - a^2$ and V becomes

$$V = \frac{2k_e qa}{x^2} \qquad [\because x >> a]$$

[redacted]

EQUIPOTENTIAL SURFACES

Any surface which has same electrostatic potential at every point, on it is called an equipotential surface. For a single charge q, the potential is given by $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$. This indicates that V is a constant, if r is constant. Thus, the equipotential surface for single point charge are spherical surfaces centred at the charge. The equipotential surfaces can be drawn through any region in which there is electric field. If all the points at same potential in the electric field are joined, then an equipotential surface is obtained.

The shape of equipotential surface due to a

(i) line charge is cylindrical (ii) point charge is spherical.

Different properties of equipotential surfaces are given as below:

(i) Equipotential surfaces do not intersect each other as it gives two directions of electric field at intersecting point which is not possible.

(ii) Equipotential surfaces are closely spaced in the region of strong electric field and widely spaced in the region of weak electric field.

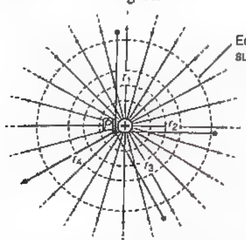
(iii) For any charge configuration, equipotential surface through a point is normal to the electric field at that point and directed from one equipotential surface at higher potential to the other equipotential surface at lower potential.

(iv) No work is required to move a test charge on an equipotential surface.

(v) For a uniform electric field E, let along X-axis, the equipotential surfaces are normal to the X-axis, i.e. planes parallel to the YZ-plane.

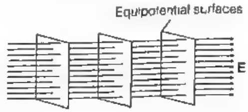
Equipotential Surfaces in Different Cases

Case I The equipotential surfaces produced by a point charge or a spherically symmetrical charge distribution is a family of concentric spheres as shown below in the figure.



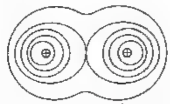
Equipotential surfaces for a point charge

Case II The equipotential surfaces for a uniform electric field are as shown below in figure by dotted lines.



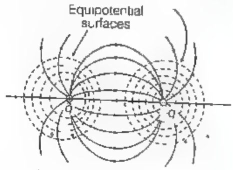
Equipotential surfaces for a uniform electric field

Case III The equipotential surfaces due to two identical positive charges are as shown below



Equipotential surfaces due to two positive charges

Case IV The equipotential surfaces for an electric dipole are as shown below in the figure by dotted lines.



Equipotential surfaces due to an electric dipole

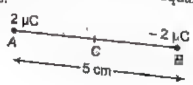
Electric field is always perpendicular to an equipotential surface and as a result, work done in moving a charge between two points on an equipotential surface is zero.

Note This topic has been frequently asked in previous years 2014, 2013, 2011, 2010.

EXAMPLE |10| Two charges 2μC and − 2 μC are placed at points A and B, 5 cm apart. Depict an equipotential surface of the system. **Delhi 2013**

Sol. Equipotential surface means the surface where potential remains same at each point.

Here, this is the system of two equal and opposite charges.



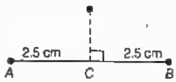
[redacted]

$\therefore$ The potential at C (mid-point of AB),

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{2\times10^{-6}}{2.5\times10^{-2}} + \frac{(-2\times10^{-6})}{2.5\times10^{-2}}\right] = 0$$

Thus, potential is zero at each point on the line which passes through the mid-point of AB and perpendicular to it. So, a plane passing through the mid-point C of AB is an equipotential surface.

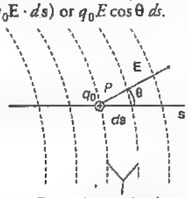


Relation between Electric Field and Electric Potential

Let us consider a positive test charge (q_0) moves a distance (ds) from one equipotential surface to another.The displacement (ds) makes an angle (θ) with the direction of the electric field (**E**).

Suppose a positive test charge (q_0) moves through a differential displacement ds from one equipotential surface to the adjacent surface.

We know that the work done by the electric field on the test charge during its movement is $-q_0\, dV$. We see that the work done by the electric field may also be written as the scalar product ($q_0\mathbf{E}\cdot d\mathbf{s}$) or $q_0 E\cos\theta\, ds$.



Two equipotential surfaces

Displacement of charge between two equipotential surfaces

Equating these two expressions for the work yields

$$-q_0 dV = q_0 E\cos\theta\, ds$$

$$\Rightarrow \quad E\cos\theta = -\frac{dV}{ds}$$

Since, $E\cos\theta$ is the component of **E** in the direction of ds, therefore

$$E_s = -\frac{\partial V}{\partial s}$$

where E_x, E_y and E_z are the x, y and z-components of **E** at any point, then

$$E = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$$

$$\because \quad E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$$

$$\therefore \quad E = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

For the simple situation in which the electric field **E** is uniform.

$$\boxed{E = -\frac{\Delta V}{\Delta s}}$$

Negative sign shows that the direction of electric field **E** in the direction of decreasing potential.

Since, ΔV is negative, then $\Delta V = -|\Delta V|$

We can rewrite this equation as given below

$$\boxed{|E| = -\frac{\Delta V}{\Delta s} = +\frac{|\Delta V|}{\Delta s}}$$

Further, the magnitude of an electric field is given by change in magnitude of potential per unit displacement normal to the equipotential surface at the point. This is called **potential gradient**, i.e.

$$\boxed{|\mathbf{E}| = -\frac{|dV|}{ds} = -\text{(Potential gradient)}}$$

We thus arrive at two important conclusions concerning the relation between electric field and potential which are as given below

(i) Electric field is in the direction in which the potential decreases steepest.

(ii) Its magnitude is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at the point.

EXAMPLE |11| **A small particle carrying a negative charge of 1.6×10^{-19} C is suspended in equilibrium between the horizontal metal plates 5 cm apart, having a potential difference of 3000 V across them. Find the mass of the particle.**

Sol. Here, $q = -1.6\times10^{-19}$ C,

$dr = 5\text{ cm} = 5\times10^{-2}$ m

and $dV = 3000$ V

$$\therefore \quad E = -\frac{\partial V}{\partial r} = \frac{-3000}{5\times10^{-2}} = -6\times10^{4}\ \text{Vm}^{-1}$$

[redacted]

As, the charged particle remains suspended in equilibrium, therefore

$$F = mg = qE$$

$$\therefore \quad m = \frac{qE}{g} = \frac{(-1.6\times10^{-19})\times(-6\times10^{4})}{9.8}$$

$$= 9.8\times10^{-16}\text{ kg}$$

EXAMPLE |12| The electric potential in a region is represented as

$$V = 2x + 3y - z.$$

Obtain expression for electric field strength.

Sol. As,
$$\mathbf{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

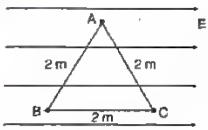
Here,
$$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}(2x + 3y - z) = 2$$

$$\frac{\partial V}{\partial y} = \frac{\partial}{\partial y}(2x + 3y - z) = 3$$

and
$$\frac{\partial V}{\partial z} = \frac{\partial}{\partial z}(2x + 3y - z) = -1$$

$\therefore$ Electric field, $\mathbf{E} = -2\hat{i} - 3\hat{j} + \hat{k}$

EXAMPLE |13| In uniform electric field, $\mathbf{E} = 10\text{ NC}^{-1}$ as shown in figure.



Find

(i) $V_A - V_B$ (ii) $V_B - V_C$

Sol. Since, electric field is directed from higher electric potential to lower electric potential,

(i) Thus, $V_B > V_A$, so $V_A - V_B$ will be negative.

Further, $d_{AB} = 2\cos 60° = 1$ m

$\therefore \quad V_A - V_B = -Ed_{AB} = (-10)(1) = -10$ V

(ii) As, $V_B > V_C$, so $V_B - V_C$ will be positive.

Further, $d_{BC} = 2.0$ m

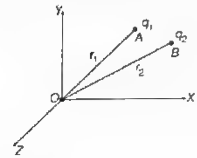
$\therefore \quad V_B - V_C = (10)(2) = 20$ V

ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF CHARGES

Electrostatic potential energy of a system of point charges is defined as the total amount of work done in bringing the different charges to their respective positions from infinitely large mutual separations.

Electrostatic Potential Energy of a System of Two Point Charges

Consider two point charges q_1 and q_2 lying at points A and B whose locations are r_1 and r_2, respectively. To find the electric potential energy of these two charges system, we must mentally build the system starting with both charges infinitely far away and at rest. First, the charge q_1 is brought from infinity to the point r_1. There is no external field against which work needs to be done, so work done in bringing q_1 from infinity to r_1 is zero. V is potential that has been set up by q_1 at the point B, where q_2 is to be placed.



$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r_{AB}}$$

where, r_{AB} is the distance between points A and B.

By definition, work done in carrying charge q_2 from ∞ to B is

$$W = \text{Potential} \times \text{Charge} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r_{AB}}\cdot q_2$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1 q_2}{r_{AB}}$$

This work is stored in the system of two point charges q_1 and q_2 in the form of electrostatic potential energy U of the system.

Thus, $\boxed{U = W = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{AB}}}$

Electrostatic potential energy is a scalar quantity. In the above formula, the values of q_1 and q_2 must be with proper signs. If $q_1, q_2 > 0$, then potential energy is positive. It means that two charges are of same sign, i.e. they repel each other. Then, in bringing closer, work is done against the force of repulsion, so that the electrostatic potential energy of the system increases.

Conversely, in separating them, work is obtained from the system, so the potential energy of the system decreases.

If $q_1 > 0, q_2 < 0$, potential energy is negative. It means that two charges are of opposite sign, i.e. they attract each other. In this case, potential energy of the system decreases in bringing them closer and increases in separating them further.

[redacted]

EXAMPLE |14| In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å.

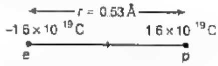
(i) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.

(ii) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (i)?

(iii) What are the answers to (i) and (ii) above, if the zero of potential energy is taken at 1.06 Å separation? **NCERT**

Hints. The potential energy of any object at any point is equal to the difference in its potential energy at infinity and at that point. Work done is equal to the total energy of the system.

Sol. Charge on electron, $q_e = -1.6 \times 10^{-19}$ C and charge on proton, $q_p = 1.6 \times 10^{-19}$ C



(i) Potential energy of the system

= Potential energy at infinity

− Potential energy at a distance of 0.53 Å

$$= 0 - \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_e q_p}{r}$$

$$= 0 - \frac{9 \times 10^9 \times (-1.6) \times 10^{-19} \times 1.6 \times 10^{-19}}{0.53 \times 10^{-10}}$$

$$= -43.47 \times 10^{-19} \text{ J} \qquad [\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}]$$

$$= -\frac{43.47 \times 10^{-19}}{1.6 \times 10^{-19}} = -27.16 \text{ eV}$$

(ii) The kinetic energy $= -\frac{1}{2} \times$ Potential energy

$$= -\frac{1}{2} \times (-27.16) = 13.58 \text{ eV}$$

Total energy = KE + PE = 13.58 − 27.16 = −13.58 eV

Thus, the minimum work done required to free the electron is 13.58 eV.

(iii) Potential energy at separation of 1.06 Å

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_e q_p}{1.06 \times 10^{-10}}$$

$$= \frac{-9 \times 10^9 \times (-1.6) \times 10^{-19} \times 1.6 \times 10^{-19}}{1.06 \times 10^{-10}}$$

$$= -21.73 \times 10^{-19} \text{ J}$$

$$= -\frac{21.73 \times 10^{-19}}{1.6 \times 10^{-19}} = -13.58 \text{ eV}$$

Thus, the potential energy of the system at 1.06 Å

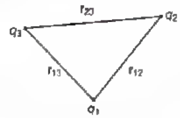
= PE at distance 1.06 Å − PE at distance 0.53 Å

= − 13.58 − (− 27.16) = 13.58 eV

Thus, on shifting the zero of potential energy, work required to free electron remains same and it is equal to 13.58 eV.

Electrostatic Potential Energy of a System of Three Point Charges

Let us now consider a system of three point charges q_1, q_2 and q_3 having position vectors $\mathbf{r}_1$, $\mathbf{r}_2$ and $\mathbf{r}_3$, respectively as from origin.



Three point charges system

To bring q_1 first from infinity to position r_1, no work is required because we bring charge q_1 from infinity to a particular location where potential is zero.

Therefore, $W_1 = 0$

The work done in bringing q_2 from infinity to position r_2 is given by $W_2 = q_2 V_1 (r_2)$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}}$$

Charges q_1 and q_2 produce a potential which at any point P is given by

$$V_{1,2} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$

Work done in bringing q_3 from infinity to position r_3 is q_3 times $V_{1,2}$ at r_3,

$$W_3 = q_3 V_{1,2}(r_3)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

The total work done in assembling the charges at the given locations (equal to the potential energy of the system) is obtained by adding the work done in different steps.

$$U = W_1 + W_2 + W_3$$

$$\Rightarrow \quad U = 0 + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$\Rightarrow \quad \boxed{U = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)}$$

This result can also be expressed in summation form as

$$\boxed{U = \left[\frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{3} \sum_{\substack{j=1 \\ j \neq i}}^{3} \frac{q_i q_j}{r_{ij}}\right]}$$

[redacted]

Due to the conservative nature of electrostatic force, the value of U is independent of the manner in which the configuration is assembled.

If we write the distance $|\mathbf{r}_i - \mathbf{r}_j|$ as r_{ij}, the above equation may be expressed as for system of n point charges system.

$$U = \frac{1}{4\pi\varepsilon_0}\left[\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{q_i q_j}{r_{ij}}\right]$$

Electrostatic potential energy of a system of N point charges is equal to the total amount of work done in assembling all the charges at the given positions from infinity.

$$U = \frac{1}{4\pi\varepsilon_0}\left[\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq 1}}^{N} q_i V_j\right]$$

where, $V_j = \sum_{\substack{j=1\\ j\neq i}}^{N}\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_j}{r_{ij}}$

= Potential at $\mathbf{r}_j$ due to all other charges

The SI unit of electrostatic potential energy is joule (J). Another convenient unit of energy is electron volt (eV).

$$\boxed{1\text{ eV} = 1.6\times10^{-19}\text{C}\times1\text{ V} = 1.6\times10^{-19}\text{ J}}$$

EXAMPLE |15| Three charges (all $q = 10\text{C}$) are placed at the edge of an equilateral triangle of side 2 m. Find the net potential energy of the system.

Sol. Given, charge, $q = 10\text{ C}$ $(q_1 = q_2 = q)$

Each side of equilateral triangle, $r = 2\text{m}$

Potential energy (PE) = ?

Potential energy between two charges is given by

$$\text{PE} = \frac{kq_1q_2}{r} \quad [\because r = \text{distance between } q_1 \text{ and } q_2]$$

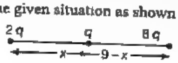
$\therefore$ PE of system will be three times the potential energy between the two charges as the equal charge is placed at the vertices of equilateral triangle.

So, $\text{PE}_{net} = \frac{3\times kqq}{r} = \frac{3kq^2}{r} = \frac{3\times9\times10^9\times10\times10}{2}$

$= 135\times10^{10}$ J

EXAMPLE |16| Three point charges q, $2q$ and $8q$ are to be placed on a 9 cm long straight line. Find the positions where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the position of the charge q due to the other two charges ?

Sol. Consider the given situation as shown in figure.



For potential energy to be minimum the bigger charges should be farthest. Let x be the distance of q from $2q$. Then potential energy of the system shown in figure would be

$$U = K\left[\frac{(2q)(q)}{x} + \frac{(8q)(q)}{(9-x)} + \frac{(2q)(8q)}{9}\right]$$

Here, $K = \frac{1}{4\pi\varepsilon_0}$

For U to be minimum $\frac{2}{x} + \frac{8}{9-x}$ should be minimum.

$$\frac{d}{dx}\left[\frac{2}{x} + \frac{8}{9-x}\right] = 0$$

$$\Rightarrow \quad \frac{-2}{x^2} + \frac{8}{(9-x)^2} = 0$$

$$\Rightarrow \quad \frac{x}{9-x} = \frac{1}{2}$$

or $\quad x = 3$ cm

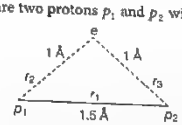
i.e. distance of charge q from $2q$ should be 3 cm.

$\therefore$ Electric field at q,

$$E = \frac{K(2q)}{(3\times10^{-2})^2} - \frac{K(8q)}{(6\times10^{-2})^2} = 0$$

EXAMPLE |17| If one of the two electrons of H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of an H_2^+, the two protons are separated by roughly 1.5 Å and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy. NCERT

Sol. Let there are two protons p_1 and p_2 with an electron e.



Distance between two protons is given by

$r_1 = 1.5\text{ Å} = 1.5\times10^{-10}$ m

Distance between proton p_1 and electron e is given by

$r_2 = 1\text{ Å} = 1\times10^{-10}$ m

Distance between proton p_2 and electron e is given by

$r_3 = 1\text{ Å} = 1\times10^{-10}$ m

The total potential energy of the system,

$$U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_{p_1}q_{p_2}}{r_1} + \frac{q_{p_1}q_e}{r_2} + \frac{q_{p_2}q_e}{r_3}\right] \quad \text{...(i)}$$

Given, $q_{p_1} = q_{p_2} = 1.6\times10^{-19}$C

and $\quad q_e = -1.6\times10^{-19}$ C

[redacted]

Putting these values in Eq. (i), we get

$$U = 9\times 10^9\left[\frac{1.6\times 10^{-19}\times 1.6\times 10^{-19}}{1.5\times 10^{-10}} + \frac{(1.6\times 10^{-19})\times(-1.6\times 10^{-19})}{10^{-10}} + \frac{1.6\times 10^{-19}\times(-1.6\times 10^{-19})}{10^{-10}}\right]$$

$$= \frac{9\times 10^9\times 1.6\times 1.6\times 10^{-38}}{10^{-10}}\left[\frac{1}{1.5}-1-1\right]$$

$$= -30.72\times 10^{-19}\ \text{J}$$

$$= \frac{-30.72\times 10^{-19}}{1.6\times 10^{-19}}\ \text{eV} = -19.2\ \text{eV}$$

Here, we use that potential energy at infinity is zero.

POTENTIAL ENERGY IN AN EXTERNAL FIELD

A single charge or a system of charges possess electrostatic potential energy in the presence of an external electric field, these are discussed as follows.

Potential Energy of a Single Charge in External Field

Potential energy of a single charge q at a point with position vector $\mathbf{r}$ in an external field $= q\cdot V(\mathbf{r})$, where $V(\mathbf{r})$ is the potential at the point due to external electric field E.

Potential Energy of a System of Two Charges in an External Field

For a system of two charges q_1 and q_2, the potential energy is given as,

$$\boxed{U = q_1\cdot V(\mathbf{r}_1) + q_2\cdot V(\mathbf{r}_2) + \frac{q_1 q_2}{4\pi\varepsilon_0 r_{12}}}$$

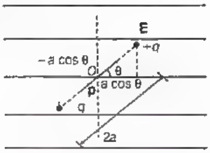
where, q_1, q_2 = two point charges at position vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, respectively

$V(\mathbf{r}_1)$ = potential at $\mathbf{r}_1$ due to the external field

and $V(\mathbf{r}_2)$ = potential at $\mathbf{r}_2$ due to the external field.

Potential Energy of a Dipole in an External Field

Consider a dipole with charges $+q$ and $-q$ placed in a uniform external electric field as shown in the figure. In a uniform electric field, the dipole experiences no force, but experiences a torque τ given by $\tau = \mathbf{p}\times\mathbf{E}$. This torque will tend to rotate the dipole. Suppose an external torque τ_{ext} is applied to the dipole so that it rotates from angle θ_1 to θ_2 with respect to the electric field (E).



Dipole in a uniform external field

The amount of work done by the external torque is given by

$$W = \int_{\theta_1}^{\theta_2}\tau_{ext}\,\theta\,d\theta = \int_{\theta_1}^{\theta_2} pE\sin\theta\,d\theta$$

$$= pE[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$= pE(\cos\theta_1 - \cos\theta_2)$$

The work done W is stored as the potential energy of the system. Therefore, the potential energy of the dipole placed in external field E is given by

$$\boxed{U(\theta) = pE(\cos\theta_1 \quad \cos\theta_2)}$$

Particular Cases

(i) When the dipole is initially aligned along the electric field, i.e. $\theta_1 = 0°$ and we have to set it at angle θ with $\mathbf{E}$, i.e. $\theta_2 = \theta$

$\therefore \quad W = -pE(\cos\theta - \cos 0°)$

$= -pE(\cos\theta - 1)$

This work done is stored in the dipole in the form of potential energy.

(ii) When the dipole is initially at right angle to $\mathbf{E}$, i.e. θ_1 90° and we have to set it at angle θ with $\mathbf{E}$, i.e. $\theta_2 = \theta$.

$\therefore \quad W = -pE(\cos\theta - \cos 90°)$

$= -pE\cos\theta$

$\therefore$ Potential energy of dipole, $U = W = -pE\cos\theta$

$U = -\mathbf{p}\cdot\mathbf{E}$

Obviously, potential energy of an electric dipole is a scalar quantity. It is measured in joule.

Important Results

Some important results related to electric dipole are as given below

(i) Electric potential at any point on the bisector of dipole is zero.

(ii) A dipole experiences a net force in a non-uniform electric field.

(iii) A dipole experiences maximum torque at the position where potential energy is zero.

[redacted]

EXAMPLE |18| An electric dipole of length 4 cm, when placed with its axis making an angle of 60° with a uniform electric field, experiences a torque of $4\sqrt{3}$ N-m. Calculate the potential energy of the dipole, if it has charge ± 8 nC.

Delhi 2014

Sol. Given, length, $2a = 4 \text{ cm} = 4 \times 10^{-2}$ m

Angle, $\theta = 60°$

torque $\tau = 4\sqrt{3}$ N-m

Charge, $Q = 8 \times 10^{-9}$ C

We know that, $\tau = Q(2a)E \sin\theta$

$\Rightarrow$ Electric field, $E = \dfrac{\tau}{Q(2a)\sin\theta}$

$$= \frac{4\sqrt{3}}{8 \times 10^{-9} \times 4 \times 10^{-2} \times \sin 60°} \text{ N/C}$$

$\therefore$ Potential energy, $U = -pE\cos\theta$

$$= -Q(2a)E\cos\theta$$

$$= -8 \times 10^{-9} \times 4 \times 10^{-2} \times \left[\frac{4\sqrt{3} \times \cos 60°}{8 \times 10^{-9} \times 4 \times 10^{-2} \times \sin 60°}\right]$$

$$= \frac{-4\sqrt{3}}{\sqrt{3}} = -4 \text{ J}$$

EXAMPLE |19| A point charge q is fixed at origin. A dipole with a dipole moment **p** is placed along the X axis far away from the origin with **p** pointing along positive X-axis. Find

(i) the kinetic energy of the dipole when it reaches a distance d from the origin.

(ii) the force experienced by the charge q at this moment.

Delhi 2003

Sol. (i) Applying energy conservation principle, increase in kinetic energy of the dipole = decrease in electrostatic potential energy of the dipole

$\therefore$ Kinetic energy of dipole at distance d from origin

$$= U_i - U_f = 0 - (-\mathbf{p}\cdot\mathbf{E}) = \mathbf{p}\cdot\mathbf{E}$$

$$= (p\hat{i})\cdot\left(\frac{1}{4\pi\varepsilon_0}\frac{q}{d^2}\hat{i}\right) = \frac{qp}{4\pi\varepsilon_0 d^2}$$

(ii) Electric field at origin due to the dipole,

$$\mathbf{E} = \frac{1}{4\pi\varepsilon_0}\frac{2p}{d^3}\hat{i} \qquad [\because \mathbf{E}_{axis} \uparrow\uparrow \mathbf{p}]$$

$\therefore$ Force on charge q,

$$\mathbf{F} = q\mathbf{E} = \frac{pq}{2\pi\varepsilon_0 d^3}\hat{i}$$

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. Which of the following is not a unit of electrostatic potential?
(a) Volt
(b) Joule/coulomb
(c) Newton / Coulomb
(d) Newton - metre / Coulomb

2. Work done by an external force in bringing a unit positive charge from infinity to a point is
(a) equal to the electrostatic potential (V) at that point
(b) equal to the negative of work done by electrostatic forces
(c) Both (a) and (b)
(d) Neither (a) nor (b)

3. To find the value of potential at a point, the external force at every point of the path is to be equal and opposite to the
(a) work done
(b) electrostatic force on the test charge at that point
(c) Both (a) and (b)
(d) Neither (a) nor (b)

4. If electrostatic potential at the surface of a sphere of 5 cm radius is 50 V, then the potential at the centre of sphere will be
(a) 10 V (b) 50 V
(c) 250 V (d) zero

5. The electrostatic potential of a uniformly charged thin spherical shell of charge Q and radius R at a distance r from the centre is
(a) $\dfrac{Q}{4\pi\varepsilon_0 r}$ for points outside and $\dfrac{Q}{4\pi\varepsilon_0 R}$ for points inside the shell
(b) $\dfrac{Q}{4\pi\varepsilon_0 r}$ for both points inside and outside the shell
(c) zero for points outside and $\dfrac{Q}{4\pi\varepsilon_0 r}$ for points inside the shell
(d) zero for both points inside and outside the shell

[redacted]

6. A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge **NCERT Exemplar**
(a) remains a constant because the electric field is uniform
(b) increases because the charge moves along the electric field
(c) decreases because the charge moves along the electric field
(d) decreases because the charge moves opposite to the electric field

7. Figure shows some equipotential lines distributed in space. A charged object is moved from point A to point B. **NCERT Exemplar**
(a) The work done in Fig. (i) is the greatest
(b) The work done in Fig. (ii) is least
(c) The work done is the same in Fig. (i), Fig.(ii) and Fig. (iii)
(d) The work done in Fig. (iii) is greater than Fig. (ii) but equal to that in

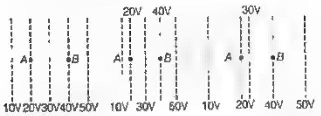


Fig (i) Fig. (ii) Fig. (iii)

8. Equipotentials at a great distance from a collection of charges whose total sum is not zero are approximately
(a) spheres (b) planes
(c) paraboloids (d) ellipsoids

9. Two similar positive point charges each of $1\,\mu C$ have been kept in air at 1m distance from each other. What will be the potential energy?
(a) 1 J (b) 1 eV
(c) 9×10^{-3} J (d) 0

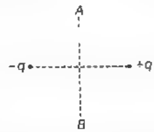
VERY SHORT ANSWER Type Questions

10. A point charge $+Q$ is placed at point O as shown in the figure. Is the potential difference $(V_A - V_B)$ positive, negative or zero? **Delhi 2016, Foreign 2016, Delhi 2011**

11. Is electrostatic potential necessarily zero at a point, where electric field strength is zero? Illustrate your answer. **Delhi 2010**

12. The potential due to a dipole at any point on its axial line is zero. Correct or Wrong? **All India 2009C**

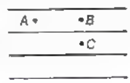
13. A charge q is moved from a point A above a dipole of dipole moment p to a point B below the dipole in equatorial plane without acceleration. Find the work done in this process. **All India 2016**



14. Why are electric field lines perpendicular at a point on an equipotential surface of a conductor? **All India 2016, 2015C**

15. Define the term potential energy for charge q at a distance r in an external field. **All India 2009**

16. For a uniform electric field given as shown below, at what point will the electric potential be maximum?



SHORT ANSWER Type Questions

17. Draw a plot showing the variation of (i) electric field (E) and (ii) electric potential (V) with distance r due to a point charge Q.

18. What is the geometrical shape of equipotential surface due to a single isolated charge? **Delhi 2013**

19. Can two equipotential surface intersect each other? Justify your answer. **Delhi 2011**

20. Give the equipotential surface at a great distance from a collection of charges whose total sum is not zero.

21. Two point charges $3\,\mu C$ and $-3\,\mu C$ are placed at points A and B, 5 cm apart.
(i) Draw the equipotential surfaces of the system.
(ii) Why do equipotential surfaces get close to each other near the point charge? **All India 2011**

[redacted]

22. Two uniformly large parallel thin plates having charge densities $+\sigma$ and $-\sigma$ are kept in the XZ-plane at a distance d apart. Sketch an equipotential surface due to electric field between the plates. If a particle of mass m and charge $-q$ remains stationary between the plates. What is the magnitude and direction of this field? **Delhi 2011**

23. Find out the expression for the potential energy of a system of three charges q_1, q_2 and q_3 located at r_1, r_2 and r_3 with respect to the common origin O. **Delhi 2010**

24. Two point charges q_1 and q_2 are located at r_1 and r_2, respectively in an external electric field **E**. Obtain the expression for the total work done in assembling this configuration. **Delhi 2014C**

25. A dipole with its charges, q and $+q$, located at the points $(0, -b, 0)$ and $(0, +b, 0)$ is present in a uniform electric field **E**. The equipotential surfaces of this field are planes parallel to the YZ-planes.

(i) What is the direction of the electric field **E**?

(ii) How much torque would the dipole experience in this field? **Delhi 2010**

26. If a point charge $+q$ is taken from A to C and then from C to B, points A and B lying on a circle drawn with another charge $+q$ at its centre, then along which path more work will be done?

27. Do free electrons travel to region of higher potential or lower potential? **NCERT Exemplar**

28. Prove that a closed equipotential surface with no charge within itself, must enclose an equipotential value.

LONG ANSWER Type I Questions

29. A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube. **NCERT**

30. (i) Derive the expression for the electric potential due to an electric dipole at a point on its axial line.

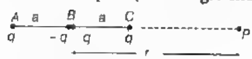
(ii) Depict the equipotential surfaces due to an electric dipole. **Delhi 2017**

31. Give the simplified expression for the following and draw the graph for variation of potential with distance.

(i) Electrostatic potential due to a point charge q at a distance r from it.

(ii) General expression for electric potential due to a dipole.

32. Given figure shows a charge array known as an electric quadrupole. For a point on the axis of the quadrupole, obtain the dependence of potential on r for $r/a >> 1$ and contrast your results with that due to an electric dipole and an electric monopole (i.e. a single charge).



NCERT

33. Define an equipotential surface. Draw equipotential surfaces

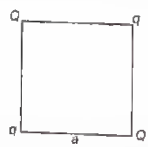
(i) in case of a single point charge

(ii) in a constant electric field in Z-direction. Why the equipotential surfaces about a single charge are not equidistant?

(iii) Can electric field exist tangential to an equipotential surface? Give reason. **All India 2016**

34. Three charges $-q$, $+Q$ and $-q$ are placed at equal distance on straight line. If the potential energy of the system of the three charges is zero, then what is the ratio of $Q : q$?

35. Four point charges Q, q, Q and q are placed at the corners of a square of side a as shown in figure



Find the

(a) resultant electric force on a charge Q and

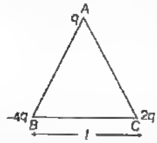
(b) potential energy of this system. **CBSE 2018**

Or

(a) Three point charges q, $-4q$ and $2q$ are placed at the vertices of an equilateral triangle ABC of side l as shown in the figure.

[redacted]

Obtain the expression for the magnitude of the resultant electric force acting on the charge q.



(b) Find out the amount of the work done to separate the charges at infinite distance. CBSE 2018

LONG ANSWER Type II Questions

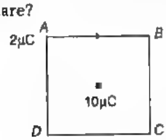
36. Three concentric metal shells A, B and C of radius a, b and c ($a < b < c$) have surface charge densities $+\sigma$, $-\sigma$ and $+\sigma$, respectively.
   (i) Find the potential of three shells at A, B and C.
   (ii) If the shells A and C are at the same potential, obtain the relation between the radii a, b and c.

37. Two metal spheres, one of radius R and the other of radius $2R$, both have same surface charge density σ. They are brought in contact and separated. What will be new surface charge densities on them? NCERT Exemplar

38. (a) Use Gauss' law to derive the expression for the electric field (E) due to a straight uniformly charged infinite line of charge density λ C/m.
   (b) Draw a graph to show the variation of E with perpendicular distance r from the line of charge.
   (c) Find the work done in bringing a charge q from perpendicular distance r_1 to $r_2 (r_2 > r_1)$. CBSE 2018

NUMERICAL PROBLEMS

39. What is the work done in moving a $2\,\mu$C point charge from corner A to corner B of a square $ABCD$, when a $10\,\mu$C charge exists at the centre of the square?



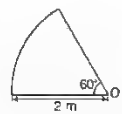
40. The electric potential at 0.1 m from a point charge is + 50 V. What is the magnitude and sign of the charge? All India 2011

41. Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point (s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero. NCERT

42. A regular hexagon of side 10 cm has a charge $5\,\mu$C at each of its vertices. Calculate the potential at the centre of the hexagon. NCERT

43. A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9}C from a point P (0, 0, 3) (in cm) to a point Q (0, 4, 0) (in cm), *via* a point R (0, 6, 9) (in cm). NCERT

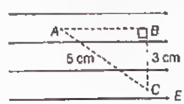
44. The circular arc is shown in the figure given below, has a uniform charge per unit length of 1×10^{-8}C/m Find the potential at the centre O of the arc.



45. A small particle carrying a negative charge of 1.6×10^{-19} C is suspended in equilibrium between the horizontal metal plates 10 cm apart, having a potential difference of 4000 V across them, find the mass of the particle.

46. An infinite plane sheet of charge density 10^{-8} C/m^2 is held in air. In this situation, how far apart are two equipotential surfaces whose potential difference is 5 V?

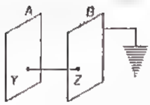
47. A test charge q is moved without acceleration from A to C along the path from A to B and then from B to C in electric field E as shown in the figure.



   (i) Calculate the potential difference between A and C.
   (ii) At which point (of the two) is the electric potential more and why? All India 2012

[redacted]

48. Two identical plane metallic surfaces A and B are kept parallel to each other in air, separated by a distance of 1 cm, surface A is given a positive potential of 10 V and the outer surface of B is earthed.



(i) What is the magnitude and direction of the electric field between the points Y and Z?

(ii) What is the work done in moving a charge of 20 μC from point Y to point Z?

HINTS AND SOLUTIONS

1. (c) From definition of potential,

$$V = \frac{W}{q} = \frac{F \cdot d}{q} \text{ volt}$$

Here, unit of force is newton, unit of distance (d) is metre and unit of chage (q) is coulomb.

Unit of potential is $\frac{\text{Joule}}{\text{Coulomb}}$ or $\frac{\text{N-m}}{\text{C}}$

2. (a) Considering potential to be zero at infinity. Work done by an external force in bringing a unit positive charge from infinity to a point without acceleration = Electrostatic potential (V) at that point

3. (b) The external force at every point of the path is to be equal and opposite to the electrostatic force on the test charge at that point

4. (b) Potential inside a conductor is same at all the points and is equal to the potential at its surface. So, potential at the centre of sphere will also be 50 V.

5. (a) If charge on a conducting sphere of radius R is Q, then potential outside the sphere.

$$V_{out} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}$$

At the surface of sphere,

$$V_s = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{R} = V_{inside}$$

6. (c) The positively charged particle experiences electrostatic force along the direction of electric field *i.e.*, from high electrostatic potential to low electrostatic potential. Thus, the work is done by the electric field on the positive charge, hence electrostatic potential energy of the positive charge decreases.

7. (c) The work done by a electrostatic force is given by $W_{12} = q(V_2 - V_1)$. Here initial and final potentials are same in all three cases and same charge is moved, so work done is same in all three cases.

8. (a) In this problem, the collection of charges, whose total sum is not zero, with regard to great distance can be considered as a point charge. The equipotentials due to point charge are spherical in shape as electric potential due to point charge q is given by

$$V = k_e \frac{q}{r}$$

This suggest that electric potentials due to point charge is same for all equidistant points. The locus of these equidistant points, which are at same potential, form spherical surface.

9. (c) Electric potential energy of the system,

$$U = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$$

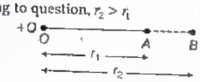
Here, $q_1 = q_2 = 1\,\mu\text{C}$
$= 1 \times 10^{-6}$ C,

$r = 1$ m and $\frac{1}{4\pi\varepsilon_0}$
$= 9 \times 10^9$ N-m^2/C^2

$$\therefore \quad U = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 1 \times 10^{-6}}{1}$$

$$= 9 \times 10^{-3} \text{ J}$$

10. According to question, $r_2 > r_1$



Potential at point A due to charge $+Q$, $(V_A) = \frac{kQ}{r_1}$

Potential at point B due to charge $+Q$, $(V_B) = \frac{kQ}{r_2}$

As $\quad V_A \propto \frac{1}{r_1}$

and $\quad V_B \propto \frac{1}{r_2}$ and $r_2 > r_1$

so, $\quad V_A > V_B$

Thus, $(V_A - V_B)$ is positive.

11. No, it is not necessary because electric field strength inside a hollow charged spherical shell is zero but potential at the point is same as that on the surface of shell.

12. Wrong, the potential due to a dipole at any point on equatorial line is zero, not on axial line.

13. As, A and B are points on the equitorial plane of dipole $V_A = V_B = 0$

Net potential $= V_A + V_B = 0$

Work done $W = \frac{V}{q}$. As $V = 0$, $W = 0$

So, the work done by the process will be zero.

[redacted]

14. Electric field is always normal to the equipotential surface at every point, because no work is done, as

$$W = q_0(V_A - V_B)$$

$\Rightarrow$ $\quad V_A - V_B = 0$

hence $\quad W = 0.$

If the field were not normal to the equipotential surface, it would have a non-zero component along the surface. So, to move a test charge against this component, a work would have to be done.

15. The electric potential energy at any point lying at a distance r from the source charge q is equal to the amount of work done in moving unit positive test charge from infinity to that point without any acceleration against electrostatic force.

16. Potential is maximum at A as potential decreases in the direction of field or we can say that $V_A > V_B = V_C$.

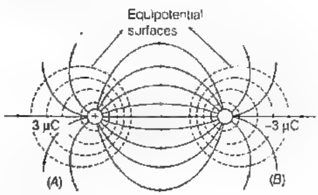
17. Refer to graph on page 64.

18. Refer to text on page 68 (case I).

19. Equipotential surfaces do not intersect each other as it gives two directions of electric field at intersecting point which is not possible.

20. As, the collection of charges at a great distance, so it has spherical equipotential surface.

21. (i) Equipotential surfaces of the system (dipole).



(ii) Equipotential surfaces get closer to each other near the point charges as strong electric field is produced there.

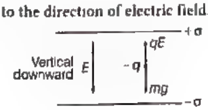
$\because \quad E = -\dfrac{\Delta V}{\Delta r}$

$\Rightarrow \quad E \propto \dfrac{1}{\Delta r}$

[for a given equipotential surface]

where, small Δr represents strong electric field and vice-versa.

22. Here, $-q$ charge experiences force in a direction opposite to the direction of electric field.



$\therefore$ $-q$ charge balances, when

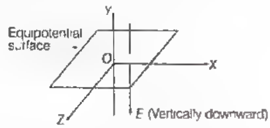
$$qE = mg$$

$\Rightarrow \quad E = \dfrac{mg}{q}$

The direction of electric field is along vertically downward direction.

Note The XZ-plane is so chosen that the direction of electric field due to two plates is along vertically downward direction otherwise weight (mg) of charged particle could not be balanced.

The sketch of equipotential surface due to electric field between the plates is shown in figure below.



23. Refer to text on page 71.

24. Refer to text on page 73.

25. (i) The direction of electric field is perpendicular to their equipotential surface. So, the direction of electric field is along X-axis as its length should be perpendicular to equipotential surface lying in YZ plane.

(ii) Length of the dipole $= 2b$

As dipole's axis is along the Y axis.

$\therefore$ Electric dipole moment, $\mathbf{p} = q(2b)\hat{\mathbf{j}}$

and electric field, $\mathbf{E} = E\,\hat{\mathbf{i}}$

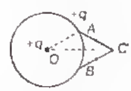
$\because \quad \boldsymbol{\tau} = \mathbf{p} \times \mathbf{E} = q(2b)\hat{\mathbf{j}} \times E\,\hat{\mathbf{i}}$

$= + 2qbE\,(\hat{\mathbf{j}} \times \hat{\mathbf{i}})$

$= 2qbE\,(-\hat{\mathbf{k}})$

$\therefore$ Torque, $|\boldsymbol{\tau}| = 2qbE$

26. Consider the situation as shown in figure.



Work done for the path AC

$$W_{AC} = +q(V_C - V_A)$$

Similarly, $W_{CB} = +q(V_B + V_C)$

$\because \quad V_A = V_B$

$\therefore \quad W_{AC} = |W_{CB}|$

27. The free electrons experience electrostatic force in a direction opposite to the direction of electric field, being of negative charge. The electric field is always directed from higher potential to lower potential.

Therefore, electrostatic force and hence, direction of travelling of electrons is from lower potential to the region of higher potential.

[redacted]

28. **Hints:** In this problem, we need to know that the electric field intensity E and electric potential V are related as $E = -\frac{dV}{dr}$ and the field lines are always perpendicular from one equipotential surface maintained at high electrostatic potential to other equipotential surface maintained at a low electrostatic potential.

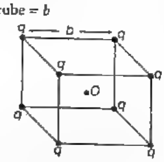
Let's assume contradicting statement that the potential is not same inside the closed equipotential surface. Let the potential just inside the surface be different to that on the surface having a potential gradient $\left(\frac{dV}{dr}\right)$.

Consequently, electric field comes into existence, which is given by, $E = -\frac{dV}{dr}$.

Consequently, field lines point inwards or outwards from the surface. These lines cannot be formed on the surface, as the surface is equipotential. It is possible only when the other end of the field lines are originated from the charges inside. This contradicts the original assumption. Hence, the entire volume inside must be equipotential.

29. Consider a cube of side b and its centre be O. The charge q is placed at each of the corners.

Side of the cube $= b$



Length of the main diagonal of the cube

$$= \sqrt{b^2 + b^2 + b^2} = \sqrt{3}b$$

Distance of centre O from each of the vertices is

$$r = \frac{b\sqrt{3}}{2} \quad \ldots\text{(i)}$$

Potential at point O due to one charge, $V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

Potential at point O due to all charges placed at the vertices of the cube,

$$V' = 8V = \frac{8 \times 1 \times q}{4\pi\varepsilon_0 r} = \frac{8q \times 2}{4\pi\varepsilon_0 \cdot b\sqrt{3}} \quad \text{[from Eq. (i)]}$$

$$= \frac{4q}{\sqrt{3}\pi\varepsilon_0 b}$$

The electric field due to one vertex is balanced by the electric field due to the opposite vertex because all charges are positive in nature. Thus, the resultant electric field at the centre O of the cube is zero.

30. (i) Refer to text on page 67.

(ii) Refer to text on page 68 (Case IV).

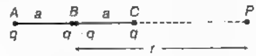
31. Refer text on page 64 for the graph.

(i) Refer to text on pages 63 and 64.

(ii) Refer to text on pages 66 and 67.

32. Given, $AC = 2a$, $BP = r$

$AP = r + a$ and $PC = r - a$



The potential at P is V.

$\therefore$ $V =$ Potential at P due to $A +$ Potential at P due to B + Potential at P due to C

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{AP} - \frac{2q}{BP} + \frac{q}{CP}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot q\left[\frac{1}{(r+a)} - \frac{2}{r} + \frac{1}{(r-a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r(r-a) - 2(r+a)(r-a) + r(r+a)}{r(r+a)(r-a)}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{r^2 - ra - 2r^2 + 2a^2 + r^2 + ra}{r(r^2 - a^2)}\right]$$

$$= \frac{q\,2a^2}{4\pi\varepsilon_0\, r(r^2 - a^2)} = \frac{q \cdot 2a^2}{4\pi\varepsilon_0 \cdot r \cdot r^2\left(1 - \frac{a^2}{r^2}\right)}$$

According to the question,

If $\frac{r}{a} >> 1$, $a << r$. Therefore, $V = \frac{q \cdot 2a^2}{4\pi\varepsilon_0 \cdot r^3}$

$$V \propto \frac{1}{r^3}$$

As, we know that electric potential at a point on axial line due to an electric dipole is

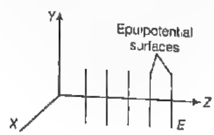
$$V \propto \frac{1}{r^2}$$

In case of electric monopole, $V \propto \frac{1}{r}$.

Then, we conclude that for larger r, the electric potential due to a quadrupole is inversely proportional to the cube of the distance r, while due to an electric dipole, it is inversely proportional to the square of r and inversely proportional to the distance r for a monopole.

33. (i) Refer to text on page 68.

(ii) Equipotential surfaces when the electric field is in Z-direction.

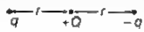


[redacted]

The equipotential surfaces due to a single point charge is represented by concentric spherical shells of increasing radius, so they are not equidistant.

(iii) No, the electric field does not exist tangentially to an equipotential surface because no work is done in moving a charge from one point to other on equipotential surface. This indicates that the component of electric field along the equipotential surface is zero. Hence, the equipotential surface is perpendicular to field lines.

34. Let the three charges be located as shown in the figure.



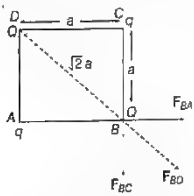
The potential energy of the system be

$$U = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)Q}{r} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q(-q)}{r} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-q)(-q)}{2r}$$

As, $\frac{1}{4\pi\varepsilon_0}\left(-\frac{qQ}{r} - \frac{qQ}{r} + \frac{q^2}{2r}\right) = 0$

$$\Rightarrow \quad \frac{2qQ}{r} = \frac{q^2}{2r} \quad \Rightarrow \quad \frac{Q}{q} = \frac{1}{4} = 1:4$$

35. (a) Force acting on charge Q placed at point B, is due to charges placed at points A, C and D.



Here, magnitude of force on charge at point B due to charge at point A is

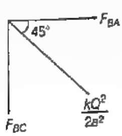
$$F_{BA} = \frac{kQq}{a^2}$$

Similarly, magnitude of force on charge at point B due to charge at point C is

$$F_{BC} = \frac{kQq}{a^2}$$

Also, the magnitude of force on charge at point B due charge at point D is

$$F_{BD} = \frac{kQ^2}{(\sqrt{2}a)^2} = \frac{kQ^2}{2a^2}$$



Let F is resultant of F_{BA} and F_{BC}.

$$\therefore \quad F = \sqrt{2}\cdot\frac{kQq}{a^2} \quad \left[\text{as } F_{BA} = F_{BC} = \frac{kQq}{a^2}\right]$$

$\therefore$ The resultant electric force on charge Q is

$$F_{net} = F + \frac{kQ^2}{2a^2} = \sqrt{2}\,\frac{kQq}{a^2} + \frac{kQ^2}{2a^2}$$

$$= \frac{kQ}{a^2}\left(\sqrt{2}q + \frac{Q}{2}\right)\text{newton}$$

(b) The potential energy of the system is given by

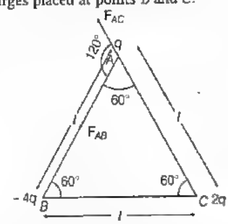
$$U = U_{AB} + U_{BC} + U_{CD} + U_{DA} + U_{AC} + U_{BD}$$

$$= \frac{kQq}{a} + \frac{kQq}{a} + \frac{kQq}{a} + \frac{kQq}{a} + \frac{kq^2}{\sqrt{2}a} + \frac{kQ^2}{\sqrt{2}a}$$

$$= \left[4\left(\frac{kQq}{a}\right) + \frac{kq^2}{\sqrt{2}a} + \frac{kQ^2}{\sqrt{2}a}\right]$$

Or

(a) Force acting on the charge q placed at A, is due to the charges placed at points B and C.



From the given figure, magnitude of force on charge at A due to charge at point C is given as

$$F_{AC} = \frac{k(q)(2q)}{l^2}, \text{ say} = F$$

Similarly, magnitude of force on charge at point A, due to charge at point B is

$$F_{AB} = \frac{k(4q)q}{l^2}, \text{ say} = 2F \qquad (\because F_{AB} = 2F_{AC})$$

$$\therefore \quad F_{res} = \sqrt{F^2 + (2F)^2 + 2(F)(2F)\cos 120^\circ}$$

$$= \sqrt{F^2 + 4F^2 + 4F^2\left(-\frac{1}{2}\right)}$$

$$\left(\because \cos 120^\circ = -\frac{1}{2}\right)$$

$$= \sqrt{F^2 + 2F^2}$$

$$= \sqrt{3}F$$

$$\therefore \quad F_{res} = \sqrt{3}\times\frac{2kq^2}{l^2}\text{ N}$$

(b) The amount of the work done to separate the charges at infinite = Potential energy of the system

$$\therefore \quad U = U_{AB} + U_{BC} + U_{AC}$$

[redacted]

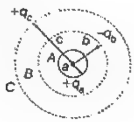
$$= \frac{k(-4q)q}{l} + \frac{k(-4q)(2q)}{l} + \frac{k(q)(2q)}{l}$$

$$= \frac{-4kq^2}{l} - \frac{8kq^2}{l} + \frac{2kq^2}{l}$$

$$U = \frac{-10kq^2}{l} \text{ J}$$

36. **(i) Potential of three shells**

At shell A



$$\text{Potential, } V_A = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_a}{a} - \frac{q_b}{b} + \frac{q_c}{c}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{4\pi a^2\sigma}{a} - \frac{4\pi b^2\sigma}{b} + \frac{4\pi c^2\sigma}{c}\right)\left[\because \sigma = \frac{q}{4\pi r^2}\right]$$

$$= \frac{\sigma}{\varepsilon_0}(a - b + c)$$

At shell B

$$\text{Potential, } V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_a}{b} - \frac{q_b}{b} + \frac{q_c}{c}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{4\pi a^2\sigma}{b} - \frac{4\pi b^2\sigma}{b} + \frac{4\pi c^2\sigma}{c}\right)\left[\because \sigma = \frac{q}{4\pi r^2}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2}{b} + c\right)$$

At shell C

$$\text{Potential, } V_C = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_a}{c} - \frac{q_b}{c} + \frac{q_c}{c}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{4\pi a^2\sigma}{c} - \frac{4\pi b^2\sigma}{c} + \frac{4\pi c^2\sigma}{c}\right)\left[\because \sigma = \frac{q}{4\pi r^2}\right]$$

$$= \frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2 + c^2}{c}\right)$$

(ii) Relation between the radii

Now, $V_A = V_C$ (given)

$$\frac{\sigma}{\varepsilon_0}(a - b + c) = \frac{\sigma}{\varepsilon_0}\frac{(a^2 - b^2 + c^2)}{c}$$

$$a - b + c = \frac{a^2 - b^2 + c^2}{c} = \frac{a^2 - b^2}{c} + c$$

$$c\,(a - b) = a^2 - b^2$$

$$\Rightarrow \quad c = a + b \quad [\because (a^2 - b^2) = (a - b)(a + b)]$$

37. Radius of sphere $A = R$

Surface charge density on sphere $A = \sigma$

Radius of sphere $B = 2R$

Surface charge density on sphere $B = \sigma$

Before contact, the charge on sphere A is

$Q_1 =$ Surface charge density $\times$ Surface area

$$\Rightarrow \quad Q_1 = \sigma \cdot 4\pi R^2 \quad \text{...(i)}$$

Before contact, the charge on sphere B is

$Q_2 =$ Surface charge density $\times$ Surface area

$$Q_2 = \sigma \cdot 4\pi(2R)^2 = \sigma \cdot 16\pi R^2 \quad \text{...(ii)}$$

Let after the contact, the charge on A be Q_1' and the charge on B be Q_2'.

According to the conservation of charge, the charge before contact is equal to charge after contact.

$$Q_1' + Q_2' = Q_1 + Q_2$$

Now, from Eqs. (i) and (ii), we get

$$Q_1' + Q_2' = 4\pi R^2\sigma + 16\pi R^2\sigma$$

$$= 20\pi R^2\sigma \quad \text{...(iii)}$$

As they are in contact. So, they have same potential.

Potential on sphere A is $V_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1'}{R}$

Potential on sphere B is $V_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_2'}{2R}$

So, $V_A = V_B$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_1'}{R} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q_2'}{2R}$$

$$\Rightarrow \quad \frac{Q_1'}{R} = \frac{Q_2'}{2R}$$

$$\Rightarrow \quad 2Q_1' = Q_2' \quad \text{...(iv)}$$

Putting the value of Q_2' in Eq. (iii), we get

$$Q_1' + 2Q_1' = 20\pi R^2\sigma \Rightarrow 3Q_1' = 20\pi R^2\sigma$$

$$\Rightarrow \quad Q_1' = \frac{20}{3}\pi R^2\sigma$$

and $$Q_2' = \frac{40}{3}\pi R^2\sigma \quad \text{[from Eq. (iv)]}$$

Let the new charge densities be σ_1 and σ_2.

$$\sigma_1 = \frac{Q_1'}{4\pi R^2} = \frac{20\pi R^2\sigma}{3 \times 4\pi R^2} = \frac{5}{3}\sigma$$

$$\sigma_2 = \frac{Q_2'}{4\pi(2R)^2} = \frac{40\pi R^2\sigma}{3 \times 4\pi \times 4R^2} = \frac{40\sigma}{16 \times 3}$$

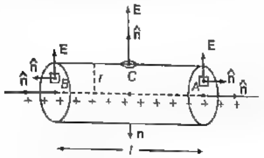
$$\sigma_2 = \frac{10\sigma}{4 \times 3} = \frac{5}{6}\sigma$$

Thus, the surface charge densities on spheres after contact are $\frac{5}{3}\sigma$ and $\frac{5}{6}\sigma$.

[redacted]

38. (a) Field due to an infinitely long thin straight charged line

Consider an infinitely long thin straight line with uniform linear charge density (λ).



Gaussian surface for a long thin straight line of uniform charge density

From symmetry, the electric field is everywhere radial in the plane cutting the wire normally and its magnitude only depends on the radial distance (r).

From Gauss' law,

$$\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

Now, $\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \oint_S \mathbf{E} \cdot \hat{\mathbf{n}}\, dS$

$$= \oint_A \mathbf{E} \cdot \hat{\mathbf{n}}\, dS + \oint_B \mathbf{E} \cdot \hat{\mathbf{n}}\, dS + \oint_C \mathbf{E} \cdot \hat{\mathbf{n}}\, dS$$

$$\therefore \oint_S \mathbf{E} \cdot dS = \oint_A \mathbf{E}\, dS \cos 90^\circ + \oint_B \mathbf{E}\, dS \cos 90^\circ + \oint_C \mathbf{E}\, dS \cos 0^\circ$$

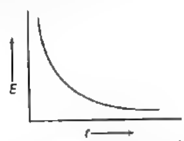
$$= \oint_C E\, dS = E(2\pi r l)$$

Charge enclosed in the cylinder, $q = \lambda l$

$$\therefore \quad E(2\pi r l) = \frac{\lambda l}{\varepsilon_0} \quad \text{or} \quad E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

The direction of the electric field is radially outward from the positive line charge. For negative line charge, it will be radially inward.

(b) Electric field (E) due to the linear charge is inversely proportional to the distance (r) from the linear charge. The variation of electric field (E) with distance (r) is shown in figure.



(c) $$V = \int \mathbf{E} \cdot d\mathbf{r} = \int_{r_1}^{r_2} \frac{\lambda}{2\pi\varepsilon_0 r}\, dr = \frac{\lambda}{2\pi\varepsilon_0} \cdot \int_{r_1}^{r_2} \frac{1}{r}\, dr$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\left[\log \frac{r_2}{r_1}\right]$$

$$\text{Work done} = qv = q\left[\frac{\lambda}{2\pi\varepsilon_0}\left(\log \frac{r_2}{r_1}\right)\right]$$

39. Work done, $W = q \times \Delta V$

But $\Delta V = 0$ as the two diagonally opposite points are at the same potential due to $10\mu C$ charge.

$\therefore \quad W = 2\mu C \times 0 = 0$

Work done $\quad W = 0$

40. Given, $\quad r = 0.1$ m, $V = +50$ V and $q = ?$

As, $\quad V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$

$\Rightarrow \quad 50 = 9 \times 10^9 \times \frac{q}{0.1}$

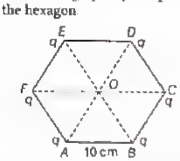
$\therefore \quad q = \frac{50 \times 0.1}{9 \times 10^9} = 5.6 \times 10^{-10}$ C

As, V is positive, therefore, q must be positive.

41. Refer to Example 6 on page 65

Ans. At 6 cm from charge -3×10^{-8} C.

42. $ABCDEF$ is a regular hexagon of side 10 cm each. At each corner, the charge $q = 5\,\mu C$ is placed. O is the centre of the hexagon.



Given, $AB = BC = CD = DE = EF = FA = 10$ cm

As, the hexagon has six equilateral triangles, so the distance of centre O from every vertex is 10 cm.

i.e. $OA = OB = OC = OD = OE = OF = 10$ cm

$\therefore$ Potential at point O = Sum of potentials at centre O due to individual point charge

i.e. $\quad V_O = V_A + V_B + V_C + V_D + V_E + V_F$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{OA} + \frac{q}{OB} + \frac{q}{OC} + \frac{q}{OD} + \frac{q}{OE} + \frac{q}{OF}\right] \quad \left[\because V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}\right]$$

Putting the values, we get

$$V_O = 9 \times 10^9 \left[\frac{5 \times 10^{-6}}{10 \times 10^{-2}} + \frac{5 \times 10^{-6}}{10 \times 10^{-2}} + \frac{5 \times 10^{-6}}{10 \times 10^{-2}} + \frac{5 \times 10^{-6}}{10 \times 10^{-2}} + \frac{5 \times 10^{-6}}{10 \times 10^{-2}} + \frac{5 \times 10^{-6}}{10 \times 10^{-2}}\right]$$

[redacted]

$$= 9 \times 10^9 \times \frac{6 \times 10^{-6} \times 5}{10 \times 10^{-2}}$$

$$= 27 \times 10^5$$

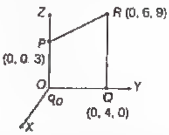
$$= 2.7 \times 10^6 \text{ V}$$

43. Charge q_O at origin $O = 8 \text{ mC} = 8 \times 10^{-3}$ C

Charge q_P at point $P = -2 \times 10^{-9}$ C

Distance $OP = r_P = 3 \text{ cm} = 0.03$ m

Distance, $OQ = r_Q = 4 \text{ cm} = 0.04$ m



Work done in bringing the charge q_P from P to Q

$= q \times$ potential difference between Q and P

$$W_{PQ} = q(V_Q - V_P)$$

$$= -2 \times 10^{-9}\left(\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_O}{OQ} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_O}{OP}\right)$$

$$= -2 \times 10^{-9}\left(\frac{9 \times 10^9 \times 8 \times 10^{-3}}{0.04} - \frac{9 \times 10^9 \times 8 \times 10^{-3}}{0.03}\right)$$

$$= \frac{18 \times 8 \times 10^{-3} \times 0.01}{0.0012}$$

$$= 1.2 \text{ J}$$

Thus, the work done in bringing the charge of -2×10^{-9}C from P to Q is 1.2 J.

44. Potential at the centre,

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}\right)$$

$$= 9 \times 10^9 \times 10^{-8} \times \frac{60}{360} \times 2\pi r$$

$$= 9 \times 10^{+9} \times 10^{-8} \times \frac{2 \times 3.14 \times 2}{6} = 188.4 \text{ V}$$

45. Refer to Example 11 on pages 69 and 70.

[Ans. 6.5×10^{-16}kg]

46. Surface charge density, $\sigma = 10^{-8}$ C/m^2

Potential difference of two equipotential surface,

$$dV = 5 \text{ V}$$

Let the separation between two equipotential surfaces be dr. Electric field intensity E due to infinite plane sheet is given by

$$E = \frac{\sigma}{\varepsilon_0}$$

The relation between E and V is given by

$$E = \frac{dV}{dr} \Rightarrow \frac{dV}{dr} = \frac{\sigma}{2\varepsilon_0}$$

$$\Rightarrow dr = \frac{2\varepsilon_0 \cdot dV}{\sigma} = \frac{2 \times (8.85 \times 10^{-12}) \times 5}{10^{-8}}$$

$$= 8.85 \times 10^{-3} \text{ m}$$

47. (i) $\because$ Electric field intensity and potential difference are related as,

$$E = -\frac{\Delta V}{\Delta r}$$

$$\Rightarrow \quad \Delta V = -E\Delta r$$

By Pythagoras law, $AC^2 = AB^2 + BC^2$

$$\Rightarrow \quad 5^2 - 3^2 = AB^2$$

$$\Rightarrow \quad AB = \Delta r = 4$$

$$\Rightarrow \quad V_A - V_C = -4E$$

$$\Rightarrow \quad V_C - V_A = 4E$$

(ii) As, $V_C - V_A = 4E$, is positive.

$\therefore \quad V_C > V_A$

Potential is greater at point C than at point A, as potential decreases along the direction of electric field.

48. (i) Electric field between the plates is given by

$$E = \frac{\Delta V}{\Delta x} = -\frac{(V_B - V_A)}{1 \times 10^{-2}}$$

$$= \frac{-(0-10)}{10^{-2}} = 10^3 \text{ V/m}$$

It is directed from A to B.

(ii) Work done in moving a charge from Y to Z is

$$W_{Y-Z} = q(\Delta V) = 20 \times 10^{-6}(V_Z - V_Y)$$

$$= 20 \times 10^{-6}(0 - 10)$$

$$= -20 \times 10^{-5} \text{ J}$$

[redacted]

|TOPIC 2|
Dielectric and Capacitance

In this topic, we are going to learn about characteristic properties of conductors and insulators. Also we will go through the concepts of capacitors and their combinations.

CONDUCTORS AND INSULATORS

Let us discuss some characteristics of conductors and insulators as discussed below.

Conductors

Conductors are the materials through which electric charge can flow easily. Most of the metals are conductors of electric charge. Silver is the best conductor of electric charge.

Under electrostatic conditions, the conductors have following properties

(i) Inside a conductor, electrostatic field is zero.

(ii) At the surface of a charged conductor, electrostatic field must be normal to the surface at every point.

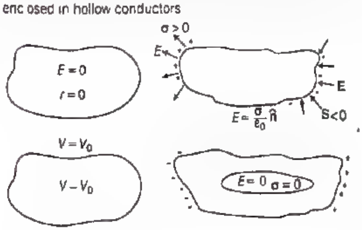
(iii) The interior of the conductor can have no excess charge in the static situation.

(iv) Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface.

(v) Surface charge density of a conductor could be different at different points.

Electrostatic Shielding

The phenomenon of protecting a certain region of space from external electric field is called electrostatic shielding. We know that inside a conductor, electric field is zero, so to protect some instruments from external field, they are enclosed in hollow conductors.



Insulators

Insulators are the materials through which electric charge cannot flow e.g. glass, rubber, wood, etc. Insulators are also called dielectrics, when an electric field is applied, induced charges appear on the surface of the dielectric. Hence, it can be said that dielectrics are the insulating materials which transmit the electric effect without conducting.

Free Charges and Bound Charges Inside the Conductor

In metallic conductors, electrons are the charge carriers. In a metal, the outer (valence) electrons part away from their atoms and are free to move, these electrons are called free electrons or conduction electrons. The electrical conductivity of a material depends upon the number of free electrons present in it. Materials which have high number of free electrons are good conductors and which have less number of free electrons are bad conductors.

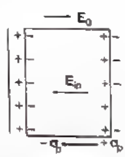
When an electron leaves an atom, atom becomes positively charged ion. The positively charged ions and bound electrons remain held in their fixed positions and are called bound charges.

Dielectrics and Polarisation

Dielectrics (or insulators) are non-conducting substances. In contrast to conductors, they have no (or negligible number of) free charges or charge carriers.

In a dielectric under the effect of an external field, a net dipole moment is induced in the dielectric. Due to molecular dipole moments, a net charge appears on the surface of the dielectric.

These induced charges (of densities $-\sigma_p$ and and $+\sigma_p$) produce a field opposing the external field. Induced field is lesser in magnitude than the external field. So, field inside the dielectric gets reduced.



Polarised dielectric slab

$$E = |E_0| - |E_{in}|$$

where, E = resultant electric field in the dielectric,

E_0 = external electric field between two plates

and E_{in} = electric field inside the dielectric.

A net dipole moment is developed by an external field in either case, whether a polar or non-polar dielectric.

[redacted]

Dielectric Constant (K)

The ratio of the strength of the applied electric field to the strength of the reduced value of the electric field on placing the dielectric between the two plates is called the dielectric constant of the dielectric medium.

It is also known as relative permittivity or specific inductive capacity and is denoted by K (or ε_r).

Therefore, dielectric constant of a dielectric medium is given by

$$\boxed{K = \frac{E_0}{E}}$$

Note The value of K is always greater than 1

Polarisation (P)

The induced dipole moment developed per unit volume in a dielectric slab on placing it in an electric field is called polarisation. It is denoted by P. If p is induced dipole moment acquired by an atom of the dielectric and N is the number of atoms per unit volume, then polarisation is given by $\boxed{P = Np}$

The induced dipole moment (p) acquired by the atom is found to be directly proportional to the reduced value of electric field (E) and is given by

$$\boxed{p = \alpha\varepsilon_0 E}$$

where, α is constant of proportionality and is called **atomic polarisability.**

Electric Susceptibility (χ)

The polarisation density of a dielectric slab is directly proportional to the reduced value of the electric field and may be expressed as $\boxed{P = \chi\varepsilon_0 E}$

The constant of proportionality χ is called electric susceptibility of the dielectric slab. It is a dimensionless constant.It describes the electrical behaviour of a dielectric. It has different values for different dielectrics.

For vacuum, $\chi = 0$

Relation between dielectric constant and electric susceptibility can be given as

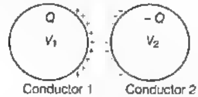
$$\boxed{K = 1 + \chi}$$

Dielectric Strength

The maximum electric field that a dielectric can withstand without breakdown (of its insulating property), is called its dielectric strength. For air, it is about 3×10^6 V/m.

Capacitors and Capacitance

A capacitor is a system of two conductors separated by an insulating medium. The conductors have charges Q and $-Q$ with potential difference, $V = V_1 - V_2$ between them. The electric field in the region between the conductors is proportional to the charge Q.



A system of two conductors or capacitors

If the potential difference (V) is the work done per unit positive charge in taking a small test charge from the conductor 2 to 1 against the field, then V is proportional to Q and the ratio $\frac{Q}{V}$ is a constant.

$$\boxed{C = \frac{Q}{V}}$$

The constant C is called the capacitance of the capacitor. Capacitance C depends on shape, size and separation of the system of two conductors.The SI unit of capacitance is farad. Its dimensional formula is $[M^{-1}L^{-2}T^4A^2]$.

1 farad = 1 coulomb/volt

A capacitor with fixed capacitance is symbolically shown as $-|\,|-$, while the one with variable capacitance is shown as $-\!\!/\!\!|\,|\!-$. In practice, farad is a very big unit, the most common units are its sub-multiples.

$$1\,\mu F = 10^{-6}\,F, 1\,nF = 10^{-9}\,F, 1\,pF = 10^{-12}\,F$$

EXAMPLE |1| When 1×10^{12} electrons are transferred from one conductor to another, a potential difference of 10 V appears between the conductors. Find the capacitance of the two conductors.

Sol. Given, number of electrons,

$$n = 1 \times 10^{12}$$

$\therefore$ Charge transferred,

$$Q = ne = 1 \times 10^{12} \times 1.6 \times 10^{-19}$$
$$= 1.6 \times 10^{-7}\,C \qquad [\because e = 1.6 \times 10^{-19}\,C]$$

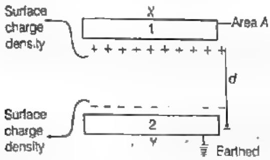
$\therefore$ Capacitance between two conductors,

$$C = \frac{Q}{V} = \frac{1.6 \times 10^{-7}}{10}$$
$$= 1.6 \times 10^{-8}\,F$$

[redacted]

PARALLEL PLATE CAPACITOR

Parallel plate capacitor consists of two thin conducting plates each of area A held parallel to each other at a suitable distance d. One of the plates is insulated and other is earthed. And also there is vacuum between the plates.



Suppose the plate X is given a charge of $+q$ coulomb. By induction, $-q$ coulomb of charge is produced on the inner surface of the plate Y and $+q$ coulomb on the outer surface. Since, the plate Y is connected to the earth, the $+q$ charge on the outer surface flows to the earth. Thus, the plates X and Y have equal and opposite charges.

Suppose the surface density of charge on each plate is σ. We know that the intensity of electric field at a point between two plane, parallel sheets of equal and opposite charges is σ/ε_0, where ε_0 is the permittivity of free space.
The intensity of electric field between the plates will be given by

$$E = \frac{\sigma}{\varepsilon_0}$$

The charge on each plate is q and the area of each plate is A. Thus,

$$\sigma = \frac{q}{A} \text{ and so, } E = \frac{q}{\varepsilon_0 A} \quad \text{...(i)}$$

Now, let the potential difference between the two plates be V volt. Then, the electric field between the plates is given by

$$E = \frac{V}{d} \text{ or } V = Ed$$

Substituting the value of E from Eq. (i), we get

$$V = \frac{qd}{\varepsilon_0 A}$$

∴ Capacitance of the parallel plate capacitor is given by

$$C = \frac{q}{V} = \frac{q}{qd/\varepsilon_0 A} \text{ or } \boxed{C = \frac{\varepsilon_0 A}{d}}$$

where, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

It is clear from this formula that in order to obtain high capacitance.

(i) A should be large, i.e. the plates of large area should be taken.

(ii) d should be small, i.e. the plates should be kept close to each other.

Note Capacity of an isolated spherical conductor is

$$\boxed{C = 4\pi\varepsilon_0 r}$$

where, r = radius of the sphere

Leakage of Charge from a Capacitor

From the formula $C = q/V$, it is clear that for large C, V is small for a given q. This means a capacitor with large capacitance can hold large amount of charge q at small V. This is very important fact, because the large amount of charge implies strong electric field around the conductor.

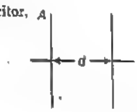
This strong electric field can ionise the surrounding air and accelerate the charges, so produced to oppositely charged plates, thereby neutralising the charge on the capacitor plates. This means the charge of the capacitor leaks away due to the reduction in an insulating power of the intervening medium.

EXAMPLE |2| What is the area of the plates of a 2F parallel plate capacitor, given that the separation between the plates is 0.5 cm? (You will realise from your answer why ordinary capacitors are in the range of μF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors). **NCERT**

Sol. Given, capacitance, $C = 2$ F

and separation between plates, $d = 0.5 \text{ cm} = 0.5 \times 10^{-2}$ m

Capacitance of a parallel plate capacitor,



$$C = \frac{\varepsilon_0 A}{d}$$

$$\text{or } A = \frac{Cd}{\varepsilon_0} = \frac{2 \times 0.5 \times 10^{-2}}{8.854 \times 10^{-12}}$$

$$= 1.13 \times 10^9 \text{ m}^2$$

$$= 1130 \text{ km}^2$$

This area is very large, so it is not possible that the capacitance of a capacitor is too large as 2 F.
So, the capacitance of any capacitor should be the range of 2 μF.

EXAMPLE |3| A parallel plate capacitor has plate area 25 cm² and a separation of 2 mm between the plates. The capacitor is connected to a battery of 12 V.

(i) Find the charge on the capacitor.

(ii) If the plate separation is decreased to 1.0 mm, then find the extra charge given by the battery to the positive plate.

[redacted]

Sol. Given, area of plate, $A = 25\text{ cm}^2 = 25\times10^{-4}\text{ m}^2$

Distance between the plates, $d = 2\text{ mm} = 2\times10^{-3}\text{ m}$

Potential difference, $V = 12\text{ V}$

(i) Charge on the capacitor, $q = CV$

$$= \frac{\varepsilon_0 A}{d}V = \frac{8.85\times10^{-12}\times25\times10^{-4}\times12}{2\times10^{-3}}$$

$$= 1.33\times10^{-10}\text{ C}$$

(ii) If the plate separation is decreased to half, the capacity becomes twice. Then, charge becomes twice as battery is still connected

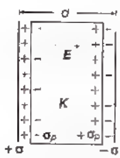
$\therefore$ Extra charge given by the battery $= q'-q$

$$= 2q - q = q = 1.33\times10^{-10}\text{ C}$$

Effect of Dielectric on Parallel Plate Capacitor

Consider a dielectric is inserted between the plates of a parallel plate capacitor and fully occupying the intervening region as shown in figure. The dielectric is polarised by the field, with surface charge densities σ_p, and $-\sigma_p$.

The electric field in the dielectric then corresponds to the case when the net surface charge density on the plates is $\pm(\sigma - \sigma_p)$.



Dielectric between the plates of a capacitor

So, net electric field between the plates, $E = \dfrac{\sigma - \sigma_p}{\varepsilon_0}$

[$\because$ dielectric is polarised in the opposite direction of external field]

$\therefore$ Potential difference between the plates,

$$V = Ed = \frac{\sigma - \sigma_p}{\varepsilon_0}d$$

For linear dielectrics, we expect σ_p to be proportional to E_0 i.e. to σ.

Thus, $(\sigma - \sigma_p)$ is proportional to σ and we can write,

$$\sigma - \sigma_p = \frac{\sigma}{K}$$

where, K is a constant characteristics of the dielectric.

Clearly, $K > 1$ [$\because \sigma_p < \sigma$]

then, $V = \dfrac{\sigma d}{\varepsilon_0 K} = \dfrac{qd}{A\varepsilon_0 K}$

$\therefore$ The capacitance C with dielectric between the plates is given by

$$\boxed{C = \frac{q}{V} = \frac{\varepsilon_0 KA}{d}}$$

The product $\varepsilon_0 K$ is called the **permittivity of the medium** and is denoted by ε.

$$\varepsilon = \varepsilon_0 K$$

For vacuum, $K = 1$ and $\varepsilon = \varepsilon_0$, where ε_0 is called the **permittivity of the vacuum**.

The dimensionless ratio,

$$K = \frac{\varepsilon}{\varepsilon_0}$$

is called the **dielectric constant** of the substance.

Similarly, $K = \dfrac{C}{C_0}$

Thus, the dielectric constant of a substance is the factor $(K > 1)$ by which the capacitance increases from its vacuum value, when the dielectric is inserted fully between the plates of a capacitor.

(i) When a dielectric slab of thickness t is inserted between the plates, then

Capacitance, $C = \dfrac{\varepsilon_0 A}{d - t + \dfrac{t}{K}}$

(ii) If several slabs of dielectric constants $K_1, K_2, K_3, \ldots$ and respective thicknesses $t_1, t_2, t_3, \ldots$ are placed in between the plates of a capacitor, then capacitance,

$$C = \frac{\varepsilon_0 A}{d - (t_1 + t_2 + t_3 + \ldots) + \dfrac{t_1}{K_1} + \dfrac{t_2}{K_2} + \dfrac{t_3}{K_3} + \ldots}$$

(iii) If a metallic slab $(K = \infty)$ of thickness t is placed between the plates of capacitor, then

Capacitance,

$$C = \frac{\varepsilon_0 A}{d - t}.$$

If metallic slab fills the entire space between the plates (i.e. $d = t$), then capacitance will become infinite.

[redacted]

EXAMPLE |4| In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3} m^2$ and the separation between the plates is 3 mm.

(i) Calculate the capacitance of the capacitor.

(ii) If this capacitor is connected to 100 V supply, what would be the charge on each plate?

(iii) How would charge on the plates be affected if a 3 mm thick mica sheet of $K = 6$ is inserted between the plates while the voltage supply remains connected?

Foreign 2014

Sol. Given, area of each plate, $A = 6 \times 10^{-3} m^2$

Distance between the plates

$$d = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$$

(i) Capacitance of parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$$

$$\therefore \quad C = 1.77 \times 10^{-11} F$$

(ii) Charge on parallel plate capacitor is given by

$$Q = CV = 1.77 \times 10^{-11} \times 100 = 1.77 \times 10^{-9} C$$

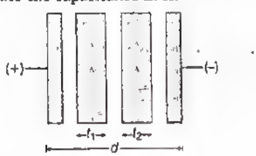
(iii) Given, $K = 6$

Now, $C' = KC$

$$\Rightarrow \quad \frac{Q'}{V} = \frac{KQ}{V}$$

$$\therefore \quad Q' = KQ = 6 \times 1.77 \times 10^{-9} = 10.62 \times 10^{-9} C$$

EXAMPLE |5| An air-cored capacitor of plate area A and separation d has a capacity C. Two dielectric slabs are inserted between its plates in two different manners as shown. Calculate the capacitance in it.



Sol. Let the charges on the plates are Q and $-Q$.

Electric field in free space is $E_0 = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$.

Electric field in first slab is $E_1 = \frac{E_0}{K_1} = \frac{Q}{A\varepsilon_0 K_1}$.

Electric field in second slab is $E_2 = \frac{E_0}{K_2} = \frac{Q}{A\varepsilon_0 K_2}$.

The potential difference between the plates is

$$V = E_0(d - t_1 - t_2) + E_1 t_1 + E_2 t_2$$

$$\Rightarrow \quad V = E_0\left(d - t_1 - t_2 + \frac{t_1}{K_1} + \frac{t_2}{K_2}\right)$$

$$\left[\because E_1 = \frac{E_0}{K_1} \text{ and } E_2 = \frac{E_0}{K_2}\right]$$

$$\therefore \quad V = \frac{Q}{A\varepsilon_0}\left(d - t_1 - t_2 + \frac{t_1}{K_1} + \frac{t_2}{K_2}\right)$$

$$\therefore \quad C = \frac{\varepsilon_0 A}{d - t_1 - t_2 + \frac{t_1}{K_1} + \frac{t_2}{K_2}}$$

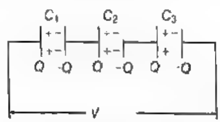
COMBINATION OF CAPACITORS

When there is a combination of capacitors in a circuit, we can sometimes replace that combination with an equivalent capacitor, i.e. single capacitor, that has the same capacitance as the actual combination of capacitors has with such a replacement, that we can simply find the circuit, affording easier solutions for unknown quantities of the circuit.

Here, we discuss two basic combinations of capacitors which can be replaced by single equivalent capacitor.

Capacitors in Series

When a potential difference (V) is applied across several capacitors connected end to end in such a way that, sum of potential differences across all the capacitors is equal to the applied potential difference V, then these capacitors are said to be connected in series.



Series combination of capacitors

The potential difference across the separate capacitors are given by

$$V_1 = \frac{Q}{C_1}, \; V_2 = \frac{Q}{C_2} \text{ and } V_3 = \frac{Q}{C_3}$$

However, the potential difference across the series combination of capacitors is V volt

where, $$V = V_1 + V_2 + V_3 \qquad \text{...(i)}$$

Let C_S represents the equivalent capacitance, then

$$V = \frac{Q}{C_S} \qquad \text{...(ii)}$$

Combining Eqs. (i) and (ii), we get

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

$$\Rightarrow \quad \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

[redacted]

The equivalent capacitance of n capacitors connected in series is equal to the sum of the reciprocals of individual capacitances of the capacitors.

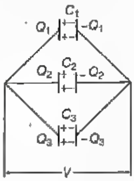
Mathematically, it is expressed as,

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots + \frac{1}{C_n}}$$

All the capacitors connected in series have same amount of charge, but potential differences between their plates are inversely proportional to their capacitances. This combination is used when a high voltage is to be divided on several capacitors. Here, capacitor with minimum capacitance has maximum potential difference between the plates.

Capacitors in Parallel

Capacitors are said to be connected in parallel when a potential difference that is applied across their combination results in the potential diffence same across each capacitor.



Parallel combination of capacitors

When a potential difference (V) is applied across several capacitors connected in parallel, then the potential difference (V) exists across each capacitor. The total charge (Q) stored on the capacitor is the sum of the charges stored on all the capacitors.

If Q is the total charge on the parallel network, then

$$Q = Q_1 + Q_2 + Q_3 \quad \ldots \text{(i)}$$

Let C_P be the equivalent capacitance of the parallel combination, then

$$Q = C_P V,\ Q_1 = C_1 V,\ Q_2 = C_2 V$$

and $$Q_3 = C_3 V \quad \ldots \text{(ii)}$$

Combining Eqs. (i) and (ii), we obtain

$$C_P V = C_1 V + C_2 V + C_3 V$$

$$\Rightarrow \quad C_P = C_1 + C_2 + C_3$$

The equivalent capacitance of n number of capacitors in parallel is equal to the algebraic sum of the individual capacitances of the capacitors.

Mathematically, it is expressed as,

$$\boxed{C_{eq} = C_1 + C_2 + C_3 + \ldots + C_n}$$

All the capacitors connected in parallel have same potential difference between their plates but the charge is distributed proportionally to their capacitances.

Capacitors are combined in parallel, when we require a large capacitance at small potential.

EXAMPLE |6| Three capacitors each of capacitance 9 pF are connected in series.

(i) What is the total capacitance of the combination?

(ii) What is the potential difference across each capacitor, if the combination is connected to a 120 V supply? NCERT

Sol. There are three capacitors each of capacitance 9 pF.

$$\therefore \quad C_1 = C_2 = C_3 = 9\,\text{pF}$$

and voltage, $V = 120\,\text{V}$

(i) The total capacitance in series combination,

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{9} + \frac{1}{9} + \frac{1}{9}$$

$$\Rightarrow \quad \frac{1}{C_S} = \frac{3}{9} \quad \Rightarrow \quad C_S = 3\,\text{pF}$$

(ii) Let the charge across the system be q and potentials across C_1, C_2 and C_3 be V_1, V_2 and V_3, respectively.

Charge, $q = C_S \cdot V = 3 \times 120 = 360\,\text{pC}$

Potential difference across C_1,

$$V_1 = \frac{q}{C_1} = \frac{360}{9} = 40\,\text{V}$$

Potential difference across C_2,

$$V_2 = \frac{q}{C_2} = \frac{360}{9} = 40\,\text{V}$$

Potential difference across C_3,

$$V_3 = \frac{q}{C_3} = \frac{360}{9} = 40\,\text{V}$$

Thus, the potential difference across each capacitor is 40 V.

EXAMPLE |7| It is required to construct a 10 μF capacitor which can be connected across a 200 V battery. Capacitors of capacitance 10 μF are available but they withstand only 50 V. Design a combination which can yield the desired result.

Sol. Capacitor of 10 μF can withstand only 50 V, therefore to be connected across a 200 V battery, four capacitors must be connected in series in a row. Capacitor C_1 of each row of four capacitors is

$$\frac{1}{C_1} = \frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{4}{10}$$

$$\Rightarrow \quad C_1 = \frac{10}{4} = 2.5\,\mu\text{F}$$

For a total capacity of 10 μF, four such rows of capacitors must be connected in parallel, so that

$$C_P = 4\,C_1$$

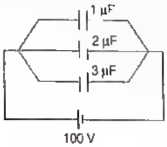
$$= 4 \times 2.5 = 10\,\mu\text{F}$$

Hence, we need 16 capacitors with 4 capacitors in series in each row and 4 such rows in parallel.

[redacted]

EXAMPLE |8| In the circuit shown in figure, find

(i) the equivalent capacitance and
(ii) the charge stored in each capacitor.



Sol. (i) The capacitors are in parallel. Hence, the equivalent capacitance is

$$C = C_1 + C_2 + C_3$$
$$= (1 + 2 + 3) = 6\,\mu F$$

(ii) Total charge drawn from the battery,

$$q = CV = 6 \times 100\,\mu C$$
$$= 600\mu C$$

This charge will be distributed in the ratio of their capacities. Hence,

$$q_1 : q_2 : q_3 = C_1 : C_2 : C_3 = 1 : 2 : 3$$

$$\therefore \quad q_1 = \left(\frac{1}{1+2+3}\right) \times 600 = 100\,\mu C$$

$$q_2 = \left(\frac{2}{1+2+3}\right) \times 600 = 200\,\mu C$$

and $$q_3 = \left(\frac{3}{1+2+3}\right) \times 600 = 300\,\mu C$$

EXAMPLE |9| Three capacitors of $1\mu F$, $2\mu F$ and $3\mu F$ are joined in series.

(i) How many times will the capacity become when they are joined in parallel ?
(ii) Determine the charge supplied by the battery of 100 V to the maximum resultant capacitor among both the arrangements.

Sol. (i) Given, $C_1 = 1\,\mu F$, $C_2 = 2\mu F$, $C_3 = 3\,\mu F$

The combined capacity (C_S) in series combination is given by

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} = \frac{11}{6}$$

$$\Rightarrow \quad C_S = \frac{6}{11}\,\mu F$$

The combined capacity (C_P) in parallel combination is given by

$$C_P = C_1 + C_2 + C_3 = 1 + 2 + 3 = 6\,\mu F$$

$$\Rightarrow \quad C_P = 11 C_S$$

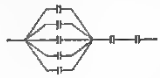
(ii) As, $C_P > C_S$

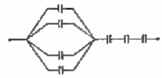
$\therefore$ The charge supplied by 100 V battery,

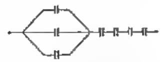
$$q_p = C_P V = 6\mu F \times 100 = 6 \times 10^{-6} \times 100$$

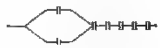
$$\Rightarrow \quad q_p = 6 \times 10^{-4}\,C = 600\,\mu C$$

EXAMPLE |10| Seven capacitors each of capacitance $2\,\mu F$ are connected in a configuration to obtain an effective capacitance $\frac{10}{11}\mu F$. Which of the following combinations will achieve the desired result?

(i) 

(ii) 

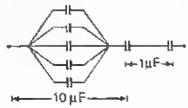
(iii) 

(iv) 

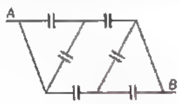
Sol. Consider the first configuration, we have

In series, $C = \frac{C_1 C_2}{C_1 + C_2}$

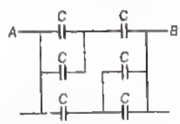
$$\therefore \quad C_{net} = \frac{(10)(1)}{10+1} = \frac{10}{11}\,\mu F$$



EXAMPLE |11| A network of six identical capacitors, each of value C is made as shown in the figure. Find the equivalent capacitance between the points A and B.



Sol. The equivalent network of the given network is shown below.



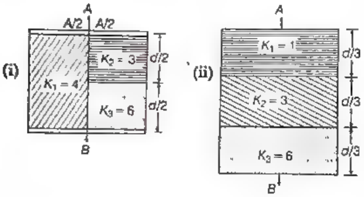
Therefore equivalent capacitance,

$$C_{eq} = [2\,C \text{ series } C] \| [C \text{ series } 2C]$$

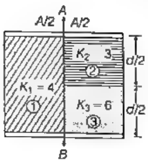
$$= 2\left[\frac{2C \times C}{2C + C}\right] = \frac{4C}{3}$$

[redacted]

EXAMPLE |12| **Find the equivalent capacitance between A and B. Given area of each plate $= A$ and separation between plate $= d$.**



Sol. (i)



$$\text{Capacitance, } C_1 = \frac{K_1 \frac{A}{2}\varepsilon_0}{d} = \frac{4\frac{A}{2}\varepsilon_0}{d} = \frac{2A\varepsilon_0}{d}$$

$$\left[\because C = \frac{KA\varepsilon_0}{d}\right]$$

$$\text{Capacitance, } C_2 = \frac{K_2 \frac{A}{2}\varepsilon_0}{\frac{d}{2}} = \frac{3A\varepsilon_0}{d}$$

$$\text{Capacitance, } C_3 = \frac{K_3 \frac{A}{2}\varepsilon_0}{\frac{d}{2}} = \frac{6A\varepsilon_0}{d}$$

$$C_2 \text{ and } C_3 \text{ are in series, } C' = \frac{C_2C_3}{C_2 + C_3} = \frac{2A\varepsilon_0}{d}$$

$$C' \text{ and } C_1 \text{ are in parallel} = \frac{4A\varepsilon_0}{d}$$

(ii) Capacitance, $C_1 = \dfrac{K_1 A\varepsilon_0}{d/3} = \dfrac{3A\varepsilon_0}{d}$,

$$C_2 = \frac{K_2 A\varepsilon_0}{d/3} = \frac{9A\varepsilon_0}{d}$$

and $$C_3 = \frac{K_3 A\varepsilon_0}{d/3} = \frac{18A\varepsilon_0}{d}$$

$\because C_1, C_2$ and C_3 are in series,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\therefore \text{Equivalent capacitance, } C_{eq} = \frac{2A\varepsilon_0}{d}$$

ENERGY STORED IN A CAPACITOR

The energy of a charged capacitor is measured by the total work done in charging the capacitor to a given potential. Let us assume that initially both the plates are uncharged. Now, we have to repeatedly move small positive charges from one plate and transfer them to the other plate.

Now, when an additional small charge (dq) is transferred from one plate to another plate, the small work done is given by $\quad dW = V'dq = \dfrac{q'}{C}dq$

[$\because$ charge on plate when dq charge is transferred be q']

The total work done in transferring charge Q is given by

$$W = \int_0^Q \frac{q'}{C}dq = \frac{1}{C}\int_0^Q q'\,dq$$

$$= \frac{1}{C}\left[\frac{(q')^2}{2}\right]_0^Q = \frac{Q^2}{2C}$$

This work is stored as electrostatic potential energy U in the capacitor.

$$U = \frac{Q^2}{2C} = \frac{1}{2}QV = \frac{(CV)^2}{2C} = \frac{1}{2}CV^2 \quad [\because Q = CV]$$

The energy stored per unit volume of space in a capacitor is called **energy density**.

Energy density, $u = \dfrac{1}{2}\varepsilon_0 E^2$

Total energy stored in series combination or parallel combination of capacitors is equal to the sum of energies stored in individual capacitors.

i.e. $\quad U = U_1 + U_2 + U_3 + \ldots$

Change in Energy on Introducing a Dielectric Slab

(i) When a dielectric slab is inserted between the plates of a charged capacitor, with battery connected to its plates. Then, the capacitance becomes K (dielectric constant) times and energy stored in the capacitor becomes KU_0.

(ii) When a dielectric slab is inserted between the plates of a charged capacitor and battery is disconnected. Then, the charge on the plates remains unchanged and energy stored in the capacitor becomes $\dfrac{U_0}{K}$, i.e. energy decreases.

Note This topic has been frequently asked in previous years 2015, 2014, 2012, 2011, 2010

[redacted]

EXAMPLE |13| A capacitor of capacity $10\,\mu F$ is subjected to charge by a battery of 10 V. Calculate the energy stored in the capacitor.

Sol. Given, capacity, $C = 10\,\mu F = 10 \times 10^{-6}$ F

Voltage, $V = 10$ V, energy, $E = ?$

$\therefore$ Energy stored in the capacitor, $E = \frac{1}{2} CV^2$

$$= \frac{1}{2} \times 10 \times 10^{-6} \times 10 \times 10 = 5 \times 10^{-4}\,\text{J}$$

EXAMPLE |14| A parallel plate capacitor has plate area A and separation d. It is charged to a potential difference V_0. This charging battery is disconnected and the plates are pulled apart to three times the initial separation. Calculate the work required to separate the plates.

Sol. $\therefore$ Capacitance, $C = \frac{\varepsilon_0 A}{d}$

Charge on plate, $Q = CV = \frac{\varepsilon_0 A}{d} V_0$

Energy stored, $U = Q^2/2C$

As d is increased 3 times, so C decreases 3 times. Battery is disconnected, so Q remains same. The difference in the energy is the work done.

Change in potential energy

$$\Delta U = U_i - U_f$$

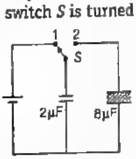
$$= \frac{Q^2}{2}\left[\frac{1}{C_i} - \frac{1}{C_f}\right] = \left(\frac{\varepsilon_0 A V_0}{d}\right)^2 \times \frac{1}{2}\left[\frac{1}{C} - \frac{1}{C/3}\right]$$

$$= -\frac{1}{2} \times \left(\frac{\varepsilon_0 A V_0}{d}\right)^2 \times \frac{2}{C} = \left(\frac{\varepsilon_0 A V_0}{d}\right) \times \frac{d}{\varepsilon_0 A V_0}$$

$$= -\frac{\varepsilon_0 A V_0}{d}$$

$\therefore$ Work done, $\Delta W = -\Delta U = \frac{\varepsilon_0 A V_0}{d}$

EXAMPLE |15| A $2\mu F$ capacitor is charged as shown in the figure. Find the percentage of its stored energy dissipated after the switch S is turned to position 2.



Sol. Initially, charge on the capacitor,

$$q_i = C_i V = 2V = q$$

This charge will remain constant after switch is shifted from position 1 to position 2.

$$U_i = \frac{1}{2}\frac{q^2}{C_i} = \frac{q^2}{2 \times 2} = \frac{q^2}{4}$$

$$U_f = \frac{1}{2}\frac{q^2}{C_f} = \frac{q^2}{2 \times 10} = \frac{q^2}{20}$$

$\therefore$ Energy dissipated $= U_i - U_f = \frac{q^2}{5}$

This energy dissipated $\left(= \frac{q^2}{5}\right)$ is 80% of the initial stored energy $\left(= \frac{q^2}{4}\right)$.

COMMON POTENTIAL

When two capacitors of different potentials are connected by a conducting wire, then charge flows from capacitor at higher potential to the capacitor at lower potential. This flow of charge continues till their potentials become equal, this equal potential is called **common potential.**

$$\boxed{\text{Common potential, } V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}}$$

where, C_1 and C_2 are capacities of two capacitors charged to potentials V_1 and V_2, respectively.

i.e. Common potential $= \dfrac{\text{Total charge}}{\text{Total capacitance}}$

$\therefore$ $C_1 V_1 + C_2 V_2 = C_1 V + C_2 V$

or $C_1 V_1 - C_1 V = C_2 V - C_2 V_2$

i.e. **Charge lost by one capacitor**

= Charge gained by the other capacitor

Note This is not true for potential, i.e. potential lost by one is not equal to potential gained by the other as their capacities are different.

Loss of Energy in Sharing Charges

When two charged capacitors are connected to each other, they share charges, till they acquire a common potential. On sharing charges, there is always some loss of energy. However, total charge of the system remains conserved. Consider two capacitors having capacitances C_1, C_2 and potentials V_1, V_2, respectively.

Then before the two capacitors are connected together, the total energy stored in the two capacitors,

$$U = U_1 + U_2 = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 \quad \text{...(i)}$$

When the two capacitors are connected together, total charge on the capacitor,

$$q = q_1 + q_2 = C_1 V_1 + C_2 V_2$$

[redacted]

Total capacitance of the two capacitors,

$$C = C_1 + C_2$$

Therefore, total energy of the two capacitors, after they are connected,

$$U' = \frac{1}{2}\cdot\frac{q^2}{C} = \frac{1}{2}\cdot\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)} \quad \text{...(ii)}$$

Subtracting Eq. (ii) from Eq. (i), we get

$$U - U' = \left(\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right) - \frac{1}{2}\cdot\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}$$

$$= \frac{\left[C_1^2V_1^2 + C_1C_2V_1^2 + C_1C_2V_2^2 + C_2^2V_2^2 - (C_1V_1 + C_2V_2)^2\right]}{2(C_1 + C_2)}$$

$$= \frac{C_1C_2(V_1^2 + V_2^2 - 2V_1V_2)}{2(C_1 + C_2)}$$

$$\Rightarrow \quad \boxed{\Delta U = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}} \text{ is a positive quantity.}$$

Since, $U - U'$ is positive, there is always a loss of energy, when two charged capacitors are connected together in the form of heat radiation due to electric current while charging.

EXAMPLE |16| A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in this process? **NCERT**

Sol. Given, $C_1 = C_2 = 600\,\text{pF} = 600 \times 10^{-12}\,\text{F}$

$$= 6 \times 10^{-10}\,\text{F}$$

$$V_1 = 200\,\text{V},\ V_2 = 0$$

$$\therefore \text{ Energy lost} = \frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)}$$

$$= \frac{(6\times10^{-10})^2(200 - 0)^2}{2\times12\times10^{-10}} = 6\times10^{-6}\,\text{J}$$

| TOPIC PRACTICE 2 |

OBJECTIVE Type Questions

1. The maximum electric field that a dielectric medium of a capacitor can withstand without break down (of its insulating property) is called its

(a) polarisation
(b) capacitance
(c) dielectric strength
(d) None of the above

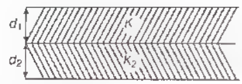
2. A parallel-plate capacitor has circular plates of radius 8 cm and plate separation 1 mm. What will be the charge on the plates if a potential difference of 100 V is applied?

(a) 1.78×10^{-8} C (b) 1.78×10^{-5} C
(c) 4.3×10^{4} C (d) 2×10^{-9} C

3. A parallel plate air capacitor has a capacitance 18 μF. If the distance between the plates is tripled and a dielectric medium is introduced, the capacitance becomes 72 μF. The dielectric constant of the medium is

(a) 4 (b) 9
(c) 12 (d) 2

4. A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant K_1 and the other has thickness d_2 and dielectric constant K_2 as shown in figure. This arrangement can be thought as a dielectric slab of thickness $d\,(= d_1 + d_2)$ and effective dielectric constant K. The K is

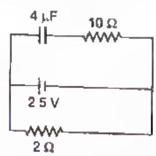


(a) $\dfrac{K_1d_1 + K_2d_2}{d_1 + d_2}$ (b) $\dfrac{K_1d_1 + K_2d_2}{K_1 + K_2}$
(c) $\dfrac{K_1K_2(d_1 + d_2)}{(K_1d_2 + K_2d_1)}$ (d) $\dfrac{2K_1K_2}{K_1 + K_2}$

5. The capacitance of a spherical conductor is 1 μF. Its radius is

(a) 1.11 m (b) 10 m
(c) 9 km (d) 1.11 cm

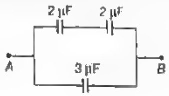
6. A capacitor of 4 μF is connected as shown in the circuit. The internal resistance of the battery is 0.5Ω. The amount of charge on the capacitor plates will be **NCERT Exemplar**



(a) 0 (b) 4 μC
(c) 16μC (d) 8μC

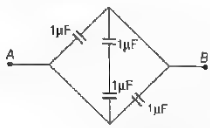
[redacted]

7. Capacitance between points A and B is



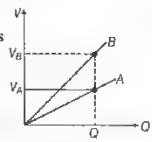
(a) 4 μF (b) $\frac{12}{7}$ μF (c) $\frac{1}{4}$ μF (d) $\frac{7}{12}$ μF

8. In the figure, the equivalent capacitance between points A and B is



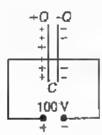
(a) 4 μF (b) 2.5 μF
(c) 2 μF (d) 0.25 μF

9. The graph shows the variation of voltage V across the plates of two capacitors A and B *versus* increase of charge Q stored in them. Which of the capacitors has higher capacitance?



(a) Capacitor A (b) Capacitor B
(c) Both (a) and (b) (d) None of these

10. A 900 pF capacitor is charged by 100 V battery in the figure. How much electrostatic energy is stored by the capacitor?



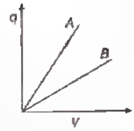
(a) 45×10^{-6} J (b) 4.5×10^{6} J
(c) 4.5×10^{-6} J (d) 0.45×10^{5} J

VERY SHORT ANSWER Type Questions

11. Distinguish between a dielectric and a conductor. **Delhi 2012**

12. Define the dielectric constant of a medium. What is its unit? **Delhi 2011**

13. The given graph shows the variation of charge q *versus* potential difference V for two capacitors C_1 and C_2. Both the capacitors have same plate separation but plate area of C_2 is greater than that C_1. Which line (A or B) corresponds to C_1 and why? **All India 2014**



14. If the difference between the radii of the two spheres of a spherical conductor is increased, state whether the capacitance will increase or decrease.

15. A metal plate is introduced between the plates of a charged parallel plate capacitor. What is its effect on the capacitance of the capacitor? **Foreign 2009**

16. A spherical shell of radius b with charge Q is expanded to a radius a. Find the work done by the electrical forces in the process.

17. Distinguish between polar and non-polar dielectrics. **All India 2010 C**

18. A sensitive instrument is to be shifted from the strong electrostatic field in its environment. Suggest a possible way.

19. The safest way to protect yourself from lightning is to be inside a car. Comment. **Delhi 2009**

20. Can the potential function have a maximum or minimum in free space? **NCERT Exemplar**

21. Why does the electric conductivity of the earth's atmosphere increase with altitude?

SHORT ANSWER Type Questions

22. A capacitor has some dielectric between its plates and the capacitor is connected to a DC source. The battery is now disconnected and then the dielectric is removed.

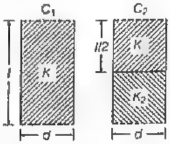
State whether the capacitance, the energy stored in it, electric field, charge stored and the voltage will increase, decrease or remain constant. **All India 2013**

23. A slab of material of dielectric constant K has the same area as that of the plates of a parallel plate capacitor, but has the thickness $d/2$, where d is the separation between the plates.

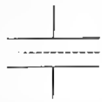
[redacted]

Find out the expression for its capacitance when the slab is inserted between the plates of the capacitor. **Delhi 2013**

24. Two identical parallel plate (air) capacitors C_1 and C_2 have capacitance C each. The space between their plates is now filled with dielectrics as shown in the figure. If the two capacitors still have equal capacitance, then obtain the relation between dielectric constants K, K_1 and K_2. **Foreign 2011**

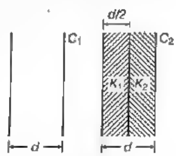


25. Figure shows a sheet of aluminium foil of negligible thickness placed between the plates of a capacitor. How will its capacitance be affected, if



(i) the foil is electrically insulated?
(ii) the foil is connected to the upper plate with a conducting wire? **Foreign 2011**

26. You are given an air filled parallel plate capacitor C_1. The space between its plates is now filled with slabs of dielectric constants K_1 and K_2 as shown in figure. Find the capacitance of the capacitor C_2 if area of the plates is A and distance between the plates is d. **Foreign 2011**

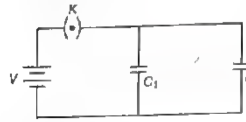


27. A parallel plate capacitor of capacitance C is charged to a potential V. It is then connected to another uncharged capacitor having the same capacitance. Find out the ratio of the energy stored in the combined system to that stored initially in the single capacitor. **All India 2014**

28. A parallel plate capacitor, each of plate area A and separation d between the two plates, is charged with charges $+Q$ and $-Q$ on the two plates. Deduce the expression for the energy stored in capacitor. **Foreign 2013**

29. Two parallel plate capacitors of capacitances C_1 and C_2 such that $C_1 = 2C_2$ are connected across a battery of V volt as shown in the figure. Initially, the key (k) is kept closed to fully charge the capacitors. The key is now thrown open and a dielectric slab of dielectric constant K is inserted in the two capacitors to completely fill the gap between the plates. Find the ratio of

(i) the net capacitance and
(ii) the energies stored in the combination before and after the introduction of the dielectric slab. **Delhi 2014C**



30. Deduce the expression for the electrostatic energy stored in a capacitor of capacitance C and having charge Q.
How will the
(i) energy stored and
(ii) the electric field inside the capacitor be affected when it is completely filled with a dielectric material of dielectric constant K? **All India 2012**

31. Guess a possible reason, why water has a much greater dielectric constant (~ 80) than mica (~ 6)?

32. A 2 m insulating slab with a large aluminium sheet of area 1 m^2 on its top is fixed by a man outside his house one evening. Will he get an electric shock, if he touches the metal sheet next morning?

33. A technician has only two capacitors. By using them in series or in parallel, he is able to obtain the capacitance of 4, 5, 20 and 25 μF. What is the capacitance of both capacitors?

LONG ANSWER Type I Questions

34. (i) How is the electric field due to a charged parallel plate capacitor affected when a dielectric slab is inserted between the plates fully occupying the intervening region?
(ii) A slab of material of dielectric constant K has the same area as the plates of a parallel plate capacitor but has thickness $\frac{1}{2}d$, where d is the separation between the plates. Find the expression for the capacitance when the slab is inserted between the plates. **Foreign 2010**

[redacted]

35. Two charged conducting spheres of radii a and b are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain, why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions? NCERT

36. Find the ratio of the potential differences that must be applied across the parallel and series combination of two capacitors C_1 and C_2 with their capacitances in the ratio 1 : 2, so that the energy stored in these two cases becomes the same. All India 2016

37. (i) Obtain the expression for the energy stored per unit volume in a charged parallel plate capacitor.

(ii) The electric field inside a parallel plate capacitor is E. Find the amount of work done in moving a charge q over a closed rectangular loop $abcda$. Delhi 2014

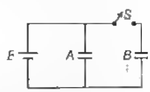
38. A parallel plate capacitor of capacitance C is charged to a potential V by a battery. Without disconnecting the battery, the distance between the plates is tripled and a dielectric medium of $K = 10$ is introduced between the plates of the capacitor. Explain giving reasons, how will the following be affected All India 2017

(i) capacitance of the capacitor

(ii) charge on the capacitor and

(iii) energy density of the capacitor?

39. Two identical parallel plate capacitors A and B are connected to a battery of V volts with the switch S is closed. The switch is now opened and the free space between the plates of the capacitors is filled with a dielectric of dielectric constant K. Find the ratio of the total electrostatic energy stored in both capacitors before and after the introduction of the dielectric.



All India 2017

40. (i) Derive the expression for the capacitance of a parallel plate capacitor having plate area A and plate separation d.

(ii) Two charged spherical conductors of radii R_1 and R_2 when connected by a conducting plate respectively. Find the ratio of their surface charge densities in terms of their radii. Delhi 2014

41. Show that the force on each plate of a parallel plate capacitor has a magnitude equal to $1/2QE$, where Q is the charge on the capacitor and E is the magnitude of electric field between the plates. Explain the origin of the factor 1/2. NCERT

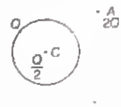
LONG ANSWER Type II Questions

42. (i) Explain, using suitable diagram, the difference in the behaviour of a

(a) conductor and

(b) dielectric in the presence of external electric field. Define the terms polarisation of a dielectric and write its relation with susceptibility.

(ii) A thin metallic spherical shell of radius R carries a charge Q on its surface. A point charge $Q/2$ is placed at its centre C and an another charge $+2Q$ is placed outside the shell at a distance x from the centre as shown in figure. Find (a) the force on the charge at the centre of the shell and at point A, (b) the electric flux through the shell.



All India 2015

43. (i) If two similar large plates, each of area A having surface charge densities $+\sigma$ and $-\sigma$ are separated by a distance d in air, find the expression for

(a) field at points between the two plates and on outer side of the plates. Specify the direction of the field in each case.

(b) the potential difference between the plates.

(c) the capacitance of the capacitor so formed.

(ii) Two metallic spheres of radii R and $2R$ are charged, so that both of these have same surface charge density σ. If they are connected to each other with a conducting wire, in which direction will the charge flow and why? All India 2016

44. (i) Derive the expression for the energy stored in parallel plate capacitor. Hence, obtain the expression for the energy density of the electric field.

(ii) A fully charged parallel plate capacitor is connected across an uncharged identical capacitor. Show that the energy stored in the combination is less than stored initially in the single capacitor. Delhi 2015

[redacted]

NUMERICAL PROBLEMS

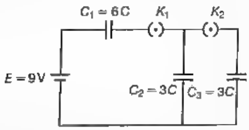
45. A capacitor of unknown capacitance is connected across a battery of V volt. The charge stored in it is $360\mu C$. When potential across the capacitor is reduced by 120 V, the charge stored in it becomes $120\mu C$.

(i) Calculate the potential V and the unknown capacitance C.

(ii) What will be the charge stored in the capacitor, if the voltage applied had increased by 120 V? **Delhi 2013**

46. In the circuit shown below, initially K_1 is closed and K_2 is opened, what are the charges on each of the capacitors? Then, K_1 was opened and K_2 was closed (order is important), what will be the charge on each capacitor now?

[Given, $C = 1\mu F$] **NCERT Exemplar**



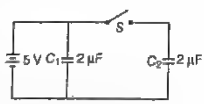
47. A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of $2.5\mu C$. The space between the concentric spheres is filled with a liquid of dielectric constant 32.

(i) Determine the capacitance of the capacitor.

(ii) What is the potential of the inner sphere?

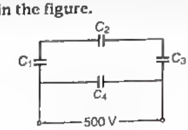
(iii) Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain, why the later is much smaller. **NCERT**

48. Figure shows two identical capacitors C_1 and C_2, each of $2\mu F$ capacitance, connected to a battery of 5 V.



Initially switch S is closed. After sometime, S is left open and dielectric slabs of dielectric constant $K = 5$ are inserted to fill completely the space between the plates of the two capacitors. How will the (i) charge and (ii) potential difference between the plates of the capacitors be affected after the slabs are inserted? **Delhi 2011**

49. A network of four capacitors each of $12\mu F$ capacitance is connected to a 500 V supply as shown in the figure.



Determine

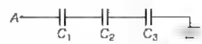
(i) the equivalent capacitance of the network and

(ii) the charge on each capacitor. **All India 2012, 2010**

50. Net capacitance of three identical capacitors in series is $1\mu F$. What will be their net capacitance, if connected in parallel?

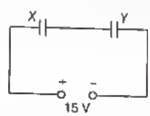
Find the ratio of energy stored in these two configurations, if they are both connected to the same source. **All India 2011**

51. Calculate the potential difference and the energy stored in the capacitor C_2 in the circuit shown in the figure. Given, potential at A is 90 V, $C_1 = 20\mu F$, $C_2 = 30\mu F$, $C_3 = 15\mu F$. **Delhi 2015**



52. A 12 pF capacitor is connected to a 50 V battery. How much electrostatic energy is stored in the capacitor? If another capacitor of 6 pF is connected in series with it with the same battery connected across the combination, find the charge stored and potential difference across each capacitor. **Delhi 2017**

53. Two parallel plate capacitors X and Y have the same area of plates and same separation between them, X has air between the plates while Y contains a dielectric medium of $\varepsilon_r = 4$.



(i) Calculate the capacitance of each capacitor, if equivalent capacitance of the combination is $4\mu F$.

(ii) Calculate the potential difference between the plates of X and Y.

(iii) Estimate the ratio of electrostatic energy stored in X and Y. **Delhi 2016**

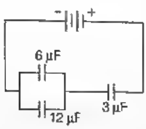
[redacted]

54. Two capacitors of unknown capacitances C_1 and C_2 are connected first in series and then in parallel across a battery of 100 V. If the energy stored in the two combinations is 0.045 J and 0.25 J respectively, then determine the value of C_1 and C_2. Also, calculate the charge on each capacitor in parallel combination. **All India 2015**

55. In the following arrangement of capacitors, the energy stored in the $6\,\mu F$ capacitor is E. Find the value of the following

(i) energy stored in 12μ F capacitor.
(ii) energy stored in 3μ F capacitor.
(iii) total energy drawn from the battery.

Foreign 2016



56. A capacitor of 200 pF is charged by a 300 V battery. The battery is then disconnected and the charged capacitor is connected to another uncharged capacitor of 100 pF. Calculate the difference between the final energy stored in the combined system and the initial energy stored in the single capacitor. **Foreign 2012**

HINTS AND SOLUTIONS

1. (c) The maximum electric field that a dielectric medium can withstand without break down (of its insulating property) is called its dielectric strength; for air it is about $3\times 10^6\ \text{Vm}^{-1}$.

2. (a) $C = \dfrac{\varepsilon_0 A}{d} = \dfrac{8.85\times 10^{-12}\times 3.14\times 0.08\times 0.08}{1\times 10^{-3}}$

$$q = CV = \frac{8.85\times 10^{-12}\times 3.14\times .08\times .08\times 100\ \text{V}}{1\times 10^{-3}}$$

$= 1.78\times 10^{-8}$ C

3. (c) $C_0 = \dfrac{\varepsilon_0 A}{d} = 18$ ...(i)

$C = \dfrac{K\varepsilon_0 A}{3d} = 72$ ...(ii)

On dividing Eq. (ii) by Eq. (i), we get

$$\frac{K}{3} = \frac{72}{18} = 4$$

$\therefore$ Dielectric constant, $K = 12$

4. (c) The capacitance of parallel plate capacitor filled with dielectric block has thickness d_1 and dielectric constant K_1 is given by

$$C_1 = \frac{K_1\varepsilon_0 A}{d_1}$$

Similarly, capacitance of parallel plate capacitor filled with dielectric block has thickness d_2 and dielectric constant K_2 is given by

$$C_2 = \frac{K_2\varepsilon_0 A}{d_2}$$

Since, the two capacitors are in series combination, the equivalent capacitance is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

or $C = \dfrac{C_1C_2}{C_1 + C_2} = \dfrac{\dfrac{K_1\varepsilon_0 A}{d_1}\ \dfrac{K_2\varepsilon_0 A}{d_2}}{\dfrac{K_1\varepsilon_0 A}{d_1} + \dfrac{K_2\varepsilon_0 A}{d_2}} = \dfrac{K_1K_2\varepsilon_0 A}{K_1 d_2 + K_2 d_1}$ ...(i)

But the equivalent capacitances is given by

$$C = \frac{K\varepsilon_0 A}{d_1 + d_2}$$

On comparing, we have, $K = \dfrac{K_1K_2(d_1 + d_2)}{K_1 d_2 + K_2 d_1}$

5. (c) Capacitance of spherical conductor, $C = 4\pi\varepsilon_0 \cdot R$

$\therefore$ Radius of conductor, $R = \dfrac{C}{4\pi\varepsilon_0} \Rightarrow C = 1\,\mu\text{F} = 1\times 10^{-6}$ F

and $\dfrac{1}{4\pi\varepsilon_0} = 9\times 10^9\ \text{N-m/C}^2$

$\therefore$ $R = 1\times 10^{-6}\times 9\times 10^9$

$\Rightarrow$ $R = 9\times 10^3\text{m} = 9$ km

6. (d) Current flows through 2Ω resistance from left to right, is given by

$$I = \frac{V}{R + r} = \frac{2.5\text{V}}{2 + 0.5} = 1\text{A}$$

The potential difference across 2Ω resistance

$$V = IR = 1\times 2 = 2\text{V}$$

Since, capacitor is in parallel with 2Ω resistance, so it also has 2V potential difference across it.

The charge on capacitor

$$q = CV = (4\,\mu\text{F})\times 2\,\text{V} = 8\,\mu\text{C}$$

Note The potential difference across $2\,\Omega$ resistance solely occurs across capacitor as no potential drop occurs across 10Ω resistance.

7. (a) Two capacitors of $2\,\mu$F capacitance are connected in series order.

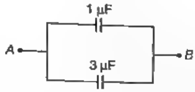
Their equivalent capacitance,

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1$$

[redacted]

$\therefore \quad C_S = 1\,\mu F$

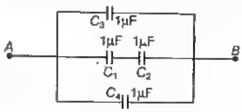
Now, $C_S = 1\,\mu F$ and $3\,\mu F$ capacitors are connected in parallel order.



Equivalent capacitance between points A and B,

$$C_{AB} = C_S + C_3 = 1 + 3 - 4\ \mu F$$

8. (b) On redrawing, the circuit is



According to the circuit, C_1 and C_2 are in series,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1} + \frac{1}{1} = 2 \Rightarrow C - \frac{1}{2}\mu F$$

Now, C, C_3 and C_4 are in parallel order.

$$\therefore \quad C_{equivalent} = C + C_3 + C_4 = \frac{1}{2} + 1 + 1 = 2.5\,\mu F$$

9. (a) From the given graphs, find the voltages, V_A and V_B, on capacitors A and B corresponding to charge Q on each of the capacitors. Clearly,

$$V_A = \frac{Q}{C_A} \quad \text{and} \quad V_B = \frac{Q}{C_B}$$

or $$\frac{V_B}{V_A} = \frac{Q/C_B}{Q/C_A} = \frac{C_A}{C_B}$$

Since, $V_B > V_A$, $C_A > C_B$ *i.e.*, the capacitor A has the higher capacitance.

10. (c) The charge on the capacitor is

$$q = CV = 900 \times 10^{-12}\ F \times 100\ V = 9 \times 10^{-8}\ C$$

The energy stored by the capacitor is

$$= (1/2)\,CV^2 = (1/2)\,qV$$

$$= 1/2 \times 9 \times 10^{-8} C \times 100\ V = 4.5 \times 10^{-6}\ J$$

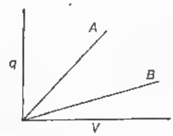
11. Dielectrics are non-conductors and do not have free electrons at all. While conductor has free electrons which makes it able to pass the electricity through it.

12. When a dielectric slab is introduced between the plates of charged capacitor or in the region of electric field, an electric field E_p induces inside the dielectric due to induced charge on dielectric in a direction opposite to the direction of applied external electric field. Hence, net electric field inside the dielectric get reduced to $E_0 - E_p$, where E_0 is external electric field.

The ratio of applied external electric field and reduced electric field is known as dielectric constant K of dielectric medium,

i.e. $K = \dfrac{E_0}{E_0 - E_p}$ and it is a dimensionless quantity

13. Line B corresponds to C_1 because slope (q/V) of B is less than slope of A.



14. Capacitance of a spherical capacitor, $C = \dfrac{4\pi\varepsilon_0 K r_1 r_2}{r_1 - r_2}$

$\Rightarrow C \propto \dfrac{1}{r_1 - r_2}$. If $r_1 - r_2$ is increased, C decreases.

15. If a metal plate is introduced between the plates of a charged parallel plate capacitor, then capacitance of parallel plate capacitor will become infinite.

16. Work done by electrical forces in the process

= Final stored energy − Initial stored energy

$$= \frac{Q^2}{2C_2} - \frac{Q^2}{2C_1} = \frac{Q^2}{2(4\pi\varepsilon_0 a)} - \frac{1}{2}\cdot\frac{Q^2}{(4\pi\varepsilon_0 b)} = \frac{Q^2}{8\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right)$$

17. **Polar dielectrics**

A polar dielectric has permanent electric dipole moment (p) in absence of electric field

Non-polar dielectrics

A non polar dielectric having zero dipole moment in its normal state

18. For this, the instrument must be enclosed fully in a metallic cover. This will provide an electrostatic shielding to the instrument.

19. The body of the car is metallic. It provides electrostatic shielding to the person in the car, because electric field inside the car is zero. The discharging due to lightning passes to the ground through the metallic body of the car.

20. No, the absence of atmosphere around conductor prevents the phenomenon of electric discharge or potential leakage and hence, potential function do not have a maximum or minimum in free space.

21. This is because of ionisation caused by highly energetic cosmic ray particles from cosmos, which are hitting the atmosphere of the earth.

22. The capacitance of the parallel plate capacitor, filled with dielectric medium of dielectric constant K is given by, $C = \dfrac{K\varepsilon_0 A}{d}$.

[redacted]

The capacitance of the parallel plate capacitor decreases with the removal of dielectric medium as for air or vacuum $K = 1$. After disconnection from battery, charge stored will remain the same due to conservation of charge. The energy stored in an isolated charge capacitor $= \frac{q^2}{2C}$.

As q is constant, energy stored $\propto 1/C$. C decrease with the removal of dielectric medium, therefore energy stored increases. Since, q is constant and $V = q/C$ and C decreases which in turn increases V and therefore E increases $E = V/d$.

23. Initially, when there is a vacuum between two plates, the capacitance of the plate is $C_0 = \frac{\varepsilon_0 A}{d}$, where A is the area of parallel plates.

Suppose that the capacitor is connected to a battery, an electric field E_0 is produced. Now, if we insert the dielectric slab of thickness $t = d/2$, the electric field reduces to E.

Now, the gap between plates is divided in two parts, for distance t, there is electric field E and for the remaining distance $(d - t)$ the electric field is E_0.

If V be the potential difference between the plates of the capacitor, then $V = Et + E_0(d - t)$

$$V = \frac{Ed}{2} + \frac{E_0 d}{2} = \frac{d}{2}(E + E_0) \qquad \left[\because t = \frac{d}{2}\right]$$

$$\Rightarrow \quad V = \frac{d}{2}\left(\frac{E_0}{K} + E_0\right) = \frac{dE_0}{2K}(K+1) \qquad \left[\text{as, } \frac{E_0}{E} = K\right]$$

Now, $\quad E_0 = \frac{\sigma}{\varepsilon_0} = \frac{q}{\varepsilon_0 A} \Rightarrow V = \frac{d}{2K} \cdot \frac{q}{\varepsilon_0 A}(K+1)$

We know that, $C = \frac{q}{V} = \frac{2K\varepsilon_0 A}{d(K+1)}$

24. After inserting the dielectric medium, let their capacitances become C_1' and C_2'.

For C_1 $\quad C_1' = KC \quad$ ...(i)

For C_2 $\quad C_2' = \frac{K_1 \varepsilon_0 (A/2)}{d} + \frac{K_2 \varepsilon_0 (A/2)}{d}$

C_2 acts as if two capacitors each of area $A/2$ and separation d are connected in parallel combination

$$C_2' = \frac{\varepsilon_0 A}{d}\left(\frac{K_1}{2} + \frac{K_2}{2}\right)$$

$$C_2' = C\left(\frac{K_1 + K_2}{2}\right) \left[\because C = \frac{\varepsilon_0 A}{d}\right] \qquad \text{...(ii)}$$

According to the problem, $C_1' = C_2'$

$$\Rightarrow \quad KC = C\left(\frac{K_1 + K_2}{2}\right)$$

$$\Rightarrow \quad K = \frac{K_1 + K_2}{2}$$

25. (i) The system will be equivalent to two identical capacitors connected in series combination in which two plates of each capacitor have separation half of the original separation.

Thus, new capacitance of each capacitor

$$C' = 2C \qquad \left[\because C \propto \frac{1}{d}\right]$$

$\because$ C and C' are in series.

$$\Rightarrow \quad C_{net} = \frac{2C \times 2C}{2C + 2C} = C$$

$$C_{net} = C \qquad \text{[original capacitor]}$$

(ii) System reduces to a capacitor whose separation reduces to half of original one.

$\therefore$ New capacitance, $C' = 2C$

26. After introducing the dielectric medium of dielectric constants K_1 and K_2, capacitor acts as if it consists of two capacitors, each having plates of area A and separation $\frac{d}{2}$ connected in series combination for

$$C_1 = \frac{\varepsilon_0 A}{d} \qquad \text{...(i)}$$

$$\Rightarrow \quad \frac{1}{C_2} = \frac{1}{\left(\frac{K_1 \varepsilon_0 A}{d/2}\right)} + \frac{1}{\left(\frac{K_2 \varepsilon_0 A}{d/2}\right)}$$

$$\Rightarrow \quad \frac{1}{C_2} = \frac{1}{\left(\frac{\varepsilon_0 A}{d}\right)}\left(\frac{1}{2K_1} + \frac{1}{2K_2}\right)$$

$$\Rightarrow \quad \frac{1}{C_2} = \frac{1}{2C_1}\left(\frac{K_2 + K_1}{K_1 K_2}\right)$$

$$\Rightarrow \quad C_2 = C_1\left(\frac{2K_1 K_2}{K_1 + K_2}\right)$$

The capacitors will be in series.

27. Let q be the charge on the charged capacitor.

$\therefore$ Energy stored in it is given by $U = \frac{q^2}{2C}$

When another uncharged similar capacitor is connected, then the net capacitance of the system is given by

$$C' = 2C$$

The charge on the system remains constant. So, the energy stored in the system is given by

$$U' = \frac{q^2}{2C'} = \frac{q^2}{4C} \qquad [\because C' = 2C]$$

Thus, the required ratio is given by $\frac{U'}{U} = \frac{q^2/4C}{q^2/2C} = \frac{1}{2}$

28. Refer to text on page 92.

29. (i) Given, $\quad C_1 = 2C_2 \quad$ ...(i)

Net capacitance before filling the gap with dielectric slab is given by

$$C_{initial} = C_1 + C_2 \qquad \text{[from Eq. (i)]}$$

$$C_{initial} = 2C_2 + C_2 = 3C_2 \qquad \text{...(ii)}$$

[redacted]

Net capacitance after filling the gap with dielectric slab of electric constant K

$C_{\text{initial}} = KC_1 + KC_2 = K(C_1 + C_2)$ [from Eq. (ii)]

$C_{\text{final}} = 3KC_1$ ...(iii)

Ratio of net capacitance is given by

$$\frac{C_{\text{initial}}}{C_{\text{final}}} = \frac{3C_2}{3KC_2} = \frac{1}{K} \quad \text{[from Eqs. (ii) and (iii)]}$$

(ii) Energy stored in the combination before introducing the dielectric slab,

$$U_{\text{initial}} = \frac{Q^2}{3C_2} \quad \text{...(iv)}$$

Energy stored in the combination after introducing the dielectric slab,

$$U_{\text{final}} = \frac{Q^2}{3KC_2} \quad \text{... (v)}$$

Ratio of energies stored

$$\frac{U_{\text{initial}}}{U_{\text{final}}} = \frac{K}{1} \quad \text{[from Eqs. (iv) and (v)]}$$

30. (i) Refer to text on page 92.
 (ii) Refer to text on pages 88 and 92.

31. Dielectric constant of water is much greater than that of mica because of the following reasons
 (i) water molecules have a symmetrical shape as compared to mica
 (ii) water molecules have permanent dipole moment.

32. Yes, the man will get an electric shock, if he touches the metal slab next morning because the steady discharging current in the atmosphere charges up the aluminium sheet. As a result, its voltage rises gradually. The rise in voltage depends on the capacitance of the capacitor formed by aluminium slab and ground.

33. Let the two capacitors be C_1 and C_2, capacitance will be maximum when connected in parallel.

i.e. $C_1 + C_2 = 25$

Capacitance will be minimum when connected in series.

i.e. $$\frac{C_1C_2}{C_1 + C_2} = 4$$

Since, we are left with only two values $5\,\mu F$ and $20\,\mu F$.
So, the value of capacitances will be $5\,\mu F$ and $20\,\mu F$.

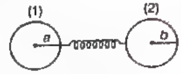
34. (i) Refer to text on page 88.
 (ii) The thickness of dielectric slab is $\frac{d}{2}$, i.e.

$$t = \frac{d}{2}$$

The capacitance of a capacitor due to dielectric slab is

$$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{k}} = \frac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2k}} = \frac{2\varepsilon_0 A}{d\left(1 + \frac{1}{k}\right)}$$

35. As the two conducting spheres are connected to each other by a wire, the charge always flows from higher potential to lower potential till both have same potential.



Capacitance of sphere (1), $C_1 = 4\pi\varepsilon_0 a$

Capacitance of sphere (2), $C_2 = 4\pi\varepsilon_0 b$

Then, Charge Q_1 on C_1, $Q_1 = C_1V$ ...(i)

and Charge Q_2 on C_2, $Q_2 = C_2V$ ...(ii)

where, V is the same potential on both the spheres

$$\therefore \quad \frac{Q_1}{Q_2} = \frac{C_1}{C_2} \quad \text{[from Eqs. (i) and (ii)]}$$

Putting the values of C_1 and C_2, we get

$$\frac{Q_1}{Q_2} = \frac{4\pi\varepsilon_0 a}{4\pi\varepsilon_0 b} = \frac{a}{b} \Rightarrow \frac{Q_1}{Q_2} = \frac{a}{b} \quad \text{...(iii)}$$

Charge density on sphere (1), $\sigma_1 = \frac{\text{Charge}}{\text{Surface area}} = \frac{Q_1}{4\pi a^2}$

Charge density on sphere (2), $\sigma_2 = \frac{Q_2}{4\pi b^2}$

$$\therefore \quad \frac{\sigma_1}{\sigma_2} = \frac{b^2}{a^2} \cdot \frac{Q_1}{Q_2} = \frac{b^2}{a^2} \cdot \frac{a}{b} \quad \text{[from Eq. (iii)]}$$

$$\text{or,} \quad \frac{\sigma_1}{\sigma_2} = \frac{b}{a} \quad \text{...(iv)}$$

The ratio of electric field on both spheres.

$$\frac{E_1}{E_2} = \frac{\sigma_1}{\sigma_2} = \frac{b}{a} \quad \text{[from Eq. (iv)]}$$

As, charge density is inversely proportional to radius. Thus, for flatter portions, the radius is more and at pointed ends, radius is less, so the charge density is more at pointed or sharp ends.

36. Total energy stored in series or parallel combination of capacitors is equal to the sum of energies stored in individual capacitors. In parallel combination, energy stored in the capacitor

$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_1^2 \quad \text{...(i)}$$

In series combination, energy stored in the capacitor

$$= \frac{1}{2}\frac{C_1C_2}{(C_1 + C_2)}V_2^2 \quad \text{...(ii)}$$

According to the question, energy in both the cases is same so,

$$\left(\frac{1}{2}C_1 + \frac{1}{2}C_2\right)V_1^2 = \frac{C_1C_2}{2(C_1 + C_2)}V_2^2$$

$$\Rightarrow \quad \frac{V_1^2}{V_2^2} = \frac{C_1C_2 \times 2}{2(C_1 + C_2)(C_1 + C_2)}$$

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{\sqrt{C_1C_2}}{C_1 + C_2}$$

[redacted]

But $\quad \dfrac{C_1}{C_2} = \dfrac{1}{2}$

$\Rightarrow \quad C_2 = 2C_1$

So, $\quad \dfrac{V_1}{V_2} = \dfrac{\sqrt{C_1 \times 2C_1}}{C_1 + 2C_1} = \dfrac{\sqrt{2}C_1}{3C_1} = \dfrac{\sqrt{2}}{3}$

37. (i) Refer to text on page 92.

(ii) Due to conservative nature of electric force, the work done in moving a charge in a close path in a uniform electric field is zero.

38. On introducing the dielectric slab to fill the gap between plates of capacitor completely when capacitor is connected with battery.

(i) The capacitance of capacitor becomes K times of original capacitor.

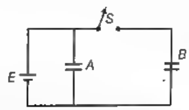
$$C' = KC = 10C$$

(ii) The potential difference V between capacitors is same due to connectivity with battery and hence, charge q' becomes K times of original charge as

$$q' = C'V' = (KC)(V) = K(CV) = Kq = 10CV$$

(iii) Refer to text on page 92.

39. The given figure is shown below.



When switch S is closed, the potential difference across capacitors A and B are same

i.e. $\quad V = \dfrac{Q_A}{C} = \dfrac{Q_B}{C}$

Initial charges on capacitors

$$Q_A = Q_B = CV$$

When the dielectric is introduced, the new capacitance of either capacitor

$$C' = KC$$

As switch S is opened, the potential difference across capacitor A remains same (V volts).

Let potential difference across capacitor B be V'. When dielectric is introduced with switch S open (i.e. battery disconnected), the charges on capacitor B remains unchanged, so

$$Q_B = CV = C'V'$$

$\Rightarrow \quad V' = \dfrac{C}{C'}V = \dfrac{V}{K}$ volt

Initial energy of both capacitors

$$U_i = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2$$

Final energy of both capacitors

$$U_f = \frac{1}{2}C'V^2 + \frac{1}{2}C'V'^2$$

$$= \frac{1}{2}(KC)V^2 + \frac{1}{2}(KC)\left(\frac{V}{K}\right)^2$$

$$= \frac{1}{2}CV^2\left[K + \frac{1}{K}\right]$$

$$= \frac{1}{2}CV^2\left(\frac{K^2+1}{K}\right)$$

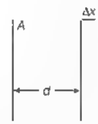
$\Rightarrow \quad \dfrac{U_i}{U_f} = \dfrac{CV^2}{\frac{1}{2}CV^2\left(\frac{K^2+1}{K}\right)} = \dfrac{2K}{K^2+1}$

40. (i) Refer to text on page 87

(ii) $\dfrac{\sigma_1}{\sigma_2} = \dfrac{R_2}{R_1}$;

Here, we can use the concept that the work done in displacing the plates against the force is equal to the increase in energy of the capacitor

41. Let the distance between the plates be increased by a very small distance Δx. The force on each plate is F. The amount of work done in increasing the separation by Δx, i.e.



$W = F \cdot \Delta x$ ...(i)

Increase in volume of capacitor

$= $ Area of plates $\times$ Increased distance

$= A \cdot \Delta x$

$u = $ Energy density $= \dfrac{\text{Energy}}{\text{Volume}}$

Increase in energy $= u \times$ volume $= u \cdot A \cdot \Delta x$ ...(ii)

As, energy = work done (W)

$\Rightarrow \quad F \cdot \Delta x = u \cdot A \cdot \Delta x$ [from Eqs. (i) and (ii)]

$\Rightarrow \quad F = u \cdot A$

$= \dfrac{1}{2}\varepsilon_0 E^2 \cdot A \quad \left[\because u = \dfrac{1}{2}\varepsilon_0 E^2 \text{ and } E = \dfrac{V}{d}\right]$

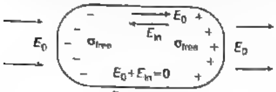
$= \dfrac{1}{2}\varepsilon_0 \cdot \dfrac{V^2}{d^2} \cdot A = \left(\dfrac{\varepsilon_0 A}{d} \cdot V\right)\dfrac{V}{d} \times \dfrac{1}{2}$

$= \dfrac{1}{2}\ E\ C\ V = \dfrac{1}{2}QE \quad \left[\because C = \dfrac{\varepsilon_0 A}{d}, CV = Q\right]$

42. (i) (a) When a capacitor is placed in an external electric field, the free charges present inside the conductor redistribute themselves in such a manner that the electric field due to induced charges opposes the external field within the conductor. This happens until a static situation is achieved, i.e. when the

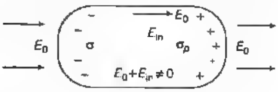
[redacted]

two fields cancel each other and the net electrostatic field in the conductor becomes zero.



Conductor

(b) In contrast to conductors, dielectrics are non-conducting substances, i.e. they have no charge carriers. Thus, in a dielectric, free movement of charges in not possible. It turns out that the external field induces dipole moment by stretching molecules of the dielectric. The collective effect of all the molecular dipole moments is the net charge on the surface of the dielectric which produces a field that opposes the external field. However, the opposing field is so induced, that does not exactly cancel the external field. It only reduces it. The extent of the effect depends on the nature of dielectric.



Dielectric

Both polar and non-polar dielectrics develop net dipole moment in the presence of an external field. The dipole moment per unit volume is called polarisation and is denoted by P for linear isotropic dielectrics

$$P = \chi E$$

where, χ is constant of proportionality and is called electric susceptibility of the electric slab.

(ii) (a) At point C, inside the shell, electric field inside a spherical shell is zero.

Thus, the force experienced by charge at centre C will also be zero.

$\because \quad F_C = qE \; (E_{inside}$ the shell $= 0)$

$\therefore \quad F_C = 0$

At point A, $|F_A| = 2Q\left[\dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{3Q/2}{x^2}\right]$

$$F = \frac{3Q^2}{4\pi\varepsilon_0 x^2}, \text{ away from shell.}$$

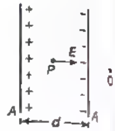
(b) Electric flux through the shell,

$\phi = \dfrac{1}{\varepsilon_0} \times$ magnitude of charge enclosed by shell

$= \dfrac{1}{\varepsilon_0} \times \dfrac{Q}{2} = \dfrac{Q}{2\varepsilon_0}$

43. (i) According to the question,

(a) Electric field due to a plate of positive charge at point



$P = \dfrac{\sigma}{2\varepsilon_0}$

Electric field due to other plate $= \dfrac{\sigma}{2\varepsilon_0}$

Since, they have same direction, so

$$E_{net} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

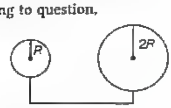
Outside the plate, electric field will be zero because of opposite direction.

(b) Potential difference between the plates is given by $\quad V = Ed = \dfrac{\sigma d}{\varepsilon_0} \qquad \left[\because E = \dfrac{\sigma}{\varepsilon_0}\right]$

(c) Capacitance of the capacitor is given by

$$C = \frac{Q}{V} = \frac{\sigma A}{\sigma d}\varepsilon_0 = \frac{\varepsilon_0 A}{d}$$

(ii) According to question,



Potential at the surface of radius R,

$$V = \frac{kq}{R} \qquad [\because q = \sigma \times 4R^2]$$

$$= \frac{k\sigma 4\pi R^2}{R} = \sigma k 4\pi R = 4k\sigma\pi R$$

Potential at the surface of radius $2R$,

$$V' = \frac{kq}{2R} \qquad [\because q = \sigma \times 4\pi(2R)^2 = 16\sigma\pi R^2]$$

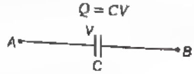
So, $\quad V' = \dfrac{k\sigma 16\pi R^2}{2R} = 8k\sigma\pi R$

Since, the potential of bigger sphere is more. So, charge will flow from sphere of radius $2R$ to sphere of radius R.

44. (i) Refer to text on page 92.

(ii) Initially, if we consider a charged capacitor, then its charge would be

$$Q = CV$$

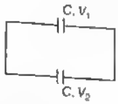


and energy stored, $\quad U_1 = \dfrac{1}{2}CV^2 \quad$ ...(i)

Then, this charged capacitor is connected to uncharged capacitor.

Let the common potential be V_1. The charge flows from first capacitor to the other capacitor unless both the capacitors attain common potential

[redacted]



$Q_1 = CV_1$ and $Q = CV_2$

Applying conservation of charge, $Q = Q_1 + Q_2$

$\Rightarrow \quad CV = CV_1 + CV_2$

$\Rightarrow \quad V = V_1 + V_2 \quad \Rightarrow, V_1 = \frac{V}{2}$

Total energy stored, $U_2 = \frac{1}{2}CV_1^2 + \frac{1}{2}CV_2^2$

$$= \frac{1}{2}C\left(\frac{V}{2}\right)^2 + \frac{1}{2}C\left(\frac{V}{2}\right)^2 \Rightarrow U_2 = \frac{1}{4}CV^2 \qquad ...(ii)$$

From Eqs. (i) and (ii), we get

$$U_2 < U_1$$

Hence, energy stored in the combination is less than that stored initially in single capacitor.

45. (i) We have initial voltage, $V_1 = V$ volt
and charge stored, $Q_1 - 360\mu C$

$$Q_1 = CV_1 \qquad ...(i)$$

Charged potential, $V_2 = V - 120$

$$Q_2 = 120\,\mu C$$

$$\Rightarrow \quad Q_2 = CV_2 \qquad ...(ii)$$

By dividing Eq. (ii) from Eq. (i), we get

$$\frac{Q_1}{Q_2} - \frac{CV_1}{CV_2} \Rightarrow \frac{360}{120} = \frac{V}{V-120}$$

$\Rightarrow \quad V = 180\text{ V}$

$\therefore \quad C = \frac{Q_1}{V_1} = \frac{360\times10^{-6}}{180} = 2\times10^{-6}\text{ F}$

$= 2\,\mu F$

Hence, the potential, $V = 180$ V and unknown capacitance is $2\,\mu F$.

(ii) If the voltage applied had increased by 120 V, then $V_3 = 180 + 120 = 300$ V

Hence, charge stored in the capacitor,

$Q_3 = CV_3 = 2\times10^{-6} \times 300 = 600\,\mu C$

46. In the circuit, when initially K_1 is closed and K_1 is opened, the capacitor C_1 and C_2 acquire potential difference V_1 and V_2, respectively. So, we have

$$V_1 + V_2 = E$$

and $\quad V_1 + V_2 = 9V$

Also, in series combination, $V \propto 1/C$

$$V_1 . V_2 = 1/6 : 1/3$$

On solving,

$\Rightarrow \quad V_1 = 3V$ and $V_2 = 6V$

$\therefore \quad Q_1 = C_1V_1 = 6\,\mu C \times 3V$

$= 18\,\mu C \qquad [\because C = 1\,\mu F]$

$\Rightarrow \quad Q_2 = C_2V_2 = 3\,\mu C \times 6V = 18\,\mu C$

and $\quad Q_3 = 0$

When K_1 was opened and K_2 was closed, the parallel combination of C_2 and C_3 in series with C_1.

[Charge on C_1 remains unchanged]

i.e. $\quad Q_1' - Q_2 = 18\,\mu C$

Charge on C_2 is shared between C_2 and C_3 in parallel

As, $\quad C_2 = C_3$

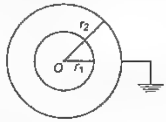
$\therefore \quad Q_2' = Q_2 = \frac{Q_2}{2} - \frac{18}{2} - 9\,\mu C \qquad [\because Q_2 = 18\,\mu C]$

47. Radius of inner sphere, $r_1 = 12$ cm

Radius of outer sphere, $r_2 = 13$ cm

and charge on inner sphere, $q = 2.5\,\mu C$

The dielectric constant, $K = 32$



(i) Capacitance of a spherical capacitor,

$$C = \frac{4\pi\varepsilon_0 K r_1 r_2}{r_1 - r_2} = \frac{1}{9\times10^9}\cdot\frac{32\times12\times13\times10^{-4}}{(13-12)\times10^{-2}}$$

$= 5.5\times10^{-9}$ F

(ii) Electric potential of inner sphere,

$= 4.5\times10^2$ V

(iii) Capacitance of an isolated sphere of radius, $r = 12$ cm

$$C - 4\pi\varepsilon_0 r = \frac{1}{9\times10^9}\times 12\times10^{-2}$$

$- 1.33\times10^{-11}$ F

The capacitance of an isolated sphere is much smaller as compared to the spherical capacitor because the outer sphere is earthed. The potential difference decreases and hence the capacitance increases.

48. Two identical capacitors C_1 and C_2 get fully charged with 5 V battery initially.

So, the charge and potential difference on both capacitors becomes

$$q = CV$$

$$= 2\times10^{-6}\times5\text{ V} = 10\,\mu C$$

and $\quad V = 5$ V

On introduction of dielectric medium of $K = 5$.

For C_1 (Continue to be connected with battery)

Potential difference of C_1, $V' = 5$ V

Capacitance, $C_1' = KC = 5\times2 = 10\,\mu F$

Charge, $q' = C'V' = 10\times5 = 50\,\mu C$

For C_2 (Disconnected from battery)

Charge, $q' = q = 10\,\mu C$

$\therefore$ Potential difference, $V' = \frac{V}{K} = \frac{5}{5} = 1$ V

[redacted]

49. (i) Here, C_1, C_2 and C_3 are in series, therefore, their equivalent capacitance

$$\frac{1}{C'}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

$$\Rightarrow \quad C'=\frac{C}{3}=\frac{12}{3}=4\ \mu F$$

Now, C' and C are in parallel combination.

$$\therefore \quad C_{net}=C'+C$$
$$=4\ \mu F+12\ \mu F=16\ \mu F$$

(ii) Being C' and C are in parallel, 500 V potential difference is applied across them.

$\therefore$ Charge on C'

$$q_1=C'V=(4\ \mu F)\times 500=2000\ \mu C$$

$\therefore$ C_1, C_2 and C_3 capacitors each will have $2000\ \mu C$ charge.

$\therefore$ Charge on C_4, $q_2=C\times V$
$$=12\times 500=6000\ \mu C$$

50. If n identical capacitors, each of capacitance C are connected in series combination give equivalent capacitance, $C_s=\frac{C}{n}$ and when connected in parallel combination, then equivalent capacitance, $C_p=nC$.

Also, for same voltage, energy stored in the capacitor is given by

$$U=\frac{1}{2}CV^2 \qquad [\text{for } V=\text{constant}]$$

$$\Rightarrow \quad U\propto C$$

In series combination, $C_s=\frac{C}{n}$

$$\Rightarrow \quad C_s=1\ \mu F \qquad [\because n=3]$$
$$\Rightarrow \quad C=nC_s=3\times 1\ \mu F=3\ \mu F$$

In parallel combination, $C_p=nC=3\times 3=9\ \mu F$

For same voltage, $U\propto C$

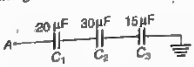
$$\Rightarrow \quad \frac{U_s}{U_p}=\frac{C_s}{C_p}$$

$$\Rightarrow \quad \frac{U_s}{U_p}=\frac{C/n}{nC}=\frac{1}{n^2}$$

$$\Rightarrow \quad \frac{U_s}{U_p}=\frac{1}{(3)^2}=\frac{1}{9}$$

or $\quad U_s:U_p=1:9$

51. Consider the given figure,



Given, $\quad C_1=20\ \mu F,\ C_2=30\ \mu F,\ C_3=15\ \mu F$

Potential at $A=90$ V

As, we can see that capacitor C_3 is earthed, therefore, potential across C_3 will be zero.

Since, capacitors C_1, C_2 and C_3 are connected in series, therefore

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$
$$=\frac{1}{20}+\frac{1}{30}+\frac{1}{15}$$

$$\Rightarrow \quad \frac{1}{C_{eq}}=\frac{3+2+4}{60}=\frac{9}{60}$$

$$\Rightarrow \quad C_{eq}=\frac{60}{9}=\frac{20}{3}\ \mu F$$

Since, charge remains same in series combination.

So, $\quad Q=C_{eq}V=\frac{20}{3}\times 90$

$$\Rightarrow \quad Q=600\ \mu C$$
$$=600\times 10^{-6}\ C$$
$$=6\times 10^{-4}\ C$$

$\therefore$ Potential difference across $C_2=\frac{Q}{V_2}$

$$\Rightarrow \quad V_2=\frac{Q}{C_2}$$

$$\Rightarrow \quad V_2=\frac{6\times 10^{-4}}{30\times 10^{-6}}=20\ V$$

$\therefore$ Energy stored in capacitor C_2 is given by

$$E=\frac{1}{2}C_2V_2^2$$
$$=\frac{1}{2}\times 30\times 10^{-6}\times (20)^2$$
$$=\frac{1}{2}\times 30\times 400\times 10^{-6}\ E$$
$$=6\times 10^{-3}\ J$$

52. Energy stored in capacitor $=\frac{1}{2}C_1V^2$

$$=\frac{1}{2}\times 12\times 10^{-12}\times (50)^2=15\times 10^{-9}\ J$$

With other capacitor 6 pF in series.

Total capacitance (C)

$$=\frac{C_1\times C_2}{C_1+C_2}=\frac{6\times 12}{6+12}=\frac{12\times 6}{18}=4\ pF$$

Charge stored in each capacitor is same and is given by

$$Q=CV$$
$$=4\times 10^{-12}\times 50\ C=2\times 10^{-10}\ C$$

Each of the capacitors will have charge equal to Q
$$=2\times 10^{-10}\ C$$

Potential on capacitors with capacitance 12 pF is

$$=\frac{Q}{C_1}=\frac{2\times 10^{-10}}{12\times 10^{-12}}\ V=16.67\ V$$

Potential on capacitor with capacitance 6 pF is

$$=\frac{2\times 10^{-10}}{6\times 10^{-12}}\ V=33.33\ V$$

[redacted]

53. According to question, let the capacitance of X be C, so capacitance of $Y = \varepsilon_r C = 4C$ $[\because \varepsilon_r = 4]$

(i) Equivalent capacitance $= \dfrac{C \times 4C}{C + 4C}$ [$\because$ X and Y are in series]

$= \dfrac{4C^2}{5C} = \dfrac{4C}{5}$ and it is given that $\dfrac{4C}{5} = 4\mu F$

So, $4C = 20\mu F =$ capacitance of Y

Capacitance of $X = C = \dfrac{20}{4} = 5\mu F$

(ii) Charge flowing through the capacitor is given by

$$q = CV = \frac{4C}{5} \times 15 = \frac{4 \times 5}{5} \times 15 = 60\,\mu C$$

Now, let the potential difference between plates of capacitors X and Y are V_x and V_y, respectively.

So, $V_x = \dfrac{q}{C_x} = \dfrac{60}{5} = 12\,V$

and $V_y = \dfrac{q}{C_y} = \dfrac{60}{20} = 3\,V$

(iii) Electrostatic energy stored in capacitance

$$X(E_x) = \frac{1}{2}CV_x^2 \quad ...(i)$$

Similarly for Y, $E_y = \dfrac{1}{2}4CV_y^2$ ...(ii)

From Eqs. (i) and (ii), we get

$$\text{Ratio} = \frac{E_x}{E_y} = \frac{\frac{1}{2}CV_x^2}{\frac{1}{2}4CV_y^2} = \frac{V_x^2}{4V_y^2} = \frac{12 \times 12}{4 \times 3 \times 3} = 4:1$$

54. When the capacitors are connected in parallel, equivalent capacitance, $C_P = C_1 + C_2$.

The energy stored in the combination of the capacitors,

$$E_P = \frac{1}{2}C_P V^2 = \frac{1}{2}(C_1 + C_2)(100)^2 = 0.25\,J$$

$\Rightarrow \quad C_1 + C_2 = 5 \times 10^{-5}$ ...(i)

When the capacitors are connected in series, equivalent capacitance,

$$C_S = \frac{C_1 C_2}{C_1 + C_2}$$

The energy stored in the combination of the capacitors,

$$E_S = \frac{1}{2}C_S V^2$$

$$\Rightarrow \quad E_S = \frac{1}{2}\frac{C_1 C_2}{C_1 + C_2}(100)^2$$

$$= \frac{1}{2} \times \frac{C_1 C_2}{5 \times 10^{-5}}(100)^2 = 0.045\,J$$

$$\Rightarrow \quad C_1 C_2 = 0.045 \times 10^{-4} \times 5 \times 10^{-5} \times 2$$

$$= 4.5 \times 10^{-10}$$

$\because \quad (C_1 - C_2)^2 = (C_1 + C_2)^2 - 4C_1C_2$

$\Rightarrow \quad (C_1 - C_2)^2 = 25 \times 10^{-10} - 4 \times 4.5 \times 10^{-10}$

$= 7 \times 10^{-10}$

$\Rightarrow \quad (C_1 - C_2) = \sqrt{7 \times 10^{-10}} = 2.64 \times 10^{-5}$

$\Rightarrow \quad C_1 - C_2 = 2.64 \times 10^{-5}$ ...(ii)

On solving Eqs. (i) and (ii), we get

$C_1 = 35\,\mu F$ and $C_2 = 15\,\mu F$

$\Rightarrow \quad Q_1 = C_1 V = 35 \times 10^{-6} \times 100$

$= 35 \times 10^{-4}\,C$

and $\quad Q_2 = C_2 V = 15 \times 10^{-6} \times 100$

$= 15 \times 10^{-4}\,C$

55. (i) As given in the question, energy of the $6\,\mu F$ capacitor is E. Let V be the potential difference along the capacitor of capacitance $6\,\mu F$.

Now, $\quad \dfrac{1}{2}CV^2 = E$

$$\frac{1}{2} \times 6 \times 10^{-6} \times V^2 = E$$

$$\Rightarrow \quad V^2 = \frac{E}{3} \times 10^6 \quad ...(i)$$

Since, potential is same for parallel connection, the potential through $12\,\mu F$ capacitor is also V. Hence, energy of $12\,\mu F$ capacitor is

$$E_{12} = \frac{1}{2} \times 12 \times 10^{-6} \times V^2 \quad \text{[from Eq. (i)]}$$

$$= \frac{1}{2} \times 12 \times 10^{-6} \times \frac{E}{3} \times 10^6 = 2E$$

(ii) Since, charge remains constant in series, the charge on $6\mu F$ and $12\,\mu F$ capacitors combined will be equal to the charge on $3\,\mu F$ capacitor.

Using the formula, $Q = CV$, we can write

$\rightarrow (6 + 12) \times 10^{-6} \times V = 3 \times 10^{-6} \times V'$

$V' = 6\,V$

Squaring on both sides, we get

$V'^2 = 36V^2$

Putting the value of V^2 from Eq. (i), we get

$$V'^2 = 36 \times \frac{E}{3} \times 10^6$$

$\Rightarrow \quad V'^2 = 12E \times 10^6$

$\therefore \quad E_3 = \dfrac{1}{2} \times 3 \times 10^{-6} \times 12E \times 10^6$

$= 18E$

(iii) Total energy drawn from battery is

$E_{total} = E + E_{12} + E_3$

$= E + 2E + 18E$

$= 21E$

56. 3×10^{-6} J; refer to Example 16 on page 94.

[redacted]

SUMMARY

- **Electrostatic Potential** It is the amount of work done (w) in moving a unit positive test charge (q) without acceleration form infinity to that point against the electrostatic force

  $\therefore \quad V = \frac{W}{q}$

  Its SI unit is volt (V) and 1 V = 1 J/C.

- **Electrostatic Potential Difference** Electrostatic potential difference between two points P and Q is equal to the work done (W_{QP}) by external force in moving a unit positive charge (q_0) against the electrostatic force from point Q to P along any path between these two points

  $\therefore \quad \Delta V = \frac{W_{QP}}{q_0}$

  Its SI unit is volt and $1V = 1\,JC^{-1}$.

Electric Potential due to a Point Charge

It can be given as, $V = \frac{q}{4\pi\varepsilon_0 r}$

Here, r is distance of the point from the charge.

Electrostatic potential at any point P due to a system of n point charges $q_1, q_2, \ldots, q_n$ whose position vectors are $\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_n$ respectively, is given by

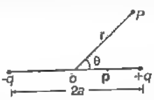
$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\mathbf{r} - \mathbf{r}_i|}$$

where, $\mathbf{r}$ is the position vector of point P w.r.t. the origin

- Electrostatic potential due to a thin charged spherical shell carrying charge q and radius R respectively, at any point P lying
  (i) inside the shell is $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$
  (ii) on the surface of shell is $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$
  (iii) outside the shell is $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$ for $r > R$

  where, r is the distance of point P from the centre of the shell.

- Electrostatic potential due to an electric dipole at any point P whose position vector is $\mathbf{r}$ w.r.t. mid-point of dipole is given by



$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p\cos\theta}{r^2} \text{ or } V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\mathbf{p}\cdot\hat{\mathbf{r}}}{|r|^2}$$

where, θ is the angle between $\hat{\mathbf{r}}$ and $\mathbf{p}$.

- **Equipotential Surfaces** Any surface which has same electrostatic potential at every point is called an equipotential surface.

- **Equipotential Surface in Different Cases** The equipotential surface can be obtained for different cases:
  (i) For a point charge, it is spherical surface
  (ii) For a uniform electric field, it is plane surface

- **Relation between Electric Field and Electrostatic Potential** It can be given as, $E = -\frac{\partial V}{\partial r} = -$ (Potential gradient)

  where, negative sign indicates that the direction of electric field is from higher potential to lower potential, i.e. in the direction of decreasing potential.

- **Electrostatic Potential Energy of a System of Charges** It is defined as, the total work done in bringing the different charges to their respective positions from infinitely large mutual separations.

- **Due to System of Two Point Charges** It can be given by

  $U = W = \frac{kq_1q_2}{r_{12}}$

- **Due to System of Three Point Charges** It can be given by

  $$U = \left[K \sum_{i=1}^{3} \sum_{\substack{j=1 \\ j \neq 1}}^{3} \frac{q_i q_j}{r_{ij}}\right]$$

- **Potential Energy of a Dipole in an External Field** Potential of a dipole in an external field can be given as

  $U = pE(\cos\theta_1 - \cos\theta_2)$

  Here, θ_1 and θ_2 are initial and final orientations of the dipole.

- **Conductors and Insulators**

  Conductors These are those materials through which electric charge can flow easily.
  The process which involves the making of a region free from electric field is known as electrostatic shielding.

  Insulators Insulators are those materials through which electric charge cannot flow.

- **Dielectric and Polarisation**

  Dielectric Constant It is the ratio of the strength of applied electric field to the strength of reduced value of electric field on placing the dielectric between the plates of a capacitor.

  Dielectric Strength The maximum electric field that a dielectric can withstand without breakdown is called its dielectric strength.

  Polarisation The induced dipole moment developed per unit volume in a dielectric slab on placing it in an electric field is called polarisation.

Electric Susceptibility Polarisation density of a dielectric slab is directly proportional to the reduced value of electric field i.e. $P = \chi \varepsilon_0 E$, where χ is called electric susceptibility

- **Capacitors and Capacitance** A capacitor is a system of two conductors separated by an insulating medium

  The capacitance of the capacitor, $C = \dfrac{Q}{V}$

  In SI system unit of capacity is farad

  Parallel Plate Capacitor Capacitance of a parallel plate capacitor can be given by, $C = \dfrac{\varepsilon_0 A}{d}$

  Effect of Dielectric on Parallel Plate Capacitor On introducing a dielectric between the parallel plates, capacitance can be given by,

  $$C = \frac{\varepsilon_0 KA}{d}, \text{ where } K \text{ is the dielectric constant}$$

- **Combination of Capacitors**

  In parallel combination,

  $$C_{eq} = C_1 + C_2 + \ldots\ldots + C_n$$

  In series combination,

  $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$$

  The energy U stored in a capacitor of capacitance C, with charge q and voltage V is,

  $$U = \frac{1}{2}qV = \frac{1}{2}CV^2 = \frac{q^2}{2C}$$

  The electrostatic energy density (energy per unit volume) in a region with electric field E is

  $$U = \frac{1}{2}\varepsilon_0 E^2$$

- **Common potential** It can be given as, $V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}$

- **Loss of Energy in Sharing Charges**

  It can be given as, $\Delta U = \dfrac{C_1C_2}{2}\dfrac{(V_1 - V_2)^2}{(C_1 + C_2)}$

[redacted]

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. If 100 J of work has to be done in moving an electric charge 4 C from a place where potential is 10 V to another place where potential is V volt, find the value of V.

(a) 5 V (b) 10 V (c) 25 V (d) 15 V

2. In an electric field with $E = 0$, the potential V varies with the distance r as

(a) $V \propto \frac{1}{r}$ (b) $V \propto r$

(c) $V \propto 1/r^2$ (d) V will not depend on r

3. A car battery is charged by a 12 V supply and energy stored in it is 7.20×10^5 J. The charge passed through the battery is CBSE 2021 (Term-I)

(a) 6.0×10^4 C (b) 5.8×10^3 J

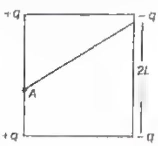
(c) 8.64×10^6 J (d) 1.6×10^5 C

4. Two charges 3×10^{-8} C and -2×10^{-8} C located 15 cm apart. At what point on the line joining the two charges is the electric potential zero?

(a) 9 cm (b) 45 cm

(c) 18 cm (d) Both (a) and (b)

5. Four charges $q, -q, +q$ and $-q$ are placed at the corners of a square of side $2L$ is shown in figure. The electric potential at point A mid-way between the two charges $+q$ and $+q$ is CBSE 2021 (Term-I)



(a) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1-\frac{1}{\sqrt{5}}\right)$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{L}\left(1+\frac{1}{\sqrt{5}}\right)$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{q}{2L}\left(1-\frac{1}{\sqrt{5}}\right)$ (d) zero

6. The electric potential V at any point (x, y, z) is given by $V = 3x^2$, where x is in metres and V in volts. The electric field at the point (1 m, 0, 2 m) is CBSE 2021 (Term-I)

(a) 6 V/m along $-X$-axis

(b) 6 V/m along $+X$-axis

(c) 1.5 V/m along X-axis

(d) 1.5 V/m along $+X$-axis

7. Which of the following is not the property of equipotential surface? CBSE SQP (Term-I)

(a) They do not cross each other.

(b) The rate of change of potential with distance on them is zero.

(c) For a uniform electric field, they are concentric spheres.

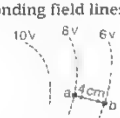
(d) They can be imaginary spheres.

8. Equipotentials at a large distance from a collection of charges, whose total sum is not zero are CBSE 2021 (Term-I)

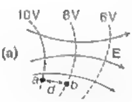
(a) spheres (b) planes

(c) ellipsoids (d) paraboloids

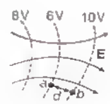
9. Three equipotential surfaces are shown in figure. Which of the following is correct one for the corresponding field lines?



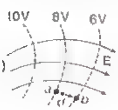
(a)



(b)



(c)



(d) None of these

[redacted]

10. The electrostatic potential on the surface of a charged conducting sphere is 100V. Two statements are made in this regard
S_1 At any point inside the sphere, electric intensity is zero.
S_2 At any point inside the sphere, the electrostatic potential is 100V.
Which of the following is a correct statement? **NCERT Exemplar**
(a) S_1 is true but S_2 is false
(b) Both S_1 and S_2 are false
(c) S_1 is true, S_2 is also true and S_1 is the cause of S_2
(d) S_1 is true, S_2 is also true but the statements are independent

11. Two charges $14\,\mu C$ and $-4\,\mu C$ are placed at $(-12\text{ cm}, 0, 0)$ and $(12\text{ cm}, 0, 0)$ in an external electric field $E = \left(\frac{B}{r^2}\right)$, where $B = 1.2 \times 10^6\ \text{N/cm}^2$ and r is in m. The electrostatic potential energy of the configuration is **CBSE 2021 (Term-I)**
(a) 97.9 J (b) 102.1 J
(c) 2.1 J (d) -97.9 J

12. A $+3.0\,nC$ charge Q is initially, at a distance of $r_1 = 10$ cm from a $+5.0\,nC$ charge q fixed at the origin. The charge Q is moved away from q to a new position at $r_2 = 15$ cm. In this process, work done by the field is **CBSE 2021 (Term-I)**
(a) 1.29×10^{-5} J (b) 3.6×10^{5} J
(c) -4.5×10^{-7} J (d) 4.5×10^{-7} J

13. On bringing an electron near to other electron, the potential energy of the system
(a) decreases (b) increases
(c) remains same (d) becomes zero

14. An electric dipole of length 1cm is placed with the axis making an angle of 30° to an electric field of strength 10^4 N/C. If it experiences a torque of $10\sqrt{2}$ Nm, the potential energy of the dipole is
(a) 0.245 J (b) 2.45 J
(c) 24.5 J (d) 245.0 J

15. What is the value of capacitance if a very thin metallic plate is introduced between two parallel plates of area A and separated at distance d?
(a) $\varepsilon_0 A/d$ (b) $\frac{2\varepsilon_0 A}{d}$ (c) $\frac{4\varepsilon_0 A}{d}$ (d) $\frac{\varepsilon_0 A}{2d}$

16. A parallel plate capacitor has a uniform electric field (Vm^{-1}) in the space between the plates. If the distance between the plates is d(m) and area of each plate is $A(\text{m}^2)$, the energy (joule) stored in the capacitor is
(a) $\frac{1}{2}\varepsilon_0 E^2$ (b) $\varepsilon_0 EAd$
(c) $\frac{1}{2}\varepsilon_0 E^2 Ad$ (d) $E^2 Ad/\varepsilon_0$

17. A variable capacitor is connected to a 200 V battery. If its capacitance is changed from $2\,\mu F$ to $X\,\mu F$, the decrease in energy of the capacitor is 2×10^{-2} J. The value of X is **CBSE 2021 (Term-I)**
(a) $1\,\mu F$ (b) $2\,\mu F$
(c) $3\,\mu F$ (d) $4\,\mu F$

18. Two parallel plate capacitors X and Y, have the same area of plates and same separation between plates. X has air and Y with dielectric of constant 2, between its plates. They are connected in series to a battery of 12 V. The ratio of electrostatic energy stored in X and Y is **CBSE SQP (Term-I)**
(a) 4 : 1 (b) 1 : 4
(c) 2 : 1 (d) 1 : 2

19. If the charge on each plate of a capacitor of $60\,\mu F$ is 3×10^{-8} C. Then, energy stored in the capacitor will be
(a) 2.5×10^{-15} J (b) 1.5×10^{-14} J
(c) 3.5×10^{-13} J (d) 7.5×10^{-12} J

20. Three capacitors $2\,\mu F$, $3\,\mu F$ and $6\,\mu F$ are joined in series with each other. The equivalent capacitance is **CBSE SQP (Term-I)**
(a) $1/2\,\mu F$ (b) $1\,\mu F$
(c) $2\,\mu F$ (d) $11\,\mu F$

21. A capacitor plates are charged by a battery with V volts. After charging battery is disconnected and a dielectric slab with dielectric constant K is inserted between its plates, the potential across the plates of a capacitor will become **CBSE SQP (Term-I)**
(a) zero (b) $V/2$
(c) V/K (d) KV

ASSERTION AND REASON

Directions (Q. Nos. 22-32) *In the following questions, two statements are given- one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*
(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

[redacted]

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

22. **Assertion** Work done by the electrostatic force in bringing the unit positive charge from infinity to the point P is positive.

Reason For $Q < 0$, the force on unit positive charge is attractive, so that the electrostatic force and the displacement (from infinity to P) are in the same direction

23. **Assertion** A and B are two conducting spheres of same radius. A being solid and B hollow. Both are charged to the same potential. Then, charge on A = charge on B.

Reason Potential on both are same.

24. **Assertion** There is no potential difference between any two points on the equipotential surface.

Reason No work is required to move a test charge on the equipotential surface from one point to other.

25. **Assertion** The expression of potential energy $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r_{12}}$, is unaltered whatever way the charges are brought to the specified locations.

Reason Path-independence of work for electrostatic force.

26. **Assertion** An electron has a high potential energy when it is at a location associated with a more negative value of potential and a low potential energy when at a location associated with a more positive potential.

Reason Electrons move from a higher potential region to lower potential region.

CBSE SQP (Term-I)

27. **Assertion** In the absence of an external electric field, the dipole moment per unit volume of a polar dielectric is zero.

Reason The dipoles of a polar dielectric are randomly oriented.

28. **Assertion** Polar molecules have permanent dipole moment.

Reason In polar molecules, the centre of positive and negative charges coincides even when there is no external field.

29. **Assertion** Charge on all the condensers connected in series is the same.

Reason Capacitance of capacitor is directly proportional to charge on it.

30. **Assertion** An electron moves from a region of lower potential to a region of higher potential.

Reason An electron has a negative charge.

31. **Assertion** A parallel plate capacitor is connected across a battery through a key. A dielectric slab of dielectric constant K is introduced between the plates. The energy which is stored becomes K times.

Reason The surface density of charge on the plate remains constant or unchanged.

32. **Assertion** If three capacitors of capacitances $C_1 < C_2 < C_3$ are connected in parallel, and in series then their equivalent capacitances.

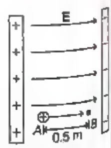
$$C_p > C_s$$

Reason $\frac{1}{C_p} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

CASE BASED QUESTIONS

Directions (Q.Nos. 33-34) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

33. **Proton in an Electric Field**



Potential difference (ΔV) between two points A and B separated by a distance x, in a uniform electric field **E** is given by $\Delta V = -Ex$, where x is measured parallel to the field lines. If a charge q_0 moves from A to B, the change in potential energy (ΔU) is given as $\Delta U = q_0\Delta V$. A proton is released from rest in uniform electric field of magnitude 8.0×10^4 Vm^{-1} directed along the positive X-axis. The proton undergoes a displacement of 0.50 m in the direction of E.

[redacted]

Mass of a proton $= 1.66 \times 10^{-27}$ kg

and charge on a proton $= 1.6 \times 10^{-19}$ C.

With the help of the passage given above, choose the most appropriate alternative for each of the following questions.

(i) As the proton moves from A to B, then

(a) the potential energy of proton decreases

(b) the potential energy of proton increases

(c) the proton loses kinetic energy

(d) total energy of the proton increases

(ii) The change in electric potential of the proton between the points A and B is

(a) 4.0×10^{4} V (b) -4.0×10^{4} V

(c) 6.4×10^{-19} V (d) -6.4×10^{-19} V

(iii) The change in electric potential energy of the proton for displacement from A to B is

(a) -6.4×10^{-19} J (b) 6.4×10^{-19} J

(c) -6.4×10^{-15} J (d) 6.4×10^{-15} J

(iv) The velocity (v_B) of the proton after it has moved 0.50 m starting from rest is

(a) 1.6×10^{5} ms^{-1} (b) 2.77×10^{6} ms^{-1}

(c) 2.77×10^{4} ms^{-1} (d) 1.6×10^{6} ms^{-1}

(v) If in place of charged plates, two similar point charges of 1 μC are kept in air at 1m distance from each other. Then, potential energy is

(a) 1 J (b) 1eV

(c) 9×10^{-3} J (d) zero

34. **Electrostatic Potential Energy**

Electrostatic potential energy of a system of point charges is defined as the total amount of work done in bringing the different charges to their respective positions from infinitely large mutual separations.

By definition, work done in carrying charge from ∞ to any point is

$$W = \text{Potential} \times \text{Charge}$$

This work is stored in the system of two point charges in the form of electrostatic potential energy U of the system.

(i) Work done in moving a charge from one point to other inside a uniformly charged conducting sphere is

(a) always zero

(b) non-zero

(c) may be zero

(d) None of the above

(ii) A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge

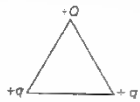
(a) remains a constant because the electric field is uniform

(b) increases because the charge moves along the electric field

(c) decreases because the charge moves along the electric field

(d) decreases because the charge moves opposite to the electric field

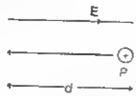
(iii) Three charges are placed at the vertex of an equilateral triangle of side l as shown in figure. For what value of Q, the electrostatic potential energy of the system is zero?



(a) $-q$ (b) $q/2$

(c) $2q$ (d) $-q/2$

(iv) In the figure, proton moves a distance d in a uniform electric field E as shown in the figure. The work done on the proton by electric field is



(a) negative (b) positive

(c) zero (d) None of these

(v) Two similar positive point charges each of 2μC have been kept in air at 3m distance from each other. What will be the potential energy?

(a) 1 J (b) 1 eV

(c) 12×10^{-3} J (d) zero

VERY SHORT ANSWER Type Questions

35. Determine the work done in moving a test charge q through the distance 1 cm along the equatorial axis of an electric dipole.

36. Why there is no work done in moving a charge from one point to another on an equipotential surface? **Foreign 2012**

37. Draw the equipotential surfaces due to an isolated point charge. **CBSE 2019**

[redacted]

38. Draw equipotential surface for an electric dipole. CBSE 2019

39. A proton released from rest in an electric field, will start moving towards a region of .... potential in the field. CBSE 2020

40. Depict equipotential surfaces due to an electric dipole. CBSE 2020

41. A charge particle (+ q) moves in a uniform electric field **E** in the direction opposite to **E**. What will be the effect on its electrostatic potential energy during its motion? CBSE 2020

42. Assume a charge starting at rest on an equipotential surface is moved off that surface and then is eventually returned to the same surface of rest after a round trip How much work did it take to do this? Explain.

43. Do electrons tend to go to regions of high potential or low potential?

44. A proton is released at rest in a uniform electric field. Does the proton's electric potential energy increase or decrease?

Does the proton move towards a location with a higher or lower electric potential?

45. What is the net charge on a charged capacitor?

46. Two circular metal plates, each of radius 10 cm, are parallel to each other at a distance of 1mm. What kind of capacitor do they make? Mention one application of this capacitor.

47. A metal plate is introduced between the plates of a charged parallel plate capacitor. What is its effect on the capacitance of the capacitor?

SHORT ANSWER Type Questions

48. Draw three equipotential surfaces corresponding to a field that uniformly increase in magnitude but remains constant along x-direction.

How are these surfaces different from that of a constant electric field along x-direction?

49. Two point charges 5 μC and -5 μC are placed at points A and B, 5 cm apart.

(i) Draw the equipotential surface of the system.

(ii) Why do equipotential surfaces get close to each other near the point charge.

50. Deduce an expression for the potential energy of a system of two point charges q_1 and q_2 located at positions r_1 and r_2, respectively in an external field (E). CBSE SQP (Term-I)

51. The plates in a parallel plate capacitor are separated by a distance d with air as the medium between the plates. In order to increase the capacity by 66% a dielectric slab of dielectric constant 5 is introduced between the plates. What is the thickness of dielectric slab?

52. A parallel plate capacitor with air as dielectric is charged by a DC source to a potential V. Without disconnecting the capacitor from the source, air is replaced by another dielectric medium of dielectric constant K. State with a reason, how does

(i) electric field between the plates and

(ii) energy stored in the capacitor change?

53. A slab of material of a dielectric constant K has the same area as that of plates of a parallel plate capacitor but has the thickness $2d/3$, where d is separation between the plates.

Find the expression of the capacitance when the slab is inserted between the plates of the capacitor.

LONG ANSWER Type I Questions

54. Two isolated metallic solid spheres of radii R and $2R$ are charged such that both of these have same charge density σ. The spheres are located far away from each other, and connected by a thin wire. Find the new charge density on the bigger sphere.

55. (a) Draw the equipotential surfaces corresponding to a uniform electric field in the z-direction.

(b) Derive an expression for the electric potential at any point along the axial line of an electric dipole. CBSE 2019

56. (a) Draw equipotential surfaces corresponding to the electric field that uniformly increases in magnitude along with the z-direction.

(b) Two charges $-q$ and $+q$ are located at points (0, 0, $-a$) and (0, 0, a). What is the electrostatic potential at the points (0, 0, $\pm z$) and (x, y, 0)? CBSE 2019

[redacted]

57. (a) Two point charges $+Q_1$ and $-Q_2$ are placed r distance apart. Obtain the expression for the amount of work done to place a third charge Q_3 at the mid-point of the line joining the two charges.

(b) At what distance from charge $+Q_1$ on the line joining the two charges (in terms of Q_1, Q_2 and r) will this work done be zero?

CBSE 2020

58. (a) Two point charges q_1 and q_2 are kept at a distance of r_{12} in air. Deduce the expression for the electrostatic potential energy of this system.

(b) If an external electric field (E) is applied on the system, write the expression for the total energy of this system. **CBSE 2020**

59. Define the following.

(i) Polarisation

(ii) Electric susceptibility (λ)

(iii) Electrostatic shielding

60. Choose the statement as wrong or right and justify.

(i) Inside a conductor, electric field is not zero because electrostatic potential is constant.

(ii) On insertion of dielectric, capacitance of capacitor increases.

(iii) When capacitors are connected in parallel, the amount of charge in each capacitor will be same.

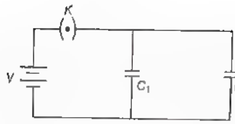
61. A parallel plate capacitor has capacitance C_0 in the absence of a dielectric. A slab of dielectric material of dielectric constant ε_r and thickness $d/3$ is inserted between the plates. What is the new capacitance when the dielectric is present?

62. Two parallel plate capacitors of capacitances C_1 and C_2 such that $C_1 - C_2/2$ are connected across a battery of V volts as shown in the figure. Initially, the key (K) is kept closed to fully charge the capacitors.

The key is now thrown open and a dielectric slab of dielectric K is inserted in the two capacitors to completely fill the gap between the plates. Find the ratio of

(i) the net capacitance and

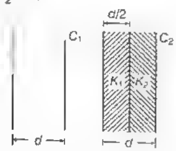
(ii) the energies stored in the combination before and after the introducing dielectric slab.



63. You are given an air filled parallel plate capacitor C_1. The space between its plates is now filled with slabs of dielectric constants K_1 and K_2 as shown in figure.

(i) Find the capacitance of the capacitor C_2 if area of the plates is A and distance between the plates is d.

(ii) What is the value of capacitance if $K_1 = K_2 = K$?



64. Derive an expression for the energy stored in a parallel plate capacitor. On charging a parallel plate capacitor to a potential V, the spacing between the plates is halved and a dielectric medium of $\varepsilon_r - 20$ is introduced between the plates, without disconnecting the DC source. Explain, using suitable expressions, how the (i) capacitance, (ii) energy density of the capacitor changes?

LONG ANSWER Type II Questions

65. (i) Establish the relation between electric field and electric potential at a point.

(ii) Draw the equipotential surface for an electric field pointing in $+z$-direction with its magnitude increasing at constant rate along $-z$-direction. **CBSE SQP (Term-I)**

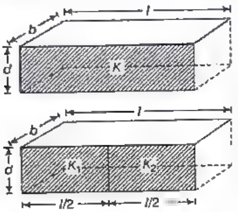
66. (i) Depict the equipotential surfaces for a system of two identical positive point charges placed at distance d apart.

(ii) Deduce the expression for the potential energy of a system of two point charges q_1 and q_2 brought from infinity to the points with positions r_1 and r_2, respectively in presence of external electric field **E**.

[redacted]

67. (i) Obtain the expression for the potential due to an electric dipole of dipole moment p at a point x on the axial line.

(ii) Two identical capacitors of plate dimensions $l \times b$ and plate separation d have dielectric slabs filled in between the space of the plates as shown in figure.



Obtain the relation between dielectric constants K, K_1 and K_2. **All India 2013**

68. (i) Describe briefly the processs of transferring the charge between the two plates of a capacitor when connected to battery. Derive an expression for the energy stored in a capacitor.

(ii) A parallel plate capacitor is charged by a battery to a potential difference V. It is disconnected from battery and then connected to another unchanged capacitor of the same capacitance. Calculate the ratio of the energy stored in the combination to the initial energy on the single capacitor. **CBSE 2019**

NUMERICAL PROBLEMS

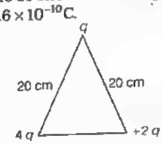
69. Calculate the potential at a point P due to a charge of 5×10^{-7} C located 11 cm away.

70. A hollow metal sphere of radius 7 cm is charged such that potential on its surface is 20 V. What is the potential at the centre of the sphere?

71. When reaching for door handle often sliding across a car seat on a dry winter day, you get a spark when your finger tip is 5 mm away from the handle. What was the potential difference between you and the door handle just before the spark? Given that dielectric strength of air $= 3 \times 10^6$ V/m.

72. Two point charges $5Q$ and Q are separated by 1m in air. At what point on the line joining the charges, is the electric field intensity zero? Also, calculate the electrostatic potential energy of the system of charges, taking the value of charge, $Q = 4 \times 10^{-7}$ C.

73. Calculate the work done to dissociate the system of three charges placed on the vertices of a triangle as shown in the figure.
Here, $q = 1.6 \times 10^{-10}$ C. **CBSE 2020**



74. Read the following passage and answer the question below it:

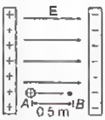
Potential difference (ΔV) between two points A and B separated by a distance x, in a uniform electric field E is given by $\Delta V = -Ex$, where x is measured parallel to the field lines. If a charge q_0 moves from A to B, the change in potential energy (ΔU) is given as $\Delta U = q_0 \Delta V$. A proton is released from rest in uniform electric field of magnitude 8×10^4 V/m directed along the positive X-axis. The proton undergoes a displacement of 0.50 m in the direction of E.

Mass of a proton $= 1.66 \times 10^{-27}$ kg and charge on a proton $= 1.6 \times 10^{-19}$ C.

With the help of the comprehension given above, choose the most appropriate alternative for each of the following questions.

(i) What will happen to the potential energy of proton, when it moves from A to B?

(ii) What will be the velocity (v_B) of the proton after it has moved 0.50 m starting from rest?

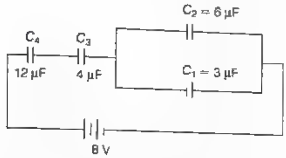


75. A 200 μF parallel plate capacitor having plate separation of 5 mm is charged by a 100 V DC source. It remains connected to the source. Using an insulated handle, the distance between the plates is doubled and a dielectric slab of thickness 5 mm and dielectric constant 10 is introduced between the plates. Explain with reason, how the (i) capacitance, (ii) electric field between the plates and (iii) energy density of the capacitor will change. **CBSE 2019**

[redacted]

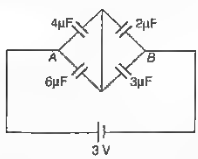
76. A $100\,\mu F$ parallel plate capacitor having plate separation of 4 mm is charged by 200 V DC. The source is now disconnected. When the distance between the plates is doubled and a dielectric slab of thickness 4 mm and dielectric constant 5 is introduced between the plates, how will (i) its capacitance, (ii) the electric field between the plates and (iii) energy density of the capacitor get affected? Justify your answer in each case.
CBSE 2019

77. In a network, four capacitors C_1, C_2, C_3 and C_4 are connected as shown in the figure.



(a) Find the net capacitance of the circuit.
(b) If the charge on the capacitor C_1 is $6\,\mu C$, (i) calculate the charge on the capacitors C_3 and C_4 and (ii) net energy stored in the capacitors C_3 and C_4 connected in series.
CBSE 2019

78. Find the total charge stored in the network of capacitors connected between A and B as shown in figure:

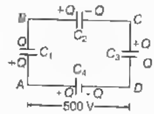


CBSE 2020

79. You are given three capacitors of $2\,\mu F$, $3\,\mu F$, $4\,\mu F$, respectively.
(a) Form a combination of all these capacitors of equivalent capacitance $\frac{13}{3}\,\mu F$.
(b) What is the maximum and minimum value of the equivalent capacitance that can be obtained by connecting these capacitors?
CBSE 2020

80. What is the area of the plates of a 2.5 F parallel plate capacitor, given that the separation between the plates is 0.2 cm? (You will realise from your answer why ordinary capacitors are in the range of μF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minimum separation between the conductors).
NCERT

81. A network of four $10\,\mu F$ capacitors is connected to a 500V supply as shown in the figure. Determine the equivalent capacitance of the network along AD.



82. If two parallel plate capacitors A and B are connected in series combination with the same supply voltage of V volt, the capacitor A has air in between its plates while B has dielectric of dielectric constant 4, then
(i) determine the capacitance of each capacitor, if the equivalent capacitance of the combination is $4\,\mu F$.
(ii) Find the ratio of electrostatic energy stored in B to A.

ANSWERS

1. (d)	2. (d)	3. (a)	4. (d)	5. (a)
6. (a)	7. (c)	8. (a)	9. (c)	10. (c)
11. (a)	12. (d)	13. (b)	14. (c)	15. (a)
16. (c)	17. (a)	18. (c)	19. (d)	20. (b)
21. (c)	22. (a)			

23. (a) A and B are two conducting spheres of same radius. A being solid and B hollow. Both are charged to the same potential. Then, charge on A = Charge on B. Because potentials on both are same

24. (a) The proof of this statement is simple. There is no potential difference between any two points on the surface and no work is required to move a test charge on the surface because work done

= potential difference × charge.

25. (a) The potential energy $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$ is unaltered whatever way the charges are brought to the specified locations, because of path independence of work for electrostatic force.

[redacted]

26. (c) Electrons move from a lower potential region to higher potential region.

27. (a) There are polar and non-polar dielectric materials. The molecules of a polar dielectric have a permanent dipole moment. However, due to random orientations net dipole moment is zero. If there is no external electric field, there is no polarisation

28. (c) The molecules of a substance may be polar or non-polar. In a non polar molecule, the centre of positive and negative charges coincides. On the other hand, a polar molecule is one is which the centres of positive and negative charges are separated, even when there is no external field. Such molecules have a permanent dipole moment

29. (c) Let two capacitors be connected in series. If $+q$ charge is installed on left plate of the first capacitor, then q charge is induced on right plate of this capacitor. This charge comes from electron drawn from the left plate of second capacitor. Thus, there will be equal charge $+q$ on the left plate of second capacitor and $-q$ charge induced on the right plate of second capacitor. Thus, each capacitor has same charge (q) when connected in series. Capacitance is quantity dependent on construction of capacitor and independent of charge.

30. (a) Electric field is set up from higher potential to lower potential. An electron is negatively charged and moves opposite to the direction of electric field, *i.e.*, from lower potential to higher potential.

31. (c) The reason is false as $\sigma' = \frac{q'}{A} = \frac{C'V'}{A} = \frac{(KC)V}{A}$

$$= \frac{Kq}{A} = K\sigma$$

(as C' KC, $V' = V$ and $CV = q$)

32. (c) Assertion is true as capacitance in parallel is greater than capacitance is series. Reason is false as $C_p = C_1 + C_2 + C_3$.

33. (i) (a) Potential energy of the proton decreases as it moves in the direction of the electric field.

(ii) (b) $\Delta V = -E\Delta x = -(8.0\times 10^4 \text{ V/m})(0.50 \text{ m})$

$$= -4\times 10^4 \text{ V}$$

(iii) (c) $\Delta U = q_0\Delta V = (1.6\times 10^{-19}\text{ C})(-4.0\times 10^4\text{ V})$

$$= -6.4\times 10^{-15} \text{ J}$$

(iv) (b) As, $\Delta K = -\Delta U = 6.4\times 10^{-15}$ J

(from conservation of energy)

$$\Delta K = \frac{1}{2}mv_B^2$$

or $$v_B = \sqrt{\frac{2\Delta K}{m}}$$

$$= \sqrt{\frac{2(6.4\times 10^{-15}\text{ J})}{(1.66\times 10^{-27}\text{ kg})}}$$

$$= 2.77\times 10^6 \text{ ms}^{-1}$$

(v) (c) Electrostatic potential energy of the system,

$$u = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = \frac{9\times10^9\times1\times10^{-6}\times1\times10^{-6}}{1}$$

$$= 9\times10^{-3}\text{ J}$$

34. (i) (a) Since, $E = 0$ inside the conductor and has no tangential component on the surface, no work is done in moving a small test charge within the conductor and on its surface.

(ii) (c) The positively charged particle experiences electrostatic force along the direction of electric field, i.e. from high electrostatic potential to low electrostatic potential. Thus, the work is done by the electric field on the positive charge, hence electrostatic potential energy of the positive charge decreases.

(iii) (d) Potential energy of the system,

$$U = \frac{KQq}{l} + \frac{Kq^2}{l} + \frac{KqQ}{l} = 0$$

$$\Rightarrow \quad \frac{Kq}{l}\times[(Q + q + Q)] = 0 \Rightarrow Q \quad -q/2$$

(iv) (a) Since the proton is moving against the direction of electric field, so work is done on the proton against electric field. It implies that electric field does negative work on the proton. Again, proton is moving in electric field from low potential region to high potential region hence, its potential energy increases.

(v) (c) Electric potential energy of the system,

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

Here, $q_1 = q_2 = 1\,\mu\text{C} = 1\times10^{-6}$ C,

$$r = 1 \text{ m and } \frac{1}{4\pi\varepsilon_0} \quad 9\times10^9 \text{ N-m}^2/\text{C}^2$$

$$\therefore \quad U = \frac{9\times10^9\times2\times10^{-6}\times2\times10^{-6}}{3}$$

$$= 12\times10^{-3}\text{ J}$$

35. Electrostatic potential at any point on the equatorial plane of dipole is zero.

$\therefore$ Work done, $\Delta W = q\Delta V = 0$

36. Change in potential is zero on an equipotential surface.

37. Refer to text and diagram on page 68 [Equipotential Surfaces in Different Cases (Case I)]

38. Refer to text and figure on page 68 [Equipotential Surfaces in Different Cases (Cave IV)]

39. decreasing

As, electric field lines starts from higher potential and ends at lower potential, so when a proton is released from rest in the field, then it moves towards the region of decreasing potential in the field.

40. Refer to text on page 68

41. When a positive charge q moves in a direction opposite to the direction of electric field, the work done by field is negative and electrostatic potential energy increases.

[redacted]

42. Work done, $\Delta W = q\Delta V = q(0) = 0$

43. High potential

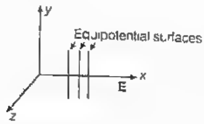
44. Proton moves from a location of higher potential to lower potential. Thus, potential energy decreases.

45. Zero

46. Parallel plate capacitor. It is used to store electrostatic energy

47. Increases

48.

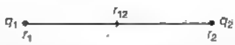


The equipotential surface are plane parallel to y-z plane As the field is increasing in magnitude along X-axis, so the spacing between the planes decreases on moving along X-axis But in case of constant electric field, the planes are spaced equally.

49. Refer to Q 21 on page 75.

50. Suppose q_1 and q_2 charges are brought from infinity at locations r_1 and r_2, respectively in an external electric field.

Let $V(\mathbf{r}_1)$ and $V(\mathbf{r}_2)$ be the potentials at positions r_1 and r_2 due to external electric field **E**. In this case, work is done in bringing these charges against their own electric fields and external electric field.



Work done in bringing q_1 from infinity to r_1 is,

$$W_1 = q_1 V(\mathbf{r}_1)$$

Similarly, for q_2, work done, $W_2 = q_2 V(\mathbf{r}_2)$

Work done on q_2 against the electric field due to q_1,

$$W_3 = \int_{\infty}^{r_{12}} \mathbf{F}_{12} \cdot d\mathbf{r} = \frac{1}{4\pi\varepsilon_0}\int_{\infty}^{r_{12}} \frac{q_1 q_2}{r^2}(-dr)$$

$$= -\frac{q_1 q_2}{4\pi\varepsilon_0}\int_{\infty}^{r_{12}} \frac{1}{r^2}dr = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

where, $r_{12} = |\mathbf{r}_2 - \mathbf{r}_1|$

$\therefore$ Potential energy of system V = Work done in assembling the configuration $= W_1 + W_2 + W_3$

$$= q_1 V(\mathbf{r}_1) + q_2 V(\mathbf{r}_2) + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

51. $\frac{\Delta C}{C} = \frac{C' - C}{C} = \frac{KC - C}{C}$ [Ans. $(d/2)$]

52. Refer to text on pages 88 and 92.

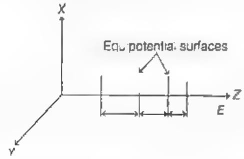
53. Refer to Q. 23 on page 95. $\left[\textbf{Ans. } \left(\frac{3K}{K+2}\right)\cdot\left(\frac{\varepsilon_0 A}{d}\right)\right]$

54. Refer to Q. 37 on page 77. [Ans. $5\sigma/6$]

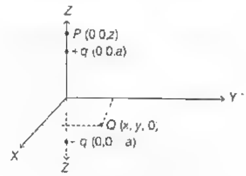
55. (a) Refer to Sol. 33 (ii) on page 80.

(b) Refer to text on page 67.

56. (a) The equipotential surface are plane parallel to X-Y plane. As the field is increasing in magnitude, the spacing between surfaces decreases.



(b) Let $P(0, 0, z)$ and $Q(x, y, 0)$ are two points on which electric potential are to be calculated.



Then, electrostatic potential at P

$$V_P = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(z-a)} - \frac{q}{(z+a)}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q \times 2a}{(z^2 - a^2)}\right]$$

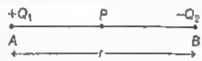
$$= \frac{1}{4\pi\varepsilon_0}\frac{p}{(z^2 - a^2)} \qquad [\because p = q \times 2a]$$

The electrostatic potential at Q is

$$V_Q = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{\sqrt{x^2 + y^2 - a^2}} - \frac{q}{\sqrt{x^2 + y^2 - a^2}}\right]$$

$$= 0$$

57. (a) The charges $+Q_1$ and $-Q_2$ are placed at A and B respectively, as shown



Let P be the mid-point of line joining A and B. The potential at P due to charge $+Q_1$ is,

$$V_1 = \frac{Q_1}{4\pi\varepsilon_0 r/2}$$

and due to charge $-Q_2$ is

$$V_2 = \frac{-Q_2}{4\pi\varepsilon_0 r/2}$$

The resultant potential at P is

[redacted]

$$V = V_1 + V_2 = \frac{Q_1}{4\pi\varepsilon_0 r/2} - \frac{Q_2}{4\pi\varepsilon_0 r/2}$$

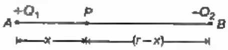
$$= \frac{2}{4\pi\varepsilon_0 r}(Q_1 - Q_2)$$

The work done to place a charge Q_3 at P is,

$$W = Q_3 V = \frac{2Q_3(Q_1 - Q_2)}{4\pi\varepsilon_0 r}$$

or $$W = \frac{Q_3(Q_1 - Q_2)}{2\pi\varepsilon_0 r}$$

(b) Let x be the distance from charge $+Q_1$ on the line joining the two charges, at which the work done will be zero as shown.



So, the potential due to charge $+Q_1$ at A is equal to potential due to charge $-Q_2$ at B at point P, i.e.

$$V_{Q_1} = V_{Q_2}$$

$$\Rightarrow \quad \frac{Q_1}{4\pi\varepsilon_0 x} = \frac{-Q_2}{4\pi\varepsilon_0(r-x)} \Rightarrow \frac{Q_1}{x} = -\frac{Q_2}{r-x}$$

$$\Rightarrow \quad rQ_1 - xQ_1 = -Q_2 x$$

or $$x = \frac{rQ_1}{(Q_1 - Q_2)}$$

58. (a) Refer to text on page 70.
(b) Refer to text on page 73.

59. Refer to text on pages 85 and 86.

60. (i) Refer to text on page 85.
(ii) Refer to text on page 88.
(iii) Refer to text on page 90.

61. Refer to Q. 23 on page 95.

62. Refer to Q. 29 page 96

63. Refer to Q. 26 page 96

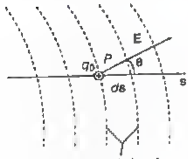
64. Refer to text on page 92.
(i) and (ii) Refer to Q. 38 on page 97.

65. (i) **Relation between Electric Field and Electric Potential**

Let us consider a positive test charge (q_0) moves a distance (ds) from one equipotential surface to another. The displacement (ds) makes an angle (θ) with the direction of the electric field $(\mathbf{E})$.

Suppose a positive test charge (q_0) moves through a differential displacement ds from one equipotential surface to the adjacent surface.

We know that, the work done by the electric field on the test charge during its movement is $-q_0\,dV$. We see that, the work done by the electric field may also be written as the scalar product $(q_0 \mathbf{E}\cdot d\mathbf{S})$ or $q_0 E\cos\theta\, ds$.



Two equipotential surfaces

(ii) Equating these two expressions for the work yields

$$-q_0 dV = q_0 E\cos\theta\, ds$$

$$\Rightarrow \quad E\cos\theta = -\frac{dV}{ds}$$

Since, $E\cos\theta$ is the component of $\mathbf{E}$ in the direction of ds, therefore

$$E_s = -\frac{\partial V}{\partial s}$$

$$\mathbf{E} = E_x\hat{\mathbf{i}} + E_y\hat{\mathbf{j}} + E_z\hat{\mathbf{k}}$$

where E_x, E_y and E_z are the x, y and z-components of E at any point.

$$\because \quad E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$$

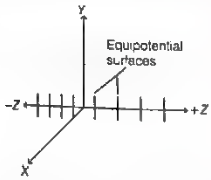
$$\therefore \quad \mathbf{E} = -\left[\frac{\partial V}{\partial x}\hat{\mathbf{i}} + \frac{\partial V}{\partial y}\hat{\mathbf{j}} + \frac{\partial V}{\partial z}\hat{\mathbf{k}}\right]$$

For the simple situation in which the electric field $\mathbf{E}$ is uniform,

$$E = -\frac{\Delta V}{\Delta s}$$

Negative sign shows that the direction of electric field $\mathbf{E}$ in the direction of decreasing potential.

The equipotential surfaces for an electric field in $+z$-direction are shown below



As, the distance between the surfaces is decreasing towards $-z$-direction, this means the electric field is increasing in magnitude towards $-z$-direction at a constant rate

66. (i) Refer to text on page 68 (Case II).
(ii) Refer to text on page 73.

67. (i) Refer to text on page 67.
(ii) Refer to Q. 24 on page 96.

[redacted]

68. (a) Refer to text on page 87 (Parallel Plate Capacitor) and page 92 (Energy stored in a Capacitor)

(b) Refer to Sol. 27 on page 101.

69. $V = \dfrac{q}{4\pi\varepsilon_0 r} = \dfrac{9\times10^9 \times 5\times10^{-7}}{11\times10^{-12}} = 40.9$ kV

70. Potential, $V = 20$ V

71. As, $E = \dfrac{dV}{dr}$

$\Rightarrow \quad 3\times10^6 = \dfrac{\Delta V}{5\times10^{-3}}$

$\Rightarrow \quad \Delta V = 15000$ V

72. Refer to Example 6 on page 65 and use $U = \dfrac{kq_1q_2}{r}$

73. Refer to Example 15 on page 72.

74. (i) Refer to text on page 73.

(ii) Apply

$$\frac{1}{2}mv^2 = eV$$

$\Rightarrow \quad v = 2.77\times10^6$ m/s

75. Given, $C = 200\,\mu$F, $d = 5$ mm, $t = 5$ mm, $V = 100$ V

(i) $C = \dfrac{\varepsilon_0 A}{d} \Rightarrow A = \dfrac{Cd}{\varepsilon_0}$

$$A = \frac{200\times10^{-6}\times5\times10^{-3}}{8.85\times10^{-12}}$$

$$= 112.99\times10^3\ \text{m}^2$$

When $d' = 2d$, then $C' = \dfrac{\varepsilon_0 A}{2d - t + \dfrac{t}{K}}$

$$= \frac{8.85\times10^{-12}\times112.99\times10^3}{\left(10-5+\dfrac{5}{10}\right)\times10^{-3}}$$

$$= 181.8\times10^{-6} = 181.8\,\mu\text{F}$$

(ii) Charge on capacitor, $q = C_0V_0$

$$= 200\times10^{-6}\times100$$

$$= 2\times10^{-2}\ \text{C}$$

$\Rightarrow \quad C_0V_0 = C'V'$

or $\quad V' = \dfrac{C_0V_0}{C'} = \dfrac{2\times10^{-2}}{181.8\times10^{-6}} = 110$ V

$$E_0 = \frac{V_0}{d} = \frac{100}{5\times10^{-3}} = 20\times10^3\ \text{V/m}$$

$$E' = \frac{V'}{2d} = \frac{110}{10\times10^{-3}} = 11\times10^3\ \text{V/m}$$

(iii) $\overline{U} = \dfrac{1}{2}\varepsilon_0 E_0^2 = \dfrac{1}{2}\times8.85\times10^{-12}\times(20\times10^3)^2$

$$= 1770\times10^{-6}\ \text{J/m}^3$$

$$(U)' = \frac{1}{2}\times\varepsilon_0(E')^2$$

$$= \frac{1}{2}\times8.85\times10^{-12}\times(11\times10^3)^2$$

$$= 535.42\times10^{-6}\ \text{J/m}^3$$

76. Given $C = 100\,\mu$F, $d = 4$ mm, $t = 4$ mm, $V = 200$ V

(i) $d' = 2d$, $k = E_r = 5$

$$C = \frac{E_0 A}{d}$$

$$\Rightarrow 100\times10^{-6} = \frac{8.85\times10^{-12}\times A}{4\times10^{-3}}$$

$\Rightarrow \quad A = 45.2\times10^3$ m^2

$$C' = \frac{E_0 A}{2d - t + \dfrac{t}{k}}$$

$$= \frac{8.85\times10^{-12}\times45.2\times10^3}{\left(8-4+\dfrac{4}{5}\right)\times10^{-3}}$$

$$= 83.33\,\mu\text{F}$$

(ii) Charge on capacitor, when 200 V is applied

$$q = C_0V_0 = 100\times10^{-6}\times200 = 2\times10^{-2}\text{C}$$

Even after the battery is removed, the charge of 2×10^{-2} C on the capacitor plate remains same.

So, $C_0V_0 = C'V'$

$$\Rightarrow V' = \frac{C_0V_0}{C'} = \frac{2\times10^{-2}}{83.33\times10^{-6}} = 240\ \text{V}$$

$$E_0 = \frac{V_0}{d} = \frac{200}{4\times10^{-3}} = 50\times10^3\ \text{V/m}$$

$$E' = \frac{V'}{2d} = \frac{240}{2\times4\times10^{-3}}$$

$$= 30\times10^{-3}\ \text{V/m}$$

(iii) $\overline{U} = \dfrac{1}{2}\varepsilon_0E_0^2 = \dfrac{1}{2}\times8.85\times10^{-12}\times(50\times10^3)^2$

$$= 11067\times10^{-6}\ \text{J/m}^3$$

$$(\overline{U})' = \frac{1}{2}\varepsilon_0(E')^2$$

$$= \frac{1}{2}\times8.85\times10^{-12}\times(30\times10^3)^2$$

$$= 3982.5\times10^{-6}\ \text{J/m}^3$$

77. C_1 and C_2 are in parallel combination, so

$$C' = C_1 + C_2 = 3 + 6 = 9\,\mu\text{F}$$

Now, C', C_3 and C_4 are in series, so net capacitance is

$$\frac{1}{C} = \frac{1}{C'} + \frac{1}{C_3} + \frac{1}{C_4} = \frac{1}{9} + \frac{1}{4} + \frac{1}{12}$$

$$= \frac{16}{36} \Rightarrow C = \frac{9}{4}\,\mu\text{F}$$

[redacted]

(b) (i) Given, $Q_1 = 6\mu C$

Now, potential across C_1,

$$V = \frac{Q_1}{C_1} = \frac{6}{3} = 2\text{ V}$$

Thus charge on C_2,

$$Q_1 = C_2 V = 6 \times 2 = 12\mu C$$

Total charge on C_1 and C_2, $Q = 12 + 6 = 18\mu C$

As charge is same in series combination,

$\therefore$ Charge on C_3 and C_4 is 18 μC each.

(ii) Total capacitance,

$$\frac{1}{C''} = \frac{1}{4} + \frac{1}{12} = \frac{1}{3}$$

$\Rightarrow$ $C'' = 3\mu F$

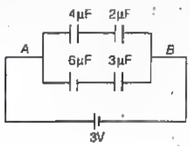
Thus, total energy stored in them is

$$U = \frac{1}{2}\frac{Q^2}{C''}$$

$$= \frac{1}{2}\frac{(18)^2}{3} \times 10^{-6}$$

$$= 54 \times 10^{-6}\text{ J}$$

78. As the given network is like a balanced Wheatstone bridge, so no current flows through the middle wire and the network becomes as shown



So, equivalent capacitance of upper arm (series combination) is

$$C_1 = \frac{4 \times 2}{4 + 2} = \frac{8}{6} = \frac{4}{3}\mu F$$

and of lower arm is

$$C_2 = \frac{6 \times 3}{6 + 3} = \frac{18}{9} = 2\mu F$$

$\therefore$ The net capacitance of the network is

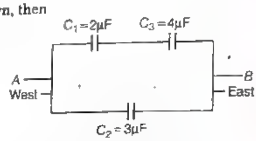
$$C_{net} = C_1 + C_2 \text{ (for parallel combination)}$$

$$= \frac{4}{3} + 2 = \frac{10}{3}\mu F$$

$\therefore$ Total charge, $Q = C_{net} \times V$

$$= \frac{10}{3} \times 3 = 10\mu C$$

79. (a) When C_1 and C_3 are in series and C_2 is in parallel as shown, then



$$C_{eq} = \frac{2 \times 4}{2 + 4} + 3 = \frac{4}{3} + 3 = \frac{13}{3}\mu F$$

(b) Maximum value of the equivalent capacitance is obtained when all capacitors are connected in parallel combination,

i.e. $C_{eq} = C_1 + C_2 + C_3 = 2 + 3 + 4 = 9\mu F$

or $C_{eq} = C_{max} = 9\mu F$

and minimum value is obtained, when all capacitors are connected in series combination,

i.e. $$C_{eq} = \frac{C_1 \times C_2 \times C_3}{C_1C_2 + C_2C_3 + C_3C_1}$$

$$= \frac{2 \times 3 \times 4}{2 \times 3 + 3 \times 4 + 4 \times 2}$$

or $C_{eq} = C_{min} = \frac{12}{13}\mu F$

80. Refer to Example 2 on page 87. [Ans. 560 km²]

81. $\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

$C_{eq.} = C' + C_4 = 133\mu F$

82. Refer to Q. 53 on page 98.

(i) 5 μF, 20 μF (ii) 1 : 4

[redacted]

03

In electrostatics, we have studied about the charges at rest. Here, we will study about charges in motion which constitute an electric current. Such electric currents occur naturally in many situations, e.g. in lightning, charges flow from the clouds to the earth through the atmosphere, which sometimes becomes disastrous, as the flow of charges is not steady. However, in our everyday life we see many devices such as a torch, a cell-driven clock, etc., where charges flow in steady manner.

CURRENT ELECTRICITY

|TOPIC 1|

Electric Current and Ohm's Law

If we maintain the constant potential difference between two conductors, we get a constant flow of charge in a metallic wire connecting the two conductors. The flow of charge in metallic wire constitutes electric current. The branch of physics which deals with the charges in motion is called current electricity.

CHAPTER CHECKLIST

- Electric Current and Ohm's Law
- Electrical Energy
- Cells, EMF and Internal Resistance
- Kirchhoff's Laws and its Applications

ELECTRIC CURRENT

It is defined as the rate of flow of electric charge through any cross-section of a conductor. It is denoted by I. Electric current can be expressed by

$$I = \frac{\text{Total charge flowing } (q)}{\text{Time taken } (t)}$$

If the charge dq flows through a conductor for small time dt, then $I = \dfrac{dq}{dt}$.

It means that the current through a conductor at a time is defined as the first derivative of charge passing through a cross-section of the conductor in a particular direction with respect to time.

If n = number of charges, e = electric charge and t = time,

then $$\boxed{I = \frac{q}{t} = \frac{ne}{t}} \qquad [\text{here, } q = ne]$$

Conventionally, the direction of electric current is along the direction of motion of positive charges and opposite to the direction of motion of negative charges.

Electric current is not always steady and hence more generally, we can define the current as follows

[redacted]

Let Δq be the net charge flowing through a cross-section of the conductor in a particular direction during the time interval Δt [i.e. between times t and $(t+\Delta t)$]. Then, at time t, the current in the conductor is given by

$$dI(t) = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t}$$

Current is a scalar quantity. Although it represents the direction of flow of positive charges, it does not follow the laws of vector addition (since the angle between the wires carrying current does not affect the total current). It follows the laws of scalar addition.

Unit of Electric Current

The SI unit of current is ampere and it is represented by A.

$$1 \text{ ampere (A)} = \frac{1 \text{ coulomb (C)}}{1 \text{ second (s)}} = 1 \text{ coulomb per second}$$

or $1\ Cs^{-1}$. Current through a wire is said to be one ampere, if a charge of one coulomb flows through any cross-section of the wire in one second.

EXAMPLE |1| How many electrons pass through a lamp in 1 min, if the current is 300 mA? Given, the charge on an electron is 1.6×10^{-19} C.

Sol. Given, current, $I = 300\text{ mA} = 300 \times 10^{-3}$ A

Charge on one electron, $e = 1.6 \times 10^{-19}$ C

Time, $t = 1\text{ min} = 60$ s

Charge passing through a lamp in 1 min, $q = I \times t$

$= 300 \times 10^{-3} \times 60$

Let n electrons pass through the lamp in 1 min

$$\therefore \quad q = ne \Rightarrow n = \frac{q}{e} = \frac{300 \times 10^{-3} \times 60}{1.6 \times 10^{-19}}$$

$$= 1.125 \times 10^{20} \text{ C}$$

> **Important Points Related to Flow of Current**
>
> As a matter of convention, the direction of flow of positive charge gives the direction of current. This is called conventional current. The direction of flow of electrons gives the direction of electronic current. Therefore, the direction of electronic current is opposite to that of conventional current.
>
> If the current varies with time, it is represented by differential limit of q, i.e. $I = \frac{dq}{dt}$. Further, I is same, even when cross-sectional area is different at different points of the conductor.
>
> Through a cross-section of the conductor in a time t, if a positive charge q_1 is flowing from A to B and a negative charge q_2 is flowing from B to A, then total current through the conductor is given by
>
> $$I = \frac{q_1}{t} + \frac{q_2}{t} = \frac{q_1 + q_2}{t}$$

The electric current, which flows during the lightning, is of the order of tens of thousands of amperes. However, the current in our nerves is in microamperes.

Current Density

The current density at a point in a conductor is the ratio of the current at that point in the conductor to the area of cross-section of the conductor at that point. If a current I is distributed uniformly over the cross-section A of a conductor, then the current density at that point is

$$\boxed{J = \frac{I}{A}}$$

It is a characteristic property of a point inside the conductor. It is a vector quantity. Its direction at a point is the direction of flow of positive charge at that point.

The SI unit of current density is Am^{-2}.

EXAMPLE |2| An aluminium wire of diameter 0.24 cm is connected in series to a copper wire of diameter 0.16 cm. The wires carry an electric current of 10A. Determine the current density in aluminium wire.

Sol. Given, diameter = 0.24 cm,

radius, $r = \frac{0.24 \times 10^{-2}}{2} = 0.12 \times 10^{-2}$ m

and current, $I = 10$ A

$\therefore$ Current density, $J = \frac{I}{A} = \frac{I}{\pi r^2}$

$$= \frac{10}{3.14 \times (0.12 \times 10^{-2})^2} = 2.2 \times 10^6 \text{ Am}^{-2}$$

Electric Current in Conductors

All metals are good conductors of electricity. The electric conduction in them can be explained by an electron theory. In an atom of a substance, the electrons in the orbits close to the nucleus are bound to it due to the strong attraction of the nuclear positive charge, but the electrons far from the nucleus experience a very weak attractive force.

Hence, the outer electrons can be removed easily from the atom (by rubbing or by heating the substance). In fact, a few outer electrons leave their atoms and move freely within the substance (in the vacant spaces between the atoms). These electrons called **free electrons** or **conduction electrons**. They carry the charge in the substance from one place to the other. Therefore, the electrical conductivity of a solid substance depends upon the number of free electrons in it. In metals, this number is quite large ($\approx 10^{29}/\text{m}^3$). Hence, metals are good conductors of electricity. Silver is the best conductor of electricity than copper, gold and aluminium, respectively.

[redacted]

In liquids and gases, electric conduction takes place by the movement of both positive and negative charge carriers unlike in metals, where the electric conduction occurs by the movement of negative charge carriers (electrons) only.

In case of a liquid conductor such as electrolytic solution, there are positive and negative charged ions, which can move on applying electric field, and hence generating the electric current. Whereas in case of a solid conductor (i.e. Cu, Fe, Ag, etc.) atoms are tightly bound to each other. They consist of large number of free electrons, which are responsible for the strong current in them when electric field is applied on them.

There are some other materials in which the electrons will be bound and they will not be accelerated, even if the electric field is applied, i.e. no current on applying electric field. Such materials are called **insulators**. e.g. Wood, plastic, rubber, etc.

In our discussions, we will focus only on solid conductors in which the positive ions are at fixed positions and the current is carried by the negatively charged electrons.

> ***Flow of Electric Charge***
>
> When no electric field is applied on a solid conductor, the free electrons in them move like molecules in a gas due to their thermal velocities. There is no preferential direction for the velocities of the electrons. The average thermal random velocity is zero. Due to this, there is no net flow of electric charge in a particular direction inside the conductor and hence no current flows in it.
>
> When an electric field is applied on a solid conductor, in the shape of a cylinder of circular cross-section by attaching positively and negatively charged circular discs of a dielectric of the same radius as that of the solid conductor, at the two ends, an electric field is generated in the conductor from positive charged disc towards negative charged disc.
>
> Electrolytic solutions are conductors of different types in which positive and negative charges can move.

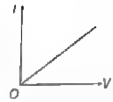
OHM'S LAW

It states that the current I flowing through a conductor is always directly proportional to the potential difference V across the ends of the conductor, provided that the physical conditions (temperature, mechanical strain, etc.) are kept constant. Mathematically,

$$I \propto V \quad \text{or} \quad V \propto I \quad \text{or} \quad \boxed{V = IR}$$

where, R is resistance of the conductor.

Its value depends upon the length, shape and the nature of the material of the conductor. It is independent of the values of V and I. Such conductors are said to obey Ohm's law. For an ohmic conductor, the graph between V and I is a straight line as shown in the figure.



EXAMPLE |3| A potential difference of 3 V is applied across a conductor through which 5 A of current is flowing. Determine the resistance of the conductor.

Sol. Given, potential difference, $V = 3\text{ V}$

Current, $I = 5\text{A}$

According to Ohm's law, $R = \dfrac{V}{I} \Rightarrow R = \dfrac{3}{5} = 0.6\,\Omega$

Resistance of a Conductor

It is defined as the ratio of the potential difference applied across the ends of the conductor to the current flowing through it.

Mathematically, $R = \dfrac{V}{I}$

The SI unit of resistance is ohm and denoted by Ω.

$$1\text{ ohm }(\Omega) = \frac{1\text{ volt (V)}}{1\text{ ampere (A)}} = 1\text{ volt/ampere (or V/A)}$$

The resistance of a conductor is said to be **one ohm**, if one ampere of current flows, when a potential difference of one volt is applied across the ends of the conductor.

Dimensional formula of electrical resistance is $[ML^2T^{-3}A^{-2}]$.

The resistance of the conductor depends upon the following factors

(i) It is directly proportional to the length of the conductor

i.e. $R \propto l$ ...(i)

(ii) It is inversely proportional to the area of the cross-section of the conductor.

i.e. $R \propto \dfrac{1}{A}$ ...(ii)

From Eqs. (i) and (ii), we get

$$R \propto \frac{l}{A} \quad \text{or} \quad \boxed{R = \rho\frac{l}{A}} \quad \text{...(iii)}$$

where, ρ is the constant of proportionality known as resistivity or specific resistance of the conductor.

(iii) It depends upon the nature of the material and temperature of the conductor.

[redacted]

EXAMPLE |4| A negligible small current is passed through a wire length 15 m and uniform cross-section $6\times 10^{-7}\,m^2$ and its resistance is measured to be 5 Ω. What is the resistivity of the material at the temperature of the experiment? **NCERT**

Sol. Given, $R = 5\Omega$, $A = 6\times10^{-7}\,m^2$ and $l = 15\,m$

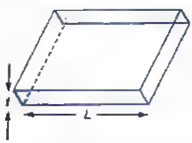
Let the resistivity of the material be ρ.

$\therefore$ Resistance of wire, $R = \rho\dfrac{l}{A}$

$$\Rightarrow \quad \rho = \frac{RA}{l} = \frac{5\times 6\times 10^{-7}}{15} = 2\times 10^{-7}\,\Omega\text{-m}$$

Thus, the resistivity of the material at the temperature of the experiment is $2\times10^{-7}\,\Omega$-m.

EXAMPLE |5| Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure. **(2010)**



Sol. Resistance between the shaded opposite faces is

$$R = \frac{\rho(L)}{A} = \frac{\rho L}{tL} = \frac{\rho}{t}$$

Note Here, R is independent of L.

Effect of Temperature on Resistance

Resistance of a metallic conductor at temperature t°C is given as

$$R_t = R_0\,(1+\alpha t+\beta t^2) \qquad \ldots(i)$$

where, R_0 = resistance of conductor at 0°C and α, β are the temperature coefficients of resistance.

If the temperature t°C is not sufficiently large which is so in most of the practical cases, then Eq. (i) can be written as

$$R_t = R_0(1+\alpha t) \text{ or } \boxed{\alpha = \frac{R_t - R_0}{R_0\times t}}$$

$$= \frac{\text{Change in resistance}}{\text{Original resistance}\times\text{Rise of temperature}}$$

Therefore, temperature coefficient of resistance is defined as the increase in resistance per unit original resistance per degree celsius or kelvin rise of temperature.

For metals, the value of α is positive, therefore, resistance of the metal increases with rise in temperature.

For insulators and semiconductors, the value of α is negative, therefore, resistance decreases with rise in temperature.

For alloys, the value of α is very small. The value of α is different at different temperatures. Temperature coefficient of resistance averaged over the temperature range t_1°C to t_2°C is given by

$$\boxed{\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}}$$

where, R_1 and R_2 are the resistances at t_1°C and t_2°C, respectively.

EXAMPLE |6| A silver wire has a resistance of 2.1 Ω at 27.5 °C and a resistance of 2.7 Ω at 100 °C. Determine the temperature coefficient of resistivity of silver. **NCERT**

Sol. Given, $t_1 = 27.5$°C, $t_2 = 100$°C

$R_1 = R_{27.5} = 2.1\,\Omega$ and $R_2 = R_{100} = 2.7\,\Omega$

$\therefore$ Temperature coefficient of silver is given by

$$\alpha = \frac{R_2 - R_1}{R_1\,(t_2 - t_1)} = \frac{2.7 - 2.1}{2.1\,(100 - 27.5)} = 0.0039\,°C^{-1}$$

DRIFT OF ELECTRONS AND THE ORIGIN OF RESISTIVITY

Drift Velocity

It is defined as the average velocity with which the free electrons in a conductor get drifted towards the positive end of the conductor under the influence of an electric field applied across the conductor. It is denoted by v_d. The drift velocity of electron is of the order of $10^{-4}\,ms^{-1}$.

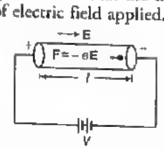
Let V be the potential difference applied across the ends of a conductor of length l, then the magnitude of electric field is

$$E = \frac{V}{l}$$

The direction of electric field is from positive to negative end of conductor as shown in figure. Since, the charge on an electron is $-e$ and each free electron in the conductor experiences a force F.

$$\therefore \quad \mathbf{F} = -e\mathbf{E}$$

Here, negative sign indicates that the direction of force is opposite to that of electric field applied.



Current in a metallic conductor

[redacted]

Acceleration of each electron is given by

$$a = -\frac{eE}{m} \quad \left[\text{from Newton's second law, } a = \frac{F}{m}\right]$$

where, m is mass of the electron.

Under the effect of applied electric field, the free electrons accelerate and acquire a velocity component in a direction opposite to the direction of electric field in addition to their thermal velocities.

However, this gain in velocity of electron due to the electric field is very small and it is lost in the next collision with the ion/atom of the conductor.

At any instant of time, the velocity acquired by electron having thermal velocity $\mathbf{u}_1$ is given by

$$\mathbf{v}_1 = \mathbf{u}_1 + \mathbf{a}\, t_1$$

where, t_1 is the time elapsed as it has suffered its last collision with ion atom of conductor.

Similarly, $\mathbf{v}_2 = \mathbf{u}_2 + \mathbf{a}\, t_2, \ldots, \mathbf{v}_n = \mathbf{u}_n + \mathbf{a}\, t_n$

$\therefore$ Average velocity, $\mathbf{v}_d = \dfrac{\mathbf{v}_1 + \mathbf{v}_2 + \ldots + \mathbf{v}_n}{n}$

$$= \frac{(\mathbf{u}_1 + \mathbf{a}\, t_1) + (\mathbf{u}_2 + \mathbf{a}\, t_2) + \cdots + (\mathbf{u}_n + \mathbf{a}\, t_n)}{n}$$

$$= \frac{\mathbf{u}_1 + \mathbf{u}_2 + \ldots + \mathbf{u}_n}{n} + \frac{\mathbf{a}[t_1 + t_2 + \ldots + t_n]}{n}$$

We know that, $\dfrac{\mathbf{u}_1 + \mathbf{u}_2 + \ldots + \mathbf{u}_n}{n} = 0$

and $\dfrac{t_1 + t_2 + \ldots + t_n}{n}$ is called average time elapsed or average relaxation time and is denoted by τ. Its value is of the order of 10^{-14} s.

$$\therefore \quad \mathbf{v}_d = 0 + \mathbf{a}\tau \quad \text{or} \quad \mathbf{v}_d = \mathbf{a}\,\tau \quad \text{or} \quad \mathbf{v}_d = -\frac{e\mathbf{E}}{m}\tau$$

Negative sign shows that $\mathbf{v}_d$ is opposite to the direction of $\mathbf{E}$.

$$\therefore \quad \boxed{\text{Average drift velocity, } \mathbf{v}_d = \frac{e\mathbf{E}}{m}\tau}$$

Average relaxation time

= Mean free path of electron/drift speed of electron.

Relation between Drift Velocity and Electric Current

Consider a conductor of length l and A be the uniform area of cross-section.

$\therefore$ Volume of conductor $= Al$

If the conductor contains free electrons n per unit volume.

Then, number of free electrons in the conductor $= nAl$

If e is the charge of an electron, then total charge on all free electrons in the conductor is given by

$$q = nAle \qquad \ldots\text{(i)}$$

Time taken by the free electrons to cross the length of the conductor

$$t = \frac{l}{v_d} \qquad \ldots\text{(ii)}$$

Since, current is the rate of flow of the charge through conductor.

$$I = \frac{q}{t}$$

Using Eqs. (i) and (ii), we get

$$I = \frac{nAle}{l/v_d} \Rightarrow \boxed{I = neAv_d} \qquad \ldots\text{(iii)}$$

Eq. (iii) gives the relation between the current flowing through the conductor and drift velocity of the electron.

Putting the value of $v_d \left(= \dfrac{eE\tau}{m}\right)$ in Eq. (iii), we get

$$\boxed{I = \frac{Ane^2\tau E}{m}}$$

EXAMPLE [7] **Estimate the average drift speed of conduction electrons in a copper wire of cross-sectional area 1.0×10^{-7} m^2 carrying a current of 1.5 A. Assume the density of conduction electrons to be 9×10^{28} m^{-3}.**

All India 2014

Sol. Given, cross-sectional area, $A = 1.0 \times 10^{-7}$ m^2

Current, $I = 1.5$A

Electron density, $n = 9 \times 10^{28}$ m^{-3}

Drift velocity, $v_d = ?$

We know that, $I = neAv_d$

$$\Rightarrow \quad v_d = \frac{I}{neA} = \frac{1.5}{9 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.0 \times 10^{-7}}$$

$$= 1.042 \times 10^{-3} \text{ m/s}$$

Deduction of Ohm's Law

We know that, $I = \dfrac{Ane^2\tau E}{m}$ and $V_d = \dfrac{eE}{m}.\tau = \left(\dfrac{eV}{ml}\tau\right)$

$$\left[\text{as } E = \frac{V}{l}\right]$$

So, $$I = Anev_d = Ane\left(\frac{eV}{ml}\tau\right) = \left(\frac{Ane^2\tau}{ml}\right)V$$

or $$\frac{V}{I} = \frac{ml}{Ane^2\tau} = R = \text{constant}$$

[redacted]

$$\therefore \quad \frac{V}{I} = R \Rightarrow \boxed{V = RI}$$

This is called Ohm's law.

Mobility

It is defined as the magnitude of drift velocity of charge per unit electric field applied. It is expressed as

$$\mu = \frac{\text{Drift velocity } (v_d)}{\text{Electric field } (E)} = \frac{qE\tau / m}{E} = \frac{q\tau}{m}$$

where, τ is the average relaxation time of the charge while drifting towards the opposite electrode and m is the mass of the charged particle.

Mobility of electrons, $\mu_e = \frac{e\tau_e}{m_e}$.

and mobility of holes, $\mu_h = \frac{e\tau_h}{m_h}$.

where, τ_e and τ_h are average relaxation time for electrons and holes, respectively. m_e and m_h refer to mass of electrons and holes, respectively. Charge on either is e.

The SI unit of mobility is $m^2s^{-1}V^{-1}$ or $m s^{-1}N^{-1}C$ and it is in the order of 10^4. The practical unit of mobility is $(Cm^2/V\text{-}s)$. Mobility is a positive quantity.

The total current in the conducting material is the sum of the currents due to the positive current carriers (holes) and negative current carriers (electrons).

EXAMPLE |8| Find the current flow through a copper wire of length 0.2 m, area of cross-section 1 mm^2, when connected to a battery of 4 V. Given that, electron mobility is $4.5 \times 10^{-6} m^2s^{-1}V^{-1}$ and charge on an electron is 1.6×10^{-19}C. The number density of electrons in copper wire is $8.5 \times 10^{28}\ m^{-3}$.

Sol. Given, length of copper wire, $l = 0.2$ m

Cross-sectional area, $A = 1\ mm^2 = 10^{-6}\ m^2$

Potential difference, $V = 4$ V

Electron mobility, $\mu = 4.5 \times 10^{-6}\ m^2s^{-1}V^{-1}$

Charge of an electron, $e = 1.6 \times 10^{-19}$ C

Number density of electrons, $n = 8.5 \times 10^{28} m^{-3}$

We know that electric field set up across the conductor,

$$E = \frac{V}{l} = \frac{4}{0.2} = 20 \text{ V/m}$$

$\therefore$ Current through the wire, $I = nAe\ v_d = nAe\mu E$ $\quad [\because \mu = v_d / E]$

$= 8.5 \times 10^{28} \times 10^{-6} \times 1.6 \times 10^{-19} \times 4.5 \times 10^{-6} \times 20$

$= 1.22$ A

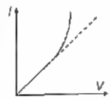
Limitations of Ohm's Law

The devices which do not obey Ohm's law are called non-ohmic devices, such as vacuum tubes, semiconductor diodes, transistors etc. The relation $\left(\frac{V}{I} = R\right)$ is valid for both, ohmic and non-ohmic devices.

For ohmic conductors, value of R is constant.

For non-ohmic devices, value of R is not constant, i.e. Ohm's law fails.

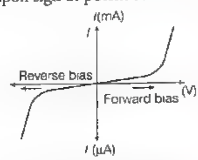
The limitations of Ohm's law are given below



Variation of potential difference with current

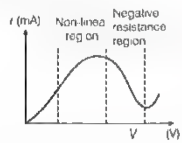
(i) Potential difference may vary non-linearly with current.

(ii) The variation of current with potential difference may depend upon sign of potential difference applied.



Variation of current according to the sign of potential difference

(iii) The relation between potential difference and current is not unique, i.e. there is more than one value of V for the same current I.



Graph of current versus potential difference for GaAs

RESISTIVITY OF VARIOUS MATERIALS

Specific resistance or resistivity of the material of a conductor is defined as the resistance of a unit length with unit area of cross-section of the material of the conductor, i.e. it is also defined as the resistance of unit cube of

[redacted]

material of the given conductor. The unit of resistivity is ohm-metre or Ω-m and its dimensional formula is $[ML^3T^{-3}A^{-2}]$.

Since, we know that $R = \rho \dfrac{l}{A} \Rightarrow \rho = \dfrac{RA}{l}$ ...(i)

Substituting the value of $R = \dfrac{ml}{ne^2 A\tau}$ in Eq. (i),

We have, $\rho = \left(\dfrac{ml}{ne^2 A\tau}\right) \cdot \dfrac{A}{l}$

∴ Resistivity of the material, $\boxed{\rho = \dfrac{m}{ne^2\tau}}$

From the above formula, it is clear that resistivity of a conductor depends upon the following factors:

(i) $\rho \propto \dfrac{1}{n}$, i.e. resistivity of a material is inversely proportional to the number density of free electrons (number of free electrons per unit volume).

As the free electron density depends upon the nature of material, so resistivity of a conductor depends on the nature of the material.

(ii) $\rho \propto 1/\tau$, i.e. resistivity of a material is inversely proportional to the average relaxation time τ of free electrons in the conductor.

As value of τ depends on the temperature of conductor, so resistivity of a conductor changes with temperature, as temperature increases, τ decreases, hence ρ increases.

Resistivity of Different Materials

Name of the materials	Resistivity at 0°C (Ω-m)	Name of the materials	Resistivity at 0°C (Ω-m)
1. Conductors		2. Semiconductors	
(i) Metals		Carbon	3.5×10^{-5}
Silver	1.6×10^{-8}	Germanium	0.46
Copper	1.7×10^{-8}	Silicon	2300
Aluminium	2.7×10^{-8}	3. Insulators	
Tungsten	5.6×10^{-8}	Glass	10^{10}-10^{14}
Iron	10×10^{-8}	Hard rubber	10^{13}-10^{16}
Platinum	11×10^{-8}	Mica	10^{11}-10^{15}
Mercury	98×10^{-8}	Wood	10^{8}-10^{11}
(ii) Alloys		Amber	5×10^{14}
Nichrome	$\sim 100 \times 10^{-8}$		
Manganin	48×10^{-8}		
Constantan	49×10^{-8}		

EXAMPLE |9| Find the time of relaxation between collision and free path of electrons in copper at room temperature.

(Given, resistivity of copper $= 1.7 \times 10^{-8}$ Ω-m, density of electrons in copper $= 8.5 \times 10^{28}\ m^{-3}$, charge on an electron $= 1.6 \times 10^{-19}$ C, mass of electron $= 9.1 \times 10^{-31}$ kg and drift velocity of free electrons $= 1.6 \times 10^{-4}\ ms^{-1}$)

Sol. Given, $\rho = 1.7 \times 10^{-8}$ Ω-m, $n = 8.5 \times 10^{28}\ m^{-3}$,

$e = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg

and $v_d = 1.6 \times 10^{-4}\ ms^{-1}$

We know that, $\rho = \dfrac{m_e}{ne^2\tau}$

∴ Relaxation time,

$$\tau = \frac{m_e}{e^2 n\rho} = \frac{9.1 \times 10^{-31}}{(1.6 \times 10^{-19})^2 \times 8.5 \times 10^{28} \times 1.7 \times 10^{-8}}$$

$$= 2.5 \times 10^{-14}\ s$$

Mean free path of electron (distance covered between two collisions)

$$= v_d\tau = 1.6 \times 10^{-4} \times 2.5 \times 10^{-14} = 4 \times 10^{-18}\ m$$

Temperature Dependence of Resistivity

Resistivity of a metal conductor is given by

$$\rho = \rho_0[1 + \alpha(T - T_0)] \quad \text{...(i)}$$

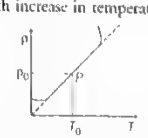
where, ρ = resistivity at temperature T,

and ρ_0 = resistivity at temperature T_0

$$\Rightarrow \quad \alpha = \frac{\rho - \rho_0}{\rho_0(T - T_0)} = \frac{d\rho}{\rho_0} \cdot \frac{1}{dT}$$

Thus, temperature coefficient of electrical resistivity is also defined as the fractional change in electrical resistivity $\dfrac{d\rho}{\rho_0}$ per unit change in temperature dT.

For metals, the value of α is positive, therefore resistivity of metal increases with increase in temperature.

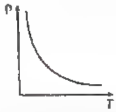


Resistivity as a function of temperature for metals

Eq.(i) implies that a graph of ρ plotted against T would be a straight line. At temperatures much lower than 0°C, the graph deviates considerably from a straight line. Eq.(i) can be used approximately over a limited range of T around any reference temperature T_0, where the graph can be approximated as a straight line.

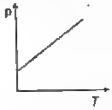
[redacted]

For semiconductors, the resistivity decreases with increase in temperature.



Resistivity as a function of temperature for semiconductors

For alloys, the resistivity is very large, but has weak dependence on temperature.



Resistivity as a function of temperature for alloys

Electric fuse is made of an alloy of zinc, copper, silver and aluminium. This is because alloys have low resistivity. This causes the wire to melt, if a current more than safe current flows through the circuit.

CONDUCTANCE AND CONDUCTIVITY

Conductance

It is defined as the reciprocal of resistance of a conductor. It is expressed as

$$\boxed{G = \frac{1}{R}}$$

Its SI unit is mho (Ω^{-1}) or siemen (S).

The dimensional formula of conductance is $[M^{-1}L^{-2}T^{3}A^{2}]$.

Conductivity

It is defined as the reciprocal of resistivity of a conductor. It is expressed as

$$\boxed{\sigma = \frac{1}{\rho}}$$

The SI unit is mho per metre ($\Omega^{-1}\,m^{-1}$) or siemen per metre (S/m).

The dimensional formula of conductivity is $[M^{-1}L^{-3}T^{3}A^{2}]$.

EXAMPLE |10| **A wire carries a current of 0.5 A, when a potential difference of 1.5 V is applied across it. What is its conductance? If the wire is of length 3m and area of cross-section 5.4 mm², then calculate its conductivity.**

Sol. Here, $I = 0.5$ A, $V = 1.5$ V, $l = 3$ m,

$$A = 5.4\ \text{mm}^2 = 5.4 \times 10^{-6}\ \text{m}^2$$

$\therefore$ New resistance, $R = \frac{V}{I} = \frac{1.5}{0.5} = 3\Omega$

$\therefore$ Conductance, $G = \frac{1}{R} = \frac{1}{3} = 0.33$ S

and electrical conductivity,

$$\sigma = \frac{1}{\rho} = \frac{l}{RA} = \frac{3}{3 \times 5.4 \times 10^{-6}} = 1.85 \times 10^{5}\ \text{Sm}^{-1}$$

Relation between J, σ and E (Microscopic form of Ohm's Law)

Since, the relation between the current flowing through the conductor and drift velocity of electron is given by

$$I = nAe\,v_d$$

$$\therefore \quad I = nAe\left(\frac{eE}{m}\tau\right) = \frac{nAe^2\tau E}{m}$$

$$\Rightarrow \quad \frac{I}{A} = \frac{ne^2\tau E}{m} \Rightarrow J = \frac{ne^2\tau E}{m} \qquad \left[\because J = \frac{I}{A}\right]$$

$$\text{or} \quad J = \frac{1}{\rho}E \qquad \left[\because \rho = \frac{m}{ne^2\tau}\right]$$

$$\therefore \quad \boxed{J = \sigma E} \qquad \left[\because \sigma = \frac{1}{\rho}\right]$$

It is a microscopic form of Ohm's law.

Classification of Materials in Terms of Conductivity

On the basis of conductivity, the materials can be classified into the following categories

Insulators

These are materials whose electrical conductivity is either very small or nil, e.g. glass, rubber, etc.

Conductors

These are materials whose electrical conductivity is very high, e.g. silver, aluminium, etc.

Semiconductors

These are those materials whose electrical conductivity lies in between that of insulators and conductors, e.g. germanium, silicon, etc.

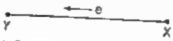
Note **Thermistor** A thermistor is a heat sensitive device whose resistivity changes very rapidly with change of temperature.
A thermistor can have a resistance in the range of 0.1 Ω to 10^{7} Ω depending upon its composition.
Superconductivity The resistivity of certain metal or alloy drops to zero, when they are cooled below a certain temperature is called superconductivity. It was observed by Prof. Kamerlingh in 1911.

[redacted]

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. Twenty million electrons reaches from point X to point Y in two micro second as shown in the figure. Direction and magnitude of the current is



(a) 1.5×10^{-10} A from X to Y
(b) 1.6×10^{-6} A from Y to X
(c) 1.5×10^{-13} A from Y to X
(d) 1.6×10^{-4} A from X to Y

2. The relation between electric current density (J) and drift velocity (v_d) is

(a) $J = nev_d$ (b) $J = \frac{ne}{v_d}$
(c) $J = \frac{v_d e}{n}$ (d) $J = nev_d^2$

where, e is the charge of electron and n is the number of electrons.

3. If drift velocity of electron is v_d and intensity of electric field is E, then which of the following relation obeys the Ohm's law?

(a) v_d = constant (b) $v_d \propto E$
(c) $v_d = \sqrt{E}$ (d) $v_d \propto E^2$

4. Which of the following characteristics of electrons determines the current in a conductor? **NCERT Exemplar**

(a) Drift velocity alone
(b) Thermal velocity alone
(c) Both drift velocity and thermal velocity
(d) Neither drift nor thermal velocity

5. The dimensional formula of resistance is

(a) $[ML^2T^{-2}A^{-2}]$ (b) $[M^2L^3T^{-2}A^{-2}]$
(c) $[ML^2T^{-3}A^{-2}]$ (d) $[ML^3T^{-3}A^{-3}]$

6. The resistance of a 10 m long wire is 10Ω. Its length is increased by 25% by stretching the wire uniformly.
The resistance of wire will change to

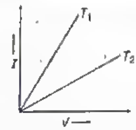
(a) 12.5 Ω (b) 14.5 Ω
(c) 15.6 Ω (d) 16.6 Ω

7. Multiplication of resistivity and conductivity of any conductor depends on

(a) cross-section (b) temperature
(c) length (d) None of these

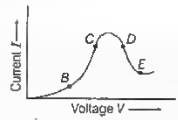
VERY SHORT ANSWER Type Questions

8. I-V graph for a metallic wire at two different temperatures T_1 and T_2 is as shown in the figure below.



Which of the two temperatures is lower and why? **Delhi 2015**

9. Graph showing the variation of current *versus* voltage for a material GaAs is shown in the figure. Identify the region of



(i) negative resistance.
(ii) where Ohm's law is obeyed. **All India 2015**

10. Why are alloys used for making standard resistance coils? **NCERT Exemplar**

11. Two materials Si and Cu, are cooled from 300 K to 60 K. What will be the effect on their resistivity? **Foreign 2013**

12. When electrons drift in a metal from lower to higher potential, does it mean that all the free electrons of the metal are moving in the same direction? **Delhi 2012**

13. Define the term mobility of charge carriers in a conductor. Write its SI unit. **Delhi 2014**

14. The relaxation time τ is nearly independent of applied E field, whereas it changes significantly with temperature T. First fact is (in part) responsible for Ohm's law, whereas the second fact leads to variation of ρ with temperature. Elaborate why? **NCERT Exemplar**

15. Is the motion of a charge across junction momentum conserving? Why or why not? **NCERT Exemplar**

[redacted]

16. Specific resistances of copper, silver and constantan are $1.78 \times 10^{-6}\ \Omega$-cm, $10^{-6}\ \Omega$-cm and $48 \times 10^{-6}\ \Omega$-cm, respectively. Which is the best conductor and why?

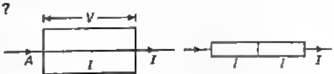
17. For wiring in the home, one uses Cu wires or Al wires. What considerations are involved in this? **NCERT Exemplar**

SHORT ANSWER Type Questions

18. What conclusion can you draw from the following observations on a resistor made of alloy manganin? **NCERT**

Current (in A)	Voltage (in V)	Current (in A)	Voltage (in V)
0.2	3.94	3.0	59.2
0.4	7.87	4.0	78.8
0.6	11.8	5.0	98.6
0.8	15.7	6.0	118.5
1.0	19.7	7.0	138.2
2.0	39.4	8.0	158.0

19. A metal rod of square cross-sectional area A having length l has current I flowing through it when a potential difference of V volt is applied across its ends (Fig. I). Now, the rod is cut parallel to its length into two identical pieces and joined as shown in Fig. II. What potential difference must be maintained across the length of $2l$, so that the current in the rod is still I?



(I) (II)

Foreign 2016

20. A conductor of length l is connected to a DC source of potential V. If the length of the conductor is tripled by gradually stretching it, keeping V constant, how will
(i) drift speed of electrons and
(ii) resistance of the conductor be affected? Justify your answer. **Foreign 2012**

21. Using the concept of drift velocity of charge carriers in a conductor, deduce the relationship between current density and resistivity of the conductor. **Delhi 2015 C**

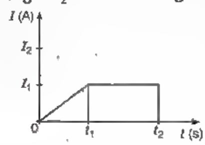
22. (i) A wire of resistivity ρ is stretched to three times its length. What will be its new resistivity?
(ii) In what manner, do the relaxation time in the good conductor change when its temperature increases?

23. Define mobility of a charge carrier. Write the relation expressing mobility in terms of relaxation time. Give its SI unit. **All India 2013**

24. Draw a plot showing the variation of resistivity of a (i) conductor and (ii) semiconductor, with the increase in temperature. How does one explain this behaviour in terms of number density of charge carriers and the relaxation time? **Delhi 2014C**

LONG ANSWER Type I Questions

25. (i) Deduce the relation between current I flowing through a conductor and drift velocity v_d of the electrons.
(ii) Figure shows a plot of current I flowing through the cross-section of a wire *versus* the time t. Use the plot to find the charge flowing in t_2 second through the wire.



26. Define relaxation time of the free electrons drifting in a conductor. How it is related to the drift velocity of free electrons? Use this relation to deduce the expression for the electrical resistivity of the material. **All India 2012**

27. Find the relation between drift velocity and relaxation time of charge carriers in a conductor.
A conductor of length L is connected to a DC source of emf E. If the length of the conductor is tripled by stretching it, keeping E constant, explain how its drift velocity would be affected. **Delhi 2015**

28. (i) Define the term of drift velocity.
(ii) On the basis of electron drift, derive an expression for resistivity of a conductor in terms of number density of free electrons and relaxation time. On what factors does resistivity of a conductor depend?

[redacted]

(iii) Why alloys like constantan and manganin are used for making standard resistors? Delhi 2016

29. Plot a graph showing temperature dependence of resistivity for a typical semiconductor. How is this behaviour explained? Delhi 2011

30. A conductor of length l is connected to a DC source of potential V. If the length of the conductor is tripled by gradually stretching it, keeping V constant, how will
   (i) drift speed of electrons and
   (ii) resistance of the conductor be affected? Justify your answer. Foreign 2012

31. (a) Define the term 'conductivity' of a metallic wire. Write its SI unit.
   (b) Using the concept of free electrons in a conductor, derive the expression for the conductivity of a wire in terms of number density and relaxation time. Hence, obtain the relation between current density and the applied electric field E. CBSE 2018

LONG ANSWER Type II Question

32. (i) Derive an expression for drift velocity of electrons in a conductor. Hence, deduce Ohm's law.
   (ii) A wire whose cross-sectional area is increasing linearly from its one end to the others, is connected across a battery of V volts. Which of the following quantities remain constant in the wire?
   (a) Drift speed (b) Current density
   (c) Electric current (d) Electric field
   Justify your answer. Delhi 2017

NUMERICAL PROBLEMS

33. Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter 1 mm. Conductor B is a hollow tube of outer diameter 2 mm and inner diameter 1 mm. Find the ratio of resistance R_A to R_B. NCERT Exemplar

34. A wire is stretched to increase its length by 5%. Calculate percentage change in its resistance.

35. At room temperature (27°C), the resistance of a heating element is 100 Ω. What is the temperature of the element, if the resistance is found to be 117 Ω, given that the temperature coefficient of the material of the resistor is $1.70\times 10^{-4}\,°C^{-1}$. NCERT

36. A heating element using nichrome connected to a 230 V supply draws an initial current of 3.2 A which settles after a few seconds to a steady value of 2.8 A. What is the steady temperature of the heating element, if the room temperature is 27°C? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is $1.70\times 10^{-4}\,°C^{-1}$. NCERT

37. A resistance coil marked 3Ω is found to have a true resistance of 3.115Ω at 300 K. Calculate the temperature at which marking is correct. Temperature coefficient of resistance of the material of coil is $4.2\times 10^{-3}\,°C^{-1}$. All India 2014

HINTS AND SOLUTIONS

1. (b) Given, number of electrons,
$$N = 20000000 = 2\times 10^7$$
Total charge on twenty million electrons is
$$q = Ne = 2\times 10^7 \times 1.6\times 10^{-19}\,C \quad (\because e = 1.6\times 10^{-19}\,C)$$
$$= 3.2\times 10^{-12}\,C$$
Now time taken by twenty million electrons to pass from point X to point Y is $t = 2\,\mu s = 2\times 10^{-6}\,s$
$$I = \frac{q}{t} = \frac{3.2\times 10^{-12}}{2\times 10^{-6}} = 1.6\times 10^{-6}\,A$$
Since, the direction of the current is always opposite to the direction of flow of electrons. Therefore due to flow of electrons from point X to point Y, the current will flow from point Y to point X.

2. (a) Current density, $J = \frac{I}{A}$, $I = neAv_d \Rightarrow J = nev_d$

3. (b) Drift velocity $v_d = -\frac{eE}{m}\tau \Rightarrow v_d \propto E$

4. (a) The relationship between current and drift speed is given by
$$I = neAv_d$$
Here, I is the current and v_d is the drift velocity.
So, $I \propto v_d$
Thus, only drift velocity determines the current in a conductor.

5. (c) Resistance, $R = \rho\frac{l}{A}$
$$= \frac{[ML^3T^{-3}A^{-2}][L]}{[L^2]} = [ML^2T^{-3}A^{-2}]$$

[redacted]

6. (c) Given, $l_1 = l + \frac{25}{100} l = \frac{5l}{4}$.

Since, volume of wire remains unchanged on increasing length, hence

$$\Rightarrow \begin{pmatrix} A_1 l_1 = Al \\ A_1 \times \frac{5l}{4} = Al \end{pmatrix} \text{ or } A_1 = 4A/5$$

Given, $R = \rho l / A = 10\Omega$ and $R_1 = \frac{\rho l_1}{A_1} = \frac{\rho 5l/4}{4A/5} = \frac{25\rho l}{16A}$

$$\therefore \quad R_1 = \frac{25}{16} \times 10 = \frac{250}{16} = 15.6\,\Omega$$

7. (d) Resistivity and conductivity of conductor depends on the nature of substance.

8. Since, slope of 1 > slope of 2

$\therefore \quad R_1 < R_2$

Also, we know that resistance is directly proportional to the temperature.

Therefore, $T_2 > T_1$.

9. (i) DE is the region, of negative resistance because the slope of curve in this part is negative.

(ii) BC is the region, where Ohm's law is obeyed because in this part, the current varies linearly with the voltage.

10. Alloys have small value of temperature coefficient of resistance with less temperature sensitivity. This keeps the resistance of wire almost constant even in small temperature change. Thus, alloy also has high resistivity for given length and cross-sectional area of conductor.

11. In silicon, the resistivity increases with decrease in temperature.

In copper, the resistivity decreases with decrease in temperature.

12. Yes, all the free electrons drift in the same direction.

13. Mobility of charge carriers inside conductor is defined as the magnitude of drift velocity of charge per unit electric field applied

SI unit of mobility is $m^2s^{-1}V^{-1}$ or $ms^{-1}N^{-1}C$.

14. Relaxation time is inversely proportional to the velocities of electrons and ions. The applied electric field produces the insignificant change in velocities of electrons at the order of 1 mm/s, whereas the change in temperature T affects velocities at the order of 10^2 m/s.

This decreases the relaxation time considerably in metals and consequently resistivity of metal or conductor increases as, $\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$

15. When an electron approaches a junction, in addition to the uniform electric field **E** facing it normally, it keeps the drift velocity fixed, as drift velocity depends on E by the relation of drift velocity, $v_d = \frac{eE\tau}{m}$.

This results into accumulation of charges on the surface of wires at the junction. These produce an additional electric field. These fields change the direction conserving momentum. Thus, the motion of a charge junction is not momentum conserving.

16. The best conductor is silver because electrical conductivity is inversely proportional to the resistivity and resistivity of silver is least

17. The Cu wires or Al wires are used for wiring in the home. The main considerations involved in this process are cost of metal and good conductivity of metal.

18. Here, Ohm's law is valid because ratio of voltage and current for different readings is same.

Also, the resistivity of alloy manganin is nearly independent of temperature.

19. From Ohm's law, we have $V = IR$

$$\Rightarrow \quad V = I\rho\frac{l}{A} \qquad \left[\because R = \rho\frac{l}{A}\right] \ldots\text{(i)}$$

When the rod is cut parallel and rejoined by length, the length of the conductor becomes $2l$, whereas the area decrease to $\frac{A}{2}$. If the current remains the same, then the potential changes as

$$V = I\rho\frac{2l}{A/2} = 4 \times I\rho\frac{l}{A} = 4V \qquad \text{[using Eq. (i)]}$$

The new potential applied across the metal rod will be four times the original potential (V).

20. The potential, V = constant, $l' = 3l$

(i) Drift speed of electrons, $v_d = \frac{V}{nel\rho}$

$$v_d \propto \frac{1}{l} \qquad [\because \text{other factors are constant}]$$

So, when length is tripled, drift velocity gets one-third.

(ii) Resistance of conductor is $R = \rho\frac{l}{A}$.

Here, wire is stretched to triple its length, that means the mass of the wire remains same in both conditions.

Before stretching mass = After stretching mass

$$M_1 = M_2$$
$$\Rightarrow \quad V_1\rho_1 = V_2\rho_2 \qquad [\because \rho_1 = \rho_2]$$
$$\text{or} \quad A_1 l_1 = A_2 l_2$$

Since, length is tripled after stretching.

$$\therefore \quad A_1 l = A_2(3l) \text{ or } A_2 = \frac{A_1}{3}$$

Hence, $R' = \rho\frac{l'}{A'} = \rho\frac{3l}{A/3} = \frac{9\rho l}{A} \Rightarrow R' = 9R$

Thus, new resistance is 9 times of its original value.

21. Refer to text on page 130.

22. (i) Resistivity is a property of the material, it does not depend on the dimensions of the wire. Thus, when the wire is stretched, then its resistivity remains same.

(ii) Refer to text on page 129.

23. Refer to text on page 128.

[redacted]

24. Refer to text on pages 128 and 129.

25. (i) Refer to text on page 127.

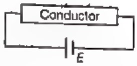
(ii) Area under I-t curve on t-axis is charge flowing through the conductor.

$$Q = \frac{1}{2}\times t_1 \times I_1 + (t_2 - t_1)\times I_1$$

26. Refer to text on pages 127 and 128.

27. Refer to text on page 127.

Source of emf E is shown in the figure below



Suppose initial length of the conductor, $l_i = l_0$.

New length, $l_f = 3l_0$

We know that,

drift velocity, $v_d \propto E_0$ [where, E_0 = electric field]

Thus, $\frac{(v_d)_f}{(v_d)_i} = \frac{(E_0)_f}{(E_0)_i}$

$$= \frac{E/l_f}{E/l_i} = \frac{l_i}{l_f} = \frac{l_0}{3l_0} = \frac{1}{3}$$

$\Rightarrow \quad (v_d)_f = \frac{(v_d)_i}{3}$

Thus, drift velocity decreases three times.

28. (i) Refer to text on page 126.

(ii) Refer to text on pages 127, 128 and 129.

(iii) Alloys like constantan and manganin are used for making standard resistor because the resistivity of these alloys here weak dependent on the temperature.

29. Refer to text on pages 129 and 130.

30. When a wire is stretched, then there is no change in the matter of the wire, hence its volume remains constant.

The potential V = constant, $l' = 3l$

(i) Drift speed of electrons $= \frac{V}{nel\rho}$

$\therefore \quad v \propto \frac{1}{l}$ [$\because$ other factors are constants]

So, when length is tripled, drift velocity gets one-third.

$$V_1 = V_2$$

$\therefore \quad A_1 l_1 = A_2 l_2$

$A_1 l = A_2 (3l)$

$\Rightarrow$ [$\because$ length is tripled after stretching]

$\therefore \quad A_2 = \frac{A_1}{3}$

i.e. When length is tripled area of cross-section is reduced to $\frac{A}{3}$.

Hence, $\quad R = \rho \frac{l'}{A'} = \rho \frac{3l}{\frac{A}{3}}$

$$= 9\rho \frac{l}{A} = 9R$$

Thus, new resistance will be 9 times of its original value.

31. (a) **Conductivity** The reciprocal of resistivity of a conductor is known as conductivity. It is expressed as

$$\sigma = \frac{1}{\rho}$$

The SI unit of conductivity is mho per metre ($\Omega^{-1}m^{-1}$).

(b) We know that, drift velocity is given by

$$v_d = \frac{eE\tau}{m} \quad \text{...(i)}$$

where, e = electric charge,

E = applied electric field,

τ = relaxation time and m = mass of electron.

But $\quad E = \frac{V}{l}$ (i.e. potential gradient)

$$\therefore \quad v_d = \left(\frac{e\tau}{m}\right)\left(\frac{V}{l}\right) \quad \text{...(ii)}$$

From the relation between current and drift velocity,

$$I = neAv_d \quad \text{...(iii)}$$

(where, n = number of density of electrons)

Putting the value of Eq. (ii) in Eq. (iii), we get

$$I = neA\left(\frac{e\tau V}{ml}\right) \text{ or } I = \left(\frac{ne^2A\tau}{ml}\right)V$$

or $$V = \left(\frac{ml}{ne^2A\tau}\right)I \quad \text{...(iv)}$$

But according to Ohm's law, $V = IR$ ...(v)

From Eqs. (iv) and (v), we get

$$R = \left(\frac{m}{ne^2\tau}\right)\frac{l}{A} \quad \text{...(vi)}$$

Also, $$R = \rho\frac{l}{A} \quad \text{...(vii)}$$

From Eqs. (vi) and (vii), we get

$\rho = \frac{m}{ne^2\tau}$ = resistivity of conductor.

As reciprocal of resistivity of conductor is known as conductivity.

$\therefore$ Conductivity, $\sigma = \frac{1}{\rho} = \frac{ne^2\tau}{m}$

Now, we know that, current density, $J = \frac{I}{A}$

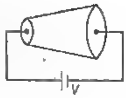
or $\quad J = \frac{neAv_d}{A} = nev_d = \left(\frac{ne^2\tau}{m}\right)E \quad \left(\because v_d = \frac{eE\tau}{m}\right)$

$\therefore \quad J = \sigma E \quad \left(\because \sigma = \frac{ne^2\tau}{m}\right)$

[redacted]

32. (i) Refer to text on pages 126, 127 and 128.

(ii) The setup is shown in the figure. Here, electric current remains constant throughout the length of the wire. Electric field also remains constant which is equal to $\frac{V}{l}$.



Current density and hence drift speed changes.

33. The resistance of first conductor, $R_A = \frac{\rho l}{\pi (0.5 \times 10^{-3})^2}$

The resistance of second conductor,

$$R_B = \frac{\rho l}{\pi[(10^{-3})^2 - (0.5 \times 10^{-3})^2]}$$

Now, the ratio of two resistors is given by

$$\frac{R_A}{R_B} = \frac{(10^{-3})^2 - (0.5 \times 10^{-3})^2}{(0.5 \times 10^{-3})^2} = 3:1$$

34. When a wire is stretched, its volume remains constant, hence

$$l_1 A_1 = l_2 A_2 = V \quad \text{[where, } V = \text{volume]}$$

Now, $R_1 = \frac{\rho l_1}{A} = \frac{\rho l_1 \times l_1}{l_1 A_1} = \frac{\rho l_1^2}{V}$, i.e. $R_1 \propto l_1^2$

Hence, $\frac{R_2}{R_1} = \frac{l_2^2}{l_1^2} = \frac{\left(l_1 + \frac{5}{100} l_1\right)^2}{l_1^2} = 1.1025$

$$\frac{R_2}{R_1} = 1.1025 \quad \text{...(i)}$$

$\therefore$ % Change in resistance $= \frac{R_2 - R_1}{R_1} \times 100$

$$= \left(\frac{R_2}{R_1} - 1\right) \times 100 = (1.1025 - 1) \times 100 \text{ [from Eq. (i)]}$$

$= 10.25\%$

35. Given, resistance of heating element at temperature $27°C$, $R_{27} = 100\,\Omega$

Resistance of heating element at temperature $t°C$,

$$R_t = 117\,\Omega$$

$$\alpha = 1.70 \times 10^{-4}\,°C^{-1}, t = ?$$

By using the formula of temperature coefficient of resistance, $\alpha = \frac{R_2 - R_1}{R_1 (t_2 - t_1)}$ ...(i)

Here, $R_2 = R_t$, $R_1 = R_{27}$, $t_2 = t$ and $t_1 = 27°C$

Such that, $\alpha = \frac{R_t - R_{27}}{R_{27}(t - 27)}$

Substituting given values in Eq. (i), we get

$$1.70 \times 10^{-4} = \frac{117 - 100}{100\,(t - 27)} \text{ or } t - 27 = \frac{17}{100 \times 1.70 \times 10^{-4}}$$

or $t = 1000 + 27 = 1027\,°C$

36. Given, potential difference $= 230\,V$

Initial current at $27\,°C = I_{27°C} = 3.2\,A$

Final current at $t\,°C = I_{t°C} = 2.8\,A$

Room temperature $= 27\,°C$

Temperature coefficient of resistance, $\alpha = 1.70 \times 10^{-4}\,°C^{-1}$

Resistance at $27\,°C$, $R_{27°C} = \frac{V}{I_{27°C}} = \frac{230}{3.2} = \frac{2300}{32}\,\Omega$

Resistance at $t°C$, $R_{t°C} = \frac{V}{I_{t°C}} = \frac{230}{2.8} = \frac{2300}{28}\,\Omega$

Temperature coefficient of resistance

$$\alpha = \frac{R_t - R_{27}}{R_{27}(t - 27)} \Rightarrow 1.70 \times 10^{-4} = \frac{\frac{2300}{28} - \frac{2300}{32}}{\frac{2300}{32}(t - 27)}$$

or $t - 27 = \frac{82.143 - 71.875}{71.875 \times 1.70 \times 10^{-4}} = 840.347$

or $t = 840.3 + 27 = 867.3\,°C$

Thus, the steady temperature of heating element is $867.3\,°C$.

37. 290.2K, refer to Sol. of Q. 35.

[redacted]

|TOPIC 2|

Electrical Energy

ELECTRICAL ENERGY AND POWER

Electrical Energy

It is defined as the total work done W by the source of emf V in maintaining the electric current I in the given circuit for a specified time t.

According to Ohm's law, we have

$$V = IR$$

Total charge that crosses the resistor is given by $q = It$

Energy gained is given by

$$E = W = Vq = V\,(It) = VIt$$

$$= [IR]\,It = I^2Rt \qquad [\because V = IR]$$

$$= \left[\frac{V}{R}\right]^2 Rt = \frac{V^2t}{R} \qquad \left[\because I = \frac{V}{R}\right]$$

$$\therefore \boxed{E = VIt = I^2Rt = \frac{V^2}{R}t}$$

The SI unit of electrical energy is joule (J),

where, 1 joule = 1 volt × 1 ampere × 1 sec = 1 watt × 1 sec

Commercial Unit of Electrical Energy

To measure the electrical energy consumed commercially, the unit of energy, i.e. joule is not sufficient. So, to express electrical energy consumed commercially, a special unit called kilowatt hour is used in place of joule.

1kWh is also called 1 unit of electrical energy. 1 kilowatt hour or 1 unit of electrical energy is the amount of energy dissipated in 1 hour in a circuit, when the electric power in the circuit is 1 kilowatt.

$$1 \text{ kilowatt hour (kWh)} = 3.6 \times 10^6 \text{ joule (J)}$$

EXAMPLE |1| A resistance coil is made by joining in parallel two resistances each of 10 Ω. An emf of 1V is applied between the two ends of coil for 5 min. Calculate the heat produced in calories.

Sol. Given, resistance, $R_1 = 10\Omega$, $R_2 = 10\Omega$

Voltage, $V = 1\text{V}$

and time, $t = 5$ min

$= 5 \times 60\text{ s} = 300\text{ s}$

Since, effective resistance in parallel combination will be

$$R_P = \frac{R_1R_2}{R_1 + R_2} = \frac{10 \times 10}{10 + 10} = 5\Omega$$

$$\therefore \text{Heat produced} = \frac{V^2t}{R_P} = \frac{1^2}{5} \times 5 \times 60 = \frac{60}{4.2} = 14.3 \text{ cal}$$

Electrical Power

It is defined as the rate of electrical energy supplied per unit time to maintain flow of electric current through a conductor.

Mathematically, $\boxed{P = VI = I^2R = \dfrac{V^2}{R}}$

The SI unit of power is watt (W).

where, 1 watt = 1 volt × 1 ampere = 1 ampere-volt.

Power of an electric circuit is said to be one watt, if one ampere current flows in it against a potential difference of one volt. The bigger units of electrical power are kilowatt (kW) and megawatt (MW).

where, 1kW = 1000 W and 1 MW = 10^6 W

Commercial unit of electrical power is horse power (HP), where 1HP = 746 W.

EXAMPLE |2| A heating element is marked 210 V, 630 W. What is the value of the current drawn by the element when connected to a 210 V DC source?

Delhi 2013

Sol. Given, $P = 630\text{W}$ and $V = 210\text{V}$

Since, $P = VI$

Therefore, $I = \dfrac{P}{V} = \dfrac{630}{210} = 3\text{A}$

|TOPIC PRACTICE 2|

OBJECTIVE Type Questions

1. A television of 200 W is used for 4h, then what is the value unit expense of electricity?

(a) 50 (b) 20 (c) 0.8 (d) 0.2

2. Two bulbs of 40W and 60W are connected to 220V line, the ratio of resistance will be

(a) 4 : 3 (b) 3 : 4 (c) 2 : 3 (d) 3 : 2

[redacted]

3. A 100 W-220 V bulb is connected to a supply of 110 V. The power dissipated in the bulb will be
(a) 100 W (b) 50 W
(c) 25 W (d) 2 W

VERY SHORT ANSWER Type Questions

4. Nichrome and copper wires of same length and same radius are connected in series. Current I is passed through them. Which wire gets heated up more? Justify your answer. **All India 2017**

5. Name the unit of electric energy used for domestic purpose.

6. What is the commercial unit of electrical energy and how is it related to joules?

SHORT ANSWER Type Questions

7. The potential difference applied across a given resistor is altered, so that the heat produced per second increases by a factor of 9. By what factor does the applied potential difference change? **All India 2017**

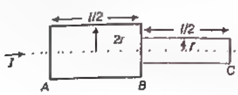
8. Power P is to be delivered to a device *via* transmission cables having resistance R_c. If V is the voltage across R and I the current through it, find the power wasted and how can it be reduced. **NCERT Exemplar**

9. When is more power delivered to a light bulb, just after it is turned on and the glow of the filament is increasing or after it has been ON for a few seconds and the glow is steady?

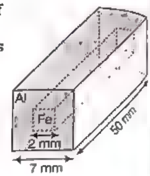
10. Two electric bulbs P and Q have their resistances in the ratio of 1 : 2. They are connected in series across a battery. Find the ratio of the power dissipation in these bulbs. **CBSE 2018**

NUMERICAL PROBLEMS

11. Two bars of radius r and $2r$ are kept in contact as shown in the figure. An electric current I is passed through the bars. Find the ratio of heat produced in bars AB and BC.



12. In an aluminium (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are $2.7 \times 10^{-8}\ \Omega$-m and $1.0 \times 10^{-7}\ \Omega$-m, respectively.



Calculate the electrical resistance between the two faces P and Q of the composite bar.

13. A room has AC run for 5 hour a day at a voltage of 220 V. The wiring of the room consists of Cu of 1 mm radius and a length of 10 m. Power consumption per day is 10 commercial units. What fraction of it goes in the joule heating in wires? What would happen, if the wiring is made of aluminium of the same dimensions? [Given, $\rho_{Cu} = 1.7 \times 10^{-8}\ \Omega$-m, $\rho_{Al} = 2.7 \times 10^{-8}\ \Omega$-m] **NCERT Exemplar**

HINTS AND SOLUTIONS

1. (c) Dissipated energy in per second,

$$P = \frac{W}{t}$$

$$W = P \times t$$

where, $P = 200$ W, $t = 4$ h

$\Rightarrow$ $W = 200 \times 4$ W-h

Unit of dissipated energy

$$= \frac{\text{watt} \times \text{hours}}{1000}$$

$$= \frac{200 \times 4}{1000} = 0.8 \text{ unit}$$

2. (d) Power, $P = \dfrac{V^2}{R}$

Given, $P_1 = 40$ W, $P_2 = 60$ W

$\therefore$ $$40 = \frac{V^2}{R_1} \quad \text{...(i)}$$

and $$60 = \frac{V^2}{R_2} \quad \text{...(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{40}{60} = \frac{R_2}{R_1} \text{ or } \frac{R_1}{R_2} = \frac{3}{2} = 3:2$$

[redacted]

3. (b) As we know, $P = \frac{V^2}{R}$ or $P = V \times I$

For 100 W bulb, $100 = 220 \times I \Rightarrow I = \frac{100}{220} = \frac{10}{22}$ A

Hence, the power dissipated for 100W bulb will be

$$P = V \times I = 110 \times \frac{10}{22} = 50 \text{ W}$$

4. For same length and same radius, resistance of wire,

$$R \propto \rho \quad \text{(where } \rho \text{ is resistivity)}$$

As $\rho_{\text{nichrome}} > \rho_{\text{copper}}$

Hence, resistance of nichrome section is more.

In series, same current flows through both sections and heat produced $= I^2Rt$. So, more heat is produced in nichrome section of wire.

5. The unit of electric energy used for domestic purpose is kilowatt hour (kWh). It is also called commercial unit of electric energy.

6. The commercial unit of electrical energy is kilowatt hour (kWh).

$1\text{kWh} = 3.6 \times 10^6 \text{J}$

7. Heat produced per second $= I^2R = \frac{V^2}{R}$.

So, when voltage is made three times, then heat produced increase nine times for same R

8. The power consumption in transmission lines is given by $P = i^2R_c$, where R_c is the resistance of transmission lines. The power is given by $P = VI$.

The given power can be transmitted in two ways namely

(i) At low voltage and high current.

(ii) At high voltage and low current.

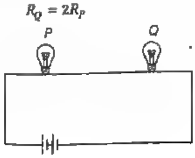
In power transmission at low voltage and high current, more power is wasted as $P \propto i^2$, whereas power transmission at high voltage and low current facilitates the power transmission with minimum power wastage.Thus, power wastage can be reduced by transmitting power at high voltage.

9. When the bulb is turned ON, the resistance of the filament is low, the current is high and a relatively large amount of power is delivered to the bulb.

As the filament warmup, its resistance increases and the current decreases. As a result, power delivered to bulb decreases.

10. Given, $\frac{R_P}{R_Q} = \frac{1}{2}$

$\therefore \quad R_Q = 2R_P \quad$ ... (i)



In series, power dissipated is given by the relation

$$P = I^2R$$

or $\quad P \propto R$

$\therefore \quad \frac{P_P}{P_Q} = \frac{R_P}{R_Q} \quad$ ... (ii)

Using Eqs. (i) and (ii), we get

$\therefore \quad \frac{P_P}{P_Q} = \frac{R_P}{2R_P} = \frac{1}{2}$

11. Current flowing through both the bars is equal.

Now, the heat produced is given by

$E = I^2Rt$

$\because \quad E \propto R$

$\therefore \quad \frac{E_{AB}}{E_{BC}} = \frac{R_{AB}}{R_{BC}} = \frac{(1/2r)^2}{(1/r)^2} \qquad \left[\because R \propto \frac{1}{A} \propto \frac{1}{r^2}\right]$

$= \frac{1}{4}$

12. Resistance between the two faces P and Q of the composite bar is given by

$$\frac{1}{R} = \frac{1}{R_A} + \frac{1}{R_{Fe}} = \left(\frac{A_{Al}}{\rho_{Al}} + \frac{A_{Fe}}{\rho_{Fe}}\right)\frac{1}{l}$$

$$\Rightarrow \quad \frac{1}{R} = \left[\frac{(7^2 - 2^2)}{27} + \frac{2^2}{10}\right]\frac{10^{-4}}{10^{-8}} \times \frac{1}{50 \times 10^{-3}}$$

$$\therefore \quad R = \frac{1875}{64} \times 10^{-6}\ \Omega = \frac{1875}{64}\ \mu\Omega$$

13. Power consumption in a day, i.e. in 5 h = 10 units

or power consumption per hour = 2 units

or power consumption = 2 units = 2kW = 2000 J/s

Also, we know that, power consumption in resistor,

$P = V \times I \Rightarrow 2000 \text{ W} = 220 \text{ V} \times I$

or $\quad I \approx 9$ A

Now, the resistance of wire is given by $R = \rho\frac{l}{A}$

where, A is cross-sectional area of conductor.

Power consumption in first current carrying wire is given by $P = I^2R$

$$= \rho\frac{l}{A}I^2 = 1.7 \times 10^{-8} \times \frac{10}{\pi \times 10^{-6}} \times 81 \text{ J/s} \approx 4 \text{ J/s}$$

The fractional loss due to the joule heating in first wire

$$= \frac{4}{2000} \times 100 = 0.2\%$$

Power loss in aluminium wire $= 4\frac{\rho_{Al}}{\rho_{Cu}} = 1.6 \times 4 = 6.4$ J/s

The fractional loss due to the joule heating in second wire

$$= \frac{6.4}{2000} \times 100 = 0.32\%$$

[redacted]

|TOPIC 3|

Cells, EMF and Internal Resistance

CELLS

An electric cell is a source of energy that maintains a continuous flow of charge in a circuit. Electric cell changes chemical energy into electrical energy.

Electromotive Force (EMF) of a Cell (E)

Electric cell has to do some work in maintaining the current through a circuit. The work done by the cell in moving unit positive charge through the whole circuit (including the cell) is called the electromotive force (emf) of the cell.

If during the flow of q coulomb of charge in an electric circuit, the work done by the cell is W, then

emf of the cell, $\boxed{E = \frac{W}{q}}$

Its unit is joule/coulomb or volt.

If $W = 1$ joule and $q = 1$ coulomb, then $E = 1$ volt, i.e. if in the flow of 1 coulomb of charge, the work done by the cell is 1 joule, then the emf of the cell is 1 volt.

Internal Resistance (r)

Internal resistance of a cell is defined as the resistance offered by the electrolyte of the cell to the flow of current through it. It is denoted by r. Its unit is ohm.

Internal resistance of a cell depends on the following factors

(i) It is directly proportional to the separation between the two plates of the cell.

(ii) It is inversely proportional to area of plate dipped into the electrolyte.

(iii) It depends on the nature, concentration and temperature of the electrolyte and increases with increase in concentration.

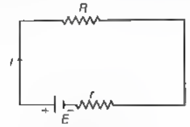
Terminal Potential Difference (V)

Terminal potential difference of a cell is defined as the potential difference between the two terminals of the cell in a closed circuit (i.e. when current is drawn from the cell). It is represented by V and its unit is volt.

Terminal potential difference of a cell is always less than the emf of the cell. In closed circuit, the current flows through the circuit including the cell, due to internal resistance of the cell there is some fall of potential. This is the amount of potential by which the terminal potential difference is less than the emf of the cell.

Relation between Terminal Potential Difference, emf of a Cell and Internal Resistance of a Cell

(i) If no current is drawn from the cell, i.e. the cell is in open circuit, so emf of the cell will be equal to the terminal potential difference of the cell.



$$I = 0 \quad \text{or} \quad V = E$$

(ii) Consider a cell of emf E and internal resistance r is connected across an external resistance R.

Current drawn from the cell,

$$I = \frac{E}{R + r} \qquad \dots\text{(i)}$$

where, E = emf of the cell,

R = external resistance

and r = internal resistance of a cell.

Now, from Ohm's law, $V = IR$

$$\Rightarrow \quad I = \frac{V}{R} \qquad \dots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{V}{R} = \frac{E}{R + r}$$

$$\Rightarrow \quad \boxed{r = \left(\frac{E}{V} - 1\right)R}$$

From definition of terminal potential difference,

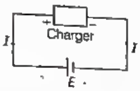
$$\boxed{V = E - Ir}$$

[redacted]

Charging of a Cell

During charging of a cell, the positive terminal (electrode) of the cell is connected to positive terminal of battery charger and negative terminal (electrode) of the cell is connected to negative terminal of battery charger. In this process, current flows from positive electrode to negative electrode of the cell.

From the given figure, $V = E + Ir$



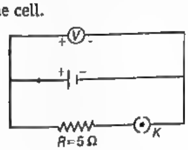
Thus, the terminal potential difference of a cell becomes greater than the emf of the cell.

The potential drop across internal resistance of the cell is called **lost voltage**, as it is not indicated by a voltmeter. Its value is equal to Ir.

Difference between EMF and Terminal Potential Difference of a Cell

S.No.	EMF	Terminal potential difference
1.	The emf of a cell is the maximum potential difference between the two electrodes (terminals) of a cell when the cell is in the open circuit.	The terminal potential difference of a cell is the potential difference between the two terminals of the cell in a closed circuit.
2.	It is independent of the resistance of the circuit and depends upon the nature of electrodes and electrolyte of the cell.	It depends upon the resistance of the circuit and current flowing through it.
3.	The term emf is used for the source of electric current.	The potential difference is measured between any two points of the electric circuit.
4.	The emf is a cause.	The potential difference is an effect.

EXAMPLE |1| The reading on a high resistance voltmeter, when a cell is connected across it, is 2.2 V. When the terminals of the cell are connected to a resistance of 5 Ω as shown in figure given below, the voltmeter reading drops to 1.8 V. Find the internal resistance of the cell.



Sol. Given, emf, $E = 2.2$ V

Terminal potential difference, $V = 1.8$ V

External resistance, $R = 5\,\Omega$

$\therefore$ Internal resistance,

$$r = \left(\frac{E-V}{V}\right)R = \left(\frac{2.2-1.8}{1.8}\right)\times 5$$

$$= \frac{10}{9}\,\Omega$$

EXAMPLE |2| A cell of emf E and internal resistance r gives a current of 0.5 A with an external resistance of 12 Ω and a current of 0.25 A with an external resistance of 25 Ω. Calculate the

(i) internal resistance of the cell (ii) emf of the cell.

Sol. Let R be external resistance in series with the cell of emf E and internal resistance r. The current in circuit is

$$I = \frac{E}{R+r}$$

Case I $I = 0.5$ A, $R = 12\,\Omega$, then

$$0.5 = \frac{E}{12+r}$$

$$\Rightarrow \quad E = 0.5\,(12+r)$$

$$\Rightarrow \quad E = 6.0 + 0.5r \qquad \text{...(i)}$$

Case II $I = 0.25$ A, $R = 25\,\Omega$, then

$$0.25 = \frac{E}{25+r}$$

$$\Rightarrow \quad E = 0.25\,(25+r)$$

$$\Rightarrow \quad E = 6.25 + 0.25r \qquad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$6.0 + 0.5\,r = 6.25 + 0.25\,r$$

$$\Rightarrow \quad r = 1\,\Omega$$

From Eq. (i), we get

$$E = 6.0 + 0.5 \times (1) = 6.5\text{ V}$$

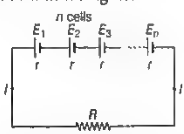
Hence, (i) internal resistance of the cell is 1 Ω.

(ii) emf of the cell is 6.5 V.

CELLS IN SERIES AND PARALLEL

Cells in Series

In this combination, n identical cells each of emf E and internal resistance r are connected in series to the external resistance R as shown in the figure.



Points to remember for series combination of cells

(i) The equivalent emf of a series combination of n cells is equal to the sum of their individual emfs.

[redacted]

(ii) The equivalent internal resistance of a series combination of n cells is equal to sum of their individual internal resistances.

Equivalent emf of n cells in series,

$E_{eq} = E_1 + E_2 + ... +$ upto n terms $= nE$

Equivalent internal resistance of n cells in series,

$r_{eq} = r_1 + r_2 + ... +$ upto n terms $= nr$

Total resistance of the circuit $= nr + R$

$\therefore$ Current in the resistance R is given by

$$\boxed{I = \frac{nE}{R + nr}}$$

where, n = number of cells,
r = internal resistance,
R = external resistance,
E = emf of cell.
and I = current flowing.

Case I When $R << nr$, then

$I = \frac{E}{r}$ = current due to a single cell

Case II When $R >> nr$, then

$I = \frac{nE}{R} = n$ times the current due to a single cell

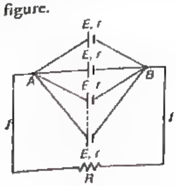
Case III When cells are of different emfs and different internal resistances, then

$$I = \frac{E_1 + E_2 + ... + E_n}{R + (r_1 + r_2 + ... + r_n)}$$

Note The maximum current can be drawn from the series combination of cells, if the value of external resistance is very high as compared to the total internal resistance of the cells

Cells in Parallel

In this combination, m cells each of emf E and internal resistance r are connected in parallel the external resistance R as shown in the figure.



Points to remember for parallel combination of cells

(i) The equivalent emf of parallel combination of cells of same emfs is equal to emf of one cell.

(ii) The reciprocal of equivalent internal resistance of parallel combination of cells is equal to the sum of the reciprocals of the internal resistance of each cell.

$\therefore \frac{1}{r_p} = \frac{1}{r_1} + \frac{1}{r_2} + ...$ upto m terms $= \frac{m}{r}$ or $r_p = \frac{r}{m}$

As, R and r_p are in series, so total resistance in the circuit $= R + \frac{r}{m}$

In parallel combination of identical cells, the effective emf in the circuit is equal to the emf due to a single cell, because in this combination; only the size of the electrodes increases but not emf.

$\therefore$ Current in the resistance R is given by

$$\boxed{I = \frac{E}{R + \frac{r}{m}}}$$

Case I When $R >> \frac{r}{m}$, then

$I = \frac{E}{R}$ = current due to a single cell

Case II When $R << \frac{r}{m}$, then $I = \frac{E}{r/m}$

$= \frac{mE}{r} = m$ times current due to a single cell

Case III When cells are of same emf and different internal resistances, then

$I = \frac{E}{R + r'}$ $\quad [\because E_1 = E_2 = ... E_n = E]$

where, $\frac{1}{r'} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + ... + \frac{1}{r_n}$ and E is emf of each cell.

Note The maximum current can be drawn from the parallel combination of cells, if the external resistance is very low as compared to the total internal resistance of the cells

EXAMPLE |3| Two identical cells, when joined together in series or in parallel give the same current, when connected to external resistance of 2 Ω. Find the internal resistance of each cell.

Sol. Let E, r be the emf and internal resistance of each cell.

External resistance, $R = 2\Omega$

[redacted]

If two cells are connected in series, then

Total emf of cells $= E + E = 2E$

Total resistance of circuit $= R + r + r = 2 + 2r$

Current in the circuit, $I_1 = \dfrac{2E}{2+2r}$

If two cells are connected in parallel, effective emf of two cells = emf of single cell $= E$

Total internal resistance of two cells $= \dfrac{r \times r}{r + r} = \dfrac{r}{2}$

Total resistance of the circuit $= R + \dfrac{r}{2} = 2 + \dfrac{r}{2}$

Current in the circuit, $I_2 = \dfrac{E}{2+\dfrac{r}{2}} = \dfrac{2E}{4+r}$

As per question, $I_1 = I_2$

$$\Rightarrow \quad \frac{2E}{2+2r} = \frac{2E}{4+r}$$

$$\Rightarrow \quad 2 + 2r = 4 + r$$

$$\therefore \quad r = 2\Omega$$

EXAMPLE |4| **When 14 cells in series, are connected to the ends of a resistance of 82.6 Ω, then the current is found to be 0.25A. When same cells after being connected in parallel are joined to the ends of a resistance of 0.053Ω, then the current is 25A. Calculate the internal resistance and the emf of each cell.**

Sol. Let E and r be the emf and internal resistance of each cell.

Case I When the cells are in series.

Total emf of cells $= 14E$

Total resistance of circuit $= 82.6 + 14r$

$\therefore$ Current in the circuit is given by

$$\frac{14E}{82.6 + 14r} = 0.25\,\text{A} \qquad \text{...(i)}$$

Case II When the cells are in parallel.

Total emf of cells $= E$

Total resistance of circuit $= 0.053 + \dfrac{r}{14}$

$\therefore$ Current in the circuit is given by

$$\frac{E}{0.053 + \dfrac{r}{14}} = 25\,\text{A} \qquad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$14\frac{\left(0.053 + \dfrac{r}{14}\right)}{(82.6 + 14r)} = 10^{-2}$$

$$\Rightarrow \quad 14 \times \frac{14 \times 0.053 + r}{14} \times 10^2 = 82.6 + 14r$$

$$\Rightarrow \quad 5.3 \times 14 + 100r = 82.6 + 14r$$

Solving, we get

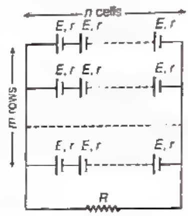
$$r = 0.097\,\Omega \approx 0.1\,\Omega$$

Substituting the value of r in Eq. (i), we get

$$E = 1.5\,\text{V}$$

Mixed Combination of Cells

In this combination, some cells are connected in series and some cells are connected in parallel as shown in the figure.



Let there be n cells in series in one row and m rows of cells are in parallel.

Suppose all the cells are identical. Let each cell be of emf and internal resistance r.

Equivalent emf of each row $= nE$

Equivalent internal resistance of each row $= nr$

Total emf of combination $= nE$

Total internal resistance of combination,

$$\frac{1}{r'} = \frac{1}{nr} + \frac{1}{nr} + \dots \text{ upto } m \text{ times}$$

$$\frac{1}{r'} = \frac{m}{nr} \text{ or } r' = \frac{nr}{m}$$

Total resistance of the circuit $= r' + R = \dfrac{nr}{m} + R$

Current in the resistance R is given by

$$\boxed{I = \frac{nE}{\dfrac{nr}{m} + R}}$$

Thus, we get the maximum current in mixed grouping of cells, if the value of external resistance is equal to the total internal resistance of all the cells, i.e. external resistance = total internal resistance of all the cells $\left(R = \dfrac{nr}{m}\right)$.

EXAMPLE |5| **36 cells, each of internal resistance 0.5 Ω and emf 1.5 V each are used to send current through an external circuit of 2 Ω resistance. Find the best mode of grouping them and the current through the external circuit.**

Sol. Here, $E = 1.5\,\text{V}$, $r = 0.5\,\Omega$, $R = 2\,\Omega$

Total number of cells, $mn = 36$ ...(i)

For maximum current in the mixed grouping,

$$\frac{nr}{m} = R$$

[redacted]

$$\Rightarrow \qquad \frac{n \times 0.5}{m} = 2 \qquad \text{... (ii)}$$

Multiplying Eqs. (i) and (ii), we get

$$0.5n^2 = 72 \Rightarrow n^2 = 144$$

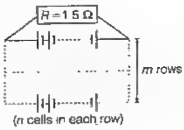
$$\therefore \qquad n = 12$$

$$\text{and} \qquad m = \frac{36}{12} = 3$$

Thus, for maximum current, there should be three rows in parallel, each containing 12 cells in series.

$$\therefore \text{ Maximum current} = \frac{mnE}{mR + nr} = \frac{36 \times 1.5}{3 \times 2 + 12 \times 0.5} = 4.5 \text{ A}$$

EXAMPLE |6| 12 cells, each of emf 1.5 V and internal resistance of 0.5 Ω, are arranged in m rows each containing n cells connected in series, as shown in the figure. Calculate the values of n and m for which this combination would send maximum current through an external resistance of 1.5 Ω.



Sol. For maximum current through the external resistance, external resistance = total internal resistance of cells

$$\text{or} \qquad R = \frac{nr}{m}$$

$$\therefore \qquad 1.5 = \frac{n \times 0.5}{\frac{12}{n}} \qquad [\because mn = 12]$$

$$\text{or} \qquad 36 = n^2$$

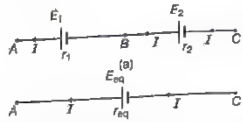
$$\therefore \qquad n = 6 \text{ and } m = 2$$

COMBINATION OF TWO CELLS IN SERIES AND PARALLEL (WITH DIFFERENT EMFS AND INTERNAL RESISTANCES)

Two Cells in Series

The two cells are said to be connected in series between two points A and C, when negative terminal of one cell is connected to positive terminal of other cell as shown in the Fig. (a).

Let E_1, E_2 be the emfs of the two cells and r_1, r_2 be their internal resistances, respectively. Let the two cells be sending the current in a circuit shown in the Fig. (a) and (b). Let V_A, V_B and V_C be the potentials at points A, B and C and I be the current flowing through them.



(a)

(b)

Potential difference between positive and negative terminals of the first cell is given by

$$V_{AB} = V_A - V_B = E_1 - Ir_1 \qquad \text{...(i)}$$

Potential difference between positive and negative terminals of second cell is given by

$$V_{BC} = V_B - V_C = E_2 - Ir_2 \qquad \text{...(ii)}$$

Potential difference between A and C of the series combination of the two cells is given by

$$\begin{aligned} V_{AC} &= V_A - V_C \\ &= (V_A - V_B) + (V_B - V_C) \\ &= (E_1 - Ir_1) + (E_2 - Ir_2) \\ &= (E_1 + E_2) - I(r_1 + r_2) \qquad \text{...(iii)} \end{aligned}$$

If the series combination of two cells is replaced by single cell between A and C of emf E_{eq} and internal resistance r_{eq} as shown in the Fig. (b), then

$$V_{AC} = E_{eq} - I\, r_{eq} \qquad \text{...(iv)}$$

Comparing Eqs. (iii) and (iv), we get

$$E_{eq} = E_1 + E_2 \qquad \text{...(v)}$$

$$\text{and} \qquad r_{eq} = r_1 + r_2 \qquad \text{...(vi)}$$

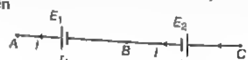
If n cells of emfs $E_1, E_2 \dots E_n$ and of internal resistances $r_1, r_2, \dots, r_n$ respectively, are connected in series between points A and C, then equivalent emf is given by

$$\boxed{E_{eq} = E_1 + E_2 + \dots + E_n}$$

Equivalent internal resistance of the cells is given by

$$\boxed{r_{eq} = r_1 + r_2 + \dots + r_n}$$

That in the series combination of two cells, if negative terminal of first cell is connected to the negative terminal of the second cell between points A and C, as shown in the Fig. (c), then



(c)

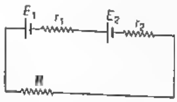
$$V_{BC} = V_B - V_C = -E_2 - Ir_2$$

Then, equivalent emf of the two cells is $E_{eq} = E_1 - E_2$

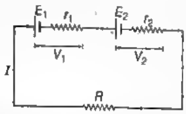
But equivalent internal resistance is $r_{eq} = r_1 + r_2$.

[redacted]

EXAMPLE |7| In the circuit shown in figure, $E_1 = 10$ V, $E_2 = 4$ V, $r_1 = r_2 = 1\,\Omega$ and $R = 2\,\Omega$. Find the potential difference across battery 1 and battery 2.



Sol. Net emf of the circuit $= E_1 - E_2 = (10-4) = 6$ V



Total resistance of the circuit $= R + r_1 + r_2 = 4\,\Omega$

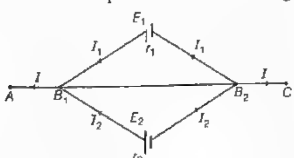
$\therefore$ Current in the circuit, $I = \dfrac{\text{Net emf}}{\text{Total resistance}} = \dfrac{6}{3} = 1.5$ A

Now, $V_1 = E_1 - Ir_1 = 10 - (1.5)(1) = 8.5$ V

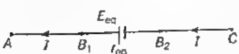
and $V_2 = E_2 + Ir_2 = 4 + (1.5)(1) = 5.5$ V

Two Cells in Parallel

The two cells are said to be connected in parallel between two points A and C, when positive terminal of each cell is connected to one point and negative terminal of each cell is connected to the other point as shown in the Fig. (a).



(a)



(b)

Let the two cells be sending the current in a circuit shown in Figs. (a) and (b). Let E_1, E_2 be the emfs of the two cells and r_1, r_2 be their internal resistances, respectively.

Let I_1, I_2 be the currents from the two cells flowing towards point B_1 and I be the current flowing out of B_1, then

$$I = I_1 + I_2 \quad \text{...(i)}$$

Let V_{B_1}, V_{B_2} be the potentials at points B_1 and B_2, respectively and V be the potential difference between B_1 and B_2. Here, the potential difference across the terminals of first cell is equal to the potential difference across the terminals of the second cell.

So, for the first cell,

V is given by $V = V_{B_1} - V_{B_2} = E_1 - I_1 r_1$ or $I_1 = \dfrac{E_1 - V}{r_1}$

For the second cell, $V = V_{B_1} - V_{B_2} = E_2 - I_2 r_2$

or $$I_2 = \frac{E_2 - V}{r_2}$$

Substituting values in Eq. (i), we get

$$I = \left(\frac{E_1 - V}{r_1}\right) + \left(\frac{E_2 - V}{r_2}\right)$$
$$= \left(\frac{E_1}{r_1} + \frac{E_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$
$$= \frac{E_1 r_2 + E_2 r_1}{r_1 r_2} - V\left(\frac{r_1 + r_2}{r_1 r_2}\right)$$

$$\Rightarrow \quad \boxed{V = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} - \frac{I\, r_1 r_2}{r_1 + r_2}} \quad \text{...(ii)}$$

If the parallel combination of cells is replaced by a single cell between B_1 and B_2 of emf E_{eq} and internal resistance r_{eq} [Fig. (b)], then

$$V = E_{eq} - Ir_{eq} \quad \text{...(iii)}$$

Comparing Eqs. (ii) and (iii), we get

$$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} \quad \text{...(iv)}$$

and $$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \quad \text{...(v)}$$

$$\Rightarrow \quad \frac{1}{r_{eq}} = \frac{r_1 + r_2}{r_1 r_2} = \frac{1}{r_1} + \frac{1}{r_2} \quad \text{...(vi)}$$

Dividing Eq. (iv) by Eq.(v), we get

$$\frac{E_{eq}}{r_{eq}} = \frac{E_1 r_2 + E_2 r_1}{r_1 r_2} = \frac{E_1}{r_1} + \frac{E_2}{r_2}$$

If n cells of emfs $E_1, E_2, \ldots, E_n$ and internal resistances $r_1, r_2, \ldots, r_n$ are connected in parallel, whose equivalent emf is E_{eq} and equivalent internal resistance is r_{eq}, then

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots + \frac{1}{r_n} \text{ and } \frac{E_{eq}}{r_{eq}} = \frac{E_1}{r_1} + \frac{E_2}{r_2} + \ldots + \frac{E_n}{r_n}$$

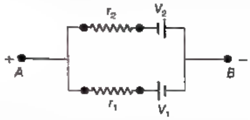
If the two cells are connected in parallel and are of the same emf E and same internal resistance r, then

From Eq. (iv), $E_{eq} = \dfrac{Er + Er}{r + r} = E$

From Eq. (vi), $\dfrac{1}{r_{eq}} = \dfrac{1}{r} + \dfrac{1}{r} = \dfrac{2}{r} \Rightarrow r_{eq} = \dfrac{r}{2}$

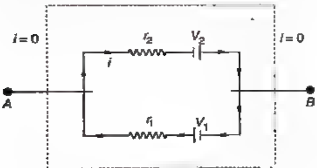
[redacted]

EXAMPLE |8| Find the emf (V) and internal resistance (r) of a single battery which is equivalent to a parallel combination of two batteries of emfs V_1 and V_2 and internal resistances r_1 and r_2 respectively, with polarities as shown in figure



Sol. (i) **Equivalent emf (V) of the battery**

Potential difference across the terminals of the battery is equal to its emf when current drawn from the battery is zero. In the given circuit,



Current in the internal circuit,

$$i = \frac{\text{Net emf}}{\text{Total resistance}} = \frac{V_1 + V_2}{r_1 + r_2}$$

Therefore, potential difference between A and B would be

$$V_A - V_B = V_1 - ir_1$$

$$= V_1 - \left(\frac{V_1 + V_2}{r_1 + r_2}\right) r_1 = \frac{V_1 r_2 - V_2 r_1}{r_1 + r_2}$$

So, the equivalent emf of the battery is

$$V = \frac{V_1 r_2 - V_2 r_1}{r_1 + r_2}$$

Note that, if $V_1 r_2 = V_2 r_1$; $V = 0$

If $V_1 r_2 > V_2 r_1$; $V_A - V_B$ = positive, i.e. A side of the equivalent battery will become the positive terminal and *vice-versa*.

(ii) **Internal resistance (r) of the battery**

r_1 and r_2 are in parallel. Therefore, the internal resistance r will be given by

$$\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$\Rightarrow \qquad r = \frac{r_1 r_2}{r_1 + r_2}$$

TOPIC PRACTICE 3

OBJECTIVE Type Questions

1. The internal resistance of a 2.1 V cell which gives a current of 0.2 A through a resistance of 10 Ω is **NEET 2013**

   (a) 0.2 Ω (b) 0.5 Ω (c) 0.8 Ω (d) 1.0 Ω

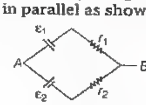
2. The cell has an emf of 2V and the internal resistance of this cell is 0.1 Ω, it is connected to resistance of 3.9 Ω, the voltage across the cell will be

   (a) 1.95 V (b) 1.5 V (c) 2 V (d) 1.8 V

3. Electromotive force of primary cell is 2.4 V. When cell is short circuited, then current becomes 4 A. Internal resistance of cell is

   (a) 60 Ω (b) 1.2 Ω
   (c) 4 Ω (d) 0.6 Ω

4. Two batteries of emf ε_1 and ε_2, ($\varepsilon_2 > \varepsilon_1$) and internal resistances r_1 and r_2 respectively are connected in parallel as shown in figure.



NCERT Exemplar

   (a) Two equivalent emf ε_{eq} of the two cells is between ε_1 and ε_2, i.e., $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$
   (b) The equivalent emf ε_{eq} is smaller than ε_1
   (c) The ε_{eq} is given by $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ always
   (d) ε_{eq} is independent of internal resistances r_1 and r_2

VERY SHORT ANSWER Type Questions

5. The emf of a cell is always greater than its terminal voltage. Why? **Delhi 2013**

6. A cell of emf E and internal resistance r is connected across an external resistance R. Plot a graph showing the variation of potential difference across R, V versus R. **NCERT Exemplar**

7. Write any two factors on which internal resistance of a cell depends. **All India 2013**

8. A cell of emf E and internal resistance r is connected across a variable load resistor R.

[redacted]

Draw the plots of the terminal voltage V *versus* (i) resistance R and

(ii) current I. **All India 2015**

9. A cell of emf E and internal resistance r is connected across a variable resistor R. Plot a graph showing variation of terminal voltage V of the cell *versus* the current I. Using the plot, show how the emf of the cell and its internal resistance can be determined. **All India 2014**

10. Two identical cells, each of emf E, having negligible internal resistance are connected in parallel with each other across an external resistance R. What is the current through this resistance? **All India 2013**

11. Which of the two emf E or potential difference V of a cell, is greater and by how much?

SHORT ANSWER Type Questions

12. First a set of n equal resistors of R each are connected in series to a battery of emf E and internal resistance R and current I is observed to flow. Then, the resistors are connected in parallel to the same battery.

It is observed that the current is increased 10 times. What is n? **NCERT Exemplar**

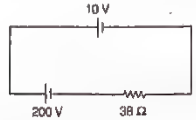
13. Write the relation between emf and potential difference for a cell. What are their respective units?

14. What is the difference between the values of potential difference across the two terminals of a cell is an open circuit and closed circuit?

15. A cell of emf E and internal resistance r is connected across a variable resistor R. Plot a graph showing the variation of terminal potential V with resistance R. Predict from the graph, the condition under which V becomes equal to E. **Delhi 2009**

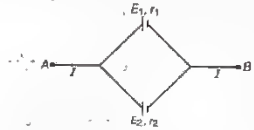
16. A low voltage supply from which one needs high currents must have very low internal resistance. Why?

17. A 10 V cell of negligible internal resistance is connected in parallel across a battery of emf 200 V and internal resistance 38 Ω as shown in the figure. Find the value of current in the circuit. **CBSE 2018**



LONG ANSWER Type I Questions

18. Two cells of emf E_1 and E_2; and internal resistances r_1 and r_2 respectively, are connected in parallel as shown in the figure.



Deduce the expressions for

(i) the equivalent emf of the combination.
(ii) the equivalent resistance of the combination.
(iii) the potential difference between the points A and B. **Foreign 2010**

19. Which type of combination of cells is used in the following three cases.

(i) If the external resistance is much larger than the total internal resistance?
(ii) If the external resistance is much smaller than the total internal resistance?
(iii) If the external resistance is equal to the total internal resistance?

20. What do you mean by terminal potential difference of a cell? Under what conditions will the terminal potential difference of a cell be greater than its emf?

LONG ANSWER Type II Question

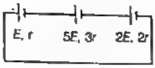
21. (i) The emf of a cell is always greater than its terminal voltage. Why? Give reason.

(ii) Plot a graph showing the variation of terminal potential difference across a cell of emf E and internal resistance r with current drawn from it. Using this graph, how does one determine the emf of the cell?

(iii) Three cells of emf E, $2E$ and $5E$ having internal resistances r, $2r$ and $3r$, variable resistance R as shown in the figure. Find the expression for the

[redacted]

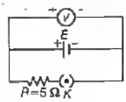
current. Plot a graph for variation of current with R.



NUMERICAL PROBLEMS

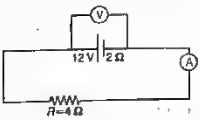
22. A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of resistor? What is the terminal voltage of the battery when the circuit is closed? NCERT

23. The reading on a high resistance voltmeter, when a cell is connected across it, is 2.2 V. When the terminals of the cell are also connected to a resistance of 5 Ω as shown in the circuit, the voltmeter reading drops to 1.8 V. Find the internal resistance of the cell All India 2013



24. It is found that when $R = 4\,\Omega$, the current is 1 A and when R is increased to 9 Ω, the current reduces to 0.5 A. Find the values of the emf E and internal resistance r. All India 2015

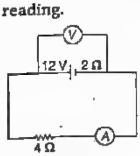
25. In the figure shown, an ammeter A and a resistor of 4 Ω are connected to the terminals of the source. The emf of the source is 12 V having an internal resistance of 2 Ω. Calculate the voltmeter and ammeter readings.



All India 2017

26. A battery of emf 12 V and internal resistance 2 Ω is connected to a 4 Ω resistor as shown in the figure.

(i) Show that a voltmeter when placed across the cell and across the resistor, in turn, gives the same reading.



(ii) To record the voltage and the current in the circuit, why is voltmeter placed in parallel and ammeter in series in the circuit? All India 2016

27. A 10 V battery of negligible internal resistance is connected across a 200 V battery and a resistance of 38 Ω as shown in the figure. Find the value of the current in the circuit. Delhi 2013

28. (i) Six lead-acid type of secondary cells each of emf 2 V and internal resistance 0.015 Ω are joined in series to provide a supply to a resistance of 8.5 Ω. What are the current drawn from the supply and its terminal voltage?

(ii) A secondary cell after long use has an emf of 1.9 V and a large internal resistance of 380 Ω. What maximum current can be drawn from the cell?
Could the cell drive the starting motor of a car? NCERT

HINTS AND SOLUTIONS

1. (b) As, $I = \dfrac{E}{R+r}$

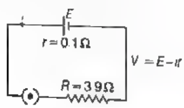
or $E = I(R+r)$

$2.1 = 0.2\,(10+r)$

$10 + r = \dfrac{2.1}{2} \times 10$

$\therefore\quad r = 10.5 - 10 = 0.5\,\Omega$

2. (a)



$\therefore\quad V = E - ir$

where, r is the internal resistance.

Also, current $i = \dfrac{E}{R+r}$

$\Rightarrow\quad V = E - \left(\dfrac{E}{R+r}\right)r$

Putting numerical values, we have

$E = 2\,\text{V},\ r = 0.1\,\Omega,\ R = 3.9\,\Omega$

$\Rightarrow\quad V = 2 - \left(\dfrac{2}{3.9 + 0.1}\right) \times 0.1$

$\Rightarrow\quad V = 1.95\ \text{V}$

[redacted]

3. (d) Electromotive force, $E = V + ir = i(R + r)$. $[\because V = iR]$

When cell is short circuited, then resistance becomes zero, i.e. $R = 0$. So, electromotive force, $E = ir$

Internal resistance of cell

$$r = \frac{E}{i} = \frac{2.4}{4} = 0.6\ \Omega$$

4. (a) The equivalent emf of this combination is given by

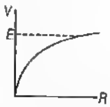
$$\varepsilon_{eq} = \frac{\varepsilon_2 r_1 + \varepsilon_1 r_2}{r_1 + r_2}$$

This suggest that the equivalent emf ε_{eq} of the two cells is given by

$$\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$$

5. The emf of a cell is greater than its terminal voltage because there is some potential drop across the cell due to its small internal resistance.

6. The graphical relationship between voltage across R and the resistance R is given below



$$\left[\because V = \frac{E}{1 + r/R}\right]$$

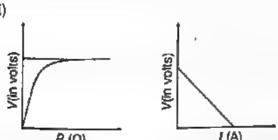
7. The high resistance voltmeter means that current will flow through it. Hence, there is no potential difference across it. So, the reading shown by the high resistance voltmeter can be taken as the emf of the cell

The internal resistance of a cell depends on

(i) the concentration of electrolyte and

(ii) distance between the two electrodes.

8. (i)



Graph between terminal voltage (V) and resistance (R)
(a)

Graph between terminal voltage (V) and current (I)
(b)

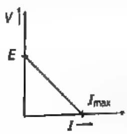
(i) The value of potential difference corresponding to zero current gives emf of the cell.

(ii) Maximum current is drawn, when terminal voltage is zero, so

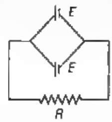
$$V = E - Ir$$
$$\Rightarrow \quad 0 = E - I_{max} r$$
$$\Rightarrow \quad r = \frac{E}{I_{max}}$$

9. We know that, $V = E - Ir$

The plot between V and I is a straight line of positive intercept and negative slope as shown below



10. The cells are arranged as shown in the circuit diagram as given below



As the internal resistance is negligible, so total resistance of the circuit = R

So, current through the resistance,

$$I = \frac{E}{R}$$

(in parallel combination, potential is same as the single cell)

11. emf E of the cell is greater than the potential difference V of the cell, by a value Ir, where I is the current flowing in the circuit and r is the internal resistance of the cell.

$$V = E - Ir$$

12. In series combination of resistors, current I is given by

$$I = \frac{E}{R + nR}$$

whereas, in parallel combination current $10\,I$ is given by

$$\frac{E}{R + \frac{R}{n}} = 10\,I \;\Rightarrow\; \frac{E}{R + \frac{R}{n}} = 10\left(\frac{E}{R + nR}\right)$$

Now, according to problem,

$$\frac{1+n}{1+\frac{1}{n}} = 10 \;\Rightarrow\; 10 = \left(\frac{1+n}{n+1}\right)n \Rightarrow n = 10$$

13. For a cell of emf E, potential difference V and internal resistance r, $V = E - Ir$, where I is the current flowing through the circuit. The SI unit of both emf and potential difference of a cell is volt (V).

14. The potential difference across the terminals of a cell is given by $V = E - Ir$.

In an open circuit, there is no current, i.e. $I = 0$

$\therefore$ $V = E$, i.e. potential difference across the terminals of a cell = emf

In a closed circuit, $V < E$.

The difference between the two values of potential difference = Ir, which is called the lost voltage.

[redacted]

15. Refer to the solution of Q. 6 for the graph.
From the graph, we can see that the value of V becomes equal to E when $I = 0$.

16. We know that, $V = E - Ir$

$\therefore$ Current in the circuit, $I = \frac{E - V}{r}$

If the value of V is small, for high value of current I, then the internal resistance r should be small as $I \propto \frac{1}{r}$.

17. Given, $\varepsilon = 10\text{ V}$, $E = 200\text{ V}$,

Now, using Kirchhoff's loop law in given figure, in loop *ABCDA*,

$$200 - 38I - 10 = 0$$
$$190 = 38I$$
$$\therefore \quad I = \frac{190}{38} = 5\text{ A}$$

18. Refer to text on page 142.

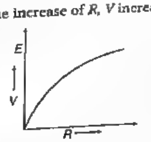
19. (i) Series combination of cells.
(ii) Parallel combination of cells.
(iii) Mixed combination of cells.

20. Refer to text on page 140.
The terminal potential difference of the cell becomes greater than the emf of the cell during charging of the cell. In this process, current flows from positive electrode to negative electrode of the cell.
Hence, $V = E + Ir$.

21. (i) The emf of a cell is greater than its terminal voltage because there is some potential drop across the cell due to its small internal resistance.

(ii) $\because \quad V = \left(\frac{E}{R + r}\right)R = \frac{E}{1 + r/R}$

i.e. with the increase of R, V increases



One can determine the emf of cell by finding terminal potential difference when current I becomes zero.

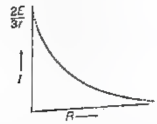
(iii) In these type of questions, we have to look out the connections of different cells, if the opposite terminals of all the cells are connected, then they support each other, i.e. these individual emf's are added up. If the same terminals of the cells are connected, then the equivalent emf is obtained by taking the difference of emf's.

Net emf of combination
$$= E - 2E + 5E = 4E$$

Net resistance of current
$$= r + 2r + 3r + R = 6r + R$$

$\therefore$ Current, $I = \frac{V}{R}$ (from Ohm's law)

$$\Rightarrow \quad I = \frac{4E}{6r + R}$$



22. Given, $E = 10\text{ V}$, $r = 3\Omega$, $I = 0.5\text{A}$

As, $\quad I = \frac{E}{R + r}$

$$\Rightarrow \quad R = \frac{E}{I} - r$$
$$= \frac{10}{0.5} - 3 = 17\,\Omega$$

and terminal voltage, $V = IR = 0.5 \times 17 = 8.5\text{ V}$

23. The emf of cell, $E = 2.2\text{ V}$

The terminal voltage across cell, when $5\,\Omega$ resistance R is connected across it, $V = 1.8\text{ V}$

Let internal resistance $= r$

$\therefore$ Internal resistance, $r = R\left(\frac{E}{V} - 1\right)$

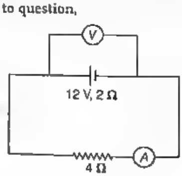
$$= 5\left(\frac{2.2}{1.8} - 1\right)$$
$$= 5 \times \frac{0.4}{1.8} = \frac{2}{1.8} = \frac{10}{9}\,\Omega$$

24. Refer to Example 2 on page 141. [Ans. 1Ω and 5 V]

25. Current in the circuit, $I = \frac{E}{R + r} = \frac{12}{4 + 2} = 2\text{ A}$

Also, terminal voltage across the cell,
$V = E - Ir = 12 - 2 \times 2 = 8\text{ V}$
So, ammeter reading $= 2$ A
and voltmeter reading $= 8$ V

26. According to question,



(i) Net current in the circuit $= \frac{12}{6} = 2\text{A}$

Voltage across the battery,
$$V_b = 12 - 2 \times 2 = 8\text{V}$$

[redacted]

Voltage across the resistance,

$$V_r = IR = 2 \times 4 = 8\text{V}$$

(ii) In order to measure the device's voltage for a voltmeter, it must be connected in parallel to that device. This is necessary because device in parallel experiences the same potential difference. An ammeter is connected in series with the circuit because the purpose of the ammeter is to measure the current through the circuit. Since, the ammeter is a low impedance device. Connecting in parallel with the circuit would cause a short circuit, damaging the ammeter of the circuit.

27. Since, the positive terminal of the batteries are connected together, so the equivalent emf of the batteries is given by

$$E = 200 - 10 = 190\text{ V}$$

Hence, the current in the circuit is given by

$$I = \frac{E}{R} = \frac{190}{38} = 5\text{ A}$$

28. (i) Six cells are joined in series.

emf of each cell, $E = 2$ V

Number of cells, $n = 6$

Total emf of circuit $= n \times E = 6 \times 2 = 12$ V

Internal resistance of each cell, $r = 0.015\ \Omega$

Total internal resistance

$$= n \times r = 6 \times 0.015 = 0.09\ \Omega$$

External load, $R = 8.5\ \Omega$

Current in the circuit, $I = \dfrac{nE}{nr + R}$

$$= \frac{12}{0.09 + 8.5} = 1.4\text{ A}$$

$\therefore$ The terminal voltage of battery,

$$V = IR = 1.4 \times 8.5 = 11.9\text{ V}$$

(ii) emf of cell, $E = 1.9$ V

Internal resistance of cell,

$$r = 380\ \Omega$$

Maximum current can be drawn from the cell, if there is zero external resistance. Therefore,

$$I_{max} = \frac{E}{r} = \frac{1.9}{380} = 0.005\text{ A}$$

Now, we see that the maximum current drawn from the cell is very low, thus the cell cannot be used to drive the starting motor of a car as the current required for this purpose is approximately 100 A for few records.

|TOPIC 4|

Kirchhoff's Laws and its Applications

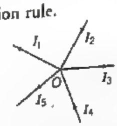
KIRCHHOFF'S RULES OR LAWS

In 1842, Kirchhoff gave the following two rules to solve complicated electrical circuits. Ohm's law is simply not adequate for the study of the circuits containing more than one source of emf. These rules are basically the expressions of conservation of electric charge and energy.

These laws were stated as follows

First Law (Junction Rule)

This law states that the algebraic sum of the currents meeting at a point in an electrical circuit is always zero. It is also known as junction rule.



Electric junction

Consider a point O in an electrical circuit at which currents I_1, I_2, I_3, I_4 and I_5 flowing through the different conductors meet, as shown in the figure.

According to Kirchhoff's first law, we have

$$I_1 + I_2 + (-I_3) + I_4 + (-I_5) = 0$$

$$\Rightarrow \quad I_1 + I_2 - I_3 + I_4 - I_5 = 0$$

$$\therefore \quad I_1 + I_2 + I_4 = I_3 + I_5$$

So, junction rule can also be stated as the sum of currents entering the junction is equal to the sum of currents leaving the junction.

Sign Convention for Kirchhoff's First Law

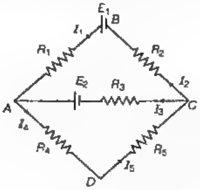
The current flowing towards the junction of conductors is considered as positive and the current flowing away from the junction is taken as negative.

Second Law (Kirchhoff's Voltage Rule)

This law states that the algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero. It means that in any closed part of an electrical circuit, the algebraic sum of the emfs is equal to the algebraic sum of the products of the resistances and currents flowing through them. It is also known as loop rule.

[redacted]

Consider a closed electrical circuit $ABCDA$ containing two cells E_1 and E_2 and five resistances R_1, R_2, R_3, R_4 and R_5.



Consider the closed loop $ABCA$. E_1 will send current in anti-clockwise and E_2 will send current in clockwise direction.

$\therefore$ Total emf of closed loop

$$ABCA = E_1 + (-E_2) = E_1 - E_2$$

But currents (I_1 and I_2) flow in anti-clockwise direction while current I_3 flows in clockwise direction.

The algebraic sum of products of resistances and current

$$= I_1R_1 + I_2R_2 + (-I_3)R_3$$
$$= I_1R_1 + I_2R_2 - I_3R_3$$

$\therefore$ According to second law, for closed part $ABCA$,

$$E_1 - E_2 = I_1R_1 + I_2R_2 - I_3R_3$$

Similarly, for closed part $ACDA$,

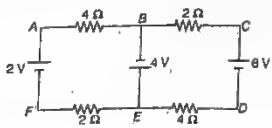
$$E_2 = I_3R_3 + I_4R_4 + I_5R_5$$

Sign Convention for Kirchhoff's Second Law

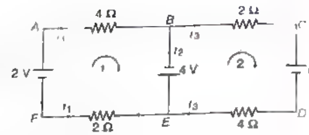
The product of resistance and current in an arm of the loop is positive, if the direction of current in that arm is in the same sense as one moves and is taken as negative, if the direction of current in an arm is opposite to the sense as one moves.

While traversing a loop, the emf of a cell is taken negative, if negative pole of the cell is encountered first, otherwise positive.

EXAMPLE |1| **Find currents in different branches of the electric circuit shown in figure.**



Sol.



Applying Kirchhoff's first law (junction law) at junction B,

$$i_1 = i_2 + i_3 \quad \text{...(i)}$$

Applying Kirchhoff's second law in loop 1 ($ABEFA$)

$$-4i_1 + 4 - 2i_1 + 2 = 0 \quad \text{...(ii)}$$

Applying Kirchhoff's second law in loop 2 ($BCDEB$),

$$2i_3 - 6 - 4i_3 - 4 = 0 \quad \text{...(iii)}$$

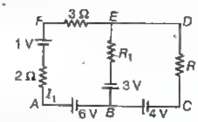
Solving Eqs. (i), (ii) and (iii), we get

$$i_1 = 1\text{ A}$$

$$\Rightarrow \quad i_2 = \frac{8}{3}\text{ A} \Rightarrow i_3 = -\frac{5}{3}\text{ A}$$

Here, negative sign of i_3 implies that current i_3 is in opposite direction of what we have assumed.

EXAMPLE |2| **Use Kirchhoff's rules to determine the potential difference between the points A and D. When no current flows in the arm BE of the electric network shown in the figure below.** **Delhi 2015**



Sol. Applying Kirchhoff's loop rule for loop $ABEFA$

$$6 + 3 + R_1 \times 0 - 3I_1 + 1 - 2I_1 = 0$$

or $\quad 10 - 5I_1 = 0$

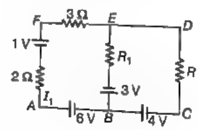
or $\quad I_1 = 2\text{A}$

For loop $BCDEB$,

$$4 - I_1 \cdot R + R_1 \times 0 - 3 = 0$$

or $\quad 1 - 2R = 0$

$\therefore \quad R = \frac{1}{2}\,\Omega$



[redacted]

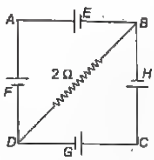
Potential difference between A and D through path $ABCD$ is

$$6+4-I_1 R = V_{AD}$$

$$\Rightarrow \quad 10 - 2\times\frac{1}{2} = V_{AD}$$

$$\therefore \quad V_{AD} = 9 \text{ volt}$$

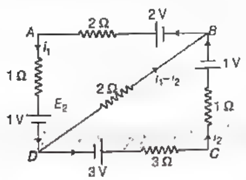
EXAMPLE |3| In the circuit shown in figure E, F, G, H are cells of emf 2, 1, 3 and 1 V respectively, and their internal resistances are 2, 1, 3 and 1 Ω, respectively. Calculate



(i) the potential difference between B and D and

(ii) the potential difference across the terminals of each cells G and H.

Sol.



Applying Kirchhoff's second law in loop $BADB$,

$$2-2i_1-i_1-1-2(i_1-i_2)=0 \quad \text{...(i)}$$

Similarly, applying Kirchhoff's second law in loop $BDCB$,

$$2(i_1-i_2)+3-3i_2-i_2-1=0 \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), we get

$$i_1=\frac{5}{13},\ i_2=\frac{6}{13}$$

and $$i_1-i_2=-\frac{1}{13}$$

(i) Potential difference between B and D,

$$V_B+2(i_1-i_2)=V_D$$

$$\therefore \quad V_B-V_D=-2(i_1-i_2)=\frac{2}{13}\text{ V}$$

(ii) $$V_G-E_G-i_2r_G=3-\frac{6}{13}\times 3=\frac{21}{13}\text{ V}$$

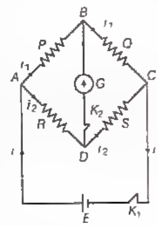
$$V_H=E_H+i_2r_H=1+\frac{6}{13}\times 1=\frac{19}{13}\text{ V}$$

WHEATSTONE BRIDGE

It is an arrangement of four resistances used to measure one of them in terms of the other three.

Consider four resistances P, Q, R and S are connected in the four arms of a quadrilateral. The galvanometer G and a tapping key K_2 are connected between points B and D. The cell of emf E and 1-way key K_1 are connected between points A and C as shown in the figure. Resistances P and Q are called ratio arms, resistance R is a variable resistance and S is unknown resistance.

The bridge is said to be balanced, when the galvanometer gives zero deflection. Thus, we have balance condition as



Wheatstone bridge

$$\boxed{\frac{P}{Q}=\frac{R}{S}}$$

Proof

In figure, four resistances P, Q, R and S are connected in the four arms of a parallelogram $ABCD$. Between B and D there is a sensitive galvanometer and a cell is connected between A and C. K_1 and K_2 are two keys. By pressing the key K_1, a current i is allowed to flow from the cell. At the point A, the current i is divided into two parts.

One part i_1 flows in the arm AB and the other part i_2 flows in the arm AD. The resistances P, Q, R and S are so adjusted that on pressing the key K_2 there is no deflection in the galvanometer G. That is, there is no current in the diagonal BD. Thus, the same current i_1 will flow in the arm BC as in the arm AB and the same i_2 will flow in the arm DC as in the arm AD.

Applying Kirchhoff 's second law for the closed loop $BADB$, we have

$$-i_1P+i_2R=0$$

$$Pi_1=Ri_2 \quad \text{...(i)}$$

[redacted]

Similarly, for the closed loop $CBDC$, we have

$$-i_1Q + i_2S = 0$$

$$Qi_1 = Si_2 \quad \quad ...(ii)$$

Dividing Eq. (i) by Eq. (ii), we have

$$\frac{i_1P}{i_1Q} = \frac{i_2R}{i_2S} \quad \text{or} \quad \frac{P}{Q} = \frac{R}{S}$$

It is clear from this formula that if the ratio of the resistances P and Q and resistance R are known, then the unknown resistance S can be calculated. This is why, the arms AB and BC are called **ratio arms**, arm AD **known arm** and arm CD **unknown arm**.

When the bridge is balanced, then on interchanging the positions of the galvanometer and the cell there is no effect on the balance condition of the bridge. Hence, the arms BD and AC are called **conjugate arms** of the bridge. (In balanced state, no current flows in the galvanometer arm, hence while computing the equivalent resistance between A and C, the resistance connected between B and D may be neglected.) The sensitivity of the bridge depends upon the values of the resistance. The bridge is maximum sensitive, when all the four resistances are of the same order.

According to Maxwell, for greater sensitivity of the bridge, the galvanometer or the battery whichever has the higher resistance should be connected across the junctions of two highest and two lowest resistances.

Note The Wheatstone bridge is most sensitive, when the resistance of all the four arms of the bridge is of same order (or same), i.e null point is obtained at the middle of bridge wire.

The advantage of null point method /zero deflection in a Wheatstone bridge is that the resistance of galvanometer does not affect the balance point, there is no need to determine current in resistances and internal resistance of a galvanometer.

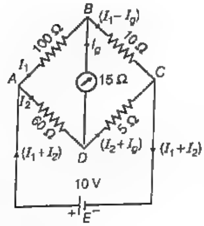
EXAMPLE |4| **In a Wheatstone bridge circuit, $P = 7\,\Omega$, $Q = 8\,\Omega$, $R = 12\,\Omega$ and $S = 7\,\Omega$. Find the additional resistance to be used in series with S, so that the bridge is balanced.**

Sol. Let the bridge be balanced when additional resistance x is put in series with S.

Then, $(S + x) = \frac{Q}{P}R$

or $\quad x = \frac{Q}{P}R - S = \frac{8}{7} \times 12 - 7 = 6.72\,\Omega$

EXAMPLE |5| **The Wheatstone bridge circuit have the resistances in various arms as shown in figure. Calculate the current through the galvanometer.**



Sol. In the closed loop $ABDA$,

$$100\,I_1 + 15\,I_g - 60\,I_2 = 0$$

$$\Rightarrow \quad 20\,I_1 + 3\,I_g - 12\,I_2 = 0 \quad ...(i)$$

In the closed loop $BCDB$,

$$10(I_1 - I_g) - 5(I_2 + I_g) - 15I_g = 0$$

$$\Rightarrow \quad 10I_1 - 30\,I_g - 5\,I_2 = 0$$

$$\Rightarrow \quad 2I_1 - 6\,I_g - I_2 = 0 \quad ...(ii)$$

In the closed loop $ADCEA$,

$$60\,I_2 + 5(I_2 + I_g) = 10$$

$$\Rightarrow \quad 65\,I_2 + 5\,I_g = 10$$

$$\Rightarrow \quad 13I_2 + I_g = 2 \quad ...(iii)$$

On solving Eqs. (i), (ii) and (iii), we get

$$I_g = 4.87\text{ mA}$$

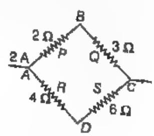
TOPIC PRACTICE 4

OBJECTIVE Type Questions

1. **Kirchhoff's current law is consequence of conservation of**

(a) energy
(b) momentum
(c) charge
(d) mass

2. **If 2 A current is flowing in the shown circuit, then potential difference $(V_B - V_D)$ in balanced condition is**



(a) 12 V
(b) 6 V
(c) 4 V
(d) zero

[redacted]

3. The Wheatstone bridge and its balance condition provide a practical method for determination of an
(a) known resistance
(b) unknown resistance
(c) Both (a) and (b)
(d) None of the above

VERY SHORT ANSWER Type Questions

4. State Kirchhoff's first law. **All India 2010**

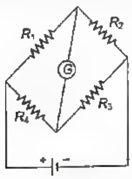
5. State Kirchhoff's second law.

6. When a Wheatstone bridge is most sensitive?

SHORT ANSWER Type Questions

7. Use Kirchhoff's rules to obtain the balance condition in a Wheatstone bridge. **Delhi 2012**

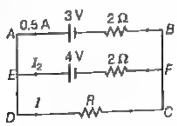
8. For the circuit diagram of a Wheatstone bridge shown in the figure, use Kirchhoff's laws to obtain its balance condition.



Delhi 2009

NUMERICAL PROBLEMS

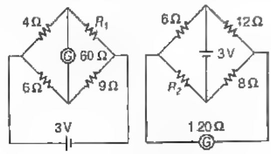
9. Using Kirchhoff's rules in the given circuit, determine



(i) the voltage drop across the unknown resistor R.
(ii) the current I in the arm EF. **All India 2011**

10. A battery of 10 V and negligible internal resistance is connected across the diagonally opposite corners of a cubical network consisting of 12 resistors each of 1 Ω resistance. Use Kirchhoff's rules to determine
(i) the equivalent resistance of the network.
(ii) the total current in the network. **All India 2010**

11. Figure shows two circuits each having a galvanometer and a battery of 3 V.
When the galvanometer in each arrangement do not show any deflection, obtain the ratio R_1/R_2. **All India 2013**



HINTS AND SOLUTIONS

1. (c) According to Kirchhoff's law, the algebraic sum of the currents is meeting at point in an electrical circuit is always zero, i.e. at any junction, the charge cannot be stored and cannot be loss. So, Kirchhoff's current law is consequence of conservation of charge.

2. (d) In Wheatstone bridge, $\frac{P}{Q} = \frac{R}{S}$

or $\frac{2}{3} = \frac{4}{6} = \frac{2}{3}$

i.e. in the balanced condition, $V_B - V_D = 0$

3. (b) In meter bridge balanced wheatstone bridge is used to determine unknown resistance.

4. Kirchhoff's first law states that the algebraic sum of currents at a junction in an electrical circuit is zero, i.e. $\Sigma I = 0$.

5. Kirchhoff's second law states that the algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero.

6. The Wheatstone bridge is most sensitive, when the resistance of all the four arms of the bridge are equal.

7. Refer to text on page 154.

8. Refer to text on page 154.
Put $P = R_1, Q = R_2, R = R_4$ and $S = R_3$.

9. (i) Applying Kirchhoff's second rule in the closed loop $ABFEA$,

$$V_B - 0.5 \times 2 + 3 = V_A$$
$$\Rightarrow \quad V_B - V_A = -2$$
$$V = V_A - V_B = +2\text{ V}$$

Potential drop across R is 2 V as R, EF and upper row are in parallel.

[redacted]

(ii) Applying Kirchhoff's first rule at E,

$$0.5 + I_2 = I$$

where, I is current through R.

Now, Kirchhoff's second rule in closed loop $FEABF$,

$$-2I_2 + 4 - 3 + 0.5 \times 2 = 0$$

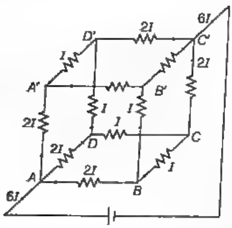
$$2I_2 - 2 = 0$$

or $\quad I_2 = 1\text{ A}$

The current in arm $EF = 1\text{ A}$

10. Let $6I$ current be drawn from the cell. Since, the paths AA', AD and AB are symmetrical, current through them is same.

As per Kirchhoff's junction rule, the current distribution is shown in the figure.



(i) Let the equivalent resistance across the combination be R.

$\therefore \quad E = V_A - V_B$

$$= (6I)\,R$$

$\Rightarrow \quad 6IR = 10 \qquad [\because E = 10\text{ V}]\ldots\text{(i)}$

(ii) Applying Kirchhoff 's second rule in loop $AA'B'C'A$,

$$-2I \times 1 - I \times 1 - 2I \times 1 + 10 = 0$$

$\Rightarrow \quad 5I = 10$

$\Rightarrow \quad I = 2\text{ A}$

Total current in the network

$$= 6I = 6 \times 2 = 12\text{ A}$$

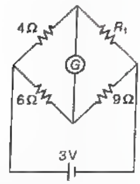
From Eq. (i), we get

$$6IR = 10$$

$\Rightarrow \quad 6 \times 2 \times R = 10$

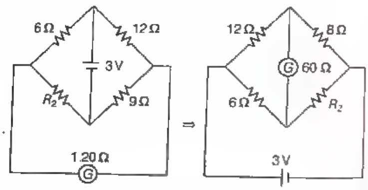
$\Rightarrow \quad R = \dfrac{10}{12} = \dfrac{5}{6}\,\Omega$

11.



For balanced Wheatstone bridge, there will be no deflection in the galvanometer.

$$\frac{4}{R_1} = \frac{6}{9} \Rightarrow R_1 = \frac{4 \times 9}{6} = 6\Omega$$



For the equivalent circuit, when the Wheatstone bridge is balanced, there will be no deflection in the galvanometer.

$\therefore \quad \dfrac{12}{8} = \dfrac{6}{R_2}$

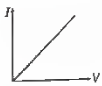
$\Rightarrow \quad R_2 = \dfrac{6 \times 8}{12} = 4\,\Omega$

$\therefore \quad \dfrac{R_1}{R_2} = \dfrac{6}{4} = \dfrac{3}{2}$

[redacted]

SUMMARY

- **Electric Current** It is defined as the rate of flow of electric charge through any cross-section of a conductor, i.e.
$$I = (dq/dt)$$
- The directed rate of flow of electric charge through any cross-section of a conductor is known as **electric current**
$$I = \frac{q}{t} = \frac{ne}{t} \qquad [\because q = ne]$$
where, n = number of charged particles constitute the current
- **Current Density** It is the ratio of the current at a point in conductor to the area of cross-section of the conductor at that point, i.e. $J = (I/A)$.
- **Ohm's Law** At constant temperature, the potential difference V across the ends of a given metallic wire (conductor) in an circuit (electric) is directly proportional to the current flowing through it.



$$V \propto I$$

The variation of current w.r.t. applied potential difference is shown with the help of given graph.
$$V = IR$$
where, R = resistance of conductor.
- **Flow of Electric Charges in Metallic Conductors** In case of solid conductor large number of free electrons causes the strong current in them.

  In the case of a liquid conductor, movement of positive and negative charged ions causes the electric current.
- **Resistance of a Conductor** Mathematically it is the ratio of potential difference applied across the ends of conductor to the current flowing through it

  $\Rightarrow$ $$R = \frac{V}{I}$$

  SI unit is ohm (Ω).

  Resistance can also be written as, $R = \rho \frac{L}{A}$.

  where, L = length of the conductor,

  A = area of cross-section

  and ρ = constant, known as resistivity of the material

  It depends upon nature of the material.
- **Effect of Temperature on Resistance**

  For metals, resistance increases with rise in temperature.

  For insulators and semiconductors, resistance decreases with rise in temperature.

  For alloys, temperature coefficient of resistance is small.
- **Temperature coefficient** of resistance is given by
$$\alpha = \frac{R_2 - R_1}{R_1(t_2 - t_1)}$$
- **Drift Velocity** It is defined as the average velocity with which the free electrons move towards the positive end of a conductor under the influence of an external electric field applied.

  $\Rightarrow$ $$v_d = \frac{eE}{m}\tau$$

  where, τ = relaxation time,

  E = electric field,

  m = mass and e = electron.
- **Relation between Drift Velocity and Electric Field**

  It is given by, $I = neAv_d$

  where, n = number density of free electrons,

  e = electronic charge

  A = cross-sectional area

  and v_d = drift velocity of an electron.
- The ratio of drift velocity of electrons and the applied electric field is known as **mobility**.

  $\Rightarrow$ $$\mu = \frac{v_d}{E} = \frac{q\tau}{m}$$

  $\therefore$ SI unit is $[m^2 s^{-1} V^{-1}]$
- **Resistivity** It is the resistance of a unit length with unit area of cross-section of the material of the conductor.
- **Relationship between resistivity and relaxation time**
$$\rho = \frac{m}{ne^2\tau}$$
where, τ = relaxation time

  Specific resistance or resistivity (ρ) depends on the material of conductor, not on the length and cross-sectional area (A), i.e geometry of conductor.
- **Effect of Temperature on Resistivity**

  For metals, resistivity increases with increase in temperature.

  For semiconductor resistivity decreases with increase in temperature.

  For alloys, resistivity is very large but has a weak dependence on temperature.
- **Classification of Materials in terms of Conductivity**

  For insulators, electrical conductivity is very small or nil.

  For conductors, electrical conductivity is very high.

  For semiconductors, electrical conductivity lies in between that of insulators and conductors.

[redacted]

- **Conductance and Conductivity**

  Conductance is the reciprocal of resistance of conductor.

  Conductivity is the reciprocal of the resistivity of conductor

- **Electrical Energy and Power** Electrical energy is the total work done in maintaining the electric current in the given circuit for a specified time.

  Electrical power is the rate of electrical energy supplied per unit time to maintain flow of electric current through conductor.

- **Internal Resistance and Electromotive Force of a Cell**

  EMF (E) It is the maximum potential difference between two terminals of circuit, when circuit is open

  Internal Resistance (r) The resistance offered by the electrolyte of the cell, to the flow of current through it

  Terminal Potential Difference (V) It is the maximum potential difference between two terminals of circuit, when the circuit is closed

- **The relationship between r, R, E and V** is

$$r = R\left(\frac{E}{V} - 1\right) \quad \ldots(i)$$

where, r = internal resistance, R = external resistance,

E = emf of cell, V = terminal voltage of cell.

Also, $V = E - Ir = \left(\frac{E}{R + r}\right)R \quad \ldots(ii)$

- **Combination of Cells**

  In series grouping, current is given by, $I = (n\varepsilon / R + nr)$

  In parallel grouping current is given by, $I = (m\varepsilon / r + mR)$

  In mixed grouping, current is given by, $I = \left(\frac{mn\varepsilon}{nr + mR}\right)$

- **Kirchhoff's Laws**

  First Law (Junction Rule) The algebraic sum of the currents meeting at a point in an electrical circuit is always zero

  Second Law (Loop Rule) The algebraic sum of changes in potential around any closed loop involving resistors and the cells in the loop is zero

- **Wheatstone Bridge** It is an arrangement of four resistances used to measure one of them in terms of another three. The bridge is said to be balanced when the galvanometer shows zero deflection

  The balance condition is $\frac{P}{Q} = \frac{R}{S}$

[redacted]

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. A potential difference V is applied to a copper wire of length l and diameter d. If V is doubled, then the drift velocity

(a) is doubled (b) is halved
(c) remains same (d) becomes zero

2. A potential difference of 100 V is applied to the ends of a copper wire one metre long. What is the average drift velocity of electrons?
(given, $\sigma = 5.81 \times 10^7 \Omega^{-1}$ or $n_{Cu} = 8.5 \times 10^{28}$ m^{-3})

(a) 0.43 ms^{-1} (b) 0.83 ms^{-1}
(c) 0.52 ms^{-1} (d) 0.95 ms^{-1}

3. Unit of specific resistance is

(a) ohm^{-1}-m^{-1} (b) ohm^{-1} m
(c) ohm-m^{-1} (d) ohm-m

4. The length of 50 Ω resistance becomes twice by stretching. The new resistance is

(a) 25 Ω (b) 50 Ω (c) 100 Ω (d) 200 Ω

5. A metal rod of length 10 cm and a rectangular cross-section of 1cm $\times \frac{1}{2}$ cm is connected to a battery across opposite faces. The resistance will be

(a) maximum when the battery is connected across 1 cm $\times \frac{1}{2}$ cm faces
(b) maximum when the battery is connected across 10 cm × 1 cm faces
(c) maximum when the battery is connected across 10 cm $\times \frac{1}{2}$ cm faces
(d) same irrespective of the three faces

6. The electric power consumed by a 220 V-100 W bulb, when operated at 110 V is **CBSE 2021 (Term-I)**

(a) 25 W (b) 30 W (c) 35 W (d) 45 W

7. If an ammeter is to be used in place of a voltmeter, then we must connect with the ammeter a **CBSE 2021 (Term-I)**

(a) low resistance in parallel
(b) low resistance in series
(c) high resistance in parallel
(d) high resistance in series

8. Kirchhoff's first rule, $\Sigma I = 0$ and second rule, $\Sigma IR = \Sigma E$ (where the symbols have their usual meanings) are respectively, based on **CBSE 2021 (Term-I)**

(a) conservation of momentum and conservation of charge
(b) conservation of energy and conservation of charge
(c) conservation of charge and conservation of momentum
(d) conservation of charge and conservation of energy

9. Which of the following has negative temperature coefficient of resistivity? **CBSE 2021 (Term-I)**

(a) Metal
(b) Metal and semiconductor
(c) Semiconductor
(d) Metal and alloy

10. If the potential difference V applied across a conductor is increased to $2V$ with its temperature kept constant, then the drift velocity of the free electrons in a conductor will **CBSE SQP (Term-I)**

(a) remain the same
(b) become half of its previous value
(c) be double of its initial value
(d) become zero

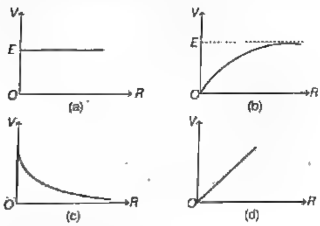
11. A constant voltage is applied between the two ends of a uniform metallic wire, heat H is developed in it. If another wire of the same material, double the radius and twice the length as compared to original wire is used, then the heat developed in it will be **CBSE SQP (Term-I)**

(a) $H/2$ (b) H
(c) $2H$ (d) $4H$

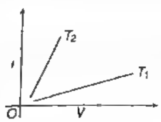
[redacted]

12. In a DC circuit, the direction of current inside the battery and outside the battery, respectively are **CBSE SQP (Term-I)**
(a) positive to negative terminal and negative to positive terminal
(b) positive to negative terminal and positive to negative terminal
(c) negative to positive terminal and positive to negative terminal
(d) negative to positive terminal and negative to positive terminal

13. A cell of emf (E) and internal resistance r is connected across a variable external resistance R. The graph of terminal potential difference V as a function of R is **CBSE 2020**



14. The current i and voltage V graph for a given metallic wire at two different temperatures T_1 and T_2 are shown in the figure. It is concluded that



(a) $T_1 > T_2$ (b) $T_1 < T_2$ (c) $T_1 = T_2$ (d) $T_1 = 2T_2$

15. The electromotive force of cell is 5V and its internal resistance is 2 Ω. This cell is connected to external resistance. If the current in the circuit is 0.4 A, then voltage of poles of cell is
(a) 5 V (b) 5.8 V (c) 4.6 V (d) 4.2 V

ASSERTION AND REASON

Directions (Q. Nos. 16-21) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

16. **Assertion** The average time of collisions τ, decreases with increasing temperature.
Reason At increased in temperature, average speed of the electrons, which act as the carriers of current, increases, resulting in more frequent collisions.

17. **Assertion** Charge carriers do not move with acceleration, with a steady drift velocity.
Reason Charge carriers under go collisions with ions and atoms during transit.

18. **Assertion** If we bend an insulated conducting wire, the resistance of the wire increases.
Reason The drift velocity of electron in bended wire remains same.

19. **Assertion** The drift velocity of electrons in a metallic wire decreases when temperature of the wire is increases.
Reason On increasing temperature, conductivity of metallic wire decreases.

20. **Assertion** Manganin and constantan are widely used in standard resistors.
Reason Manganin and constantan resistances values would change very little with temperatures.

21. **Assertion** Higher the range, lower is the resistance of an ammeter.
Reason To increase the range of an ammeter, additional shunt is added in series to it. **CBSE 2021 (Term-I)**

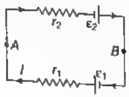
CASE BASED QUESTIONS

Directions (Q.Nos. 22-23) *This question is case study based question. Attempt any 4 sub-parts from this question. Each question carries 1 mark.*

22. **Potential Difference**
The potential difference (V) across a source in a circuit is not equal to its emf (ε). This is due to the reason that every source of electric energy has some internal resistance (r). Further, ε, V and r are related to each other as $V = \varepsilon - Ir$. A single battery shown in figure, consists of two

[redacted]

cells of emf's ε_1 and ε_2 and internal resistances r_1 and r_2, respectively in series.



(i) The current in the internal circuit is

(a) zero (b) $\dfrac{\varepsilon_2 - \varepsilon_1}{r_1 + r_2}$

(c) $\dfrac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2}$ (d) $\dfrac{\varepsilon_1 - \varepsilon_2}{r_1 + r_2}$

(ii) The equivalent emf of the battery is

(a) $(\varepsilon_1 + \varepsilon_2)$ (b) $(\varepsilon_1 - \varepsilon_2)$

(c) $(\varepsilon_2 - \varepsilon_1)$ (d) $\dfrac{(\varepsilon_1 r_2 - \varepsilon_2 r_1)}{r_1 + r_2}$

(iii) For the terminal A to be positive

(a) $\varepsilon_1 > \varepsilon_2$ (b) $\varepsilon_2 > \varepsilon_1$

(c) $\varepsilon_1 r_1 = \varepsilon_2 r_2$ (d) $\varepsilon_1 r_2 > \varepsilon_2 r_1$

(iv) The internal resistance of the battery is

(a) $(r_1 + r_2)$ (b) $\dfrac{(r_1 + r_2)}{r_1 r_2}$

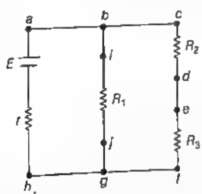
(c) $\dfrac{r_1 r_2}{(r_1 + r_2)}$ (d) $\dfrac{r_2}{(r_1 + r_2)}$

(v) The algebraic sum of changes in potential around any closed loop involving resistor and cells in the loop is

(a) more than zero (b) less than zero

(c) zero (d) constant

23. An experiment was set-up with the circuit diagram shown in figure. Given that, $R_1 = 10\,\Omega$, $R_2 = R_3 = 5\Omega$, $r = 0\,\Omega$ and $E = 5$ V
CBSE 2021 (Term-I)



(i) The points with the same potential are

(a) b, c, d (b) f, h, j (c) d, e, f (d) a, b, j

(ii) The current through branch bg is

(a) 1 A (b) $\dfrac{1}{3}$ A (c) $\dfrac{1}{2}$ A (d) $\dfrac{2}{3}$ A

(iii) The power dissipated in R_1 is

(a) 2 W (b) 2.5 W

(c) 3 W (d) 4.5 W

(iv) The potential difference across R_3 is

(a) 1.5 V (b) 2 V (c) 2.5 V (d) 3 V

FILL IN THE BLANK

24. A copper wire of non-uniform area of cross-section is connected to a DC battery. The physical quantity which remains constant along the wire is ....

VERY SHORT ANSWER Type Questions

25. What is the significance of direction of electric current?

26. Describe how the resistivity of the conductor depends upon

(i) number density (n) of free electrons and

(ii) relaxation time (τ).

27. Two conducting wires A and B of the same length but of different materials are joined in series across a battery. If the number density of electrons in A is twice than that in B, find the ratio of drift velocities of electrons in two wires.

28. How does the mobility of electrons in a conductor change, if the potential difference applied across the conductor is doubled, keeping the length and temperature of the conductor constant? **CBSE 2019**

29. When a potential difference is applied across the ends of a conductor, how is the drift velocity of the electrons related to the relaxation time? **CBSE 2019**

30. How is the drift velocity in a conductor affected with the rise in temperature? **CBSE 2019**

31. Show variation of resistivity of copper as a function of temperature in graph.

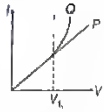
32. On what basic conservation laws, are Kirchhoff's laws based?

33. Define the conductivity of a conductor. Write its SI unit. **All India 2017 C**

[redacted]

SHORT ANSWER Type Questions

34. Figure below shows a plot of current *versus* voltage for two different materials P and Q. Which of the two materials satisfies Ohm's law? Explain.



35. Derive the expression for the resistivity of a good conductor in terms of the relaxation time of electrons.

36. Write the expression for the resistivity of a metallic conductor showing its variation over a limited range of temperatures.

37. Car batteries are often rated in unit ampere hours. Does this unit designate the amount of current, energy, power or charge that can be drawn from the battery? Explain.

38. Two bulbs are rated (P_1, V) and (P_2, V). If they are connected (i) in series and (ii) in parallel across a supply V, find the power dissipated in the two combinations in terms of P_1 and P_2. **CBSE 2020**

39. A wire of length L_0 has a resistance R_0. It is gradually stretched till its length becomes $2L_0$. **CBSE 2020**

(a) Plot a graph showing variation of its resistance R with its length L during stretching.

(b) What will be its resistance when its length becomes $2\,L_0$?

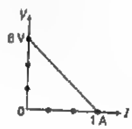
40. A wire of length L_0 has a resistance R_0. It is gradually stretched till its length becomes 1.5 L_0.

(a) Plot a graph showing variation of its resistance R with its length l during stretching.

(b) What will be its resistance when its length becomes $1.5\,L_0$?

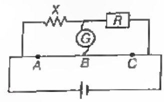
41. Is there some net field inside the cell, when the circuit is closed and a steady current passes through? Explain.

42. The plot of the variation of potential difference across a combination of three identical cells in series *versus* current is as shown in the figure. What is the emf of each cell?



43. R_1, R_2 and R_3 are three different values of resistor R. Such that $R_1 > R_2 > R_3$

A, B and C are the null points obtained corresponding to R_1, R_2 and R_3, respectively. For which resistor, the value of X will be most accurate and why?

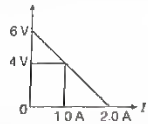


44. The figure shows a plot of terminal voltage V *versus* the current I of a given cell.

Calculate from the graph

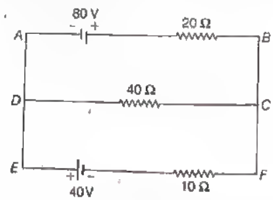
(i) emf of the cell.

(ii) internal resistance of the cell. **All India 2017C**



LONG ANSWER Type I Questions

45. Using Kirchhoff's rules, calculate the current through the 40 Ω and 20 Ω resistors in the following circuit. **CBSE 2019**



46. Show on a plot, variation of resistivity of (i) a conductor and (ii) a typical semiconductor as a function of temperature.

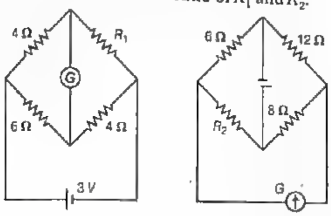
Using the expression for the resistivity in terms of number density and relaxation time between the collisions, explain how resistivity in the case of a conductor increases while it decreases in a semiconductor, with the rise of temperature.

CBSE 2019

[redacted]

47. With the help of a suitable diagram, explain in brief about the sensitivity of Wheatstone bridge?

48. Define the term current sensitivity of a galvanometer. In the circuits shown in the figures, the galvanometer shows no deflection in each case. Find the ratio of R_1 and R_2.



All India 2017C

LONG ANSWER Type II Questions

49. A variable resistor R is connected across a cell of emf E and internal resistance r.

CBSE SQP (Term-I)

(a) Draw the circuit diagram.
(b) Plot the graph showing variation of potential drop across R as function of R.
(c) At what value of R, current in circuit will be maximum?

50. A storage battery is of emf 8V and internal resistance 0.5 Ω is being charged by DC supply of 120 V using a resistor of 15.5 Ω.

CBSE SQP (Term-I)

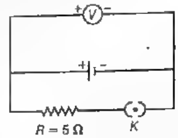
(a) Draw the circuit diagram.
(b) Calculate the potential difference across the battery.
(c) What is the purpose of having series resistance in this circuit?

NUMERICAL PROBLEMS

51. At 20°C, the carbon resistor in an electric circuit connected to a 5 V battery has a resistance of 200 Ω. What is the current in the circuit when the temperature of the carbon rises to 80° C?

52. A semiconductor has electron concentration 0.45×10^{12} m^3 and hole concentration 5×10^{20} m^3. Find its conductivity. Given, electron mobility = 0.135 m^2/Vs and hole mobility = 0.048 m^2/ Vs, $e = 1.6 \times 10^{-19}$C.

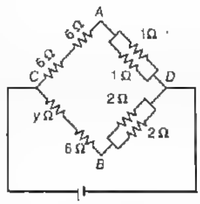
53. Write any two factors on which internal resistance of a cell depends. The reading on a high resistance voltmeter when a cell is connected across it is 2.2 V.



When the terminals of the cell are also connected to a resistance of 5 Ω as shown in the circuit, the voltmeter reading drops to 1.8 V. Find the internal resistance of the cell.

54. The emf of a battery is 2 V and its internal resistance is 2 Ω. Its potential difference is measured by a voltmeter of resistance 998 Ω. Calculate the percentage error in the reading of emf shown by the voltmeter.

55. For what value of unknown resistance y, the potential difference between A and B is zero in the arrangement as shown in figure given below?



56. The resistance of a potentiometer wire of length 10 m is 20 Ω. A resistance box and a 2V accumulator are connected in series with it. What resistance should be introduced in the box to have a potential drop of 1 μV/mm of the potentiometer wire?

ANSWERS

1. (*a*)	2. (*a*)	3. (*d*)	4. (*d*)	5. (*a*)
6. (*a*)	7. (*a*)	8. (*d*)	9. (*c*)	10. (*c*)
11. (*c*)	12. (*c*)	13. (*b*)	14. (*a*)	15. (*d*)

[redacted]

16. (a) With increase in temperature, average speed of the electrons, (which acts as the carriers of current) increases resulting in more frequent collisions. The average time of collisions τ, thus decreases with temperature.

17. (a) Charge carriers do not move with acceleration but with a steady drift velocity. This is because of the collisions with ions and atoms during transit.

18. (d) Bending will not increase the resistance of the conducting wire. Also drift velocity of electron is independent of bending of conductor.

19. (b) Increasing the temperature of a conductor, the kinetic energy of free electrons increases.

On account of this, they collide more frequently with each other (and with the ions of the conductor) and consequently their drift velocity decreases.

20. (a) Manganin and constantan have very low temperature coefficient resistance.

21. (a)

22. (i) (c) Net current, $I = \dfrac{\text{Net emf}}{\text{Total resistance}} = \dfrac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2}$

(ii) (d) Equivalent emf of battery

$$\varepsilon - V_A - V_B = \varepsilon_1 - Ir_1$$
$$= \varepsilon_1 - \left(\frac{\varepsilon_1 + \varepsilon_2}{r_1 + r_2}\right) r_1 = \frac{(\varepsilon_1 r_2 - \varepsilon_2 r_1)}{(r_1 + r_2)}$$

(iii) (d) Terminal A is positive, if $V_A > V_B$ or $V_A - V_B > 0$

or $(\varepsilon_1 r_2 - \varepsilon_2 r_1) > 0$

or $\varepsilon_1 r_2 > \varepsilon_2 r_1$

(iv) (a) Since, r_1 and r_2 are in series, so resultant resistance is

$$r = r_1 + r_2$$

(v) (c) **Loop Rule** The algebraic sum of changes in potential around any closed loop involving resistors and cells in the loop is zero. This rule is also obvious, since, electric potential is dependent on the location of the point. Thus, starting with any point, if we come back to the same point, the total change must be zero. In a closed loop, we do come back to the starting point and hence the rule. $\Sigma Ir = \Sigma V$

23. (i) (b), (ii) (c), (iii) (b), (iv) (c)

24. Current

25. Electric current is caused by the flow of electrons in a conductor. But the direction of electric current is taken as the opposite direction of movement of electrons.

26. Resistivity of a conductor is given by $\rho = \dfrac{m}{ne^2\tau}$

(i) Resistivity $\rho \propto \dfrac{1}{n}$, where n is the number density of free electrons.

(ii) Resistivity $\rho \propto \dfrac{1}{\tau}$, where τ is the relaxation time.

27. As the wires A and B are joined in series, the current through them is same.

$$I_A = I_B$$

$$(neAv_d)_A = (neAv_d)_B \quad [\text{as } I = neAv_d]$$

$$\Rightarrow \quad n_A v_{d_A} = n_B v_{d_B}$$

$$\frac{v_{d_A}}{v_{d_B}} = \frac{n_B}{n_A} = \frac{1}{2}$$

28. The mobility of electrons in a conductor is given by

$$\mu = \frac{e\tau}{m}$$

where, e = charge on electron, m = mass of electrons and τ = relaxation time.

Also, $\tau \propto T$.

But here temperature (T) is kept constant. As mobility is independent of potential difference, so there is no change in it.

29. Average drift velocity, $v_d = \dfrac{eE}{m}\tau$

where, e = charge on electron,

m = mass of electron,

E = electric potential or field across conductor

and τ = relaxation time.

30. The average drift velocity, $v_d = \dfrac{eE}{m}\tau$

where, τ = relaxation time

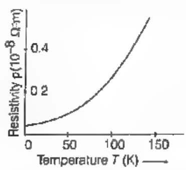
The relaxation time is directly proportional to the temperature of conductor i.e.

$$\tau \propto T$$

$$\therefore \quad v_d \propto T$$

So, the drift velocity increases with rise in temperature.

31. Graph of resistivity of copper as a function of temperature is given below (resistivity of metals increases with increase in temperature).



32. Kirchhoff's current law is based on law of conservation of charge and Kirchhoff's voltage law is based on law of conservation of energy.

33. Refer to text on page 130.

34. The plot of V *versus* I is a straight line for materials that obey Ohm's law. So, from the figure, material P obeys Ohm's law.

35. Refer to text on pages 128 and 129.

36. Refer to text on pages 129 and 130.

37. Ampere hours is the unit of charge as ampere is the unit of current and hours is the unit of time.

Charge = Current × Time.

[redacted]

38. The resistance P_1 is $R_1 = \frac{V^2}{P_1}$

and that P_2 is $R_2 = \frac{V^2}{P_2}$

(i) In series, $R = R_1 + R_2$

$\Rightarrow \quad I = \frac{V}{R} = \frac{V}{R_1 + R_2}$

and $\quad P = I^2 (R_1 + R_2)$

$$= \frac{V^2}{(R_1 + R_2)^2}(R_1 + R_2)$$

$$= \left(\frac{1}{\frac{R_1}{V^2} + \frac{R_2}{V^2}}\right) = \frac{1}{\frac{1}{P_1} + \frac{1}{P_2}} = \frac{P_1 P_2}{P_1 + P_2}$$

(ii) In parallel, $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{V^2}{R} = \frac{V^2}{R_1} + \frac{V^2}{R_2}$

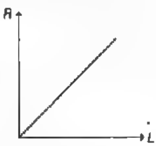
$$\boxed{P = P_1 + P_2}$$

39. (a) Initially, the resistance R_0 of a wire of length L_0 is given by

$$R_0 = \rho \frac{L_0}{A_0} \qquad \text{...(i)}$$

$\Rightarrow \quad R_0 \propto L_0$

The variation of resistance R with length L during stretching is shown as



(b) When a wire is stretched to $2L_0$, the area of wire becomes $\frac{A_0}{2}$. So, the new resistance will be

$$R = \frac{\rho 2L_0}{A_0/2} = 4\rho\frac{L_0}{A_0} = 4R_0 \qquad [\because \text{using Eq. (i)}]$$

$\therefore$ The resistance of wire becomes 4 times of previous.

40. (a) Refer to Q. 39 on page 162.

(b) 2.25 R_0; Refer to Q. 39 on page 162.

41. Refer to text on page 140.

42. When three identical cells are connected in series, the equivalent emf is given by

$$E_{eq} = E_1 + E_2 + E_3 = 3E$$

From the graph, $3E = 6V$

$\Rightarrow \quad E = \frac{6}{3} = 2V$

$\therefore$ Emf of each cell = 3V

43. The figure given is a potentiometer. The sensitivity of potentiometer can be increased by reducing the current in the circuit. This can be done by increasing the value of R. So, the value of X will be most accurate for R_1.

44. $V = E - ir$

(i) When $i = 0$, then $V = E$.

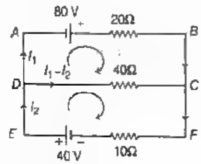
When $i = 0$, then $V = 6V$ (from the graph)

$\therefore$ emf of the cell $(E) = 6V$

(ii) When $i = 2A$ then $V = 0$ (from the graph)

$\therefore E - ir \Rightarrow r = \frac{E}{i} = \frac{6}{2} = 3\Omega$

45. Taking loops clockwise as shown in figure.



Using KVL in $ABCDA$,

$-80 + 20I_1 + 40(I_1 - I_2) = 0$

$\Rightarrow \quad 3I_1 - 2I_2 - 4 \qquad \text{...(i)}$

Using KVL in $DCFED$,

$-40(I_1 - I_2) + 10I_2 - 40 = 0$

$\Rightarrow -4I_1 + 5I_2 = 4 \qquad \text{...(ii)}$

From Eqs. (i) and (ii), we get

$I_1 = 4$ A

and $\quad I_2 = 4$ A

Thus, $\quad I_{40} = I_1 - I_2 = 0$ A

$I_{20} = I_1 = 4$ A

46. (i) and (ii) Refer to text on pages 129 and 130. (Temperature Dependence of Resistivity)

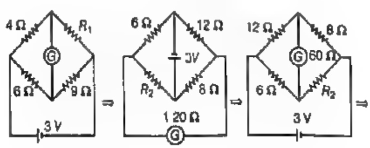
Refer to text on pages 128 and 129. (Resistivity of Various Materials)

47. Refer to text on pages 153 and 154.

48. Current sensitivity of a galvanometer is defined as the deflection per unit current.

For balanced Wheatstone bridge, there will be no deflection in the galvanometer.

$\Rightarrow \quad \frac{4}{R_1} = \frac{6}{4} \Rightarrow R_1 = \frac{4 \times 4}{6} = \frac{8}{3}\,\Omega$



[redacted]

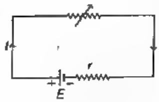
For the equivalent circuit, when the wheatstone bridge is balanced, there will be no deflection in the galvanometer.

$\therefore \quad \dfrac{12}{8} = \dfrac{6}{R_2}$

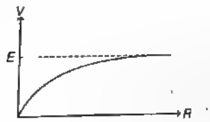
$\Rightarrow \quad R_2 = \dfrac{6 \times 8}{12} = 4\Omega$

$\therefore \quad \dfrac{R_1}{R_2} = \dfrac{8/3}{4} = \dfrac{2}{3}$

49. (a) Circuit diagram



(b) Variation of potential drop across R as function of R is $\left[\because V = E\left(1 - \dfrac{1}{1 + r/R}\right)\right]$



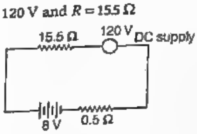
(c) The maximum value of current is obtained, when the resistance of external resistance R is zero,

i.e. $\quad I_{max} = \dfrac{E}{r}$

50. Given, $E = 8$ V, $\quad r = 0.5\,\Omega$,

$V = 120$ V and $R = 15.5\,\Omega$

(a)



(b) Potential across battery in charging

$V = E + Ir$

Effective voltage, $V' = V - E = 120 - 8 = 112$ V

The current in circuit, $I = \dfrac{12}{15.5 + 0.5} = 7$ A

$\Rightarrow V = 8 + 7 \times 0.5$

$= 8 + 3.5 = 11.5$ V

(c) Series resistor in the charging circuit limits the current drawn from the external source.

51. Refer to text on page 129. [Ans. 26 mA]

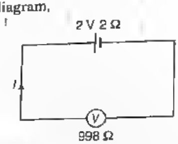
52. Refer to text on page 130. [Ans. 3.84 Sm^{-1}]

53. Refer to Example 1 on page 141. $\left[\text{Ans. } \dfrac{1}{9}\,\Omega\right]$

54. Given, $E = 2$V

$r = 2\Omega$

From the diagram,



$I = \dfrac{2}{2 + 998} = \dfrac{2}{1000}$ A

$V = E - Ir = 2 - \dfrac{2}{1000} \times 2$

$= 2 - 0.004 = 1.996$ V

$\therefore$ % error $= \dfrac{0.004}{2} \times 100 = 0.2\%$

55. As, $V_A - V_B = 0$

Thus, it is a balanced Wheatstone bridge. [Ans. 18 Ω]

56. Apply the balancing condition of a potentiometer

Given, $\quad E = 2$ volt

$V = 0.01$ volt

$R = 2\,\Omega/\text{m}$

Let r is resistance introduced in the box

$\therefore \quad r = \left(\dfrac{E}{V} - 1\right)R$

$= \left(\dfrac{2}{0.01} - 1\right) \times 2$

or $\quad r = 398\,\Omega$ [Ans. 398 Ω]

[redacted]

04

Electricity and magnetism have been known to us for more than 2000 years and we treated them as two separate subjects. The first evidence for the existence of relationship between electricity and magnetism was observed in 1820 by Hans Oersted, the man who himself used to demonstrate that electricity and magnetism had got no relationship with each other.

MOVING CHARGES AND MAGNETISM

So, in this chapter, we will be going to study magnetism produced by a moving charge and further we will proceed with Ampere's circuital law and its applications and at last, the chapter will be ended with magnetic force and torque between two parallel conductors. All the topics mentioned above are discussed in detail, so it will be more interesting to understand them very carefully after going through each and every sentence very thoroughly.

CHAPTER CHECKLIST
- Magnetic Field and Its Applications
- Ampere's Circuital Law and Moving Charges
- Magnetic Force and Torque Experienced by a Current Loop

|TOPIC 1|
Magnetic Field and Its Applications

MAGNETIC FIELD

In electrostatics, we studied that a static charge produces an electric field. Similarly, a moving charge or current flowing through a conductor produces a magnetic field.

The space in the surroundings of a magnet or a current carrying conductor in which its magnetic influence can be experienced is called magnetic field.

The SI unit of magnetic field is tesla (T) or weber/metre^2 (Wbm^{-2}) or $NA^{-1}m^{-1}$ and its CGS unit is gauss (G).

$$1 \text{ tesla} = 10^4 \text{ gauss}$$

Oersted's Experiment

In the summer of 1820, HC Oersted by his experiment concluded that a current carrying conductor deflects magnetic compass needle placed near it. He found that the alignment of magnetic needle is tangential to an imaginary circle

[redacted]

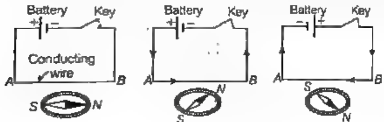
which has the straight wire as its centre and has its plane perpendicular to the wire as shown in Fig. (a).

If the current is reversed, the needle is deflected in opposite direction as shown in Fig. (b). The deflection of the needle indicates that a magnetic field is established around a current carrying wire. On increasing the current in the wire or bringing the needle closer to the wire, the deflection of the needle increases. He also found that the iron filings sprinkled around the wire arrange themselves in concentric circles with the wire as the centre as shown in Fig. (c).

This experiment shows that the magnetic field is produced due to electric current. Electric current means moving charge, so it can be concluded that moving charges produce magnetic field in the surroundings.

Note A current or field (electric or magnetic) emerging out of the plane of the paper is represented by a dot (⊙) and going into the plane of the paper is represented by a cross (⊗).

Consider a conducting wire AB be placed over the magnetic needle parallel to it. It will be found that the North pole of needle gets deflected towards the West as shown in Fig. (e). If the direction of current is reversed, then the North pole of needle gets deflected towards East as shown in Fig. (f).



Direction of deflection of magnetic needle

The direction of deflection of magnetic needle due to current in the wire is given by Ampere's swimming rule.

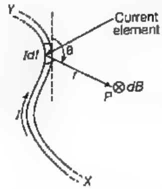
Ampere's Swimming Rule

According to this rule, if we imagine a man is swimming along the wire in the direction of current with his face turned towards the magnetic needle, so that the current enters through his feet and leaves at his head, then the North pole of the needle will be deflected towards his left hand. This rule can be recollected with the help of the word SNOW. It means, current from South to North, in a wire over the magnetic needle, the North pole of the needle is deflected towards West.

MAGNETIC FIELD DUE TO A CURRENT ELEMENT : BIOT-SAVART'S LAW

Biot-Savart's law is an experimental law predicted by Biot and Savart. This law deals with the magnetic field induction at a point due to a small current element (a part of any conductor, carrying current).

Let XY be current carrying conductor, I be current in the conductor, dl be infinitesimal small element of the conductor, dB be magnetic field at point P at a distance r from the element.



According to Biot-Savart's law, the magnitude of magnetic field induction (dB) at a point P due to a current element depends on the following factors

(i) $dB \propto I$ (i.e. magnetic field is directly proportional to the current flowing through the conductor).

(ii) $dB \propto dl$ (i.e. magnetic field is directly proportional to the length of the element).

(iii) $dB \propto \sin\theta$ (i.e. magnetic field is directly proportional to the sine of angle between the length of element and line joining the element to point (P).

(iv) $dB \propto \frac{1}{r^2}$ (i.e. magnetic field is inversely proportional to the square of distance between the element and point P). Combining all the above relations,

$$dB \propto \frac{Idl\sin\theta}{r^2}$$

This relation is called **Biot-Savart's law.**

If conductor is placed in air or vacuum, then magnetic field is given by

$$\boxed{dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl\sin\theta}{r^2}}$$

where, $\frac{\mu_0}{4\pi}$ is a proportionality constant, μ_0 is the permeability of free space. $\mu_0 = 4\pi \times 10^{-7}$ Tm/A (or Wb/A-m), its dimensions are $[MLT^{-2}A^{-2}]$.

[redacted]

In vector form, Biot-Savart's law can be written as

$$d\mathbf{B} \propto \frac{Id\mathbf{l}\times\mathbf{r}}{r^3} = \frac{\mu_0}{4\pi}\cdot\frac{Id\mathbf{l}\times\mathbf{r}}{r^3} \quad \text{...(i)}$$

From Eq. (i), the direction of $d\mathbf{B}$ would be the direction of the cross-product vector ($d\mathbf{l}\times\mathbf{r}$), which is represented by the right handed screw rule or right hand thumb rule.

Here, $d\mathbf{B}$ is perpendicular to the plane containing $d\mathbf{l}$ and $\mathbf{r}$ is directed inwards (since, point P is to the right of the current element).

Magnetic field induction at point P due to current through entire wire is

$$\mathbf{B} = \int \frac{\mu_0}{4\pi}\cdot\frac{Id\mathbf{l}\times\mathbf{r}}{r^3}$$

or $$B = \int \frac{\mu_0}{4\pi}\cdot\frac{Idl\sin\theta}{r^2}$$

Biot-Savart's law in terms of current density J can be written as

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\cdot\frac{\mathbf{J}\times\mathbf{r}}{r^3}dV \quad \left[\because J = \frac{I}{A} = \frac{Idl}{Adl} = \frac{Idl}{dV}\right]$$

where, J = current density at any point on the current element and dV = volume of the element.

Biot-Savart's law in terms of charge (q) and its velocity (v) can be written as

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\cdot\frac{q(\mathbf{v}\times\mathbf{r})}{r^3}\left[\because Id\mathbf{l} = \frac{q}{dt}d\mathbf{l} = q\frac{d\mathbf{l}}{dt} = q\mathbf{v}\right]$$

Biot-Savart's law in terms of magnetising force or magnetising intensity (H) of the magnetic field is

in SI or MKS system,

$$d\mathbf{H} = \frac{d\mathbf{B}}{\mu_0} = \frac{1}{4\pi}\cdot\frac{Id\mathbf{l}\times\mathbf{r}}{r^3} = \frac{1}{4\pi}\cdot\frac{Id\mathbf{l}\times\hat{\mathbf{r}}}{r^2}$$

$$\therefore \quad d\mathbf{H} = \frac{1}{4\pi}\cdot\frac{Idl\sin\theta}{r^2}$$

In CGS units, $d\mathbf{H} = \frac{Id\mathbf{l}\times\mathbf{r}}{r^3}$ and $dH = \frac{Idl\sin\theta}{r^2}$

Features of Biot-Savart's Law

Some important features of Biot-Savart's law are as follows

(i) This law is analogous to Coulomb's law in electrostatics.

(ii) The direction of $d\mathbf{B}$ is perpendicular to both $Id\mathbf{l}$ and $\mathbf{r}$. It is given by right hand thumb rule.

(iii) If $\theta = 0°$, i.e. the point P lies on the axis of the linear conductor carrying current (or on the wire carrying current), then

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{Idl\sin 0°}{r^2} = 0$$

It means that there is no magnetic field induction at any point on the thin linear current carrying conductor.

(iv) If $\theta = 90°$, i.e. the point P lies at a perpendicular position with respect to current element, then

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{Idl}{r^2}, \text{ which is maximum.}$$

If $\theta = 180°$, then $dB = 0$, which is minimum.

Similarities and Differences between Biot-Savart's Law and Coulomb's Law

The Biot-Savart's law for the magnetic field has certain similarities as well as differences with the Coulomb's law for the electrostatic field. Some of these are as follows

(i) Both are long range, since both depend inversely on the square of distance from the source to the point of interest. The principle of superposition applies to both fields. (In this connection, note that the magnetic field is linear in the source $Id\mathbf{l}$ just as the electrostatic field is linear in its source, the electric charge.)

(ii) The electrostatic field is produced by a scalar source, namely, the electric charge. The magnetic field is produced by a vector source $Id\mathbf{l}$.

(iii) The electrostatic field is along the displacement vector joining the source and the field point. The magnetic field is perpendicular to the plane containing the displacement vector $\mathbf{r}$ and the current element $Id\mathbf{l}$.

(iv) There is an angle dependence in the Biot-Savart's law, which is not present in the electrostatic case. The magnetic field at any point in the direction of $d\mathbf{l}$ is zero. Along this line, $\theta = 0°$, $\sin 0° = 0$, so $|d\mathbf{B}| = 0$.

Permittivity and Permeability

Electric permittivity ε_0 is the physical quantity that determines the degree of interaction of electric field with medium. However, magnetic permeability μ_0 is the physical quantity that measures the ability of a substance to acquire magnetisation in magnetic field, i.e. the degree of penetration of matter by B.

The relation between ε_0 and μ_0 is always a constant, i.e.

$$\mu_0\varepsilon_0 = (4\pi\varepsilon_0)\times\left(\frac{\mu_0}{4\pi}\right)$$

$$= \frac{1\times10^{-7}}{9\times10^9}$$

$$= \frac{1}{(3\times10^8)^2} = \frac{1}{c^2}$$

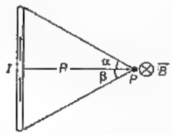
[redacted]

MAGNETIC FIELD DUE TO A CURRENT CARRYING CONDUCTOR

Radial magnetic field created by a current element is perpendicular to both current element dl and position vector r.

Magnetic field B for a straight wire of finite length is given by

$$B = \frac{\mu_0 I}{4\pi R}(\sin\alpha + \sin\beta)$$



According to right hand thumb rule, the direction of magnetic field in this case is perpendicular to the plane of paper and directed inwards.

Magnetic field B for infinitely long wire,

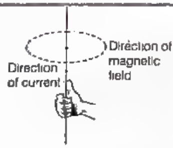
As, $\alpha = \beta = \pi/2$

then $\boxed{B = \frac{\mu_0 I}{2\pi R}}$

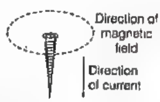
The direction of the magnetic field associated with a current carrying conductor can be determined by right hand thumb rule or Maxwell's cork screw rule.

Right Hand Thumb Rule

According to this rule, if we imagine a linear wire conductor to be held in the grip of the right hand such that the thumb points in the direction of current, then the curvature of the fingers around the conductor will give the direction of magnetic field lines.



Maxwell's Cork Screw Rule



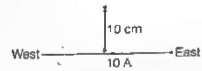
According to this rule, if we imagine a right handed cork screw placed along the current carrying wire conductor, rotated such that the screw moves in the direction of current, then the direction of rotation of the screw gives the direction of magnetic field lines.

EXAMPLE |1| A current 10 A is flowing East to West in a long wire kept horizontally in the East-West direction. Find the magnitude and direction of magnetic field in a horizontal plane at a distance of 10 cm North.

Sol. Given, current, $I = 10$ A (East to West)

Distance, $r = 10 \text{ cm} = 10 \times 10^{-2}$ m

Magnetic field, $|B| = ?$



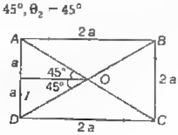
The magnitude of magnetic field $|B|$ for infinite length of wire $= \frac{\mu_0 I}{2\pi r}$

$$\Rightarrow |B| = \frac{4\pi \times 10^{-7} \times 10}{2 \times \pi \times 10 \times 10^{-2}} = 2 \times 10^{-5} \text{T}$$

The direction of magnetic field is given by right hand thumb rule or Maxwell's cork screw rule. So, the direction of magnetic field at point 10 cm North due to flowing current is perpendicularly inwards to the plane of paper.

EXAMPLE |2| Find an expression for the magnetic field at the centre of a coil bent in the form of square of side $2a$, carrying current I as shown in the figure.

Sol. Given, $\theta_1 = 45°, \theta_2 = 45°$



Total magnetic field due to each side at point O is given by

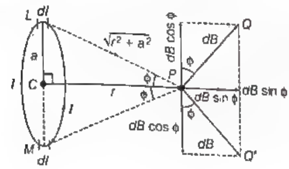
$$B = 4\frac{\mu_0 I}{4\pi a}(\sin\theta_1 + \sin\theta_2) = 4\frac{\mu_0 I}{4\pi a}(\sin 45° + \sin 45°)$$

$$= \frac{\mu_0 I}{\pi a}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\sqrt{2}\mu_0 I}{\pi a}$$

MAGNETIC FIELD ON THE AXIS OF A CIRCULAR CURRENT CARRYING LOOP

Let us consider a circular loop of radius a with centre C. Let the plane of the coil be perpendicular to the plane of the paper and current I be flowing in the direction shown. Suppose P is any point on the axis of a coil at a distance r from the centre C.

[redacted]



Now, consider a current element Idl on top (L), where current comes out of paper normally, whereas at bottom (M), current enters into the plane paper normally.

$\because$ $\quad LP \perp dl$

Also, $\quad MP \perp dl$

$\therefore$ $\quad LP = MP = \sqrt{r^2 + a^2}$

The magnetic field at point P due to the current element Idl, according to Biot-Savart's law is given by

$$dB = \frac{\mu_0}{4\pi} \cdot \frac{Idl \sin 90^\circ}{(r^2 + a^2)} = \frac{\mu_0}{4\pi} \cdot \frac{Idl}{(r^2 + a^2)}$$

where, a = radius of circular loop

and $\quad r$ = distance of point P from the centre C along the axis.

According to right hand screw rule, the direction of dB is perpendicular to LP and along PQ, where $PQ \perp LP$. Similarly, the same magnitude of magnetic field is obtained due to current element Idl at the bottom and direction is along PQ', where $PQ' \perp MP$.

Now, resolving dB due to current element at L and M. So, $dB \cos \phi$ components balance each other and net magnetic field is given by

$$B = \oint dB \sin \phi = \oint \frac{\mu_0}{4\pi} \left(\frac{Idl}{r^2 + a^2} \right) \cdot \frac{a}{\sqrt{r^2 + a^2}}$$

$$\left[\because \text{In } \Delta PCL, \sin \phi = \frac{a}{\sqrt{r^2 + a^2}} \right]$$

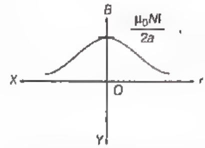
$$= \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} \oint dl = \frac{\mu_0}{4\pi} \frac{Ia}{(r^2 + a^2)^{3/2}} (2\pi a)$$

or $$\boxed{B = \frac{\mu_0 I a^2}{2(r^2 + a^2)^{3/2}}} \quad \ldots \text{(ii)}$$

For N turns, the net magnetic field is given by

$$\boxed{B = \frac{\mu_0 N I a^2}{2(r^2 + a^2)^{3/2}}}$$

The direction of B is along the axis and away from the loop, when current in the coil is in anti-clockwise direction.



Variation of magnetic field induction (B) with distance r

EXAMPLE |3| A circular coil of 120 turns has a radius of 18 cm and carries a current of 3 A. What is the magnitude of the magnetic field at a point on the axis of the coil at a distance from the centre equal to the radius of the circular coil?

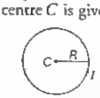
Sol. Given, number of turns $N = 120$, current $I = 3$ A, radius of coil, $r = 18$ cm $= 0.18$ m and distance from the centre to a point on axis, $a = r = 0.18$ m

As, $$B = \frac{\mu_0 N I a^2}{2(a^2 + r^2)^{3/2}} = \frac{4\pi \times 10^{-7} \times 120 \times 3 \times (0.18)^2}{2[(0.18)^2 + (0.18)^2]^{3/2}}$$

$\Rightarrow$ $B = 4.4 \times 10^{-4}$ T

MAGNETIC FIELD AT THE CENTRE OF A CURRENT CARRYING CIRCULAR LOOP

Consider a circular loop of radius R carrying current I. Magnetic field at its centre C is given by



$$B = \frac{\mu_0 I}{2R}$$

is obtained by setting $r = 0$ in previous relation (ii).

If we take a coil having N number of turns, then magnetic field at its centre is

$$\boxed{B = \frac{\mu_0 N I}{2R}}$$

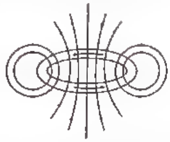
The direction of magnetic field at the centre of circular loop is given by **right hand rule.**

Similarly, magnetic field at the centre of a semi-circular wire of radius R, carrying current I is given by, $B = \dfrac{\mu_0 I}{4R}$

[redacted]

Right Hand Rule

According to this rule, if we hold the thumb of right hand mutually perpendicular to the grip of fingers such that the curvature of fingers depicts the direction of current in circular wire loop, then the thumb will point in the direction of magnetic field near the centre of loop.



Note As current carrying loop has the magnetic field lines around it thus, it behaves as a magnet with two mutually opposite poles

The anti-clockwise flow of current behaves like a North pole whereas clockwise flow as South pole.

EXAMPLE |4| A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries a current of 0.40 A. What is the magnitude of the magnetic field B at the centre of the coil? **NCERT**

Sol. Here, $n = 100$, $r = 8\text{cm} = 8\times10^{-2}\text{m}$ and $I = 0.40\text{A}$

$\therefore$ Magnetic field B at the centre,

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2\pi In}{r} = \frac{10^{-7}\times 2\times 3.14\times 0.40\times 100}{8\times10^{-2}} = 3.1\times10^{-4}\text{T}$$

EXAMPLE |5| The magnetic field B due to a current carrying circular loop of radius 12cm at its centre is 0.5×10^{-4}T. Find the magnetic field due to this loop at a point on the axis at a distance of 5.0 cm from the centre.

Sol. Magnetic field at the centre of a circular loop,

$$B_1 = \frac{\mu_0 I}{2R}$$

and that at an axial point, $B_2 = \dfrac{\mu_0 IR^2}{2(R^2+x^2)^{3/2}}$

Thus, $\dfrac{B_2}{B_1} = \dfrac{R^3}{(R^2+x^2)^{3/2}}$ or $B_2 = B_1\left[\dfrac{R^3}{(R^2+x^2)^{3/2}}\right]$

Substituting the values, we have

$$B_2 = (0.5\times10^{-4})\left[\frac{(12)^3}{(144+25)^{3/2}}\right] = 3.9\times10^{-5}\text{ T}$$

EXAMPLE |6| An electric current is flowing in a circular coil of radius a. At what distance from the centre on the axis of the coil will the magnetic field be $\frac{1}{8}$th of its value at the centre?

Sol. Magnetic field induction at a point on the axis at distance x from the centre of the circular coil carrying current is

$$B_1 = \frac{\mu_0}{4\pi}\frac{2\pi nIa^2}{(a^2+x^2)^{3/2}}$$

Magnetic field induction at the centre of the circular coil carrying current is

$$B_2 = \frac{\mu_0}{4\pi}\cdot\frac{2\pi nI}{a}$$

But as per question, $B_1 = \dfrac{B_2}{8}$

$$\Rightarrow \frac{\mu_0}{4\pi}\frac{2\pi nIa^2}{(a^2+x^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{2\pi nI}{a}\times\frac{1}{8}$$

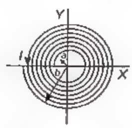
$$\Rightarrow \frac{a^2}{(a^2+x^2)^{3/2}} = \frac{1}{8a} \Rightarrow 8a^3 = (a^2+x^2)^{3/2}$$

$$\Rightarrow 2a = (a^2+x^2)^{1/2}$$

$$\Rightarrow 4a^2 = a^2+x^2$$

$$\Rightarrow x = \sqrt{3}a$$

EXAMPLE |7| A long insulated copper wire is closely wound as a spiral of N turns. The spiral has inner radius a and outer radius b. The spiral lies in the XY-plane and a steady current I flows through the wire. Find the Z-component of the magnetic field at the centre of the spiral.



Sol. If we take a small strip of dr at distance r from centre, then number of turns in this strip would be

$$dN = \left(\frac{N}{b-a}\right)dr$$

Magnetic field due to this element at the centre of the coil will be

$$dB = \frac{\mu_0(dN)I}{2r} = \frac{\mu_0 NI}{2(b-a)}\frac{dr}{r}$$

$$\therefore \quad B = \int_{r=a}^{r=b} dB$$

$$= \frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$$

[redacted]

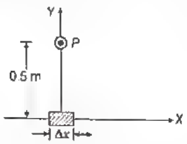
TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. A magnetic field can be produced
(a) only by moving charge
(b) only by changing electric field
(c) Both (a) and (b)
(d) None of the above

2. Biot-Savart law indicates that the moving electrons (velocity v) produce a magnetic field **B** such that **NCERT Exemplar**
(a) **B** is perpendicular to v
(b) **B** is parallel to v
(c) it obeys inverse cube law
(d) it is along to the line joining the electron and point of observation

3. An element $\Delta l = \Delta x\,\hat{\mathbf{i}}$ is placed at the origin and carries a current $I = 10$ A.



If $\Delta x = 1$ cm, magnetic field at point P is
(a) $4 \times 10^{-8}\,\hat{\mathbf{k}}$ T (b) $4 \times 10^{-8}\,\hat{\mathbf{i}}$ T
(c) $4 \times 10^{-8}\,\hat{\mathbf{j}}$ T (d) $-4 \times 10^{-8}\,\hat{\mathbf{j}}$ T

4. There is a thin conducting wire carrying current. What is the value of magnetic field induction at any point on the conductor itself?
(a) 1 (b) Zero
(c) -1 (d) Either (a) or (b)

5. A helium nucleus moves in a circle of 0.8 m radius in one second. The magnetic field produced at the centre of circle will be
(a) $\mu_0 \times 10^{-19}$ (b) $\mu_0 \times 10^{+19}$
(c) $2\mu_0 \times 10^{-19}$ (d) $\dfrac{2 \times 10^{-19}}{\mu_0}$

VERY SHORT ANSWER Type Questions

6. In what respect does a wire carrying a current differ from a wire, which carries no current?

7. How can you justify that a current carrying wire produces magnetic field?

8. Give the dependence of magnetic field produced by a current conductor.

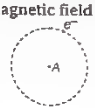
9. State Biot-Savart's law and express this law in the vector form. **All India 2017**

10. Among Biot-Savart's law and Coulomb's law, which one is angle dependent?

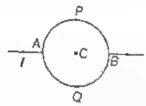
11. Name the kind of magnetic field produced by an infinitely long current carrying conductor.

12. Draw the magnetic field lines due to a current carrying loop. **Delhi 2013**

13. An electron is revolving around a circular loop as shown in the figure. What will be the direction of magnetic field at the point A?

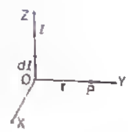


14. There is a circuit given below, where APB and AQB are semi-circles. What will be the magnetic field at the centre C of the circular loop?

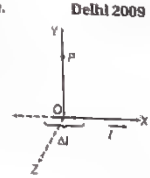


SHORT ANSWER Type Questions

15. State Biot-Savart's law. A current I flows in a conductor placed perpendicular to the plane of the paper. Indicate the direction of the magnetic field due to a small element dl at a point P situated at a distance r from the element as shown in the figure. **Delhi 2009**

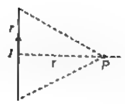


16. An element $\Delta l = \Delta x\,\hat{\mathbf{i}}$ is placed at the origin (as shown in figure) and carries a current $I = 2$A. Find out the magnetic field at a point P on the Y-axis at a distance of 1.0 m due to the element $\Delta x = w$ cm. Also, give the direction of the field produced. **Delhi 2009**



17. Find the magnetic field at point P due to the current carrying conductor of current I as shown in the figure.

[redacted]



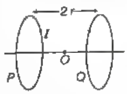
18. If a current loop of radius R carrying an anti-clockwise current I is placed in a plane parallel to YZ-plane.Then, what will be the magnetic field at a point on the axis of the loop?

19. A circular coil of closely wound N turns and radius r carries a current I in the clockwise direction. Find
    (i) the direction of magnetic field at its centre.
    (ii) the magnitude of magnetic field at the centre. All India 2012

20. A straight wire of length L is bent into a semi-circular loop. Use Biot-Savart's law to deduce an expression for the magnetic field at its centre due to the current I passing through it. Delhi 2011C

21. A wire of length L is bent round in the form of a coil having N turns of same radius. If a steady current I flows through it in clockwise direction, then find the magnitude and direction of the magnetic field produced at its centre. Foreign 2009

22. Two identical circular loops P and Q, each of radius r and carrying equal currents are kept in the parallel planes having a common axis passing through O. The direction of current in P is clockwise and in Q is anti-clockwise as seen from O, which is equidistant from the loops P and Q. Find the magnitude of the net magnetic field at O. Delhi 2012



LONG ANSWER Type I Questions

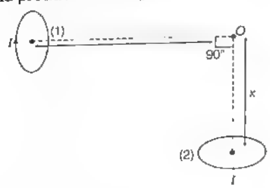
23. Use Biot-Savart's law to derive the expression for the magnetic field on the axis of a current carrying circular loop of radius R.
    Draw the magnetic field lines due to a circular wire carrying current (I). Delhi 2016

24. Using Biot-Savart's law, write the expression for the magnetic field **B** due to an element dl carrying current I at a distance **r** from it in a vector form.
    Hence, derive the expression for the magnetic field due to a current carrying loop of radius R at a point P and distance x from its centre along the axis of the loop. Delhi 2015

LONG ANSWER Type II Questions

25. State Biot-Savart's law expressing it in the vector form. Use it to obtain the expression for the magnetic field at an axial point distance d from the centre of a circular coil of radius a carrying current I. Also, find the ratio of the magnitudes of the magnetic field of this coil at the centre and at an axial point for which $d = a\sqrt{3}$. Delhi 2013C

26. Two very small identical circular loops (1) and (2) carrying equal current I are placed vertically (with respect to the plane of the paper) with their geometrical axes perpendicular to each other as shown in the figure. Find the magnitude and direction of the net magnetic field produced at the point O. Delhi 2014



NUMERICAL PROBLEMS

27. A current of 5 A is flowing from South to North in a straight wire. Find the magnetic field due to a 1cm piece of wire at a point 1 m North-East from the piece of wire. All India 2011

28. An element $dl = dx\,\hat{\mathbf{i}}$ (where, $dx = 1$ cm) is placed at the origin and carries a large current $I = 10$ A What is the magnetic field on the Y-axis at a distance of 0.5 m? NCERT

29. A long straight wire in the horizontal plane carries a current of 50 A in North to South direction.Give the magnitude and direction of **B** at a point 2.5 m East of the wire. NCERT

30. Two wires A and B have the same length equal to 44 cm and carry a current of 10 A each. Wire A is bent into a circle and wire B is bent into a square.

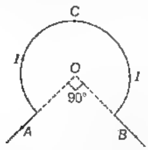
[redacted]

(i) Obtain the magnitudes of the fields at the centres of the two wires.

(ii) Which wire produces a greater magnetic field at its centre?

31. A tightly wound 100 turns coil of radius 10 cm is carrying a current of 1 A. What is the magnitude of the magnetic field at the centre of the coil? **NCERT**

32. The wire shown in the figure carries a current of 10 A. Determine the magnitude of magnetic field induction at the centre O. Given the radius of bent coil is 3 cm.



33. Two identical circular coils, P and Q each of radius R, carrying currents 1 A and $\sqrt{3}$ A respectively, are placed concentrically and perpendicular to each other lying in the XY and YZ-planes. Find the magnitude and direction of the net magnetic field at the centre of the coils. **All India 2017**

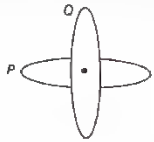
34. A straight wire carrying a current of 10 A is bent into a semi-circular arc of radius 2.0 cm as shown in the figure. What is the magnetic field at O due to

(i) straight segments

(ii) the semi-circular arc?

35. Two concentric circular coils x and y of radii 16 cm and 10 cm respectively lie in the same vertical plane containing the North to South direction. Coil x has 20 turns and carries a current of 16 A, coil y has 25 turns and carries a current of 18 A. The sense of the current in x is anti-clockwise and clockwise in y, for an observer looking at the coils facing West. Find the magnitude and direction of the net magnetic field due to the coils at their centre. **NCERT**

36. Two identical loops P and Q each of radius 5 cm are lying in perpendicular planes such that they have a common centre as shown in the figure. Find the magnitude and direction of the net magnetic field at the common centre of the two coils, if they carry currents equal to 3 A and 4 A, respectively. **Delhi 2017**



HINTS AND SOLUTIONS

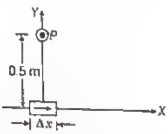
1. (a) Electric current or moving charges produce magnetic field around them.

2. (a) In Biot-Savart's law, magnetic field $\mathbf{B} \parallel id\mathbf{l} \times \mathbf{r}$ and $id\mathbf{l}$ due to flow of electron is in opposite direction of $\mathbf{v}$ and by direction of cross product of two vectors, i.e.

$$\mathbf{B} \perp \mathbf{v}$$

3. (a) The magnitude of magnetic field,

$$|d\mathbf{B}| = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$



i.e., $$|d\mathbf{B}| = \frac{10^{-7} \times 10 \times 10^{-2}}{25 \times 10^{-2}} = 4 \times 10^{-8}\ \text{T}$$

As, $$Id\mathbf{l} \times \mathbf{r} = \Delta x\hat{\mathbf{i}} \times y\hat{\mathbf{j}} = y\,\Delta x(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) = y\,\Delta x\hat{\mathbf{k}}$$

So, the direction of the field is in the $+Z$-direction.

4. (b) $$|d\mathbf{B}| = \frac{\mu_0}{4\pi}\left|\frac{Id\mathbf{l} \times \mathbf{r}}{r^3}\right| = \frac{\mu_0}{4\pi} \times \frac{Idl\sin\theta}{r^2}$$

If point lies on the conductor, then $\theta = 0°$ or $180°$. So, $\sin\theta = 0$, thus $dB = 0$. Hence, the magnetic field induction at any point on the conductor itself is zero.

5. (c) The magnetic field at the centre of circle, $B = \frac{\mu_0 i}{2r}$

The charge on helium nucleus is $2e$, so

Current, $$i = \frac{q}{t} = \frac{2e}{t}$$

$$\Rightarrow \quad B = \frac{\mu_0 \times 1.6 \times 10^{-19} \times 2}{2 \times 0.8}$$

$$= 2\mu_0 \times 10^{-19}\ \text{N/A-m}$$

[redacted]

6. A current carrying wire produces magnetic field but wire which does not carry current has no magnetic field.

7. It can be justified by placing a magnetic needle around current carrying wire, which shows deflection of needle.

8. Magnetic field produced by a current conductor is
(i) Directly proportional to the current flowing through the conductor, length of the element and sine of the angle between the length of the element and line joining the element to the point
(ii) Inversely proportional to the square of the distance between the element and the point.

9. Biot-Savart's law states that, the magnitude of magnetic field intensity (dB) at a point P due to current element is given by

$$dB \propto \frac{I\,dl \sin\theta}{r^2}$$

or $$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

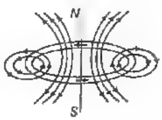
Thus, in vector notation,

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{l}\times\mathbf{r}}{r^3}$$

10. Biot-Savart's law is an angle dependent law.

11. An infinitely long current carrying conductor produces magnetic field in the form of concentric circular loops in a plane of straight conductor.

12. Magnetic field lines due to a current carrying loop is given by



13. As, electron is revolving clockwise, therefore conventional current due to the motion of electron will be in anti-clockwise direction.
So, according to right hand rule, magnetic field at point A will be in outward direction.

14. Magnetic field due to loop APB at the centre is given by

$$B_1 = \frac{\mu_0 I}{4a}\odot$$

Magnetic field due to loop AQB at the centre is given by

$$B_2 = \frac{\mu_0 I}{4a}\otimes$$

So, net magnetic field at centre $= B_1 + B_2 = 0$ (zero)

15. Refer to text on pages 168 and 169.

16. Biot-Savart's law states that

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\cdot\frac{I\,d\mathbf{l}\times\hat{\mathbf{r}}}{|r|^2}$$

Here $\Delta x = w$ cm

$\because$ $\Delta l = \Delta x \hat{i}$

$\Rightarrow$ $I = 2\text{A}, r = 1\text{ m}$

$\therefore$ $$dB = \frac{\mu_0}{4\pi}\frac{(2w\hat{i}\times\hat{j})}{(1)^2}$$

$I\,dl = 2\times w\hat{i}$

$\because$ $\hat{r} = \hat{j} \Rightarrow |r| = 1$ m

$\therefore$ $$d\mathbf{B} = \frac{\mu_0 w}{2\pi}\hat{k}$$

$\Rightarrow$ $$|d\mathbf{B}| = \frac{\mu_0 w}{2\pi}$$

and direction along $+Z$-axis.

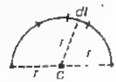
17. Refer to text on page 170.

$$B_P = \frac{\mu_0 I}{4\pi r}(\sin 45^\circ + \sin 45^\circ) = \frac{\mu_0 I}{4\pi r}\times\sqrt{2} = \frac{\mu_0 I}{2\sqrt{2}\pi r}$$

18. Refer to text on pages 170 and 171.

19. (i) Inward
(ii) Refer to text on page 171.

20. According to the questions the wire will now look like,



$\because$ Length L is bent into semi-circular loop
$\therefore$ Length of wire = Circumference of semi-equal circular wire

$\Rightarrow$ $L = \pi r$

$\Rightarrow$ $$r = \frac{L}{\pi} \quad \text{...(i)}$$

Considering a small element dl on current loop. The magnetic field dB due to small current element Idl at centre C. Using Biot Savart's law, we have

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{Idl\sin 90^\circ}{r^2} \quad [\because Idl \perp \mathbf{r}, \therefore \theta = 90^\circ]$$

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{Idl}{r^2}$$

$\therefore$ Net magnetic field at C due to semi-circular loop,

$$B = \int_{\text{semi-circle}}\frac{\mu_0}{4\pi}\frac{Idl}{r^2} = \frac{\mu_0}{4\pi}\frac{I}{r^2}\int_{\text{semi-circle}} dl$$

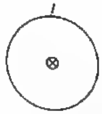
$$= \frac{\mu_0}{4\pi}\frac{I}{r^2}L \quad \left[\text{but } r = \frac{L}{\pi}\right]$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{IL}{(L/\pi)^2} = \frac{\mu_0}{4\pi}\times\frac{IL}{L^2}\times\pi^2 = \frac{\mu_0 I\pi}{4L}$$

which is the required expression

21. When a straight wire is bent into the form of a circular coil of N turns, then the length of the wire is equal to circumference of the coil multiplied by the number of turns. Let the radius of coil be r

[redacted]



As, the wire is bent round in the form of a coil having N turns.

$\therefore$ $N \times$ circumference of the coil = Length of the wire

$\Rightarrow \quad (2\pi r) \times N = L$

$\Rightarrow \quad r = \dfrac{L}{2\pi N}$ ... (i)

Magnetic field at the centre due to N turns of a coil is given by

$$B = \frac{\mu_0 (NI)}{2r} = \frac{\mu_0 (NI)}{2\left(\frac{L}{2\pi N}\right)} \quad \text{[from Eq. (i)]}$$

$$= \frac{\mu_0 \pi N^2 I}{L}$$

The direction of magnetic field is perpendicular to the plane of loop and entering into it.

22. Magnetic field at O due to two loops will be in same direction ($Q \to P$, along the axis) and of equal magnitude.

$$B = B_1 + B_2 \text{ but } B_2 = B_1$$

$$\Rightarrow \quad B = 2B_1 = 2\left[\frac{\mu_0 I r^2}{2(r^2 + r^2)^{3/2}}\right]$$

$$= \frac{\mu_0 I r^2}{(2r^2)^{3/2}} = \frac{\mu_0 I r^2}{2^{3/2} r^3}$$

$$= \frac{\mu_0 I}{2^{3/2} r}$$

23. Refer to text on pages 170 and 171.

For magnetic field lines Refer to Sol. 12.

24. Refer to text on pages 168, 169, 170 and 171.

25. Refer to text on pages 168, 169, 170 and 171.

In this answer, put $r = d$.

Magnetic field induction at the centre of the circular coil carrying current is

$$B_2 = \frac{\mu_0}{4\pi} \frac{2\pi I}{a}, \quad B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi a^2 I}{(a^2 + d^2)^{3/2}}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{a^2 \times a}{(a^2 + d^2)^{3/2}} = \frac{a^3}{(a^2 + d^2)^{3/2}}$$

$$= \frac{a^3}{(a^2 + 3a^2)^{3/2}} \quad [\because d = a\sqrt{3}]$$

$$= \frac{a^3}{(4a^2)^{3/2}} = \frac{a^3}{8a^3}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{1}{8}$$

26. The magnetic field at a point due to a circular loop is given by

$$B = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(a^2 + r^2)^{3/2}}$$

where, I = current through the loop,

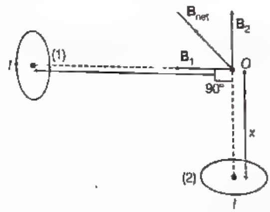
a = radius of the loop

and r = distance of O from the centre of the loop.

Since I, a and $r = x$ are the same for both the loops, the magnitude of B will be the same and is given by

$$B_1 = B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I a^2}{(a^2 + x^2)^{3/2}}$$

The direction of magnetic field due to loop (1) will be away from O and that of the magnetic field due to loop (2) will be towards O as shown. The direction of the net magnetic field will be as shown below



The magnitude of the net magnetic field is given by

$$B_{net} = \sqrt{B_1^2 + B_2^2}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2\pi \sqrt{2} I a^2}{(a^2 + x^2)^{3/2}}$$

27. Here, $I = 5$ A, $dl = 1$ cm $= 0.01$ m, $r = 1$ m, $\theta = 45°$

[$\because$ direction is North-East]

$$\therefore \quad dB = \frac{\mu_0}{4\pi} \cdot \frac{I dl \sin\theta}{r^2}$$

$$= 10^{-7} \times \frac{5 \times 0.01 \times \sin 45°}{(1)^2} = 3.54 \times 10^{-9} \text{ T}$$

Its direction is vertically downwards.

28. Here, $dl = dx = 1$ cm $= 10^{-2}$ m

$I = 10$ A, $r = 0.5$ m

Using Biot-Savart's law,

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{I d\mathbf{l} \times \mathbf{r}}{r^3} = \frac{\mu_0}{4\pi} \cdot \frac{I dx}{r^2} (\hat{\mathbf{i}} \times \hat{\mathbf{j}})$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I dx}{r^2} \hat{\mathbf{k}} \quad [\because \hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}]$$

$$= \frac{10^{-7} \times 10 \times 10^{-2}}{(0.5)^2} \hat{\mathbf{k}}$$

$$= 4 \times 10^{-8} \hat{\mathbf{k}} \text{ T}$$

[redacted]

29. Refer to Example 1 on page 179. [Ans. 4×10^{-6} T]

30. Given, $I = 10$ A, length of each wire = 44 cm

(i) Let r be the radius of wire A when it is bent into a circle.

$$\Rightarrow \quad 2\pi r = 44 \Rightarrow r = \frac{7}{100} \text{ m}$$

Magnetic field at the centre of the circular coil carrying current is given by

$$B = \frac{\mu_0}{4\pi}\frac{2\pi I}{r} = 10^{-7}\times 2\times\frac{22}{7}\times 10\times\frac{100}{7} = 9\times10^{-5} \text{ T}$$

When another wire is bent into a square of each side L, then

$$4L = 44 \Rightarrow L = 11 \text{cm} = 0.11 \text{m}$$

Since, magnetic field induction at a point, at perpendicular distance a from the linear conductor carrying current is given by

$$B = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 + \sin\theta_2)$$

$$B = 4\times\frac{\mu_0 I}{4\pi a}(\sin 45^\circ + \sin 45^\circ)$$

$$= 4\times10^{-7}\times\frac{10}{(11/100)}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right)$$

$$= 10.3\times10^{-5} \text{T}$$

(ii) The magnetic field due to a square will be more than that due to a circle of same perimeter.

31. Refer to Example 4 on page 172. [Ans. 6.28×10^{-4} T]

32. Here, $I = 10$ A, $r = 3$ cm, $r = 3\times10^{-2}$ m

Angle subtended by coil at the centre,

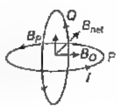
$$\theta = 360^\circ - 90^\circ = 270^\circ = \frac{3\pi}{2} \text{ rad}$$

Magnetic field induction at O due to current through circular path ACB is

$$B = \frac{\mu_0}{4\pi}\frac{I}{r}\theta = 10^{-7}\times\frac{10}{(3\times10^{-2})}\times\frac{3\pi}{2}$$

$$= 1.57\times10^{-4} \text{ T}$$

33. Magnetic field due to circular wire P,



$$B_P = \frac{\mu_0}{4\pi}\times\frac{2\pi I_1}{R}$$ [along vertically upwards]

$$= \frac{\mu_0 I_1}{2R}$$

Magnetic field due to circular wire Q,

$$B_Q = \frac{\mu_0}{4\pi}\times\frac{2\pi I_2}{R}$$ [along horizontal towards left]

$$= \frac{\mu_0 I_2}{2R}$$

Net magnetic field at the common centre of the two coils,

$$B = \sqrt{B_P^2 + B_Q^2}$$

$$= \sqrt{\left(\frac{\mu_0 I_1}{2R}\right)^2 + \left(\frac{\mu_0 I_2}{2R}\right)^2}$$

$$= \sqrt{\left(\frac{\mu_0}{2R}\right)^2(I_1^2 + I_2^2)}$$

$$= \frac{\mu_0}{2R}\sqrt{I_1^2 + I_2^2}$$

$$= \frac{4\pi\times10^{-7}}{2\times R}\sqrt{(1)^2 + (\sqrt{3})^2}$$

$$= \frac{4\pi\times10^{-7}}{R} \text{ T}$$

Resultant magnetic field makes an angle θ with direction of B_Q, which is given by

$$\tan\theta = \frac{B_P}{B_Q} = \frac{1}{\sqrt{3}}$$

$$\therefore \quad \theta = 30^\circ$$

34. (i) Magnetic field due to straight segment is

$$\mathbf{B} = \int\frac{\mu_0}{4\pi}\cdot\frac{I\, d\mathbf{l}\times\mathbf{r}}{r^3}$$

For point O, $d\mathbf{l}$ and $\mathbf{r}$ for each element of straight segments PQ and RS are parallel.

Therefore, $d\mathbf{l}\times\mathbf{r} = 0$

Thus, magnetic field due to straight segment is zero.

(ii) Magnetic field at centre O due to semi-circular arc

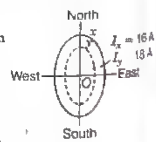
$$= \frac{\text{Magnetic field at centre of circular coil}}{2}$$

$$= \frac{1}{2}\left(\frac{\mu_0 I}{2r}\right) = \frac{\mu_0 I}{4r} = \frac{(4\pi\times10^{-7})\times10}{4\times2\times10^{-2}}$$

[given, $I = 10$ A and $r = 20$ cm $= 2\times10^{-2}$ m]

$$= 5\pi\times10^{-5} \text{ T}$$

35. For coil x



Radius of coil, $r_x = 16$ cm = 0.16m

Number of turns, $n_x = 20$

Current in the coil, $I_x = 16$ A (anti-clockwise)

For coil y

Radius of coil, $r_y = 10$ cm = 0.1 m

Number of turns, $n_y = 25$

Current in the coil, $I_y = 18$ A (clockwise)

The magnitude of the magnetic field at the centre of coil x,

$$B_x = \frac{\mu_0}{4\pi}\frac{2I_x\pi n_x}{r_x}$$

$$= \frac{10^{-7}\times2\times16\times\pi\times20}{0.16} = 4\pi\times10^{-4} \text{T}$$

[redacted]

The direction of magnetic field due to the coil x at centre O is towards right, i.e. East, according to right hand thumb rule. The magnitude of the magnetic field at the centre of coil y,

$$B_y = \frac{\mu_0}{4\pi} \cdot \frac{2\pi I_y n_y}{r_y} = \frac{10^{-7} \times 2 \times \pi \times 18 \times 25}{0.1}$$

$$= 9\pi \times 10^{-4}\,\text{T}$$

The direction of magnetic field due to coil y at centre O is towards left, i.e. West, according to right hand thumb rule. Here, the magnitude of B_y is greater than B_x, so the resultant magnetic field will be in the direction of B_y, i.e. left (West).

Net magnetic field at the centre, $B = B_y - B_x$

$$= (9\pi - 4\pi)\,10^{-4}\,\text{T}$$

$$= 5\pi \times 10^{-4}\,\text{T}$$

[$\because B_y$ and B_x are opposite to each other]

$$= 1.6 \times 10^{-3}\,\text{T}$$ [towards West]

36. $$B_{net} = \sqrt{B_P^2 + B_Q^2} = \sqrt{\left(\frac{\mu_0 i_P}{2r}\right)^2 + \left(\frac{\mu_0 I_Q}{2r}\right)^2}$$

$$= \frac{\mu_0}{2r}\sqrt{i_P^2 + i_Q^2} = \frac{4\pi \times 10^{-7}}{2 \times 5 \times 10^{-2}} \times 5$$

$$= 2\pi \times 10^{-5}\,\text{T}$$

Resultant magnetic field makes an angle θ with B_Q which is given by,

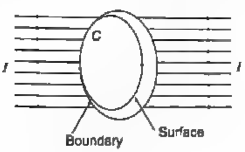
$$\tan\phi = \frac{B_P}{B_Q} = \frac{i_P}{i_Q} = \frac{3}{4}$$

|TOPIC 2|

Ampere's Circuital Law and Moving Charges

AMPERE'S CIRCUITAL LAW

According to this law, the line integral of a magnetic field **B** around any closed path in vacuum is μ_0 times the net current I_{net} enclosed by the curve.



Mathematically,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{net}$$

Ampere's law is applicable only for an Amperian loop as the Gauss's law is used for Gaussian surface in electrostatics.

The choice of an Amperian loop has to be such that, at each point of the loop either

(i) **B** is tangential to the loop and is a non-zero constant

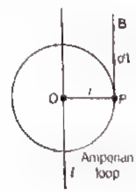
(ii) **B** is normal to the loop

(iii) **B** vanishes

Ampere's circuital law has same content as the Biot-Savart's law. Both of these relate magnetic field and current and express the same physical consequences of a steady electrical current. Ampere's circuital law holds for any loop but does not always facilitate. Ampere's circuital law can be conveniently applied in situations of high symmetry. e.g. To find magnetic field of a straight wire, magnetic field of solenoid and toroid as discussed in coming sections.

Magnitude of Magnetic Field of a Straight Wire using Ampere's Law

Magnetic field due to a straight conductor at a point P at a distance (r) is in the form of a circle of radius (r) which is taken as closed path for Amperian loop.



Angle between **B** and $d\mathbf{l}$ is zero, everywhere in this path. Hence, on applying Ampere's law to this closed path, we get

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \quad \text{or} \quad \oint B\,dl \cos 0^\circ = \mu_0 I$$

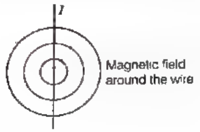
[redacted]

$$\Rightarrow \quad B\oint dl = \mu_0 I$$

$$\Rightarrow \quad B \times 2\pi r = \mu_0 I \text{ or } \boxed{B = \frac{\mu_0 I}{2\pi r}}$$

From the result, some important points can be derived.

(i) The magnetic field at every point on a circle of radius r is same in magnitude. The magnetic field around a wire possesses cylindrical symmetry.



(ii) The field direction at any point on this circle is tangential to it. The lines of constant magnitude of magnetic field forms concentric circles. The circular lines are called magnetic field lines.

(iii) Even though the wire is of infinite length, the field due to it at a non-zero distance is not infinite.

EXAMPLE |1| A straight wire carries a current of 3 A. Calculate the magnitude of the magnetic field at a point 15 cm away from the wire.

Sol. Here, current, $I = 3$ A, point where magnetic field is to be determined, $a = 15$ cm $= 0.15$ m

$\therefore$ Magnitude of magnetic field, $B = \dfrac{\mu_0 2I}{4\pi a}$

$$= \frac{10^{-7} \times 2 \times 3}{0.15}$$

$$= 4 \times 10^{-6} \text{ T}$$

THE SOLENOID AND THE TOROID

A solenoid is an insulated long wire closely wound in the form of a helix. Its length is very long as compared to its diameter. The toroid is a hollow circular ring on which a large number of insulated turns of a metallic wire are closely wound. The solenoid and the toroid are two equipments which are used to produce magnetic fields.

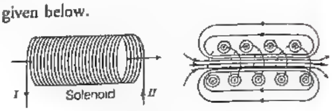
In television, we make use of solenoid to generate magnetic field needed for the deflection of electrons in picture tube.

Toroid is used in devices such as synchroton in which high magnetic field is required, which is generated by either a toroid or combination of both solenoid and toroid.

Both solenoid and toroid have symmetrically geometric shapes, therefore we can apply Ampere's law conveniently to find the magnetic field.

Magnetic Field of a Solenoid

A long coil of wire consisting of closely packed loops is called solenoid, whose magnetic field resembles that of a bar magnet of south(S) and north(N) poles as shown in the figure given below.



Magnetic field of a solenoid

Inside the solenoid, magnetic field is uniform and parallel to the solenoid axis. Outside the solenoid, magnetic field is assumed to be zero.

Consider an air cored solenoid having closely packed coils in which I is current, n is number of turns per unit length and B is magnetic field inside the solenoid.

Applying Ampere's circuital law to determine magnetic field (B) inside the solenoid, we choose rectangular closed path $PQRS$, where $PQ = L$ and the line integral of **B** over closed path $PQRS$ is

$$\oint_{PQRS} \mathbf{B} \cdot d\mathbf{l} = \int_P^Q \mathbf{B} \cdot d\mathbf{l} + \int_Q^R \mathbf{B} \cdot d\mathbf{l} + \int_R^S \mathbf{B} \cdot d\mathbf{l} + \int_S^P \mathbf{B} \cdot d\mathbf{l}$$

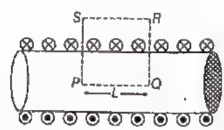
Now, $\int_Q^R \mathbf{B} \cdot d\mathbf{l} = \int_S^P \mathbf{B} \cdot d\mathbf{l} = 0$

[because along QR and PS, the field B is at right angles to dl, so that $\mathbf{B} \cdot d\mathbf{l} = Bdl \cos 90° = 0$]

$\int_R^S \mathbf{B} \cdot d\mathbf{l} = 0$ [$\because$ B is zero at points outside the solenoid]

$$\therefore \oint_{PQRS} \mathbf{B} \cdot d\mathbf{l} = \int_P^Q \mathbf{B} \cdot d\mathbf{l} = \int_P^Q B\,dl \cos 0°$$

$$= \int_P^Q B\,dl = BL$$



Hence, from Ampere's law,

$$\oint_{PQRS} \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times (\text{current enclosed by } PQRS)$$

[redacted]

Here, number of turns per unit length is n, the number of turns in length L is nL. The current in each turn is I, so net current enclosed by the loop is nLI.

Total current, $I_{net} = nLI$

$$\therefore \quad \oint_{PQRS} \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times nIL$$

$$\Rightarrow \quad BL = \mu_0 nIL$$

$$\Rightarrow \quad \boxed{B = \mu_0 nI}$$

From the expression, it is clear that B is independent of the length and diameter of the solenoid and is uniform over the cross-section of the solenoid.

If a material of permeability μ_r is used as a core, then B inside the solenoid is $\mu_0 \mu_r n I$.

At points near the end of air closed solenoid,

$$\boxed{B = \frac{1}{2}\mu_0 nI}$$

Note The formula for magnetic field $B = \mu_0 nI$ is only valid when length of the solenoid (l) is much larger than its radius(r), i.e. $l >> r$

EXAMPLE |2| **The length of a solenoid is 0.2 m and it has 120 turns. Find the magnetic field in its interior, if a current of 2.5 A is flowing through it.**

Sol. Here, $l = 0.2$ m, $N = 120$, $I = 2.5$A

Magnetic field in the interior of the solenoid,

$$B = \mu_0 nI = \mu_0 \frac{N}{l} I$$

$$= 4\pi \times 10^{-7} \times \frac{120}{0.2} \times 2.5$$

$$= 1.85 \times 10^{-3} \text{ T}$$

EXAMPLE |3| **A solenoid of length 0.5 m has a radius of 1 cm and is made up of 500 turns. It carries a current of 5 A. What is the magnitude of magnetic field inside the solenoid?** NCERT

Sol. Given, total number of turns, $N = 500$

Length of solenoid, $l = 0.5$ m

Current, $I = 5$ A

and radius $r = 1 \text{ cm} = 10^{-2}$ m

Here, $\frac{l}{r} = \frac{0.5}{10^{-2}} = 50$

$\Rightarrow \quad l >> r$

$$\therefore \quad B = \mu_0 nI = \frac{\mu_0 NI}{l}$$

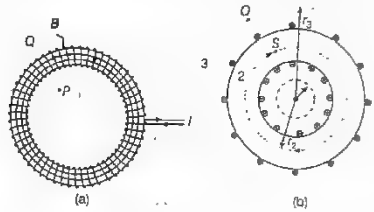
$$= 4\pi \times 10^{-7} \times \frac{500}{0.5} \times 5$$

$$= 6.28 \times 10^{-3} \text{ T}$$

Magnetic Field of a Toroid

An endless solenoid in the form of a ring is called a toroid. Magnetic field lines inside the toroid are circular, concentric with the centre of toroid.

Let I be the current, r be the mean radius, n be the number of turns per unit length and B be the magnetic field inside the toroid.



(a) A toroid carrying a current I (b) A sectional view of the toroid. The magnetic field can be obtained at an arbitrary distance r from the centre O of the toroid by Ampere's circuital law. The dashed lines labelled 1, 2 and 3 are three circular Amperian loops

The line integral of magnetic field around closed path of circle of radius r is

$$\oint \mathbf{B} \cdot d\mathbf{l} = \oint B\, dl \cos 0° = B \times 2\pi r$$

Now, from Ampere's law,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times \text{current enclosed by closed path}$$

$$\Rightarrow \quad B \times 2\pi r = \mu_0 n (2\pi r) I \quad \text{or} \quad \boxed{B = \mu_0 nI}$$

If the toroid is a material cored of relative permeability μ_r, then magnetic field inside the toroid,

$$\boxed{B = \mu_0 \mu_r n I}$$

FORCE ON A MOVING CHARGE IN A UNIFORM MAGNETIC FIELD

When a charged particle (q) moves with velocity ($\mathbf{v}$) inside a uniform magnetic field $\mathbf{B}$, then force acting on it is

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

Force due to magnetic field depends on $q, \mathbf{v}, \mathbf{B}$. The magnetic force will be zero, if the particle is at rest. The reason is that for the charged particle at rest $|\mathbf{v}| = 0$, which will turn the expression for magnetic force, $q(\mathbf{v} \times \mathbf{B})$, into zero. Only moving charges feel the magnetic force. The magnetic force is at its maximum value, when $\mathbf{v}$ and $\mathbf{B}$ are perpendicular to each other because in this case the angle $\theta = 90°$, in the expression $F = qvB \sin\theta$ and the maximum force will be

$$F_{max} = qvB \sin 90° = qvB \quad [\because \sin 90° = 1]$$

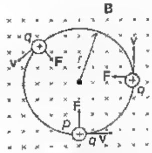
[redacted]

The magnetic force includes the cross product of velocity (**v**) of the particle and magnetic field (**B**). Thus, the magnetic force will be zero, if the velocity vector and magnetic field vector are either parallel or anti-parallel to each other.

The force ($\mathbf{F}_{magnetic}$) acting on a charged particle moving with velocity (**v**) through a magnetic field (**B**) is always perpendicular to **v** and **B**.

From right hand thumb rule, the force F is perpendicular to velocity (v) and magnetic field (**B**). Hence, it changes its path continuously.

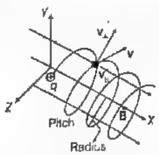
In case of motion of a charge in a magnetic field, the magnetic force is perpendicular to the velocity of the particle. So, no work is done and no change in the magnitude of the velocity is produced.



Circular motion

Magnetic force acts as a centripetal force and produces a circular motion perpendicular to the magnetic field. If v and B are perpendicular to each other, then particle will describe a circle.

If a charged particle has a velocity not perpendicular to B, then component of velocity along B remains unchanged as the motion along the magnetic field will not be affected by the magnetic field. Then, the motion of the particle in a plane perpendicular to B is as before a circular one, thereby producing a helical motion.



Helical motion

As, centripetal force required for circular motion is provided by magnetic force, so

$$\frac{mv_\perp^2}{r} = qBv_\perp \text{ or } \boxed{r = \frac{mv_\perp}{qB} = \frac{p}{qB}} \quad \text{...(i)}$$

where, m = mass of charged particle

and r = radius of circular path.

where, $v_\perp$ and $v_\parallel$ are perpendicular and parallel components of the velocity **v**.

Radius of the path of the charged particle is proportional to the momentum ($p = mv$) of the particle and inversely proportional to the magnitude of charge (q) and magnetic field (B).

In terms of kinetic energy (K), the equation may be expressed as, $r = \dfrac{\sqrt{2mK}}{qB}$ $\quad [\because p = \sqrt{2mK}]$

Time period (T) of the motion is given by

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \times \frac{mv}{qB}$$

or $$\boxed{T = \frac{2\pi m}{qB}} \quad \text{...(ii)}$$

Angular frequency, $\omega = \dfrac{2\pi}{T} = \dfrac{Bq}{m}$

Frequency, $\nu = \dfrac{Bq}{2\pi m}$ $\quad [\because \omega = 2\pi\nu]$

The angular speed ω of the particle is given by

$$\omega = v/r \text{ or } \omega = (qB)/m$$

For helical path, the distance moved along the magnetic field in one rotation is called pitch (P).

$$\boxed{P = v_\parallel T = \frac{2\pi m v_\parallel}{qB}}$$

Note One tesla (1 T) is defined as the field which produces a force of one newton (1 N) when a charge of one coulomb (1 C) moves perpendicularly in the region of the magnetic field at a velocity of 1 ms^{-1}

Aurora Borealis

During a solar flare, a large number of electrons and protons are ejected from the sun. Some of them get trapped in the earth's magnetic field and move in helical paths along the field lines which come closer near magnetic poles and collide with atoms and molecules of the atmosphere. Excited oxygen atoms emit green light and excited nitrogen atoms emit pink light and this phenomenon is called Aurora Borealis in Physics.

Note This topic is very important as it has been asked frequently in the previous years 2017, 2016, 2015, 2014, 2012, 2011, 2010.

EXAMPLE |4| A proton and an α-particle, accelerated through same potential difference, enter in a region of uniform magnetic field with their velocities perpendicular to the field. Compare the radii of circular paths followed by them.

Sol. Let mass of proton = m, charge of proton = e

[redacted]

Now, mass of α-particle = 4 m, charge of α-particle = $2e$

When a charge q is accelerated by V volts, it acquires a kinetic energy $E_K = qV$

$\therefore$ Momentum is given by $mv = \sqrt{2mE_K} = \sqrt{2mqV}$

Radius, $r = \dfrac{mv}{qB}$ or $r = \dfrac{\sqrt{2mqV}}{qB} = \sqrt{\dfrac{2mV}{qB^2}}$

Thus, $\dfrac{r_p}{r_\alpha} = \sqrt{\dfrac{2mV}{eB^2}} \times \sqrt{\dfrac{2eB^2}{2(4m)V}} = \dfrac{1}{\sqrt{2}}$

EXAMPLE |5| A beam of protons with a velocity of 4×10^5 ms^{-1} enters in a region of uniform magnetic field of 0.3 T. The velocity makes an angle of 60° with the magnetic field. Find the radius of the helical path taken by the proton beam and the pitch of the helix.

Sol. Velocity component along the field

$$v_{\parallel} = 4 \times 10^5 \times \cos 60° = 2 \times 10^5 \text{ ms}^{-1}$$

and velocity component perpendicular to the field

$$v_\perp = (4 \times 10^5) \sin 60° = 2\sqrt{3} \times 10^5 \text{ ms}^{-1}$$

Proton will describe a circle in plane perpendicular to magnetic field with radius,

$$r = \frac{mv_\perp}{qB} = \frac{(1.67 \times 10^{-27} \text{ kg}) \times (2\sqrt{3} \times 10^5 \text{ ms}^{-1})}{(1.6 \times 10^{-19} \text{C}) \times (0.3 \text{ T})} = 1.2 \text{ cm}$$

Time taken to complete one revolution is

$$T = \frac{2\pi r}{v_\perp} = \frac{2 \times 3.14 \times 0.012}{2\sqrt{3} \times 10^5} \text{ s}$$

Because of $v_{\parallel}$ protons will also move in the direction of magnetic field.

$\therefore$ Pitch of helix $= v_{\parallel} \times T$

$$= \frac{2 \times 10^5 \times 2 \times 3.14 \times 0.012}{2\sqrt{3} \times 10^5} \text{ m} = 0.044 \text{ m} = 4.4 \text{ cm}$$

FORCE ON A MOVING CHARGE IN A UNIFORM MAGNETIC AND ELECTRIC FIELD (LORENTZ FORCE)

Suppose a point charge (q) is moving in the presence of both electric and magnetic fields. Let q be the magnitude of the charge, **v** be velocity of the point charge, **B** be the magnetic field and **E** be the electric field. We have studied two kinds of forces that can be exerted on an electrically charged particle. The electric force is given by $\mathbf{F} = q\mathbf{E}$ and the magnetic force is $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$.

The sum of these forces represents the net force that can be exerted on a particle due to its electric charge (q), this sum is called the Lorentz force and is given by

$$\mathbf{F}_{\text{Lorentz}} = \mathbf{F}_{\text{electric}} + \mathbf{F}_{\text{magnetic}}$$
$$= q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$$
$$= q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$$

Force on negative charge is opposite to that of positive charge.

MOTION OF CHARGED PARTICLE IN COMBINED ELECTRIC AND MAGNETIC FIELD

Velocity Selector

Net force in presence of magnetic and electric field is

$$\mathbf{F} = q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$$

Consider that electric and magnetic fields are perpendicular to each other and also perpendicular to the velocity of particle. Suppose we have

$$\mathbf{E} = E\,\hat{\mathbf{j}},\ \mathbf{B} = B\,\hat{\mathbf{k}} \text{ and } \mathbf{v} = v\,\hat{\mathbf{i}}$$

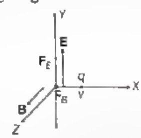
Then, $\mathbf{F}_E = q\mathbf{E} = qE\,\hat{\mathbf{j}}$

and $\mathbf{F}_B = q(\mathbf{v} \times \mathbf{B}) = q(v\hat{\mathbf{i}} \times B\hat{\mathbf{k}}) = -qvB\,\hat{\mathbf{j}}$

$\therefore$ $\mathbf{F} = \mathbf{F}_E + \mathbf{F}_B = qE\,\hat{\mathbf{j}} + (-qvB\hat{\mathbf{j}})$

$= q(E - vB)\,\hat{\mathbf{j}}$

Thus, electric and magnetic forces are in opposite directions as shown in the figure given below.



Point charge q is moving in the presence of perpendicular electric and magnetic fields

If we adjust the value of **E** and **B** such that magnitude of the two forces are equal, then total force on the charge is zero and the charge will move in the fields undeflected. This happens, when

$$qE = qvB$$

$$\Rightarrow \quad \boxed{v = \frac{E}{B}}$$

The above condition can be used to select a charged particle of a particular velocity from charges moving with different speeds. Therefore, it is called velocity selector.

EXAMPLE |6| A proton beam passes without deviation through a region of space, where there are uniform transverse mutually perpendicular electric and magnetic fields with $E = 220$ kV/m and $B = 50$ mT. Then, the beam

[redacted]

strikes a grounded target. Find the force imparted by the beam on the target, if the beam current is equal to $I = 0.80$ mA.

Sol. Since, proton is moving in a straight line, hence net force is zero.

$$\therefore \quad qE = Bqv \Rightarrow v = \frac{E}{B}$$

Also, current associated with the beam,

$$I = ne$$

$$\Rightarrow \quad n = I/e$$

where, n is number of protons/time.

Momentum of a proton $= mv$

$\therefore$ Force, $F = n\,m\,v$

$$= \frac{I\,m\,E}{eB} = \frac{0.80\times 10^{-3}\times 1.67\times 10^{-27}\times 220\times 10^{3}}{1.6\times 10^{-19}\times 50\times 10^{-3}}$$

$$= 3.6\times 10^{-5}\text{ N}$$

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. For a toroid, magnetic field strength in the region enclosed by wire turns is given by
   (a) $B = \mu_0 n I$, where n = number of turns
   (b) $B = \mu_0 I/n$, where n = number of turns per metre
   (c) $B = \frac{\mu_0 I}{2r}$, where r = mean radius
   (d) $B = \frac{\mu_0 NI}{2\pi r}$, $\begin{cases}\text{where, } N = \text{number of turns}\\ \text{and } r = \text{radius of toroid.}\end{cases}$

2. The value of force F acting on charge q moving with velocity perpendicular to the magnetic field B will be
   (a) $F = qvB$ (b) $F = \frac{qv}{B}$
   (c) $F = \frac{qB}{v}$ (d) $F = \frac{Bv}{q}$

3. An electron of charge (e) is moving parallel to uniform magnetic field B with constant velocity v. The force acting on electron is
   (a) Bev (b) Be/v
   (c) B/ev (d) zero

4. In a uniform magnetic field, an electron (or charge particle) enters perpendicular to the field. The path of electron will be
   (a) ellipse (b) circular
   (c) parabolic (d) linear

5. If the velocity of charged particle is doubled and value of magnetic field is reduced to half, then the radius of path of charged particle will be
   (a) 8 times (b) 4 times (c) 3 times (d) 2 times

6. An electron is projected with uniform velocity along the axis of a current carrying long solenoid. Which of the following is true? NCERT Exemplar
   (a) The electron will be accelerated along the axis
   (b) The electron path will be circular about the axis
   (c) The electron will experience a force at 45° to the axis and hence execute a helical path
   (d) The electron will continue to move with uniform velocity along the axis of the solenoid

VERY SHORT ANSWER Type Questions

7. Magnetic field lines can be entirely confined within the core of toroid, but not within a straight solenoid. Why?

8. An electron does not suffer any deflection while passing through a region of uniform magnetic field, what is the direction of the magnetic field? All India 2009

9. A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction, and comes out of it following a complicated trajectory. Would its final speed equal to the initial speed, if it suffered no collisions with the environment? NCERT

10. A narrow beam of protons and deuterons, each having the same momentum, enters a region of uniform magnetic field directed perpendicular to their direction of momentum. What would be the ratio of the radii of the circular path described by them? Foreign 2011

11. A loop of irregular shape carrying current is located in an external magnetic field.
    If the wire is flexible, why does it change to a circular shape?

12. A solenoid tends to contract when a current passes through it. Justify the given statement.

13. A proton and an electron travelling along parallel paths enter a region of uniform magnetic field, acting perpendicular to their paths. Which of them will move in a circular path with higher frequency? CBSE 2018

[redacted]

SHORT ANSWER Type Questions

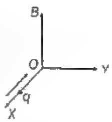
14. A long solenoid of length L having N turns carries a current I. Deduce the expression for the magnetic field in the interior of the solenoid. **All India 2011C**

15. Obtain with the help of a necessary diagram, the expression for the magnetic field in the interior of a toroid carrying current. **All India 2011C**

16. Define one tesla using the expression for the magnetic force acting on a particle of charge q moving with velocity v in a magnetic field **B**. **Foreign 2014**

17. A particle of charge q and mass m is moving with velocity v. It is subjected to a uniform magnetic field B directed perpendicular to its velocity. Show that it describes a circular path. Write the expression for its radius. **Foreign 2012**

18. A charge q moving along the X-axis with a velocity **v** is subjected to a uniform magnetic field **B** acting along the Z-axis as it crosses the origin O.



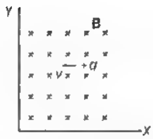
(i) Trace trajectory.
(ii) Does the charged particle gain kinetic energy as it enters the magnetic field? Justify your answer. **Delhi 2009**

19. Write the expression in the vector form for the Lorentz magnetic force **F** due to a charge moving with velocity **v** in a magnetic field **B**. What is the direction of the magnetic force? **Delhi 2014**

20. Write the expression for Lorentz magnetic force on a particle of charge q moving with velocity **v** in a magnetic field **B**. Show that no work is done by this force on the charged particle. **All India 2011**

21. Find the condition under which the charged particles moving with different speeds in the presence of electric and magnetic field vectors can be used to select charged particles of a particular speed. **All India 2017**

22. A point charge is moving with a constant velocity perpendicular to a uniform magnetic field as shown in the figure. What should be magnitude and direction of the electric field so that the particle moves undeviated along the same path? **Foreign 2009**



23. An electron and a proton moving with the same speed enter the same magnetic field region at right angles to the direction of the field. Show the trajectory followed by the two particles in the magnetic field. Find the ratio of the radii of the circular paths which the particles may describe. **Foreign 2010**

24. An iron ring of relative permeability μ_r has windings of insulated copper wire of n turns per metre. When the current in the windings is I, find the expression for the magnetic field in the ring. **CBSE 2018**

LONG ANSWER Type I Questions

25. Explain, how Biot-Savart's law enables one to express the Ampere's circuital law in the integral form, *viz*.

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

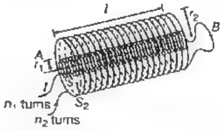
where, I is the total current passing through the surface. **Delhi 2015**

26. A long straight solid metal wire of radius R carries a current I uniformly distributed over its circular cross-section. Find the magnetic field at a distance r from the axis of wire (i) inside (ii) outside the wire.

27. (i) State Ampere's circuital law expressing it in the integral form.
(ii) Two long coaxial insulated solenoids, S_1 and S_2 of equal lengths are wound one over the other as shown in the figure. A steady current I flows through the inner solenoid S_1 to the other end B, which is connected to the outer solenoid S_2 through which the same current I flows in the opposite direction, so as to come out at end A. If n_1 and n_2 are the number of turns per unit length, find the magnitude and direction of the net magnetic

[redacted]

field at a point (a) inside on the axis and (b) outside the combined system. **Delhi 2014**



28. (i) Write the expression for the force **F** acting on a particle of mass m and charge q moving with velocity v in a magnetic field **B**. Under what conditions will it move in
(a) a circular path and
(b) a helical path?

(ii) Show that the kinetic energy of the particle moving in magnetic field remains constant. **Delhi 2017**

29. (i) Write the expression for the magnetic force acting on a charged particle moving with velocity v in the presence of magnetic field **B**.

(ii) A neutron, an electron and an alpha particle moving with equal velocities, enter a uniform magnetic field going into the plane of the paper as shown in the figure. Trace their paths in the field and justify your answer.

Delhi 2016

30. A uniform magnetic field **B** is set up along the positive X-axis. A particle of charge q and mass m moving with a velocity v enters the field at the origin in XY-plane such that it has velocity components both along and perpendicular to the magnetic field **B**. Trace, giving reason, the trajectory followed by the particle. Find out the expression for the distance moved by the particle along the magnetic field in one rotation. **Delhi 2015**

LONG ANSWER Type II Questions

31. (i) Using Ampere's circuital law, derive the expression for the magnetic field in the vector form at a point on the axis of a solenoid.

(ii) What does a toroid consist of? Find out the expression for the magnetic field inside a toroid for N turns of the coil having the average radius r and carrying a current I. Show that the magnetic field in the open space interior and exterior to the toroid is zero.

32. Answer the following questions.

(i) A magnetic field that varies in magnitude from point to point but has a constant direction (East to West) is set up in a chamber. A charged particle enters the chamber and travels undeflected along a straight path with constant speed. What can you say about the initial velocity of the particle?

(ii) A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction, and comes out of it following a complicated trajectory. Would its final speed equal to the initial speed, if it suffered no collisions with the environment?

(iii) An electron travelling West to East enters a chamber having a uniform electrostatic field in North to South direction. Specify the direction in which a uniform magnetic field should be set up to prevent the electron from deflecting its straight line path. **NCERT**

NUMERICAL PROBLEMS

33. A solenoid of length 50 cm having 100 turns carries a current of 2.5 A. Find the magnetic field (i) in the interior of the solenoid, (ii) at one end of the solenoid. **Delhi 2010**

34. A solenoid of length 1.0 m and 3.0 cm diameter has 5 layers of windings of 850 turns each and carries a current of 5 A. What is the magnetic field at the centre of solenoid? Also, calculate the magnetic flux from a cross-section of the magnetic flux solenoid at the centre of solenoid. **All India 2011**

35. A magnetic field of 100 G $(1\text{ G} = 10^{-4}\text{T})$ is required which is uniform in a region of linear dimension about 10 cm and area of cross-section about 10^{-3} m^2. The maximum current carrying capacity of a given coil of wire is 15 A and the number of turns per unit length that can be wound round a core is at most 1000 turns m^{-1}. Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is not ferromagnetic. **NCERT**

[redacted]

36. An electron of energy 2000 eV describes a circular path in magnetic field of flux density 0.2 T. What is the radius of path? Take, $e = 1.6 \times 10^{-19}$ C, $m = 9 \times 10^{-31}$ kg.

HINTS AND SOLUTIONS

1. (d) For toroid, applying Ampere's circuital law,

$$B(2\pi r) = \mu_0 NI \Rightarrow B = \frac{\mu_0 NI}{2\pi r}$$

where, B = magnetic field of a toroid,

N = number of turns of toroidal coil

and r = radius of toroid.

2. (a) The force on charge q, $F = qvB$.

3. (d) The force on electron, $F = qvB\sin\theta$.

The electron is moving parallel to the magnetic field,

so $\theta = 0°$

$\therefore \quad F = qvB\sin 0° = 0$

4. (b) When the charged particle enters in the magnetic field perpendicular to it, then the force due to magnetic field,

$$F = qvB\sin 90° = qvB$$

The direction of this force is always perpendicular to movement of charged particle. The charged particle is moving under the influence of constant force but its direction is continuously changing. So, the particle will move in a circular path with constant velocity v.

5. (b) In first case, the radius of path, $r = \dfrac{mv}{qB}$

In second case, the radius of path, $r' = \dfrac{m'v'}{q'B'} = \dfrac{m \times 2v}{q \times B/2}$

$= 4r$

6. (d) Magnetic Lorentz force on electron projected with uniform velocity along the axis of a current carrying long solenoid $F = -evB\sin 180° = 0\ (\theta = 0°)$ as magnetic field and velocity are parallel. So, the electron will continue to move with uniform velocity along the axis of the solenoid.

7. The magnetic field lines always form closed loops. As, the turns of the wires in a toroidal solenoid are wound over its core in circular form, the field lines are confined within the core of toroid. In a straight solenoid, the magnetic field lines cannot form closed loops within the solenoid.

8. As $\quad |\mathbf{F}| = qvB\sin\theta$

$\therefore$ If $\quad \theta = 0°$ or $180°$, then $F = 0$

When the particle moves parallel or anti-parallel to the magnetic field, then it does not experience any deflection.

9. Yes, the final speed is equal to its initial speed as the magnetic force acting on the charged particle only changes the direction of velocity of charged particle but cannot change the magnitude of velocity of charged particle.

10. For given momentum of charged particle, radius of circular path depends on charge and magnetic field as

$$r = \frac{mv}{qB} \Rightarrow r \propto \frac{1}{qB}$$

For given momentum, $r_{proton} : r_{deuteron} = 1:1$

As, they have same momentum, charges are moving in small magnetic field.

11. Forces on the loop due to magnetic field act is in all directions. Thus, the loop attains a circular shape.

12. The turns of the solenoid are parallel to each other and carry current in the same direction. As we know that two parallel current carrying conductors in the same direction attract each other. Thus, the solenoid tends of contract.

13. As we know that in a circular path, frequency of a charged particle is given by

$$\nu = \frac{qB}{2\pi m}$$

or $\quad \nu \propto \dfrac{1}{m}$

Since, $m_p > m_e$, therefore electron will move in circular path with higher frequency.

14. Refer to text on pages 180 and 181.

15. Refer to text on page 181.

16. Refer to text on page 183.

$\because \quad F = qvB$

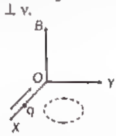
$\Rightarrow \quad B = \dfrac{F}{qv}$

$\Rightarrow \quad 1\text{ T} = \dfrac{1\text{N}}{(1\text{C})(1\text{ms}^{-1})}$

17. Refer to text on page 182.

18. (i) As, the charged particle is moving perpendicularly to the magnetic field. So, it will perform circular motion in XY-plane.

(ii) No, the charged particle does not gain any KE as Lorentz force acting on it does not perform any work as $\mathbf{F}_m \perp \mathbf{v}$.



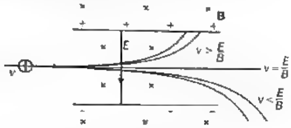
19. The expression in vector form is given by $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$. The direction of the magnetic force is in the direction of $(\mathbf{v} \times \mathbf{B})$, i.e. perpendicular to the plane containing $\mathbf{v}$ and $\mathbf{B}$.

20. Refer to text on page 183.

Lorentz magnetic force always acts perpendicular to the direction of motion of the particle. Thus, work done by this force is zero.

[redacted]

21. A diagram in which particle moves in magnetic and electric field is shown below



Forces on a charged particle are

F_e = electric force = qE,

F_m = magnetic force = Bqv

For a particle to go straight without any deflection

$$F_e = F_m \Rightarrow qE = Bqv \rightarrow v = \frac{E}{B}$$

In this way, particles having speed, $v = \frac{E}{B}$ are separated.

22. As, $\mathbf{v} = -v\hat{\mathbf{i}}$

[$\because$ the particle is moving along X-direction]

$\mathbf{B} = -B\hat{\mathbf{k}}$

[$\because$ the magnetic field is perpendicular to the plane of the paper directed inwards, i.e. negative Z-direction]

$\therefore$ Force acting due to magnetic field,

$\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B}) = q[-v\hat{\mathbf{i}} \times (-B\hat{\mathbf{k}})]$.

$\mathbf{F}_m = -qvB\hat{\mathbf{j}}$ $\quad$ [$\because \hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$]

$\Rightarrow$ Magnitude of $\mathbf{F}_m = |\mathbf{F}_m| = qvB$

The direction of Fm is along negative Y-direction.For the undeflected motion of particle,

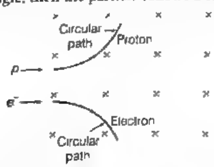
Force due to electric field = Force due to magnetic field,

$$q\mathbf{E} = q(\mathbf{v} \times \mathbf{B})$$

$$\therefore \quad \mathbf{E} = \mathbf{v} \times \mathbf{B}$$

Magnitude of electric field, $|E| = |\mathbf{v} \times \mathbf{B}|$ and direction of magnetic field will be perpendicular to both $\mathbf{v}$ and $\mathbf{B}$, i.e. along Y-axis.

23. When a charged particle enters in the magnetic field at right angle, then the particle follows a circular path.



Radius of the circular path, $r = \frac{mv}{qB}$

For same speed v, magnitude of charge and magnetic field

$$r \propto m$$

$$\Rightarrow \quad \frac{r_e}{r_p} = \frac{m_e}{m_p}$$

where, m_e and m_p are masses of electron and proton, respectively.

$\because \quad m_e < m_p$

i.e. Proton is much heavier than electron.

$\Rightarrow \quad r_e < r_p$

The curvature of path of electron is much more than curvature of path of proton

24. An iron ring having insulated copper winding is also called a toroid. Magnetic field lines inside the toroid are circular, concentric with the centre of toroid.

Let I be the current, r be the mean radius, n be the number of turns per unit length and B be the magnetic field inside the toroid.

The line integral of magnetic field around closed path of circle of radius r is

$$\oint \mathbf{B} \cdot d\mathbf{l} = \oint B\,dl \cos 0° = B \times 2\pi r$$

Now, from Ampere's law,

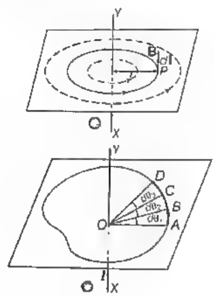
$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \times$ current enclosed by closed path

$$B \times 2\pi r = \mu_0 n(2\pi r) I \quad \text{or} \quad B = \mu_0 nI$$

If the toroid has a material core of relative permeability μ_r, then magnetic field is given by

$$B = \mu_0 \mu_r n I$$

25.



Consider any arbitrary closed path perpendicular to the plane of paper around a long straight conductor XY carrying current from X to Y, lying in the plane of paper

Let the closed path be made of large number of small elements, where

$$AB = dl_1, BC = dl_2, CD = dl_3$$

Let $d\theta_1, d\theta_2, d\theta_3$, be the angles subtended by the various elements at point O through which conductor is passing. Then

$$d\theta_1 + d\theta_2 + d\theta_3 + \ldots = 2\pi$$

[redacted]

Suppose these small elements $AB, BC, CD, \ldots$ are small circular arcs of radii $r_1, r_2, r_3 \ldots$ respectively.

Then $d\theta_1 = \frac{dl_1}{r_1}, d\theta_2 = \frac{dl_2}{r_2}, d\theta_3 = \frac{dl_3}{r_3}$

If $\mathbf{B}_1, \mathbf{B}_2, \mathbf{B}_3$ are the magnetic field inductions at a point along the small elements $d\mathbf{l}_1, d\mathbf{l}_2, d\mathbf{l}_3, \ldots$, then from Biot-Savart's law we know that for the conductor of infinite length, magnetic field is given by

$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r_1}, B_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r_2}, B_3 = \frac{\mu_0}{4\pi} \cdot \frac{2I}{r_3}$$

In case of each element, the magnetic field induction B and current element vector $d\mathbf{l}$ are in the same direction.

Line integral of B around closed path is

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mathbf{B}_1 \cdot d\mathbf{l}_1 + \mathbf{B}_2 \cdot d\mathbf{l}_2 + \mathbf{B}_3 \cdot d\mathbf{l}_3 + \ldots$$

$$= B_1(dl_1) + B_2(dl_2) + B_3(dl_3) + \ldots$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2I}{r_1} dl_1 + \frac{\mu_0}{4\pi} \cdot \frac{2I}{r_2} dl_2 + \frac{\mu_0}{4\pi} \cdot \frac{2I}{r_3} dl_3 + \ldots$$

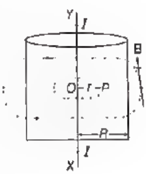
$$= \frac{\mu_0 2I}{4\pi} \left[\frac{dl_1}{r_1} + \frac{dl_2}{r_2} + \frac{dl_3}{r_3} + \ldots \right]$$

$$= \frac{\mu_0 2I}{4\pi} [d\theta_1 + d\theta_2 + d\theta_3 + \ldots]$$

$$= \frac{\mu_0}{4\pi} 2I \times 2\pi = \mu_0 I$$

which is an expression of Ampere's circuital law.

26. (i) Let the point P be lying inside the wire at a perpendicular distance r from the axis of the wire. Consider a circular path of radius r around the axis of the wire. By symmetry, the magnetic field produced due to current flowing in the wire at any point over this path is tangential to it and equal in magnitude at all points on this path.



Current enclosed by the closed path,

$$I' = \frac{I}{\pi R^2} \times \pi r^2 = \frac{I r^2}{R^2}$$

Applying Ampere's circuital law,

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \mu_r I'$$

$$\Rightarrow B(2\pi r) = \mu_0 \mu_r \frac{I r^2}{R^2}$$

$$\Rightarrow \quad B = \frac{\mu_0 \mu_r I r^2}{2\pi R^2 r}$$

$$\Rightarrow \quad B = \frac{\mu_0 \mu_r I r}{2\pi R^2}$$

(ii) When point P is outside the wire, $r > R$, so that the current enclosed by closed path $= I$

Using Ampere's circuital law, $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$

$$B \times 2\pi r = \mu_0 I \text{ or } B = \frac{\mu_0 I}{2\pi r}$$

27. (i) Refer to text on page 179.

(ii) According to Ampere's circuital law, the net field is given by $B = \mu_0 n I$

(a) The net magnetic field is given by

$$B_{net} = B_2 - B_1 = \mu_0 n_2 I - \mu_0 n_1 I \qquad [\because I_2 = I_1 = I]$$
$$= \mu_0 I(n_2 - n_1)$$

The direction is from B to A.

(b) As the magnetic fields due to S_1 is confined solely inside S_1 as the solenoids are assumed to be very long. So, there is no magnetic field outside S_1 due to current in S_1, similarly, there is no field outside S_2.

$\therefore \qquad B_{net} = 0$

28. (i) Refer to text on pages 181 and 182.

(ii) Since, force always adjusts itself in a direction which becomes perpendicular to velocity, so only direction of velocity changes not the magnitude. Hence, the kinetic energy of the particle always remains constant.

29. (i) Refer to text on pages 181 and 182.

(ii) According to question, magnetic force on a charge particle is given by

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

The direction of force on the charged particle is given by $(\mathbf{v} \times \mathbf{B})$ with the sign of charged particle, i.e. for α-particle, charge is positive and direction of $\mathbf{v}$ is $+\hat{\mathbf{i}}$ and direction of $\mathbf{B}$ is $-\hat{\mathbf{k}}$.

So, direction of force is $+(\hat{\mathbf{i}} \times -\hat{\mathbf{k}})$, i.e. $+\hat{\mathbf{j}}$.

It describes a circle with anti clockwise motion.

For neutron

It is a neutral particle so, it goes undeviated.

As $\qquad \mathbf{F} = q(\mathbf{v} \times \mathbf{B}) = 0$

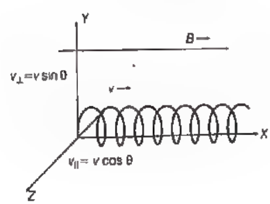
For electron

Force is given by $\mathbf{F} = -e(\mathbf{v} \times \mathbf{B})$

So, direction $= -(\hat{\mathbf{i}} \times -\hat{\mathbf{k}}) \Rightarrow -\hat{\mathbf{j}}$

e^- describes a circle with clockwise motion.

30.



[redacted]

The path of the charged particle will be helix. As, the charge moves linearly in the direction of the magnetic field with velocity $v\cos\theta$ and also describe the circular path due to velocity $v\sin\theta$.

Time taken by the charge to complete one circular rotation,

$$T = \frac{2\pi r}{v_\perp} \quad \text{...(i)}$$

$$\Rightarrow \quad f = qv_\perp B$$

and

$$\frac{mv_\perp^2}{r} = qv_\perp B$$

$$\Rightarrow \quad \frac{v_\perp m}{qB} = r \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\Rightarrow \quad T = \frac{2\pi v_\perp m}{qB\, v_\perp} = \frac{2\pi m}{Bq}$$

Distance moved by the particle along the magnetic field in one rotation (pitch of the helix path)

$$= v_{||} \times T \qquad [\because v_{||} = v_{\text{parallel}}]$$

$$= v\cos\theta \times \frac{2\pi m}{Bq}$$

$$P = \frac{2\pi mv\cos\theta}{qB}$$

31. (i) Refer to text on pages 180 and 181.

(ii) Refer to text on page 181.

Magnetic field inside the open space interior of the toroid Let the loop 2 be shown in the figure, experience magnetic field B.

No current threads the loop 2 which lie in the open space inside the toroid.

$\therefore$ By Ampere's circuital law,

$$\oint_{\text{loop 2}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 (I) = 0$$

$$\Rightarrow \quad \mathbf{B} = 0$$

Magnetic field in the open space exterior of the toroid Let us consider a coplanar loop 3 in the open space of exterior of toroid. Here, each turn of toroid threads the loop two times in opposite directions.

Therefore, net current threading the loop

$$= NI - NI = 0$$

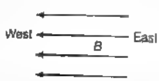
$\therefore$ By Ampere's circuital law,

$$\oint_{\text{loop 3}} \mathbf{B} \cdot d\mathbf{l} = \mu_0 (NI - NI) = 0$$

$$\Rightarrow \quad \mathbf{B} = 0$$

Thus, there is no magnetic field in the open space interior and exterior of the toroid.

32. (i) The magnetic field is in constant direction from East to West. According to the question, a charged particle travels undeflected along a straight path with constant speed. It is only possible, if the magnetic force experienced by the charged particle is zero.

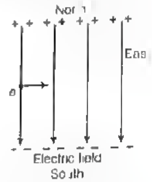


The magnitude of magnetic force on a moving charged particle in a magnetic field is given by $F = qv\,B\sin\theta$ (where θ is the angle between $\mathbf{v}$ and $\mathbf{B}$). Here $F = 0$, if and only if $\sin\theta = 0$ (as $v \neq 0$, $q \neq 0$, $B \neq 0$). This indicates the angle between the velocity and magnetic field is $0°$ or $180°$.

Thus, the charged particle moves parallel or anti-parallel to the magnetic field **B**.

(ii) Yes, the final speed be equal to its initial speed as the magnetic force acting on the charged particle only changes the direction of velocity of charged particle but cannot change the magnitude of velocity of charged particle.

(iii) As the electric field is from North to South, that means the plate in North is positive and in South is negative. Thus, the electrons (negatively charged) attract towards the positive plate that means move towards North. If we want that there should be no deflection in the path of electron, then the magnetic force should be in South direction.



By $\mathbf{F} = -e(\mathbf{v} \times \mathbf{B})$, the direction of velocity is West to East, the direction of force is towards South, by using the Fleming's left hand rule, the direction of magnetic field (**B**) is perpendicularly inwards to the plane of paper.

33. Here, $I = 2.5$ A, $n = \frac{100}{0.50} = 200$

(i) $B = \mu_0 nI = 4\pi \times 10^{-7} \times 200 \times 2.5$

$B = 6.28 \times 10^{-4}$ T

(ii) $B = \frac{\mu_0 nI}{2} = \frac{4\pi \times 10^{-7} \times 200 \times 2.5}{2}$

$= 3.14 \times 10^{-4}$ T

34. Number of turns, $N = 850 \times 5$, $l = 1$ m, $I = 5$ A

Area of cross-section, $A = \pi r^2 = \frac{22}{7}\left(\frac{3}{2} \times 10^{-2}\right)^2 m^2$

Magnetic field at the centre of solenoid, $B = \mu_0 NI/l$

$$= 4\pi \times 10^{-7} \times (850 \times 5) \times 5/1$$

$$= 2.671 \times 10^{-2}\ \text{T}$$

$\therefore$ Magnetic flux $= BA$

$$= 2.671 \times 10^{-2} \times \frac{22}{7} \times \left(\frac{3}{2} \times 10^{-2}\right)^2$$

$$= 1.89 \times 10^{-5}\ \text{Wb}$$

[redacted]

35. Magnetic field, $B = 100\text{ G} = 100\times10^{-4}\text{ T} = 10^{-2}\text{ T}$

To design the solenoid, let us find the product of current and number of turns in the solenoid.

The magnitude of magnetic field, $B = \mu_0 nI$

or $nI = \frac{B}{\mu_0} = \frac{10^{-2}}{4}\times3.14\times10^{-7} \Rightarrow nI = 7961 \approx 8000$

Here, the product of nI is 8000.

Current, $I = 8$ A and number of turns, $n = 1000$

The other design is $I = 10$ A and $n = 800$/m. This is the most appropriate design as per the requirement.

36. Here, energy of electron, $E' = 2000$ eV

$$E' = 2000\times1.6\times10^{-19}\text{ J} = 3.2\times10^{-16}\text{ J}$$

Now, $B = 0.2$ T, $r = ?$, $E' = \frac{1}{2}mv^2$

$$\Rightarrow \quad v = \sqrt{\frac{2E'}{m}}$$

$$Bev = \frac{mv^2}{r}$$

$$\therefore \quad r = \frac{mv}{Be} = \frac{m}{Be}\sqrt{\frac{2E'}{m}} = \frac{\sqrt{2E'm}}{Be}$$

$$= \frac{\sqrt{2\times3.2\times10^{-16}\times9\times10^{-31}}}{0.2\times1.6\times10^{-19}}$$

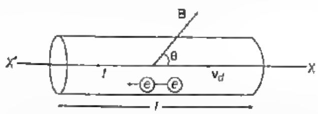
$$= 7.5\times10^{-4}\text{ m}$$

|TOPIC 3|

Magnetic Force and Torque Experienced by a Current Loop

FORCE ON A CURRENT CARRYING CONDUCTOR IN A UNIFORM MAGNETIC FIELD

When a current carrying conductor is placed in a uniform magnetic field, then due to motion of free electrons inside the conductor, a magnetic force acts on it.



Current carrying conductor in a uniform magnetic field

Let us consider a portion of length l and cross-sectional area A of a straight conductor carrying a current I.

Let the magnetic field **B** be in the plane of the paper directed upwards and making an angle θ with the direction of velocity of electrons.

Let n be the number of free electrons per unit volume in the conductor and v_d be the drift velocity of the electrons. From the relation $\mathbf{F} = q(\mathbf{v}\times\mathbf{B})$, where $(q = e)$ is the charge of an electron. If however, the conductor makes an angle θ with the magnetic field **B** measured from the conductor towards the field **B**, then the magnitude of the force on each electron is

$$F' = ev_d B\sin\theta$$

The number of electrons in the length l of the conductor is

$$N = nAl$$

The total force F on the free electrons and hence on the length l of the conductor is, therefore

$$F = F'\times N = (ev_d B\sin\theta)(nAl)$$

$$= (neAv_d)Bl\sin\theta$$

But current flowing through a conductor, $I = neAv_d$

$$\therefore \quad F = IBl\sin\theta$$

or $$\mathbf{F} = I(\mathbf{l}\times\mathbf{B})$$

If $\theta = 0°$ or $180°$, then $F = IBl\sin 0° = 0$

[$\because \sin 0° = 0$ and $\sin 180° = 0$]

It means a conductor placed parallel to direction of magnetic field, experiences no force due to magnetic field.

If $\theta = 90°$, then force is maximum.

$$F_{max} = IBl\sin 90°$$

$$= IBl \qquad [\because \sin 90° = 1]$$

It means a conductor placed perpendicular to direction of magnetic field, experiences maximum force.

Rules to Find Out the Direction of Force

The direction of the force acting on a current carrying conductor in a magnetic field can be found by any of the following two rules.

[redacted]

(i) **Right Hand Palm Rule** If we stretch our right hand palm such that the thumb points in the direction of the current (I) and the stretched fingers in the direction of the magnetic field **B**, then the force **F** on the conductor will be perpendicular to the palm in the direction of pushing by the palm as shown in the Fig. (a).

(a) (b)

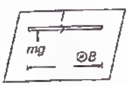
(ii) **Fleming's Left Hand Rule** If the fore-finger, the middle-finger and the thumb of the left hand are stretched mutually at right angles to one another such that the fore-finger points in the direction of the magnetic field **B** and the middle-finger in the direction of the current I, then the thumb will point in the direction of the force **F** on the conductor as shown in the Fig. (b).

EXAMPLE |1| **A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire?**

Sol. Magnetic force on the wire,

$$F = BIl\sin\theta = BIl\sin 90^\circ$$
$$= 0.27 \times 10 \times 3 \times 10^{-2}$$
$$= 81 \times 10^{-3}\,\text{N} \qquad [\because \theta = 90^\circ]$$

EXAMPLE |2| **A straight wire of mass 200 g and length 1.5 m carries a current of 4 A. It is suspended in mid air by a uniform horizontal magnetic field B. What is the magnitude of the magnetic field?**



Sol. Applying Fleming's rule, we find that upward force F of magnitude IlB acts. For mid air suspension this must be balanced by the force due to gravity.

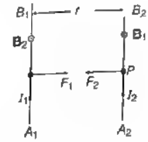
$$\therefore \quad mg = IlB \Rightarrow B = \frac{mg}{Il}$$

Given, $m = 200\,\text{g} = 0.2\,\text{kg}$, $g = 9.8\,\text{ms}^{-2}$, $I = 4\,\text{A}$, $l = 1.5\,\text{m}$

We have, $$B = \frac{0.2 \times 9.8}{4 \times 1.5} = 0.325\,\text{T}$$

FORCE BETWEEN TWO PARALLEL CURRENT CARRYING CONDUCTORS

To find the force on a current carrying wire due to a second current carrying wire, first find the magnetic field due to second wire at the site of first wire. Then, find the force on the first wire due to that field.



Two parallel current carrying conductors

Let us consider A_1B_1 and A_2B_2 are two infinite long straight conductors.

I_1 and I_2 are the currents flowing through them and these are at r distance apart.

Magnetic field induction at a point P on conductor A_2B_2 due to current

I_1 passing through A_1B_1 is $B_1 = \dfrac{\mu_0 2I_1}{4\pi r}$

The unit length of A_2B_2 will experience a force as

$$F_2 = B_1 I_2 \times l = B_1 I_2 l$$

or $$F_2 = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r} \cdot l$$

Conductor A_1B_1 also experiences the same amount of force, directed towards the wire A_2B_2.

Therefore, force between two current carrying parallel conductors per unit length is

$$\boxed{F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r}}$$

Two linear parallel conductors carrying currents in the same direction attract each other while carrying currents in opposite direction they repel each other.

Definition of Ampere (In terms of the force)

One ampere is the current which flows through each of the two parallel uniform long linear conductors, which are placed in free space at a distance of 1 m from each other and which attract or repel each other with a force of 2×10^{-7} N/m of their lengths.

[redacted]

EXAMPLE |3| Calculate the force per unit length on a long straight wire carrying current of 4 A due to a parallel wire carrying 6 A current, if the distance between the wires is 3 cm.

Sol. Given, $I_1 = 4\text{ A}, I_2 = 6\text{ A}, r = 3\text{ cm} = 0.03\text{ m}$

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r}$$

$$= \frac{10^{-7} \times 2 \times 4 \times 6}{0.03}$$

$$= 1.6 \times 10^{-4}\text{ N/m}$$

EXAMPLE |4| A short conductor of length 5 cm is placed parallel to a long conductor of length 1.5 m near its centre. The conductors carry currents 4 A and 3 A respectively in the same direction. What is the total force experienced by the long conductor when they are 3 cm apart?

Sol. Force on long conductor is equal and opposite to the force on small conductor $= \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2 l}{r}$

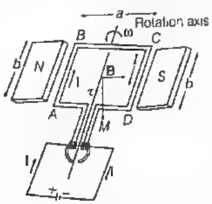
Given, $I_1 = 4\text{ A}, I_2 = 3\text{ A}, r = 3 \times 10^{-2}\text{ m}, l = 5 \times 10^{-2}\text{ m}$

$$\Rightarrow \quad F = \frac{4\pi \times 10^{-7} \times 2 \times 4 \times 3 \times 5 \times 10^{-2}}{4\pi \times 3 \times 10^{-2}}$$

$$= 4 \times 10^{-6}\text{ N}$$

TORQUE EXPERIENCED BY A CURRENT LOOP IN UNIFORM MAGNETIC FIELD (MAGNETIC DIPOLE)

Consider a rectangular loop *ABCD* be suspended in a uniform magnetic field **B**. Let $AB = CD = b$ and $AD = BC = a$. Let I be the current flowing through the loop.



A rectangular current carrying coil in uniform magnetic field

Case I The rectangular loop is placed such that the uniform magnetic field B is in the plane of loop.

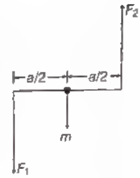
No force is exerted by the magnetic field on the arms AD and BC ($\because$ they are parallel to the magnetic field).

Magnetic field exerts a force F_1 on arm AB,

$$\therefore \quad F_1 = IbB$$

Magnetic field exerts a force F_2 on arm CD,

$$F_2 = IbB = F_1$$



F_1 and F_2 are equal and opposite, so net force on the loop is zero. But line of action of F_1 and F_2 are opposite and parallel, so they form a couple.

The torque produced due to couple on the loop rotates the loop in anti-clockwise direction.

Torque, $\tau = \mathbf{r} \times \mathbf{F}$

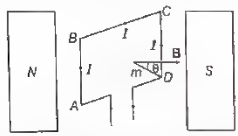
So, $\tau = F_1 \dfrac{a}{2} + F_2 \dfrac{a}{2}$ $\quad [\because \sin 90° = 1]$

($\because$ Torque = Force × Perpendicular distance of line of action)

$$\Rightarrow \quad \tau = IbB\frac{a}{2} + IbB\frac{a}{2} = I\,(ab)\,B = IAB$$

where, b be breadth of the rectangular coil, a be length of the rectangular coil and $A = ab$ (area of the coil).

Case II The plane of the loop is not along the magnetic field, but makes an angle with it.



The area vector of the loop *ABCD* makes an arbitrary angle θ with the magnetic field

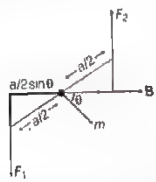
Angle between the field and the normal to the coil is θ.

Forces on BC and DA are equal and opposite and they cancel each other as they are collinear.

Force on AB is F_1 and force on CD is F_2.

and $\quad F_1 = F_2 = IbB$

Magnitude of torque on the loop is as shown in figure below:



Top view of the loop. The forces F_1 and F_2 acting on the area *AB* and *CD* are indicated

$$\tau = F_1 \frac{a}{2} \sin\theta + F_2 \frac{a}{2} \sin\theta$$

$$\tau = IabB\sin\theta = IAB\sin\theta$$

[redacted]

where, $A = ab$ (area of coil)

The torque on the loop can be expressed as the vector product of the magnetic moment of the coil and the magnetic field

$$\boxed{\tau = MB\sin\theta\hat{n} = \mathbf{M}\times\mathbf{B}}$$

where, $\mathbf{M} = NI\mathbf{A}$ is magnetic moment of the loop, its unit is ampere-metre^2.

This is analogous to the electrostatic case (electric dipole of dipole moment p in an electric field E)

When **M** and **B** are parallel, then current loop is in stable equilibrium. Any small rotation of the loop produces a torque which brings it back to its original position. When **M** and **B** are anti-parallel, then current loop is in unstable equilibrium.

Magnetic moment of the loop of N turns,

$$M = NIA$$

The total torque on the coil is given by

$$\tau = NIAB\sin\theta = (NIA)\,B\sin\theta = BINA\sin\theta$$

The presence of this torque is also the reason why a small magnet or any magnetic dipole aligns itself with the external magnetic field.

EXAMPLE |5| A circular coil of 20 turns and radius 10 cm carries a current of 5 A. It is placed in a uniform magnetic field of 0.10 T. Find the torque acting on the coil, when the magnetic field is applied in the plane of coil.

Sol. Given, total number of turns, $N = 20$

Radius, $r = 10\text{ cm} = 10\times10^{-2}\text{ m}$

Current, $I = 5\text{ A}$

Angle, $\theta = 90^\circ$

External uniform magnetic field $(B) = 0.10\text{ T}$

Torque, $\tau = ?$

As, torque, $\tau = BINA\sin\theta$

$\Rightarrow \quad = 0.10\times5\times20\times0.0314\times\sin90^\circ$

$= 0.314\text{ N-m}$

EXAMPLE |6| Calculate the torque of a 100 turns rectangular coil of length 40 cm and breadth 20 cm, carrying a current of 10 A, when placed making an angle of 60° with a magnetic field of 5 T.

Sol. Given, $I = 10\text{A}$, $N = 100$, $l = 40\text{cm}$, $b = 20\text{ cm}$

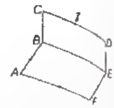
$B = 5\text{ T}, \theta = 60^\circ$

$A = l\times b = 40\times20 = 800\text{ cm}^2 = 8\times10^{-2}\text{ m}^2$

$\because \quad \tau = NBIA\sin\theta = 100\times5\times10\times8\times10^{-2}\times\sin60^\circ$

$= 346.41\text{ N-m}$

EXAMPLE |7| Find the magnitude of magnetic moment of the current carrying loop *ABCDEFA*. Each side of the loop is 10 cm long and current in the loop is $I = 2.0$ A.



Sol. By assuming two equal and opposite currents in *BE*, two current carrying loops (*ABEFA* and *BCDEB*) are formed. Their magnetic moments are equal in magnitude but perpendicular to each other. Hence,

$$M_{net} = \sqrt{M^2 + M^2} = \sqrt{2}M$$

$\because \quad M = IA = 2\times(l\times b)$

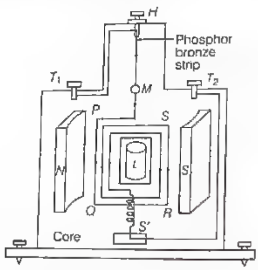
$= 2\times(0.1)(0.1) = 0.02\text{ A-m}^2$

$\therefore \quad M_{net} = (\sqrt{2})(0.02) = 0.028\text{ A-m}^2$

MOVING COIL GALVANOMETER

Principle

Its working is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a torque.



Schematic arrangement of moving coil galvanometer

Construction

The moving coil galvanometer consists of a coil with many turns free to rotate about a fixed axis, in a uniform radial magnetic field. There is a cylindrical soft iron core which not only makes the field radial but also increases the strength of the magnetic field. When a current flows through the coil, a torque acts on it.

Working

Suppose the coil *PQRS* is suspended freely in the magnetic field. Let l be length *PQ* or *RS* of the coil, b be breadth *QR* or *SP* of the coil, N be number of turns in the coil and area of each turn of the coil, $A = l\times b$. Let B be strength of the

[redacted]

magnetic field in which coil is suspended and I is current passing through the coil in the direction $PQRS$.

Let at any instant, α be the angle, which normal drawn on the plane of the coil makes with the direction of magnetic field. The rectangular coil carrying current when placed in the magnetic field experiences a torque whose magnitude is given by $\tau = NIBA \sin \alpha$. Due to deflecting torque, the coil rotates and suspension wire gets twisted. A restoring torque is set up in the suspension wire.

Let θ be the twist produced in the phosphor bronze strip due to rotation of the coil and k be the restoring torque per unit twist of the phosphor bronze strip. Then,

Total restoring torque produced $= k\theta$

In equilibrium position of the coil,

Deflecting torque = Restoring torque

$$NIBA = k\theta$$

$$\Rightarrow \quad I = \frac{k}{NBA}\theta = G\theta$$

where, $\dfrac{k}{NBA} = G$

It is known as galvanometer constant.

i.e. $\theta \propto I$. It means that the deflection produced is proportional to the current flowing through the galvanometer.

Current sensitivity of the galvanometer is the deflection per unit current flowing through it.

It is given by $I_S = \dfrac{\theta}{I} = \dfrac{NAB}{k}$

Its unit is rad/A or div/A.

Voltage sensitivity is the deflection per unit voltage.

It is given by

$$V_S = \frac{\theta}{V} = \left(\frac{NAB}{k}\right)\frac{I}{V}$$

or $$V_S = \frac{NAB}{k} \times \frac{I}{IR} = \frac{NAB}{kR}$$

[$\because$ according to Ohm's law, $V = IR$]

Its unit is rad/V or div/V.

Note Dead beat galvanometer is one in which the coil comes to rest at once after the passage of current through it. The deflection can be noted in no time.

EXAMPLE |8| **In order to increase the current sensitivity of a moving coil galvanometer by 50%, its resistance is increased so that the new resistance becomes twice its initial resistance. By what factor does its voltage sensitivity change?**

Sol. Increased current sensitivity, $I_S' = I_S + \dfrac{50\, I_S}{100} = \dfrac{150\, I_S}{100}$

$$= \frac{3\, I_S}{2},\ R' = 2R$$

Initial voltage sensitivity, $V_S = \dfrac{I_S}{R}$ ...(i)

New voltage sensitivity, $V_S' = \dfrac{I_S'}{R'} = \dfrac{\frac{3}{2} I_S}{2R}$

$\Rightarrow \quad V_S' = \dfrac{3}{4} V_S$ [from Eq. (i)]

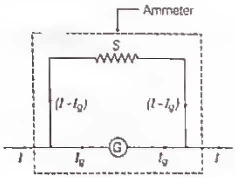
$\therefore$ % decrease in voltage sensitivity $= \dfrac{V_S - V_S'}{V_S} \times 100$

$$= \left(1 - \frac{3}{4}\right) \times 100 = 25\%$$

Note This topic has been frequently asked in the previous year exams, i.e. in year 2016, 2015, 2014, 2010, 2009

Conversion of a Galvanometer into Ammeter

To convert a galvanometer into ammeter, its resistance needs to be lowered, so that maximum current can pass through it and it can give exact reading.



A shunt (low resistance) is connected in parallel with the galvanometer.

$$S = \left(\frac{I_g}{I - I_g}\right)G$$

where, I = total current in circuit,

G = resistance of the galvanometer,

S = resistance of the shunt (low resistance)

and I_g = current through the galvanometer.

EXAMPLE |9| **A galvanometer of resistance 15 Ω gives full scale deflection for a current of 2 mA. Calculate the shunt resistance needed to convert it to an ammeter of range 0 to 5 A.**

Sol. Given,

$$G = 15\ \Omega,\ I_g = 2\text{ mA}$$
$$= 2 \times 10^{-3}\text{ A},\ I = 5\text{ A}$$

[redacted]

$\therefore$ Shunt resistance, $S = \dfrac{I_g G}{I - I_g}$

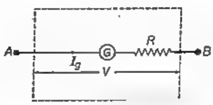
$$= \frac{2\times10^{-3}\times15}{5-2\times10^{-3}}$$

$$= 0.006\ \Omega$$

This resistance $S = 0.006\ \Omega$ is connected in parallel with the galvanometer. The small resistance is connected in parallel, because we have to decrease the resistance of the galvanometer so that most of the current passes through it and it gives the exact value of the current.

Conversion of a Galvanometer into Voltmeter

To convert a galvanometer into voltmeter, its resistance needs to be increased, so that there is no potential drop across it because with high resistance no current passes through it.



Voltmeter

A high resistance is connected in series with the galvanometer, then the value of R is given by

$$R = \frac{V}{I_g} - G$$

where, V = potential difference across the terminals A and B

I_g = current through the galvanometer (full scale deflection current),

R = high resistance

and G = resistance of the galvanometer.

EXAMPLE |10| **The full scale deflection current of a galvanometer of resistance 1 Ω is 5 mA. How will you convert it into a voltmeter of range 5 V?**

Sol. From the relation, $V = I_g (G + R)$, we have

$$R = \frac{V}{I_g} - G$$

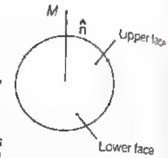
$$= \left(\frac{5}{5\times10^{-3}}\right) - 1$$

$$= 999\ \Omega$$

i.e., a resistance of 999 Ω should be connected in series with the galvanometer to convert it into a voltmeter of desired range.

CIRCULAR CURRENT LOOP AS A MAGNETIC DIPOLE

A current loop behaves as a magnetic dipole. If we look at the upper face, current is anti-clockwise, so it has North polarity. If we look at lower face, current is clockwise, so it has South polarity.



That means current loop behaves as a system of two equal and opposite magnetic poles hence, it acts as a magnetic dipole.

Magnetic dipole moment of loop, $M = NIA$

where, I = current flowing through the loop

A = area enclosed by the loop

and N = number of turns in the coil

The magnitude of magnetic field on the axis of a circular loop of radius R, carrying steady current I is given by

$$B = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$$

For $x >> R$,

$$B = \frac{\mu_0 I R^2}{2x^3} = \frac{\mu_0 I A}{2\pi x^3} \qquad [\because A = \pi R^2]$$

$$\Rightarrow \quad B = \frac{\mu_0 M}{2\pi x^3} \qquad [\because M = IA]$$

MAGNETIC DIPOLE MOMENT OF A REVOLVING ELECTRON

An electron being a charged particle, constitutes a current while moving in its circular orbit around the nucleus ($\because$ Moving charge constitutes a current as well as magnetic field). If T is the time period of revolution, then current constituted by electron is $I = \dfrac{e}{T}$ ... (i)

where, e = charge of electron.

If r is the orbital radius of electron and its orbital speed is v, then

$$T = \frac{2\pi r}{v} \Rightarrow I = \frac{e}{\frac{2\pi r}{v}} = \frac{ev}{2\pi r} \qquad \text{[from Eq. (i)]}$$

Magnetic moment of revolving electron, $M = IA$

$$M = \frac{ev}{2\pi r}\pi r^2 = \frac{evr}{2}$$

The direction of this magnetic moment is into the plane of the paper.

[redacted]

$$M = \frac{e}{2m_e}(m_e vr) = \frac{e}{2m_e} l \text{ or } M = \frac{-el}{2m_e}$$

where, $l = m_e vr$ is angular momentum of the electron.

$\frac{M}{l} = \frac{e}{2m_e}$ is a constant, called **gyromagnetic ratio**, its value is 8.8×10^{10} C/kg for an electron.

From Bohr's hypothesis, angular momentum can have only some discrete values, $l = \frac{nh}{2\pi}$

where, h = Planck's constant and n is natural number

i.e. $n = 1, 2, 3, \ldots \Rightarrow M = \frac{e}{2m_e} \cdot \frac{nh}{2\pi} = \frac{e}{4\pi m_e} nh$

For $n = 1$, M will be minimum.

$$\therefore \quad M_{min} = \frac{eh}{4\pi m_e}$$

It is Bohr's magneton, which is defined as magnetic moment of revolving electron in its first orbit. Its value is 9.27×10^{-24} A-m^2.

EXAMPLE |1| An electron in a hydrogen atom is moving with a speed of 2.3×10^6 ms^{-1} in an orbit of radius 0.53 Å. Calculate the magnetic moment of the revolving electron.

Sol. Given, $v = 2.3 \times 10^6$ ms^{-1},

$r = 0.53$ Å $= 0.53 \times 10^{-10}$ m

Equivalent current, $I = \frac{e}{T} = \frac{e}{\frac{2\pi r}{v}} = \frac{ev}{2\pi r}$

$$= \frac{1.6 \times 10^{-19} \times 2.3 \times 10^6}{2 \times 3.14 \times 0.53 \times 10^{-10}}$$

$$= 1.105 \times 10^{-3} \text{ A}$$

$\therefore$ Magnetic moment,

$M = IA = I(\pi r^2) = 1.105 \times 10^{-3} \times 3.14 \times (0.53 \times 10^{-10})^2$

$= 9.75 \times 10^{-24}$ A-m^2

| TOPIC PRACTICE 3 |

OBJECTIVE Type Questions

1. Two parallel wires are placed 1 m apart and 1 A and 3 A currents are flowing in the wires in opposite direction. The force acting per unit length of both the wires will be
   (a) 6×10^{-7} N/m attractive
   (b) 6×10^{-5} N/m attractive
   (c) 6×10^{-7} N/m repulsive
   (d) 6×10^{-5} N/m repulsive

2. A circular loop of area A, carrying current I, is placed in a magnetic field B perpendicular to the plane of the loop. The torque on the loop due to magnetic field is
   (a) BIA (b) $2\,BIA$ (c) $\frac{1}{2}\,BIA$ (d) zero

3. The area of a circular ring is 1 cm^2 and current of 10 A is passing through it. If a magnetic field of intensity 0.1 T is applied perpendicular to the plane of the ring. The torque due to magnetic field on the ring will be
   (a) zero (b) 10^{-4} N-m (c) 10^{-2} N-m (d) 1 N-m

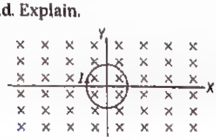
4. A circular current loop of magnetic moment M is in an arbitrary orientation in an external magnetic field B. The work done to rotate the loop by 30° about an axis perpendicular to its plane is **NCERT Exemplar**
   (a) MB (b) $\sqrt{3}\frac{MB}{2}$
   (c) $\frac{MB}{2}$ (d) zero

5. The current i is flowing in a coil of area A with the number of turns N, then the magnetic moment of the coil M will be
   (a) NiA (b) Ni/A
   (c) $Ni/\sqrt{A}$ (d) $N^2 Ai$

6. A galvanometer of resistance 25 Ω shows full scale deflection for current of 10 mA. To convert it into 100 V range voltmeter, the required series resistance is
   (a) 9975 Ω (b) 10025 Ω
   (c) 10000 Ω (d) 975 Ω

VERY SHORT ANSWER Type Questions

7. A conducting loop carrying a current I is placed in a uniform magnetic field, pointing into the plane of the paper as shown in the figure, then the loop will have a tendency to expand. Explain.



8. Give the magnitude of torque which acts on a coil carrying current placed in a uniform radial magnetic field.

[redacted]

9. Write the underlying principle of a moving coil galvanometer. **Delhi 2016**

10. Why should the spring/suspension wire in a moving coil galvanometer have low torsional constant?

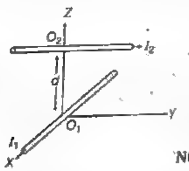
11. Why is a coil wrapped on a conducting frame in a galvanometer?

12. The coils in certain galvanometers, have a fixed core made of a non-magnetic metallic material. Why does the oscillating coil come to rest so, quickly in such a core?

13. A voltmeter, an ammeter and a resistance are connected in series with a lead accumulator. The voltmeter gives some deflection but the deflection of ammeter is zero. Comment.

SHORT ANSWER Type Questions

14. Two long wires carrying currents I_1 and I_2 are arranged as shown in the figure One carrying current I_1 is along the X-axis. The other carrying current I_2 is along a line parallel to Y-axis, given by $x = 0$ and $z = d$. Find the force exerted at point O_2 because of the wire along the X-axis.



NCERT Exemplar

15. Two long parallel wires carrying a current I, separated by a distance r are exerting a force F on each other. If the distance between them is increased to $2r$ and current in each wire is reduced from I to $I/2$, then what will be the force between them?

16. (i) Two long straight parallel conductors a and b carrying steady currents I_a and I_b respectively are separated by a distance d. Write the magnitude and direction, what is the nature and magnitude of the force between the two conductors?
(ii) Show with the help of a diagram, how the force between the two conductors would change when the currents in them flow in the opposite directions. **Foreign 2014**

17. A rectangular coil of sides l and b carrying a current I is subjected to a uniform magnetic field B acting perpendicular to its plane. Obtain the expression for the torque acting on it. **Delhi 2014C**

18. Define current sensitivity and voltage sensitivity of galvanometer. Increasing the current sensitivity may not necessarily increase the voltage sensitivity of a galvanometer, justify your answer. **All India 2009**

19. How is a moving coil galvanometer converted into a voltmeter? Explain giving the necessary circuit diagram and the required mathematical relation used. **All India 2011C**

20. A galvanometer gives full scale deflection with the current I_g.
Can it be converted into an ammeter of range $I < I_g$?

LONG ANSWER Type I Questions

21. Draw a labelled diagram of a moving coil galvanometer and explain its working. What is the function of radial magnetic field inside the coil? **Foreign 2012**

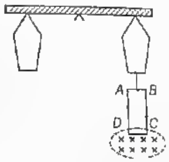
22. Answers the following questions.
(i) Write two reasons why a galvanometer cannot be used as such to measure the current in a given circuit. Name any two factors on which the current sensitivity of a galvanometer depends.
(ii) Why is it necessary to introduce a cylindrical soft iron core inside the coil of a galvanometer?

23. State the principle of working of a galvanometer.
A galvanometer of resistance G is converted into a voltmeter to measure upto V volts by connecting a resistance R_1 in series with the coil. If a resistance R_2 is connected in series with it, then it can measure upto $V/2$ volts. Find the resistance, in terms of R_1 and R_2, required to be connected to convert it into a voltmeter that can read upto 2V. Also, find the resistance G of the galvanometer in terms of R_1 and R_2. **All India 2015**

24. A 100 turns rectangular coil $ABCD$ (in XY-plane) is hung from one arm of a balance figure. A mass 500 g is added to the other arm to balance

[redacted]

the weight of the coil. A current 4.9 A passes through the coil and a constant magnetic field of 0.2 T acting inward (in XZ-plane) is switched ON such that only arm CD of length 1 cm lies in the field. How much additional mass m must be added to regain the balance? **NCERT Exemplar**



LONG ANSWER Type II Questions

25. Explain using a labelled diagram the principle and working of a moving coil galvanometer. What is the function of

(i) uniform radial magnetic field (ii) soft iron core?

Also, define the terms

(iii) current sensitivity and

(iv) voltage sensitivity of a galvanometer?

Why does increasing the current sensitivity not necessarily increase voltage sensitivity? **Delhi 2015**

26. (i) Draw a labelled diagram of a moving coil galvanometer. Describe briefly its principle and working.

(ii) Answer the following are as follows

(a) Why is it necessary to introduce a cylindrical soft iron core inside the coil of a galvanometer?

(b) Increasing the current sensitivity of a galvanometer may not necessarily increase its voltage sensitivity. Explain giving reason. **All India 2014**

27. (i) Explain giving reasons, the basic difference in converting a galvanometer into

(a) a voltmeter and (b) an ammeter

(ii) Two long straight parallel conductors carrying steady currents I_1 and I_2 are separated by a distance d. Explain briefly, with the help of a suitable diagram, how the magnetic field due to one conductor acts on the other. Hence, deduce the expression for the force acting between the two conductors. Mention the nature of this force. **All India 2012**

NUMERICAL PROBLEMS

28. What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A making an angle of 30° with the direction of a uniform magnetic field of 0.15 T? **NCERT**

29. A long straight wire carrying current of 25 A rests on a table shown in the figure. Another wire PQ of length 1 m, mass 2.5 g carries the same current but in the opposite direction. The wire PQ is free to slide up and down. To what height will PQ rise? **NCERT Exemplar**

30. A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire? **NCERT**

31. A uniform magnetic field of 1.5 T exists in a cylindrical region of radius 10.0 cm, its direction parallel to the axis along East to West. A wire carrying current of 7.0 A in the North to South direction passes through this region. What is the magnitude and direction of the force on the wire, if

(i) the wire intersect the axis?

(ii) the wire is turned from North-South to North East-North West direction?

(iii) the wire in the North-South direction is lowered from the axis by a distance of 6.0 cm? **NCERT**

32. A solenoid 60 cm long and of radius 4.0 cm has 3 layers of winding of 300 turns each. A 2.0 cm long wire of mass 2.5 g lies inside the solenoid (near its centre) normal to its axis, both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6.0 A in the wire. What value of current (with appropriate sense of circulation) in the windings of the solenoid can support the weight of the wire? ($g = 9.8\ m/s^2$) **NCERT**

[redacted]

33. A conductor of length 2 m carrying current of 2 A is held parallel to an infinitely long conductor carrying current of 10 A at a distance of 100 mm. Find the force on a small conductor. Delhi 2010

34. Two long and parallel straight wires A and B carrying currents of 8.0 A and 5.0 A in the same direction are separated by a distance of 4.0 cm. Estimate the force on a 10 cm section of wire A. NCERT

35. A wire AB is carrying a steady current of 12 A and is lying on the table. Another wire CD carrying 5 A is held directly above AB at a height of 1 mm.
Find the mass per unit length of the wire CD, so that it remains suspended at its position, when left free. Give the direction of the current flowing in CD with respect to that in AB. (Take the value of $g = 10\ \text{ms}^{-2}$) All India 2013

36. A circular coil of 100 turns, radius 10 cm carries a current of 5 A. It is suspended vertically in a uniform magnetic field of 0.5 T, the field lines making an angle of 60° with the plane of the coil. Calculate the magnitude of the torque that must be applied to it to prevent it from turning.

37. A square coil of side 10 cm consists of 20 turns and carries current of 12 A. The coil is suspended vertically and normal to the plane of the coil makes an angle of 30° with the direction of a uniform horizontal magnetic field of magnitude 0.80 T. What is the magnitude of torque experienced by the coil? NCERT

38. (i) A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle 60° with the normal of the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.
(ii) Would your answer change, if the circular coil were replaced by a planar coil of some irregular shape that encloses the same area? All other particulars are also unaltered. NCERT

39. A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.1 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the (i) total torque on the coil, (ii) total force on the coil (iii) average force on each electron in the coil due to the magnetic field?
(The coil is made of copper wire of cross-sectional area $10^{-5}\ \text{m}^2$ and the free electron density in copper is given to be about $10^{29}/\text{m}^3$). NCERT

40. A rectangular coil of area $2\times10^{-4}\ \text{m}^2$ and 40 turns is pivoted about one of its vertical sides. The coil is in a radial horizontal field of 60 G. What is the torsional constant of the hair springs connected to the coil, if a current of 4.0 mA produces an angular deflection of 16°?

41. Two moving coil meters M_1 and M_2 having the following particulars
$R_1 = 10\ \Omega$, $N_1 = 30$, $A_1 = 3.6\times10^{-3}\text{m}^2$, $B_1 = 0.25\ \text{T}$
$R_2 = 14\ \Omega$, $N_2 = 42$, $A_2 = 1.8\times10^{-3}\text{m}^2$,
$B_2 = 0.50\ \text{T}$ (The spring constants are identical for the two meters). Determine the ratio of (i) current sensitivity and (ii) voltage sensitivity of M_2 and M_1. NCERT

42. A galvanometer coil has a resistance of 15 Ω and the meter shows full scale deflection for a current of 4 mA. How will you convert the meter into an ammeter of range 0 to 6 A? NCERT

43. When a galvanometer having 30 division scale and 100 Ω resistance is connected in series to the battery of emf 3 V through a resistance of 200 Ω, shows full scale deflection. Find the figure of merit of the galvanometer in microampere.

44. A galvanometer coil has a resistance of 12 Ω and the meter shows full scale deflection for a current of 3 mA. How will you convert the meter into a voltmeter of range 0 to 18 V? NCERT

HINTS AND SOLUTIONS

1. (c) The force acting per unit length,
$$\frac{F}{L} = \frac{\mu_0}{2\pi}\cdot\frac{I_1 I_2}{r} = 2\times10^{-7}\times\frac{1\times3}{1} = 6\times10^{-7}$$
If the currents are in opposite direction, then the wires will repel each other.

2. (d) Torque experienced by a current loop in a uniform magnetic field, $\tau = NIBA\sin\theta$
When $\theta = 0°$, $\tau = 0$ $(\because N = 1 \text{ and } \sin 0° = 0)$

[redacted]

3. (a) Given, $A = 1\text{ cm}^2 = 1\times 10^{-4}\text{ m}^2$,
$I = 10\text{ A}, B = 0.1\text{ T}, \theta = 0°$

The moment of force or torque acting on the circular ring,

$\tau = IBA\sin\theta$

$= 10\times 0.1\times 1\times 10^{-4}\times \sin 0° = 0$

4. (d) The rotation of the loop by 30° about an axis perpendicular to its plane make no change in the angle made by axis of the loop with the direction of magnetic field, therefore, the work done to rotate the loop is zero.

5. (a) The magnetic moment of a current-carrying coil $M = iA$. If there are N turns, then $M = NiA$.

6. (a) To convert a galvanometer into a voltmeter by connecting a high resistance in series, required series resistance will be, $R = \frac{V}{I_g} - G$

which restricts the current to safe limit I_g.

where, G = resistance of galvanometer = 25 Ω,

I_g = current with which galvanometer gives full scale deflection = 10 mA,

$= 10\times 10^{-3}\text{ A}$

V = required range of voltmeter = 100 V

$\Rightarrow \quad R = \frac{100}{10\times 10^{-3}} - 25 = 9975\ \Omega$

7. We can see that magnetic field is perpendicular to paper and current in the loop is in clockwise direction. So, by Fleming's left hand rule, force on each element of the loop is radially outwards, so loop will have a tendency to expand.

8. Torque, $\tau = NBIA\sin\theta$, where the terms have their usual meanings.

9. The principle of moving coil galvanometer is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a torque.

10. Low torsional constant facilitates greater deflection θ in coil for given value of current and hence, sensitivity of galvanometer increases.

11. In order to produce electromagnetic damping i.e., by producing eddy currents in conducting frame which helps in stopping the coil soon.

12. Due to eddy currents produced in core which opposes the cause (deflection of coil), that produces it. This further helps in stopping the coil so on, i.e. in making the galvanometer dead beat.

13. Voltmeter and resistance being very high when connected in series, makes the effective resistance of the circuit very high. Due to this, current in the circuit becomes extremely small.

14. Here, first we have to find the direction of magnetic field at point O_2 due to the wire carrying current I_1. Use Maxwell's right hand grip (cork screw) rule, the direction of magnetic field at point O_2 due to current I_1 is along Y-axis.

Here, the wire at point O_2 is placed along Y-axis. Now, by the formula, $\mathbf{F} = I_2(\mathbf{l}\times\mathbf{B})$

Angle between $\mathbf{l}$ and $\mathbf{B}$ is 0°, both are at Y-axis, i.e.

$$F = IlB\sin 0° = 0$$

So, the force exerted at point O_2 because of wire along X-axis is zero.

15. Force per unit length is

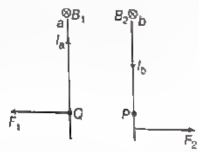
$$F = \frac{\mu_0 2I^2}{4\pi r} \qquad [\because I_1 = I_2 = I]$$

If r is increased to $2r$ and I is reduced to $\frac{I}{2}$, then new force per unit length is $F' = \frac{\mu_0}{4\pi}\times\frac{2(I/2)^2}{2r}$

$$= \frac{1}{8}\left(\frac{\mu_0}{4\pi}\cdot\frac{2I^2}{r}\right) \Rightarrow F' = \frac{F}{8}$$

$\therefore$ Force per unit length between them is $\frac{F}{8}$.

16. Refer to text on page 192.



Now, let the direction of current in conductor b be reversed. The magnetic field B_2 at point P due to current I_a flowing through a will be downwards. Similarly, the magnetic field B_1 at point Q due to current I_b passing through b will also be downward as shown. The froce on a will be therefore towards the left. Also, the force on b will be towards the right. Hence, the two conductors will repel each other as shown.

17. Equivalent magnetic moment of the coil, $\mathbf{M} = IA\hat{\mathbf{n}}$

$\therefore \quad \mathbf{M} = Ilb\,\hat{\mathbf{n}}$

where, $\hat{\mathbf{n}}$ = unit vector ⊥ to the plane of the coil.

$\therefore$ Torque $= \mathbf{M}\times\mathbf{B} = Ilb(\hat{\mathbf{n}}\times\mathbf{B}) = 0$

As $\hat{\mathbf{n}}$ and $\mathbf{B}$ are parallel or anti-parallel to each other.

18. Refer to text on pages 195.

Increasing the current sensitivity may not necessarily increase the voltage sensitivity, because the current sensitivity increases with the increase of number of turns of the coil but the resistance of coil also increases which affect adversely on voltage sensitivity.

19. Refer to text on page 196.

20. The resistance of an ideal ammeter is zero or very low in practical condition, so to convert a galvanometer into ammeter its resistance needs to be decreased which can be done by connecting a low resistance in its parallel order.

[redacted]

A moving coil galvanometer of range I_g and resistance G can be converted into ammeter by connected very low resistance shunt in parallel with galvanometer.

21. Refer to text on pages 194 and 195.

22. (i) The galvanometer cannot be used to measure the current because

(a) all the currents to be measured passes through coil and it gets damaged easily.

(b) its coil has considerable resistance because of length and it may affect original current.

Current sensitivity of galvanometer depends on

(a) the magnetic field

(b) the value of torsional constant.

(ii) It is necessary to introduce a cylindrical soft iron core inside the coil of a galvanometer because magnetic field is increased, so its sensitivity increases and magnetic field becomes radial. So, angle between the plane of coil and magnetic line of force is zero in all orientations of coil.

23. According to the principle of working of a moving coil galvanometer, when a current carrying coil is placed in a magnetic field, it experiences a torque.

A high resistance that is connected in series with the galvanometer to convert into voltmeter. The value of the resistance is given by, $R = \frac{V}{I_g} - G$

where, V – potential difference across the terminals of the voltmeter, I_g = current through the galvanometer and G = resistance of the galvanometer.

When resistance R_1 is connected in series with the galvanometer, then

$$R_1 = \frac{V}{I_g} - G \quad \text{...(i)}$$

When resistance R_2 is connected in series with the galvanometer, then $R_2 = \frac{V}{2I_g} - G$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{V}{2I_g} = R_1 - R_2$$

and $G = R_1 - 2R_2$

The resistance R_3 required to convert the given galvanometer into voltmeter of range 0 to 2 V is given by

$$R_3 = \frac{2V}{I_g} - G$$

$$\Rightarrow \quad R_3 = 4(R_1 - R_2) - (R_1 - 2R_2) = 3R_1 - 2R_2$$

G in terms of R_1 and R_2 is given by $G = R_1 - 2R_2$

24. For equilibrium balance, net torque should also be equal to zero. When the field is off, $\Sigma\tau = 0$ considering the separation of each hung from mid-point be l.

$\because$ The magnetic force applied on CD by magnetic field must balance the weight.

$\therefore \quad Mgl = W_{coil}\, l$

$\Rightarrow \quad 500\, g\, l = W_{coil}\, l \Rightarrow W_{coil} = 500 \times 9.8\ N$

Taking moment of force about mid-point, we have the weight of coil. When the magnetic field is switched ON

$$Mgl + mgl = W_{coil}\, l + IBL \sin 90°\, l \Rightarrow mgl = BILl$$

$\therefore$ Additional mass,

$$m = \frac{BIL}{g} = \frac{0.2 \times 4.9 \times 1 \times 10^{-2}}{9.8}$$

$$= 10^{-3}\ \text{kg} = 1\ \text{g}$$

Thus, 1g of additional mass must be added to regain the balance.

25. For principle and working of galvanometer,

Refer to text on pages 194 and 195.

(i) **Cylindrical soft iron** core which not only makes the field radial but also increases the strength of the magnet.

(ii) **Radial magnetic field** is a field in which coil of the galvanometer always remains parallel to the field even on large deflection.

(iii) and (iv) refer to text on page 195.

Current sensitivity does not depend upon resistance (R), whereas voltage sensitivity does, as evident from their expression. Current sensitivity can be increased by increasing the number of turns of the coil. However, this increases the resistance of the coil. since voltage sensitivity decreases with increase in the resistance of the coil the effect of increase in number of turns is nullified in the case of voltage sensitivity

26. (i) Refer to text on pages 194 and 195.

(ii) (a) Refer to Sol. 22 (ii).

(b) Refer to Sol. 25 (iv).

27. (i) A galvanometer of range I_g and resistance G can be converted into

(a) a voltmeter of range V, by connecting a high resistance R in series with galvanometer whose value is given by

$$R = \frac{V}{I_g} - G$$

(b) an ammeter of range I, by connecting a very low resistance (shunt) in parallel with galvanometer whose value is given by

$$S = \frac{I_g G}{I - I_g}$$

(ii) Refer to text on page 192.

Thus, the nature of force is attractive.

When direction of flow of current is in opposite direction, the nature of force becomes repulsive.

[redacted]

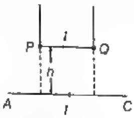
28. Here, $I = 8$ A, $\theta = 30°$, $B = 0.15$ T, $F = ?$, $l = 1$ m

We know that, $F = BIl \sin\theta$

$$\frac{F}{l} = BI\sin\theta = 0.15 \times 8 \times \sin 30°$$

$$= 0.15 \times 8 \times (1/2) = 0.6 \text{ N m}^{-1}$$

29. Mass of wire PQ, $m = 2.5 \text{ g} = 2.5 \times 10^{-3}$ kg



Length of wire PQ, $l = 1$ m

Current in wire PQ and AC, $I = 25$ A

Let the wire PQ rises upto a height h.

The magnetic field on wire PQ due to wire AC is B.

By using the formula of magnetic field due to an infinite length of wire,

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} = \frac{\mu_0}{4\pi} \times \frac{2 \times 25}{h}$$

$$= \frac{10^{-7} \times 50}{h} = \frac{50 \times 10^{-7}}{h} \quad \text{...(i)}$$

The direction of magnetic field B on wire PQ is perpendicularly inwards to the plane of paper (by using Maxwell's right hand rule).

Force on wire PQ, $\mathbf{F} = I(\mathbf{l} \times \mathbf{B})$

[$\because$ angle between $\mathbf{l}$ and $\mathbf{B}$ is 90°]

$$\Rightarrow F = IlB \sin 90° = 25 \times 1 \times \frac{50 \times 10^{-7}}{h} \times 1 \text{ [From Eq.(i)]}$$

$$\Rightarrow F = \frac{1250 \times 10^{-7}}{h} \quad \text{...(ii)}$$

The wire will lift, if the weight of the wire is balanced by force due to wire AC.

i.e. $F = mg$

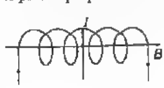
$$\Rightarrow \frac{1250 \times 10^{-7}}{h} = 2.5 \times 10^{-3} \times 9.8 \quad \text{[from Eq. (ii)]}$$

$$\therefore \quad h = \frac{1250 \times 10^{-7}}{2.5 \times 9.8 \times 10^{-3}} = 51.02 \times 10^{-4} \text{ m}$$

$$= 51.02 \times 10^{-2} \text{ cm} = 0.51 \text{ cm}$$

Thus, the wire PQ will rise upto a height of 0.51 cm.

30. Here, the angle between the magnetic field and the direction of flow of current is 90°. Because the magnetic field due to a solenoid is along the axis of the solenoid and the wire is placed perpendicular to the axis.



Given, $l = 3 \text{ cm} = 3 \times 10^{-2}$ m

$I = 10$ A, $B = 0.27$ T

The magnitude of magnetic force on the wire,

$$F = IlB\sin 90°$$

$$= 10 \times 3 \times 10^{-2} \times 0.27 \times \sin 90°$$

$$= 8.1 \times 10^{-2} \text{ N}$$

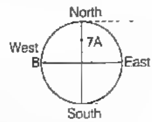
According to right hand palm rule, the direction of magnetic force is perpendicular to plane of paper inwards.

31. (i) Uniform magnetic field,

$B = 1.5$ T

Radius, $r = 10.0 \text{ cm} = 0.1$ m

Current in the wire, $I = 7.0$ A



The magnitude of force on the wire,

$$\mathbf{F} = I(\mathbf{l} \times \mathbf{B}) = IlB \sin 90°$$

[$\because$ angle between $\mathbf{l}$ and $\mathbf{B}$ is 90° and the length of wire is equal to the diameter of the cylindrical region]

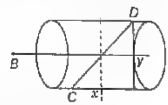
$\therefore$ Force on the wire,

$$F = I \times 2r \times B$$

$$= 7 \times 2 \times 0.1 \times 1.5 = 2.1 \text{ N}$$

According to Fleming's left hand rule, the direction of force is vertically inwards to the plane of paper.

(ii) Now, we take the component of length of wire. The horizontal component experiences no force as B is parallel to length.



The vertical component,

y = Diameter of the cylinder

So, force $F = IlB \sin 90°$

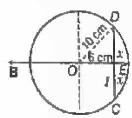
$$= 7 \times 0.1 \times 1.5 \times 2 \times 1$$

$$= 2.1 \text{ N}$$

According to the Fleming's left hand rule, the direction of force is perpendicularly inwards to the plane of paper.

(iii) Let the wire be shifted by 6 cm and the position of wire is CD.

$$OE = 6 \text{ cm}, OD = 10 \text{ cm}, DE = EC = x$$



In ΔODE, $OD^2 = OE^2 + DE^2$

$$\Rightarrow \quad 100 = 36 + DE^2$$

$$\Rightarrow \quad DE^2 = 64 \quad \text{or} \quad DE = 8 \text{ cm}$$

and $\quad l' = CD = 2DE = 16 \text{ cm} = 0.16$ m

[redacted]

Magnitude of force,

$\mathbf{F}' = I(\mathbf{l} \times \mathbf{B}) = 7\ (0.16 \times 1.5 \times \sin 90°) = 1.68$ N

According to Fleming's left hand rule, the direction of force is vertically downwards to the plane of the paper.

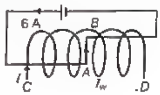
32. **For solenoid**

Given, length $l = 60$ cm,

Radius = 4 cm

Number of layers = 3

Number of turns in each layer = 300



For wire

Given, length, $l_w = 2$ cm

Mass, $m = 2.5$ g, current $I_w = 6$ A

Let I be the current passing through the solenoid, so the magnetic field due to the solenoid.

$$B = \mu_0 nI \qquad \left[\because n = \frac{\text{Number of turns}}{\text{length}} = \frac{300 \times 3}{0.6}\right]$$

$$= 4\pi \times 10^{-7} \times \frac{300 \times 3}{0.6} \times I \qquad \text{...(i)}$$

Force on the wire, $\mathbf{F} = I_w(\mathbf{l}_w \times \mathbf{B}) = I_w\ (l_w B \sin\theta)$

[$\because$ angle between l_w and B is 90°]

This force balances by the weight of wire = mg

$$\therefore \qquad I_w l_w B \sin 90° = mg$$

$$6 \times 0.02 \times \frac{4\pi \times 10^{-7} \times 300 \times 3}{0.6} I = 2.5 \times 10^{-3} \times 9.8$$

[from Eq. (i)]

$$\text{Current, } I = \frac{2.5 \times 10^{-3} \times 9.8 \times 0.6}{108 \times 4\pi \times 10^{-7}} = 108.36 \text{ A}$$

33. 8×10^{-5} N ; refer to Example 4 on page 193.

34. 2×10^{-5} N; refer to Example 3 on page 192.

35. Force per unit length between the current carrying wires is given as

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r}$$

where, I_1 = current in wire $AB = 12$ A

I_2 = current in wire $CD = 5$ A

and $\quad r$ = distance between wires = 1 mm = 1×10^{-3} m

$$\therefore \qquad \frac{\mu_0}{4\pi} \cdot \frac{2I_1 I_2}{r} = mg$$

where, m = mass per unit length.

$$\Rightarrow \quad 10^{-7} \times \frac{2 \times 12 \times 5}{1 \times 10^{-3}} = m \times 10$$

$$\Rightarrow \qquad m = 10^{-7} \times \frac{2 \times 12 \times 5}{1 \times 10^{-3}} \times \frac{1}{10}$$

$$m = 1.2 \times 10^{-3} \text{ kg/m}$$

Current in CD should be in opposite direction to that in AB.

36. Refer to Example 5 on page 194. **[Ans.** 3.927 N-m**]**

37. Given, $N = 20$, $I = 12$ A, $B = 0.80$ T,

$l = 10\text{cm} = 10 \times 10^{-2}$ m, $\theta = 30°$

$\because$ Area, $A = l^2 = (10 \times 10^{-2})^2 = 100 \times 10^{-4}$ m^2

As, $\qquad \tau = NBIA \sin\theta$

$\Rightarrow \qquad \tau = 20 \times 0.80 \times 12 \times 100 \times 10^{-4} \times \sin 30°$

$= 9600 \times 10^{-4} = 0.96$ N-m

38. Here, $N = 30$, $R = 8.0\text{cm} = 8 \times 10^{-2}$ m,

$I = 6.0$ A, $\theta = 60°$ and $B = 1.0$ T

(i) The magnitude of the counter torque

= magnitude of the deflecting torque

$= NAIB \sin\theta = N\ (\pi R^2)\ IB \sin\theta$

$= 30 \times 3.14 \times (8 \times 10^{-2})^2 \times 6.0 \times 1.0 \times \sin 60°$

$= 3.14$ N-m

(ii) The answer would not change as area enclosed by the coil as well as all other particulars remain unaltered and the formula, $\tau = NAIB \sin\theta$ is true for planar coil for any shape

39. Given, number of turns, $N = 20$

Radius of circular coil, $r = 10$ cm = 0.1 m

Magnitude of magnetic field, $B = 0.1$ T

The angle between the area vector and magnetic field is 0°.

$\Rightarrow \qquad \theta = 0°$

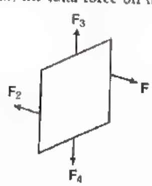
Current in the coil, $I = 5.0$ A

(i) Torque on the coil, $\tau = NIAB \sin\theta$

$= 20 \times 5 \times \pi\,(0.1)^2 \times 0.1 \times \sin 0° = 0$

[$\because \sin 0° = 0$]

(ii) The forces on the planar loop are in pairs, i.e. the forces on two opposite sides are equal and opposite to each other and on the other two opposite sides, they are same. Thus, the total force on the coil is zero



($\because \mathbf{F}_1 = -\mathbf{F}_2$ and $\mathbf{F}_3 = -\mathbf{F}_4$)

[redacted]

(iii) Number density of electrons, $N = 10^{29}/\text{m}^3$

Area of cross-section of copper wire, $A = 10^{-5}\,\text{m}^2$

The magnitude of magnetic force, $\mathbf{F} = e\,(\mathbf{v}_d \times \mathbf{B})$

$\because \quad I = neAv_d$

$\therefore \quad v_d = \dfrac{I}{neA} \Rightarrow F = e \cdot \dfrac{I}{NeA} \cdot B \sin 90^\circ$

$= \dfrac{0.1 \times 5}{10^{-5} \times 10^{29}}\,\text{N} \qquad [\because \sin 90^\circ = 1]$

$= 5 \times 10^{-25}$

40 Here, $B = 60\,\text{G}$, $A = 2\times10^{-4}\,\text{m}^2$, $N = 40$

$I = 4\,\text{mA} = 4\times10^{-3}\,\text{A}$, $\theta = 16^\circ$

$\because \quad I = \dfrac{k}{NBA}\theta \Rightarrow k = \dfrac{NBAI}{\theta}$

$= \dfrac{40\times60\times2\times10^{-4}\times4\times10^{-3}}{16}$

$= 1.2\times10^{-4}$ N-m per degree

41. Given, $R_1 = 10\,\Omega$, $N_1 = 30$, $A_1 = 3.6\times10^{-3}\,\text{m}^2$,

$B_1 = 0.25\,\text{T}$, $R_2 = 14\,\Omega$, $N_2 = 42$,

$A_2 = 1.8\times10^{-3}\,\text{m}^2$, $B_2 = 0.50\,\text{T}$

$k_1 = k_2$ (spring constants are same) ...(i)

(i) Using the formula of current sensitivity, $I = \dfrac{NAB}{k}$

$\therefore \quad \dfrac{I_{S_2}}{I_{S_1}} = \dfrac{N_2B_2A_2k_1}{N_1B_1A_1k_2} = \dfrac{42\times0.50\times1.8\times10^{-3}}{30\times0.25\times3.6\times10^{-3}}$

$= 1.4$ [From Eq. (i)]

(ii) Using the formula of voltage sensitivity,

$V = \dfrac{NAB}{kR}$

$\therefore \quad \dfrac{V_{S_2}}{V_{S_1}} = \dfrac{N_2B_2A_2k_1R_1}{k_2R_2N_1B_1A_1}$

$= \dfrac{42\times0.50\times1.8\times10^{-3}\times10}{14\times30\times0.25\times3.6\times10^{-3}}$

$= 1$ [from Eq. (i)]

42. Refer to Example 9 on page 195. [Ans. $0.01\,\Omega$]

43. Here, $n = 30$, $G = 100\,\Omega$, $E = 3\,\text{V}$, $R = 200\,\Omega$, $k = ?$

Total resistance $= G + R = 100 + 200 = 300\,\Omega$

$I_g = \dfrac{E}{G+R} = \dfrac{3}{300} = \dfrac{1}{100}\,\text{A}$

$k = \dfrac{I_g}{n} = \dfrac{1/100}{30} = \dfrac{1}{3}\times10^{-3}\,\text{A/div}$

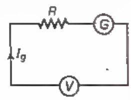
$k = \dfrac{1}{3}\times10^{-3}\times10^{6}\,\mu\text{A/div}$

Figure of merit (Restoring torque per unit twist)

$k = 333.3\,\mu\text{A/div}$

44. Given, resistance of galvanometer coil, $G = 12\,\Omega$

Current in galvanometer,



$I_g = 3\,\text{mA} = 3\times10^{-3}\,\text{A}$

and potential difference,

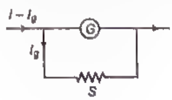
$V = 18\,\text{V}$

We can convert the galvanometer into voltmeter by using a large resistance R in series. The resistance can be calculated using the formula,

$R = \dfrac{V}{I_g} - G$

$R = \dfrac{18}{3\times10^{-3}} - 12 = 5988\,\Omega$

This resistance ($R = 5988\,\Omega$) is connected in series with the galvanometer. The resistance is connected in series because we have to increase the resistance of the galvanometer, so that almost no current flows through it and it gives an exact value of potential difference.



[redacted]

SUMMARY

- **Magnetic Field** The space in the surroundings of a magnet or a current carrying conductor in which its magnetic influence can be experienced is called magnetic field
- **Oersted's Experiment** HC Oersted by his experiment observed that a current carrying conductor deflects magnetic compass needle placed near it.
- **Ampere's Swimming Rule** If a man is swimming along the wire in the direction of current with his face always turned towards the needle so that the current enters through his feet and leaves at his head, then the *N*-pole of the magnetic needle will be deflected towards his left hand
- **Biot-Savart's Law** This law deals with the magnetic field induction at a point due to a small current element, i e
$$dB \propto \frac{Idl\sin\theta}{r^2}$$
- **Permittivity and Permeability** Electric permittivity (ε_0), the degree of interaction of electric field with medium. Magnetic permeability the ability of a substance to acquire magnetisation in a magnetic field
- **Right Hand Thumb Rule** When the thumb of right hand is placed along the direction of current the fingers curl around the conductor in the direction of magnetic field lines
- Magnetic field at any point along the **axis of circular current carrying conductor** is $B = \frac{\mu_0 I a^2}{2(r^2 + a^2)^{3/2}}$
- Magnetic field at the **centre of a circular current carrying conductor/ coil**
$$B = \frac{\mu_0 I}{2r}$$
- **Ampere's Circuital Law** According to this law, the line integral of the magnetic field **B** around any closed path in vacuum is equal to μ_0 times the net current enclosed by the curve,
i e $\oint \mathbf{B} \times d\mathbf{l} = \mu_0 I$
- **Magnitude of a Magnetic Field of a Straight Wire** It is given by $B = \frac{\mu_0 I}{2\pi r}$
- **Solenoid** It is an insulated long wire closely wound in the form of a helix
- **Magnetic Field due to Straight Solenoid** At any point inside the solenoid, $B = \mu_0 nI$
At points near the end of air closed solenoid, $B = (\mu_0 nI/2)$
- **Force on Moving Charge in a Uniform Magnetic Field** When a charged particle (q) moves with a velocity (v) inside a uniform magnetic field then force acting on it is given by $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$
- **Magnetic Force On a Charged Particle** It is given by
$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$
- When charged particle enters into a magnetic field perpendicularly, then
(i) $\frac{mv^2}{r} = qvB$ (ii) $r = \frac{mv}{qB}$ (iii) $T = \frac{2\pi m}{qB}$
(iv) $v = \frac{qB}{2\pi m}$ (v) $\text{KE} = \frac{q^2B^2r^2}{2m}$
- **Lorentz Force** The sum of the electric force and magnetic force that can be exerted on a charged particle due to its electric charge (q) is called Lorentz force
It is $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$
- **Force on a Current Carrying Conductor in a Uniform Magnetic Field** It is given by, $F = IlB\sin\theta$
- **Fleming's Left Hand Rule** If the forefinger, middle finger and the thumb of the left hand are stretched mutually at right angles to one another such that the forefinger points in the direction of magnetic field middle finger in the direction of current then thumb will point in the direction of force on the conductor
- **Force between Two Parallel Current Carrying Conductors**
It is given by $F = \left[\frac{\mu_0}{4\pi} \frac{2 I_1 I_2}{r}\right]$
- **Torque Experienced by a Current Loop in a Uniform Magnetic Field** It is given by $\tau = BINA\sin\theta$
- **Moving Coil Galvanometer** It is an instrument which is based on the fact that when a current carrying coil is placed in a magnetic field, then it experiences a torque
- **Circular Current Loop as Magnetic Dipole** The magnitude of the magnetic field on the axis at a distance x from the centre of a circular loop of radius R carrying a steady current I is
$$B = \frac{\mu_0 IR^2}{2(x^2 + R^2)^{3/2}}$$
- Torque on a bar magnet in a uniform magnetic field is
$$\tau = MB\sin\theta = \mathbf{M} \times \mathbf{B}$$

[redacted]

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. Vector form of Biot-Savart's law is

(a) $d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{\mathbf{I} \times d\mathbf{l}}{r^2}$

(b) $d\mathbf{B} = \frac{I\,d\mathbf{l} \times \mathbf{r}}{r^3}$

(c) $d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{l} \times \mathbf{r}}{r^3}$

(d) $d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{l} \times \mathbf{r}}{r^2}$

2. A polygon shaped wire is inscribed in a circle of radius R. The magnetic induction at the centre of polygon, when current flows through the wire is

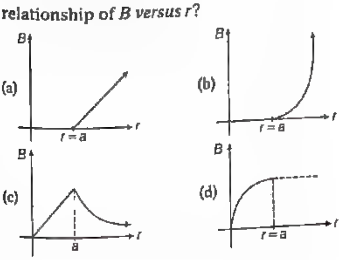
(a) $\frac{\mu_0 n I}{2\pi R}\tan\left(\frac{2\pi}{n}\right)$

(b) $\frac{\mu_0 n I}{2\pi R}\tan\left(\frac{4\pi}{n}\right)$

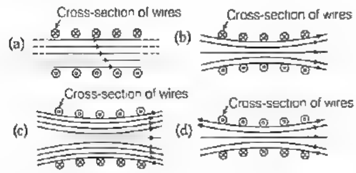
(c) $\frac{\mu_0 n I}{2\pi R}\tan\left(\frac{\pi}{n}\right)$

(d) $\frac{\mu_0 n I}{2\pi R}\tan\left(\frac{\pi}{n^2}\right)$

3. For a cylindrical conductor of radius a, which of the following graphs shows a correct relationship of B *versus* r?



4. Which of the following represent a correct figure to display of magnetic field lines due to a solenoid?



5. A long solenoid has 20 turns cm^{-1}. The current necessary to produce a magnetic field of 20 mT inside the solenoid is approximately

(a) 1A (b) 2 A (c) 4 A (d) 8 A

6. An electron is travelling horizontally towards East. A magnetic field in vertically downward direction exerts a force on the electron along

(a) East (b) West (c) North (d) South

7. Which of the following statements is correct? **CBSE 2021 (Term-I)**

(a) Magnetic field lines do not form closed loops.

(b) Magnetic field lines start from North pole and end at South pole of a magnet.

(c) The tangent at a point on a magnetic field line represents the direction of the magnetic field at that point

(d) Two magnetic field lines may intersect each other

8. The magnetic field at the centre of a current carrying circular loop of radius R is B_1. The magnetic field at a point on its axis at a distance R from the centre of the loop is B_2, then the ratio $\left(\frac{B_1}{B_2}\right)$ is **CBSE 2021 (Term-I)**

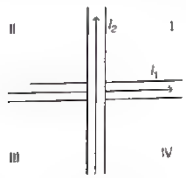
(a) $2\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) 2

[redacted]

9. A current carrying wire kept in a uniform magnetic field will experience a maximum force, when it is **CBSE 2021 (Term-I)**

(a) perpendicular to the magnetic field
(b) parallel to the magnetic field
(c) at an angle of 45° to the magnetic field
(d) at an angle of 60° to the magnetic field

10. Two wires carrying currents I_1 and I_2 lie, one slightly above the other, in a horizontal plane as shown in figure. The region of vertically upward strongest magnetic field is **CBSE 2021 (Term-I)**



(a) I (b) II
(c) III (d) IV

11. Two parallel conductors carrying current of 4.0 A and 10.0 A are placed 2.5 cm apart in vacuum. The force per unit length between them is

(a) 6.4×10^{-5} N/m (b) 6.4×10^{-2} N/m
(c) 4.6×10^{-4} N/m (d) 3.2×10^{-4} N/m

12. A straight conducting rod of length l and mass m is suspended in a horizontal plane by a pair of flexible strings in a magnetic field of magnitude B. To remove the tension in the supporting strings, the magnitude of the current in the wire is **CBSE 2021 (Term-I)**

(a) $\frac{mgB}{l}$ (b) $\frac{mgl}{B}$
(c) $\frac{mg}{lB}$ (d) $\frac{lB}{mg}$

13. A proton and an alpha particle move in circular orbits in a uniform magnetic field. Their speeds are in the ratio of 9 : 4. The ratio of radii of their circular orbits $\left(\frac{r_p}{r_{alpha}}\right)$ is **CBSE 2021 (Term-I)**

(a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{8}{9}$ (d) $\frac{9}{8}$

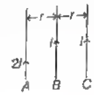
14. The SI unit of magnetic field intensity is **CBSE SQP (Term-I)**

(a) A-mN^{-1} (b) $NA^{-1}\,m^{-1}$
(c) $NA^{-2}\,m^{-2}$ (d) $NA^{-1}\,m^{-2}$

15. The coil of a moving coil galvanometer is wound over a metal frame in order to **CBSE SQP (Term-I)**

(a) reduce hysteresis
(b) increase sensitivity
(c) increase moment of inertia
(d) provide electromagnetic damping

16. Three infinitely long parallel straight current carrying wires A, B and C are kept at equal distance from each other as shown in the figure . The wire C experiences net force F .The net force on wire C, when the current in wire A is reversed will be **CBSE SQP (Term-I)**



(a) zero (b) $F/2$
(c) F (d) $2F$

17. In a hydrogen atom, the electron moves in an orbit of radius 0.5 Å making 10 rps, the magnetic moment associated with the orbital motion of the electron will be **CBSE SQP (Term-I)**

(a) 2512×10^{-38} A-m^2 (b) 1.256×10^{38} A-m^2
(c) 0.628×10^{-38} A-m^2 (d) zero

18. The current sensitivity of a galvanometer increases by 20%. If its resistance also increases by 25%, then the voltage sensitivity will **CBSE SQP (Term-I)**

(a) decreased by 1% (b) increased by 5%
(c) increased by 10% (d) decreased by 4%

19. Two wires of the same length are shaped into a square of side a and a circle with radius r. If they carry same current, the ratio of their magnetic moment is **CBSE SQP (Term-I)**

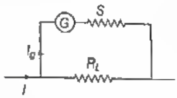
(a) $2:\pi$ (b) $\pi:2$
(c) $\pi:4$ (d) $4:\pi$

20. The wire which connects the battery of a car to its starter motor carries current of 300 A during starting. Force per unit length between wires (wires are 0.7 m long and 0.015 m distant apart) is

[redacted]

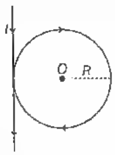
(a) $1.2\ Nm^{-1}$ repulsive
(b) $1.2\ Nm^{-1}$ attractive
(c) $2.4\ Nm^{-1}$ repulsive
(d) $2.4\ Nm^{-1}$ attractive

21. For the voltmeter circuit given,



(a) $\frac{I_g}{I} = \frac{G}{S}$
(b) $\frac{I}{I_g} = \frac{R_L + G}{S}$
(c) $(I - I_g)R_L = I_g(G+S)$
(d) $IR_L = I_g G$

22. A current I flows through a long straight conductor which is bent into a circular loop of radius R in the middle as shown in the figure.



The magnitude of the net magnetic field at point O will be **CBSE 2020 (Term-I)**

(a) zero
(b) $\frac{\mu_0 I}{2R}(1+\pi)$
(c) $\frac{\mu_0 I}{4\pi R}$
(d) $\frac{\mu_0 I}{2R}\left(1-\frac{1}{\pi}\right)$

23. A current of 10A is flowing from east to west in a long straight wire kept on a horizontal table. The magnetic field developed at a distance of 10 cm due north on the table is **[CBSE 2020]**

(a) 2×10^{-5} T, acting downwards
(b) 2×10^{-5} T, acting upwards
(c) 4×10^{-5} T, acting downwards
(d) 4×10^{-5} T, acting upwards

24. An electron and a proton are moving along the same direction with the same kinetic energy. They enter a uniform magnetic field acting perpendicular to their velocities. The dependence of radius of their paths on their masses is **CBSE 2020**

(a) $r \propto m$
(b) $r \propto \sqrt{m}$
(c) $r \propto \frac{1}{m}$
(d) $r \propto \frac{1}{\sqrt{m}}$

25. A current of 5 A is flowing from east to west in a long straight wire kept on a horizontal table. The magnetic field developed at a distance of 10 cm due south on the table is **CBSE 2020**

(a) 1×10^{-5} T, acting downwards
(b) 1×10^{-5} T, acting upwards
(c) 2×10^{-5} T, acting downwards
(d) 2×10^{-5} T, acting upwards

26. There are uniform electric and magnetic fields in a region pointing along X-axis. An α-particle is projected along Y-axis with a velocity v. The shape of the trajectory will be **CBSE 2020**

(a) circular in XZ-plane
(b) circular in YZ-plane
(c) helical with its axis parallel to X-axis
(d) helical with its axis parallel to Y-axis

27. The coil of a galvanometer consists of 100 turns and effective area of $1\ cm^2$. The restoring couple is $10^{-8}\ Nm\ rad^{-1}$. The magnetic field between poles is of 5 T. Current sensitivity of this galvanometer is

(a) 5×10^4 rad/μ amp
(b) 5×10^6 per amp
(c) 2×10^{-7} per amp
(d) 5 rad/μ amp

ASSERTION AND REASON

Directions (Q. Nos. 28-33) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

[redacted]

28. **Assertion** If a proton and an α-particle enter a uniform magnetic field perpendicularly with the same speed, the time period of revolution of α-particle is double than that of proton.
Reason In a magnetic field, the period of revolution of a charged particle is directly proportional to the mass of the particle and inversely proportional to the charge of particle.

29. **Assertion** The magnetic field produced by a current-carrying solenoid is independent of its length and cross-sectional area.
Reason The magnetic field inside the solenoid is uniform.

30. **Assertion** An electron and a proton enter a magnetic field with equal velocities, then the force experienced by the proton will be more than electron.
Reason The mass of proton is 1837 times more than that of electron.

31. **Assertion** A proton and an electron, with same momenta, enter in a magnetic field in a direction at right angles to the lines of the force. The radius of the paths followed by them will be same.
Reason Electron has less mass than the proton. CBSE SQP (Term1)

32. **Assertion** On increasing the current sensitivity of a galvanometer by increasing the number of turns, may not necessarily increase its voltage sensitivity.
Reason The resistance of the coil of the galvanometer increases on increasing the number of turns. CBSE SQP (Term1)

33. **Assertion** When radius of a current carrying loop is doubled, its magnetic moment becomes four times.
Reason The magnetic moment of a current carrying loop is directly proportional to the area of the loop. CBSE 2021 (Term1)

CASE BASED QUESTIONS

Directions (Q.Nos. 34-35) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

34. **Electron Moving in Magnetic Field**

An electron with a speed $v_0 << c$ moves in a circle of radius r_0 in a uniform magnetic field. This electron is able to traverse a circular path as magnetic field is perpendicular to the velocity of the electron. The time required for one revolution of the electron is T_0. The speed of the electron is now doubled to $2v_0$.

(i) The radius of the circle will change to
(a) $4r_0$ (b) $2r_0$
(c) r_0 (d) $r_0/2$

(ii) The time required for one revolution of the electron will change to
(a) $4T_0$ (b) $2T_0$
(c) T_0 (d) $T_0/2$

(iii) A charged particle is projected in a magnetic field $\mathbf{B} = (2\hat{i} + 4\hat{j}) \times 10^2$ T. The acceleration of the particle is found to be $\mathbf{a} = (x\hat{i} + 2\hat{j})\,\text{ms}^{-2}$. Find the value of x.
(a) $4\ \text{ms}^{-2}$ (b) $-4\ \text{ms}^{-2}$
(c) $-2\ \text{ms}^{-2}$ (d) $2\ \text{ms}^{-2}$

(iv) If the given electron has a velocity not perpendicular to **B**, then the trajectory of the electron is
(a) straight line (b) circular
(c) helical (d) zig-zag

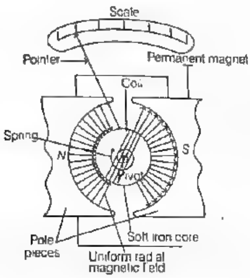
(v) If this electron of charge (e) is moving parallel to uniform magnetic field with constant velocity v. The force acting on the electron is
(a) Bev (b) $\frac{Be}{v}$
(c) $\frac{B}{ev}$ (d) zero

35. **Moving Coil Galvanometer**

Moving coil galvanometer operates on Permanent Magnet Moving Coil (PMMC) mechanism and was designed by the scientist D'arsonval.
Moving coil galvanometers are of two types
(i) Suspended coil
(ii) Pivoted coil type or tangent galvanometer.
Its working is based on the fact that when a current carrying coil is placed in a magnetic field, it experiences a torque.

[redacted]

This torque tends to rotate the coil about its axis of suspension in such a way that the magnetic flux passing through the coil is maximum



(i) A moving coil galvanometer is an instrument which
(a) is used to measure emf
(b) is used to measure potential difference
(c) is used to measure resistance
(d) is a deflection instrument which gives a deflection when a current flows through its coil

(ii) To make the field radial in a moving coil galvanometer,
(a) number of turns of coil is kept small
(b) magnet is taken in the form of horse-shoe
(c) poles are of very strong magnets
(d) poles are cylindrically cut

(iii) The deflection in a moving coil galvanometer is
(a) directly proportional to torsional constant of spring
(b) directly proportional to the number of turns in the coil
(c) inversely proportional to the area of the coil
(d) inversely proportional to the current in the coil

(iv) In a moving coil galvanometer, having a coil of N-turns of area A and carrying current I is placed in a radial field of strength B.
The torque acting on the coil is
(a) NA^2B^2I (b) $NABI^2$
(c) N^2ABI (d) $NABI$

(v) To increase the current sensitivity of a moving coil galvanometer, we should decrease
(a) strength of magnet
(b) torsional constant of spring
(c) number of turns in coil
(d) area of coil

VERY SHORT ANSWER Type Questions

36. State the rule that is used to find the direction of magnetic field at a point near a current carrying straight conductor.

37. What will be the magnetic field at the centre of a circular coil carrying current, when the current through the coil is doubled and the radius of the coil is halved?

38. What is the force on a charge moving along the direction of the magnetic field?

39. Name the force which is experienced by a moving charged particle in electric and magnetic field.

40. Under what condition does an electron moving through a magnetic field experience maximum force?

41. A charged particle moves through a magnetic field. Is the momentum of the particle affected?

42. An electron beam projected along $+X$-axis experiences a force due to a magnetic field along the $+Y$-axis. What is the direction of the magnetic field?

43. In a certain arrangement, a proton does not get deflected while passing through a magnetic field region. Under what condition is it possible?

44. Write the expression for the force between parallel current carrying conductors.

45. An electron with charge $-e$ and mass m travels at a speed v in a plane perpendicular to a magnetic field of magnitude B. The electron follows a circular path of radius R. In a time t, the electron travels half-way around the circle. What is the amount of work done by the magnetic field?

[redacted]

46. A solenoid with n loops of wire tightly wrapped around an iron core is carrying an electric current I. If the current through this solenoid is reduced to half, then what change would you expect in inductance L of the solenoid.?

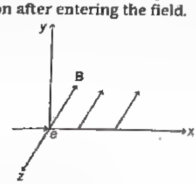
47. A proton is accelerated through a potential difference V, subjected to a uniform magnetic field acting normal to the velocity of the proton. If the potential difference is doubled, how will the radius of the circular path described by the proton in the magnetic field change? CBSE 2019

48. Write the relation for the force acting on a charged particle q moving with velocity **v** in the presence of a magnetic field **B**. CBSE 2019

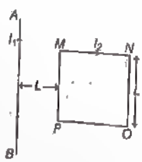
49. When a charge q is moving in the presence of electric (E) and magnetic (B) fields which are perpendicular to each other and also perpendicular to the velocity v of the particle, write the relation expressing v in terms of E and B. CBSE 2019

50. Define the term current sensitivity of a moving coil galvanometer. CBSE 2020

51. An electron moves along $+x$ direction. It enters into a region of uniform magnetic field **B** directed along z direction as shown in figure. Draw the shape of trajectory followed by the electron after entering the field. CBSE 2020



52. A square shaped current carrying loop $MNOP$ is placed near a straight long current carrying wire AB as shown in the figure. The wire and the loop lie in the same plane. If the loop experiences a net force F towards the wire, find the magnitude of the force on the side NO of the loop. CBSE 2020



SHORT ANSWER Type Questions

53. A deuteron and an alpha particle having same momentum are in turn allowed to pass through a magnetic field **B**, acting normal to the direction of motion of the particles. Calculate the ratio of the radii of the circular paths described by them. CBSE 2019

54. A charged particle q is moving in the presence of a magnetic field B which is inclined to an angle 30° with the direction of the motion of the particle. Draw the trajectory followed by the particle in the presence of the field and explain how the particle describes this path. CBSE 2019

55. An α-particle and a proton of the same kinetic energy are in turn allowed to pass through a magnetic field **B**, acting normal to the direction of motion of the particles. Calculate the ratio of radii of the circular paths described by them. CBSE 2019

56. Two similar coils are placed mutually perpendicular such that their centres coincide. At centre, what will be the ratio of the magnitudes of magnetic fields due to one coil and the resultant magnetic field?

57. In what way, current carrying solenoid behaves like a bar magnet. Find the magnetic field induction at the axis of solenoid due to current flowing through it.

58. What is Lorentz force? Give some important characteristics of this force.

59. Equal currents are flowing through two infinitely long parallel wires in the same direction. What will be the magnetic field at a point mid-way between the two wires?

60. Deduce an expression for the torque on a current carrying loop suspended in a uniform magnetic field.

61. In a moving coil galvanometer having a coil of N turns of area A and carrying current I and is placed in a radial field of strength B. What will be the torque acting on the coil?

62. Is it possible to decrease or increase the range of given voltmeter? Explain.

[redacted]

LONG ANSWER Type I Questions

63. Using Ampere's circuital law, find an expression for the magnetic field at a point situated at a normal distance R from an infinitely long current carrying straight wire.

64. What is the force that a conductor of length dl carrying a current I experiences, when placed in a magnetic field B? What is the direction of this force?

65. An electron being accelerated through a potential difference of V enters a uniform magnetic field of B perpendicular to the direction of motion. Find the radius of path described by the electron.

66. (a) Derive the expression for the torque acting on a current carrying loop placed in a magnetic field.
(b) Explain the significance of a radial magnetic field when a currrent carrying coil is kept in it. **CBSE 2019**

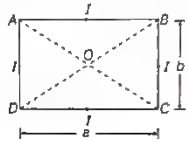
67. (a) State the underlying principle of a moving coil galvanometer.
(b) Give two reasons to explain why a galvanometer cannot as such be used to measure the value of the current in a given circuit.
(c) Define the terms (i) voltage sensitivity and (ii) current sensitivity of a galvanometer. **CBSE 2019**

68. Two infinitely long straight wires A_1 and A_2 carrying currents I and $2I$ flowing in the same directions are kept d distance apart. Where should a third straight wire A_3 carrying current 1.5 I be placed between A_1 and A_2, so that it experiences no net force due to A_1 and A_2? Does the net force acting on A_3 depend on the current flowing through it?

LONG ANSWER Type II Questions

69. Three wires of equal lengths are bent into the form of three loops. One of the loops is square-shaped, second loop is triangular-shaped and third loop is circular. These are suspended in a uniform magnetic field and the same current is passed through them. Which loop will experience greater torque? Give reasons.

70. A rectangular current carrying loop of length a and breadth b is shown in the figure. Find the magnetic field at the centre of the loop.



71. Two straight infinitely long wires are fixed in space, so that the current in the left wire is 2 A and directed out of the plane of the page and the current in the right wire is 3 A and directed into the plane of the page. In which region(s) is/are there a point on the X-axis, at which the magnetic field is equal to zero due to these currents carrying wires? **CBSE SQP (Term-I)**
Justify your answer.

72. (a) Show that a current carrying solenoid behaves like a small bar magnet. Obtain the expression for the magnetic field at an external point lying on its axis.
(b) A steady current of 2A flows through a circular coil having 5 turns of radius 7 cm. The coil lies in xy-plane with its centre at the origin. Find the magnitude and direction of the magnetic dipole moment of the coil. **CBSE 2020**

Or

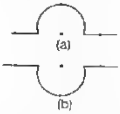
(a) Derive the expression for the force acting between two long parallel current carrying conductors. Hence, define 1A current.
(b) A bar magnet of dipole moment 3 Am^2 rests with its centre on a frictionless pivot. A force F is applied at right angles to the axis of the magnet, 10 cm from the pivot. It is observed that an external magnetic field of 0.25 T is required to hold the magnet in equilibrium at an angle of 30° with the field. Calculated the value of F. How will the equilibrium be effected if F is withdrawn? **CBSE 2020**

[redacted]

NUMERICAL PROBLEMS

73. A copper coil of 100 turns, radius 8×10^{-2} m carries a current of 0.40 A. What will be the magnitude of magnetic field at the centre of coil ?

74. A straight wire carrying a current of 12 A is bent into a semi-circular arc of radius 2.0 cm as shown in Fig. (a). Consider the magnetic field B at the centre of the arc.



(i) What is the magnetic field due to the straight segments?

(ii) In what way the contribution to B from the semi-circle differs from that of a circular loop and in what way does it resemble?

(iii) Would your answer be different, if the wire was bent into a semi-circular arc of the same radius but in the opposite way as shown in Fig. (b)?

75. A closely wound solenoid 0.80 m long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8×10^{-2}m. If the current carried is 0.8A, what will be the magnitude of field near the centre?

76. A beam of protons passes undeflected with a horizontal velocity v, through a region of electric and magnetic fields, mutually perpendicular to each other and normal to the direction of beam. If the magnitudes of electric and magnetic fields are 100 kV/m and 50 mT respectively, calculate the

(i) velocity of the beam and

(ii) force with which it strikes the target on a screen, if the proton beam current is equal to 0.80 mA.

77. Two concentric circular wire loops of radii 20 cm and 30 cm are located in an XY-plane, each carries a clockwise current of 7 A.

(i) Find the magnitude of the net magnetic dipole moment of the system.

(ii) Repeat for reversed current in the inner loop.

78. The coil of galvanometer consists of 100 turns and effective area of 1 cm^2. The restoring couple is 10^{-8} N-m/rad The magnetic field between poles is of 5 T. What will be the current sensitivity of galvanometer?

79. The current sensitivity of a MCG increases by 20% when its resistance is increased by a factor of 2. Calculate by what factor the voltage sensitivity changes?

80. A galvanometer with a coil of resistance 12.0 Ω shows full scale deflection for a current of 2.5 mA. How will you convert this meter into

(i) an ammeter of range 0 to 7.5 A?

(ii) a voltmeter of range 0 to 10 V? Determine the net resistance of the meter in each case. When an ammeter is put in a circuit, does it read less or more than the actual current in the original circuit? When a voltmeter is put across a part of the circuit, does it read less or more than the required voltage drop? Explain.

81. A galvanometer having 30 divisions has a current sensitivity of 20 μA/ div. It has a resistance of 25 Ω.

(i) How will you convert it into an ammeter of range 0-1 A?

(ii) How will you convert this ammeter into a voltmeter of range 0-1 V?

ANSWERS

1. (c)	2. (c)	3. (c)	4. (c)	5. (d)
6. (d)	7. (c)	8. (a)	9. (a)	10. (b)
11. (d)	12. (c)	13. (d)	14. (b)	15. (d)
16. (a)	17. (b)	18. (d)	19. (c)	20. (a)
21. (c)				

22. (d) The magnetic field due to the long straight conductor at O is given by

$$B_1 = \frac{\mu_0 I}{2\pi R} \otimes$$

and that due to circular loop of radius R is,

$$B_2 = \frac{\mu_0 I}{2R} \odot$$

As,

$$B_2 > B_1$$

[redacted]

$\therefore$ The magnitude of net magnetic field at point O is

$$B_{net} = B_2 - B_1 = \frac{\mu_0 I}{2R} - \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{2R}\left(1 - \frac{1}{\pi}\right)$$

23. (a) Refer to example-1 given on Page-179

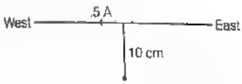
24. (i) Since, the angle between their velocities and uniform magnetic field is 90° So, the path followed by them will be circular in nature.

The radius of circular path followed by charged particle in a uniform magnetic field,

$$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$$

As kinetic energy K is same, so $r \propto \sqrt{m}$.

25. (b) The situation is as shown,



Magnetic field due to long straight wire is

$$B = \frac{\mu_0 I}{2\pi r}$$

Here, $I = 5$A and $r = 10$ cm $= 0.1$ m

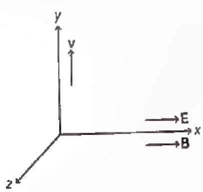
$$\therefore \quad B = \frac{4\pi \times 10^{-7} \times 5}{2\pi \times 0.1} = 1 \times 10^{-5} \text{ T}$$

The direction of magnetic field is given by right hand thumb rule. So, the direction of magnetic field of point 10 cm is upward due south on the table.

26. (a) The net force acting on α-particle is given by

$$F = q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$$

The direction of E, v and B are shown as,



$$\therefore \quad \mathbf{F} = q[E\hat{\mathbf{i}} + (v\hat{\mathbf{j}} \times B\hat{\mathbf{i}})]$$
$$= q\,[E\hat{\mathbf{i}} + vB(-\hat{\mathbf{k}})]$$

So, the shape of trajectory will be circular in xz- plane.

27. (b) 28. (a) 29. (b) 30. (d)

31. (b) 32. (a) 33. (a)

34. (i) (b) As, $r_0 = \frac{mv}{qB} \Rightarrow r' = \frac{m(2v_0)}{qB} = 2r_0$

(ii) (c) As, $T = \frac{2\pi m}{qB}$

Thus, it remains same as it is independent of velocity.

(iii) (b) As, $F \perp B$

Hence, $a \perp B$

$$\therefore \quad \mathbf{a} \cdot \mathbf{B} = 0$$
$$\Rightarrow (x\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) \cdot (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) = 0$$
$$2x + 8 = 0$$
$$\Rightarrow \quad x = -4 \text{ ms}^{-2}$$

(iv) (c) If the charged particle has a velocity not perpendicular to B, then component of velocity along B remains unchanged as the motion along the B will not be affected by B.

Then, the motion of the particle in a plane perpendicular to B is as before circular one. Thereby, producing helical motion.

(v) (d) The force on electron, $F = qvB \sin\theta$

The electron is moving parallel to the B, so $\theta = 0°$.

$$\Rightarrow \quad F = qvB\sin 0° = 0$$

35. (i) (d) A moving coil galvanometer is a sensitive instrument which is used to measure a deflection when a current flows through its coil.

(ii) (d) Uniform field is made radial by cutting pole pieces cylindrically.

(iii) (b) The deflection in a moving coil galvanometer, $\phi = \frac{NAB}{k} \cdot I$ or $\phi \propto N$, where N is number of turns in a coil, B is magnetic field and A is area of cross-section.

(iv) (d) The deflecting torque acting on the coil,

$$\tau_{\text{deflection}} = NIAB$$

(v) (b) Current sensitivity of galvanometer

$$\frac{\phi}{I} = S_i = \frac{NBA}{k}$$

Hence, to increase (current sensitivity) S_i, (torsional constant of spring) k must be decreased.

36. Right hand thumb rule states that, if we imagine a linear wire conductor to be need in the grip of the right hand such that the thumb points in the direction of current, then the curvature of the fingers around the conductor will give the direction of magnetic field lines.

37. Magnetic field at the centre of the coil is given by

$$B = \frac{\mu_0 I}{2R}$$
$$B' = \frac{\mu_0 (2I)}{2(R/2)} = 4B$$

38. Force on a moving charge in magnetic field is given as,

$$F = qvB\sin\theta$$

Here, $\theta = 0° \Rightarrow F = 0$

39. Lorentz force

40. Magnetic force, $\mathbf{F} = q(\mathbf{v} \times \mathbf{B}) = qvB\sin\theta$

Maximum force, $F_{max} = qvB$

When, $\sin\theta = 1$ or $\theta = 90°$

41. No, its momentum does not get affected.

[redacted]

42. Direction of magnetic field is in Z-axis direction.

43. When it is along the magnetic field.

44. Force between the parallel current carrying conductors is $F = \frac{\mu_0}{4\pi}\frac{2I_1I_2}{r}$

45. When a charged particle move in a plane, perpendicular to the direction of magnetic field, then a magnetic force acts on it. The direction of this force is perpendicular to the velocity of the charged particle.

So, no work is done, i.e. work done is zero.

46. The inductance (L) of a solenoid does not depend on the current flowing through it, but depends on the magnetic field and permeability of material of iron core. So, on reducing the current to half, the L remains same

47. The kinetic energy of proton due to potential V is given by

$$K = eV$$

where, e = charge on proton

The radius of circular path of proton in a magnetic field is

$$r = \frac{\sqrt{2mK}}{qB} = \frac{\sqrt{2meV}}{qB}$$

If potential is doubled, i.e. $V' = 2V$, then

$$r' = \frac{\sqrt{2me \times 2V}}{qB} = \sqrt{2}r$$

Thus, radius becomes $\sqrt{2}$ times of previous value.

48. When a charged particle q moves with velocity $\mathbf{v}$ in a uniform magnetic field $\mathbf{B}$, then the force acting on it is given by

$$\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$$

49. $F_{lorentz} = F_{electric} + F_{magnetic}$

$$= q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) = q[\mathbf{E} + (\mathbf{v} \times \mathbf{B})]$$

50. Current sensitivity of the galvanometer is the deflection per unit current flowing through it.

It is given as, $I_C = \frac{\theta}{I} = \frac{NAB}{R}$

where, R is the restoring torque per unit twist of phosphor bronze strip.

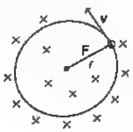
51. The magnetic force acting on the electron is given as,

$$\mathbf{F} = e(\mathbf{v} \times \mathbf{B}) = evB\sin\theta$$

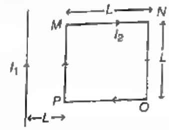
If the electron moves along $+x$ direction and $\mathbf{B}$ is directed along $-z$ direction, then $\theta = 90°$.

$$\Rightarrow \quad F = evB$$

So, the trajectory followed by the electron after entering the field will be circular as shown below



52. The given loop can be shown below as



The force acting on the arms MN and PO of the given loop are equal, mutually opposite and collinear. Hence, they balance each other.

Force on arm PM,

$$F_1 = \frac{\mu_0 I_1 I_2 L}{2\pi L}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi}, \text{ attractive in nature} \quad \text{...(i)}$$

Force on arm NO,

$$F_2 = \frac{\mu_0 I_1 I_2 L}{2\pi(2L)}$$

$$= \frac{\mu_0 I_1 I_2}{4\pi}, \text{ repulsive in nature} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$F_{net} = F = F_1 - F_2 = \frac{\mu_0 I_1 I_2}{2\pi} - \frac{\mu_0 I_1 I_2}{4\pi}$$

$$= \frac{\mu_0 I_1 I_2}{4\pi}, \text{ attractive in nature} \quad \text{...(iii)}$$

So, from Eqs. (ii) and (iii), we can conclude that the magnitude of the force on side NO of the loop is $F = \left(\frac{\mu_0 I_1 I_2}{4\pi}\right)$ when the net force F is towards the wire.

53. Mass on deuteron, $m_d = 2m$

Charge on deuteron, $q_d = e$

Mass on α-particle, $m_\alpha = 4m$

Charge on α-particle, $q_\alpha = 2e$

The radius of circular path is given by

$$r = \frac{mv}{qB}$$

Momentum of particle, $p = mv$

$$\therefore \quad r \propto \frac{1}{qB} \quad [\because \text{ momentum is same}]$$

So, $\frac{r_d}{r_\alpha} = \frac{q_\alpha}{q_d} = \frac{2e}{e} = \frac{2}{1}$

or $r_d : r_\alpha = 2:1$

54. When an charged particle q enters a uniform magnetic field at an angle of 30°, then its path becomes helix of radius

$$r = \frac{mv\sin 30°}{eB} = \frac{mv}{2eB}$$

[redacted]

For diagram and discription Refer to text on pages 181 and 182 (Force on a Moving Charge in a Uniform Magnetic Field)

55. Refer to sol. of Example 4 on page 182.

56. Ratio $= \dfrac{B_1}{\sqrt{B_1^2 + B_2^2}} = \dfrac{1}{\sqrt{2}}$

57. Refer to text on pages 180 and 181.

58. Refer to text on page 183.

59. Zero

60. Refer to text on pages 193 and 194.

61. Refer to text on pages 194 and 195

62. Refer to text on page 196.

63. Refer to text on pages 179 and 180.

64. Refer to text on page 191.

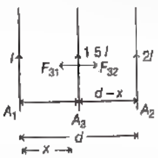
65. $K = \dfrac{1}{2}mv^2 = eV$

$$r = \frac{mv}{qB} = \frac{mv}{eB} = \sqrt{\frac{2mV}{eB^2}}$$

66. (a) Refer to text on pages 193 and 194 (Torque Experienced by a Current Loop in Uniform Magnetic Field).
(b) In a radial magnetic field, the magnetic torque remains maximum for all positions of the coils.

67. (a) Refer to text on page 194 [Moving Coil Galvanometer (Principle)].
(b) Refer to page 195.
(c) Refer to text on page 195 (Working of Moving Coil Galvanometer)

68. Let the current in the third wire A_3 be in same direction as that of A_1 and A_2, so it will experience attractive force due to both.



The force on A_3 due to A_1 is $F_{31} = \dfrac{\mu_0}{2\pi} \cdot \dfrac{I \times 1.5\, I \times l}{x}$

where, l = unit length of conductor wire A_2
and x = distance between A_1 and A_3.
Similarly, force on A_3 due to A_2 is

$$F_{32} = \frac{\mu_0}{2\pi} \cdot \frac{1.5 I \times 2I \times l}{(d - x)}$$

According to question, $F_{31} = F_{32}$

$$\Rightarrow \quad \frac{\mu_0}{2\pi} \cdot \frac{1.5 I^2 l}{x} = \frac{\mu_0}{2\pi} \frac{3I^2}{(d - x)} l \Rightarrow \frac{1.5}{x} = \frac{3}{d - x}$$

$$\Rightarrow \quad d - x = 2x \text{ or } x = \frac{d}{3}$$

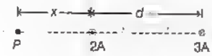
Yes, the net force acting on A_3 depends on the current flowing through it.

69. For magnetic moment refer Q. 17 on page 198.
Now, apply the formula, $\tau = MB$
Square will experience maximum torque.

70. Refer to Example 2 on page 170.

$$\left[\text{Ans. } B = \frac{2\mu_0 I \sqrt{a^2 + b^2}}{\pi ab}\right]$$

71. Let d be the distance between two current carrying wires, then the magnetic field in the region I at a point P at a distance x can be calculated using figure given below.

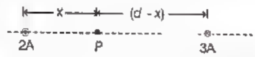


Due to 2A, $B_1 = \dfrac{\mu_0 \times I_1}{2\pi x} = \dfrac{\mu_0 \times 2}{2\pi x}$, downward

Due to 3A, $B_2 = \dfrac{\mu_0 \times I_2}{2\pi (x + d)} = \dfrac{\mu_0 \times 3}{2\pi (x + d)}$, upward

$\therefore$ Net magnetic field, $B_P = \dfrac{\mu_0}{2\pi}\left(\dfrac{2}{x} - \dfrac{3}{x + d}\right)$, downward ...(i)

The magnetic field in region II is



Due to 2A, $B_1 = \dfrac{\mu_0 \times 2}{2\pi x}$, upward

Due to 3A, $B_2 = \dfrac{\mu_0 \times 3}{2\pi (d - x)}$, upward

$\therefore$ Net magnetic field, $B_P = \dfrac{\mu_0}{2\pi}\left(\dfrac{2}{x} + \dfrac{3}{d - x}\right)$, upward

The magnetic field in region III is

Due to 2A, $B_1 = \dfrac{\mu_0 \times 2}{2\pi (x + d)}$, upward

Due to 3A, $B_2 = \dfrac{\mu_0 \times 3}{2\pi d}$, downward

$\therefore$ Net magnetic field, $B_P = \dfrac{\mu_0}{2\pi}\left(\dfrac{3}{d} - \dfrac{2}{x + d}\right)$, downward

As, the current and hence the magnetic field, due to 2A is less than that due to 3A.

[redacted]

So, for zero magnetic field, $\frac{\mu_0}{2\pi}\left(\frac{2}{x}-\frac{3}{x+d}\right)=0$

$\Rightarrow \quad 2x+2d=3x$ or $x=2d$

So, the point lies in region I.

72. (a) Refer to text pages 180 and 181.

(b) Given, $I=2$A and $N=5$

and $r=7$ cm $=0.07$ m

Magnetic dipole moment of a coil,

$M=NIA=5\times 2\times\pi(0.07)^2=0.154$ Am^2

The direction of magnetic dipole moment is perpendicular to the plane of coil, i.e. it is along Z-axis.

Or

(a) Refer to text on page 192.

(b) Given, $M=3$Am^2, $d=10$ cm $=0.1$ m

$B=0.25$ T and $\theta=30^\circ$

The torque acting on the bar magnet,

$\tau=MB\sin\theta\hat{n}=\mathbf{M}\times\mathbf{B}$

Also, $\tau=F\cdot d\Rightarrow F\cdot d=MB\sin\theta$

$\Rightarrow \quad F=\frac{MB\sin\theta}{d}=\frac{3\times 0.25\times\sin 30^\circ}{0.1}$

$=\frac{0.75}{2\times 0.1}=3.75$ N

If F is withdrawn from the bar magnet, then it will rotate, due to the torque (τ) and align itself along the field direction.

73. Refer to Example 3 on page 171.

[Ans. $B=\frac{\mu_0 I}{2r}=3.1\times10^{-4}$ T]

74. (i) Zero, magnetic field due to a semi-circular wire at its centre is half of magnetic field due to a circular loop.

(ii) Now, refer to text on page 170.

$B_{\text{semi-circle}}=\frac{\mu_0 I}{4r}=37.68\times10^{-5}$ T

(iii) The magnitude of the magnetic field remains same but the direction will be opposite.

75. Refer to Example 3 on page 181.

[Ans. $B=\mu_0 nI=2.5\times10^{-2}$ T]

76. For undeflected beam, $v=\frac{E}{B}$

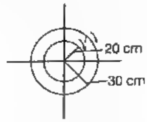
(i) 2×10^6 m/s

(ii) $\mathbf{F}=q(\mathbf{E}+\mathbf{v}\times\mathbf{B})=1.675\times10^{-5}$ N

77. (i) $M_1=N_1I_1A_1 \otimes$

$M_2=N_2I_2A_2 \otimes$

$\therefore M=M_1+M_2=2.86$ A-m^2



(ii) $M=|M_1-M_2|=1.10$ A-m^2

78. Current sensitivity, $I_s=\frac{NAB}{K}=\frac{100\times1\times10^{-4}\times5}{10^{-8}}$

$=5\times10^6$ A^{-1}

79. Refer to Example 8 on page 195.

[Ans. Decreased by a factor 0.4]

80. Refer to Example 9 and 10 on pages 195 and 196.

[Ans. (i) Resistance of ammeter $4\times10^{-3}\,\Omega$

(ii) Resistance of voltmeter $=4000\,\Omega$

81. Refer to Example 9 on pages 195 and 196.

[Ans. (i) Shunt $=0.815\Omega$

(ii) Resistance in series 0.985Ω

[redacted]

A naturally occurring ore of iron, magnetite attracts small pieces of iron towards it. The phenomenon of attraction of small bits of iron, steel, cobalt, nickel, etc., towards the ore, is called magnetism.

MAGNETISM AND MATTER

|TOPIC 1|

Bar Magnet and Magnetic Dipole

A **magnet** is a material or an object that produces a magnetic field. The magnetic field is invisible but is responsible for most notable property of a magnet.

Magnets are of two types

(i) Natural magnets (ii) Artificial magnets

Natural magnets are generally irregular in shape and weak in strength. On the other hand, artificial magnets have desired shape and desired strength. A bar magnet, a horse shoe magnet, compass needle, etc., all are the examples of artificial magnets.

Some commonly known ideas about magnetism are given below

(i) The earth behaves as a magnet with the magnetic field pointing approximately from the geographic South to North.

(ii) When a bar magnet is suspended freely, it points in the North-South direction. The tip of the magnet which points to the geographic North is called the **North pole** and the tip which points to the geographic South is called the **South pole** of the magnet.

(iii) There is a **repulsive force**, when North poles (or South poles) of two magnets are brought close together. Conversely, there is an attractive force between the North pole of one magnet and the South pole of the other.

CHAPTER CHECKLIST

- Bar Magnet and Magnetic Dipole
- Magnetic Properties of Materials

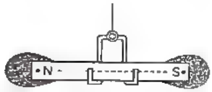
[redacted]

(iv) It is possible to make magnets out of iron and its alloys. When a piece of a substance, such as soft iron, steel, cobalt, nickel, etc., is placed near a magnet, it acquires magnetism.

THE BAR MAGNET

The bar magnet has two poles similar to the positive and negative charges of an electric dipole. One pole is designated as North pole (N) and the other as South pole (S). When a bar magnet is suspended freely, these poles point approximately towards the geographic North and South poles, respectively.

Like magnetic poles repel each other and **unlike magnetic poles attract** each other. If a bar magnet is dropped into a pile of iron-filings, then the maximum amount of filings get **deposited near** the ends of the magnet and almost nil in the middle.



The pattern suggests that attraction is maximum at the two ends of the bar magnet. These ends are called **poles** of the magnet. The poles of a magnet can never be separated.

Magnetic Length of a Bar Magnet

The distance between two poles of a bar magnet is known as **magnetic length** of a magnet. Its direction is from S-pole of the magnet to N-pole and is represented by $2l$. It is sometimes also known as effective length (L_e) of the magnet and is less than its geometric length (L_g).

For a bar magnet, $L_e = \left(\frac{5}{6}\right) L_g$.

Bar magnet

EXAMPLE |1| **Consider a short magnetic dipole of magnetic length 20 cm. Find its geometric length.**

Sol. Geometric length of a magnet is $\frac{6}{5}$ times its magnetic length.

$\therefore$ Geometric length $= \frac{6}{5} \times 20$

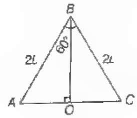
$= 24$ cm

EXAMPLE |2| **A thin bar magnet of length $4L$ is bent at the mid-point, so that the angle between them is $60°$. Find the new length of the bar magnet.**

Sol. On bending the bar magnet, the length of the bar magnet,

$$AC = AO + OC = 2L\sin\left(\frac{60°}{2}\right) + 2L\sin\left(\frac{60°}{2}\right)$$

$$= 4L\sin 30° = 4L \times \frac{1}{2} = 2L$$



The Non-existance of Magnetic Monopole

We cannot isolate the North or South pole of a magnet, i.e. magnetic poles exist in pairs. If a bar magnet is broken into two halves, we get two similar bar magnets with somewhat weaker properties. Unlike electric charges, isolated magnetic North and South poles are known as magnetic monopoles which do not exist.

If a bar magnet is broken, each piece behaves as a small magnet

Pole Strength

It is defined as the strength of a magnetic pole to attract magnetic materials towards itself. It is a scalar quantity and its SI unit is ampere-metre (A-m). The strength of N and S-pole of a magnet is conventionally represented by $+m$ and $-m$, respectively. It depends on the nature of material and area of cross-section of the magnet.

Strength of N and S-pole of a magnet is always equal and opposite ($+m$ and $-m$).

Force between Two Magnetic Poles

The force of attraction or repulsion F between two magnetic poles of strengths m_1 and m_2 separated by a distance r is directly proportional to the product of pole strengths and inversely proportional to the square of the distance between their centres, i.e.

$$F \propto \frac{m_1 m_2}{r^2} \quad \text{or} \quad F = K\frac{m_1 m_2}{r^2}$$

where, K is magnetic force constant.

[redacted]

In SI unit,

$$K = \frac{\mu_0}{4\pi} = 10^{-7}\ \text{Wb A}^{-1}\ \text{m}^{-1}$$

where, μ_0 is absolute magnetic permeability of free space (air/vacuum).

$$\boxed{F = \frac{\mu_0}{4\pi} \cdot \frac{m_1 m_2}{r^2}} \quad \dots \text{(i)}$$

This is called **Coulomb's law** of magnetic force. SI unit of magnetic pole strength is ampere-metre.

Suppose $\quad m_1 = m_2 = m$ (say), $r = 1$ m

and $\quad F = 10^{-7}$ N

From Eq. (i), we have

$$10^{-7} = 10^{-7} \times \frac{(m)(m)}{1^2}$$

or $\quad m^2 = 1$ or $m = \pm 1$ A-m

Therefore, strength of a magnetic pole is said to be **one ampere-metre,** if it repels an equal and similar pole, when placed in vacuum (or air) at a distance of one metre from it, with a force of 10^{-7} N.

EXAMPLE |3| **Two poles one of which is 5 times as strong as the other, exert on each other a force equal to 0.8×10^{-3} kg-wt, when placed 10 cm apart in air. Find the strength of each pole.**

Sol. Let m and $5m$ be the pole strength of the two poles.

Here, $\quad F = 0.8 \times 10^{-3}\text{kg-wt} = 0.8 \times 10^{-3} \times 9.8\ \text{N},$

$$r = 10\ \text{cm} = 0.1\ \text{m}$$

$$\therefore \quad F = \frac{\mu_0}{4\pi} \cdot \frac{m_1 m_2}{r^2}$$

$$\Rightarrow 0.8 \times 10^{-3} \times 9.8 = \frac{10^{-7} \times m \times 5m}{(0.1)^2}$$

$$\Rightarrow \quad m = 12.52\ \text{A-m}$$

and $\quad 5m = 5 \times 12.52$ A-m $= 62.6$ A-m

EXAMPLE |4| **Two identical magnets with a length 100 cm are arranged freely with their like poles facing in a vertical glass tube. The upper magnet hangs in air above the lower one so that the distance between the nearest poles of the magnet is 3 mm. If the pole strength of the pole of these magnets is 6.64 A-m, then determine the force between the two magnets.**

Sol. Given, pole strength, $m = 6.64$ A-m

$$r = 3\text{mm} = 3 \times 10^{-3}\ \text{m}$$

Since, force, $F = \frac{\mu_0}{4\pi} \times \frac{m_1 m_2}{r^2}$

$$\therefore \quad F = \frac{\mu_0}{4\pi} \times \frac{m^2}{r^2} \qquad [\because m_1 = m_2 = m]$$

$$= \frac{\mu_0}{4\pi} \times \frac{(6.64)^2}{(3 \times 10^{-3})^2}$$

$$= \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{44.0896}{9 \times 10^{-6}} = 10^{-1} \times 4.8988$$

$$= 0.49\ \text{N}$$

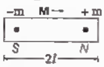
MAGNETIC DIPOLE

A **magnetic dipole** is an arrangement of two magnetic poles of equal and opposite strengths $(-m, +m)$ separated by a small distance. e.g. A bar magnet, a compass needle, etc., are magnetic dipoles.

The two poles of a magnetic dipole (or a magnet), called North pole and South pole, are always of equal strength and of opposite nature. Further, such two magnetic poles always exist in pair.

Magnetic Dipole Moment

It is the product of the strength of either pole and the magnetic length of the magnet. It is represented by M.



The direction of magnetic dipole moment is same as that of 2l. Therefore,

$$\boxed{\mathbf{M} = m(2\mathbf{l})}$$

SI unit of magnetic dipole moment is ampere-metre^2 (A-m^2)

EXAMPLE |5| **A magnetic wire of dipole moment 4π A-m^2 is bent in the form of semicircle. Find the new magnetic moment.**

Sol. If length of wire is $2l$, then magnetic moment

$$M = m \times 2l = 4\pi\ \text{A-m}^2 \qquad \text{[given]}$$

As wire is bent in the form of semicircle, effective distance between the ends is $2r$.

So, new dipole moment

$$M' = m \times 2r = m \times 2 \times \frac{2l}{\pi} = \frac{2}{\pi}(m \times 2l) \qquad [\because \pi r = 2l]$$

$$= \frac{2}{\pi} M = \frac{2}{\pi} 4\pi$$

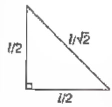
$$= 8\ \text{A-m}^2$$

[redacted]

EXAMPLE |6| The length of a magnetised steel wire is l and its magnetic moment is M. It is bent into the shape of L with two sides equal. What will be the new magnetic moment?

Sol. If m is strength of each pole, then magnetic moment $M = m \times l$

When the wire is bent into L shape, effective distance between the poles



$$= \sqrt{\left(\frac{l}{2}\right)^2 + \left(\frac{l}{2}\right)^2} = \frac{l}{\sqrt{2}}$$

∴ New magnetic moment,

$$M' = m \times \frac{l}{\sqrt{2}} = \frac{M}{\sqrt{2}} \quad [m \text{ will remain unchanged}]$$

Magnetic Field Lines

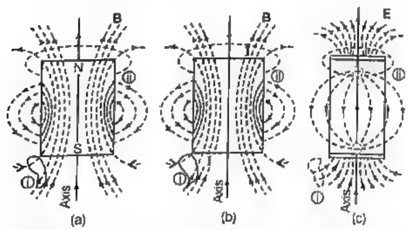
The magnetic field lines are a visual and intuitive realisation of the magnetic field. The magnetic field lines in a magnetic field are those imaginary lines which continuously represent the direction of the magnetic field. The tangent drawn at any point on magnetic field line shows the direction of magnetic field at that point.

Properties of Magnetic Field Lines

Important properties of magnetic field lines are given below

(i) The magnetic field lines of a magnet (or a solenoid) form continuous closed loops. This is unlike the electric dipole, where these field lines begin from a positive charge and end on the negative charge or escape to infinity.

(ii) The tangent to the field line at a given point represents the direction of the net magnetic field **B** at that point.



The field lines of (a) a bar magnet, (b) a current carrying finite solenoid and (c) an electric dipole. At large distances, their field lines are very similar. The curves labelled (i) and (ii) are closed to Gaussian surfaces

(iii) The larger the number of field lines crossing per unit area, the stronger is the magnitude of the magnetic field **B**.

(iv) The magnetic field lines do not intersect, for if they did, the direction of the magnetic field would not be unique at the point of intersection.

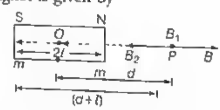
MAGNETIC FIELD STRENGTH AT A POINT DUE TO A BAR MAGNET

The strength of magnetic field at any point is defined as, the force experienced by a hypothetical unit North pole placed at that point. It is a vector quantity. The direction of magnetic field **B** is the direction along which hypothetical unit North pole would tend to move, if free to do so.

We have used the word hypothetical unit North pole in the above discussion because an isolated magnetic pole does not exist.

When Point Lies on Axial Line of a Bar Magnet

Let $2l$ be the magnetic length of a bar magnet with centre O. The magnetic dipole moment of the magnet is M, where $M = m \times 2l$, $OP = d$, is the distance of the point P on the axial line from the centre of the magnet. If m is the strength of each pole, then magnetic field strength at P due to N-pole of magnet is given by



$$B_1 = \frac{\mu_0}{4\pi} \times \frac{m \times 1}{(NP)^2} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(d-l)^2}, \text{ along } NP \text{ produced.}$$

Magnetic field strength at P due to S-pole of magnet is given by

$$B_2 = \frac{\mu_0}{4\pi} \times \frac{m \times 1}{(SP)^2} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(d+l)^2}, \text{ along } PS \text{ produced.}$$

∴ Magnetic field strength at P due to the bar magnet

$$B = B_1 - B_2 = \frac{\mu_0}{4\pi} \cdot \frac{m}{(d-l)^2} - \frac{\mu_0}{4\pi} \cdot \frac{m}{(d+l)^2}$$

$$= \frac{\mu_0 m}{4\pi}\left[\frac{1}{(d-l)^2} - \frac{1}{(d+l)^2}\right]$$

$$= \frac{\mu_0 m}{4\pi}\left[\frac{(d+l)^2 - (d-l)^2}{(d^2 - l^2)^2}\right] \quad \left[\because (a-b)(a+b) = a^2 - b^2\right]$$

[redacted]

$$= \frac{\mu_0 m.4ld}{4\pi(d^2 - l^2)^2} = \frac{\mu_0 (m \times 2l) 2d}{4\pi(d^2 - l^2)^2}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2Md}{(d^2 - l^2)^2} \qquad [\because M = m \times 2l]$$

When the magnet is short, $l^2 << d^2$, such that l^2 is neglected.

$$\therefore \qquad B = \frac{\mu_0}{4\pi} \cdot \frac{2Md}{d^4}$$

$$\Rightarrow \qquad \boxed{B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3}}$$

The direction of **B** is along NP produced.

EXAMPLE |7| What is the magnitude of the axial fields due to a bar magnet of length 5 cm at a distance of 50 cm from its mid-point? The magnetic moment of the bar magnet is 0.40 A -m^2.

Sol. Given, magnetic length of bar magnet, $2l = 5$ cm

$$\Rightarrow \qquad l = 2.5 \text{ cm} = 2.5 \times 10^{-2} \text{ m}$$

Distance, $d = 50 \text{cm} = 0.5 \text{m}$

Magnetic moment, $M = 0.40 \text{A} \cdot \text{m}^2$

$$\therefore \qquad B = \frac{\mu_0}{4\pi} \cdot \frac{2Md}{(d^2 - l^2)^2}$$

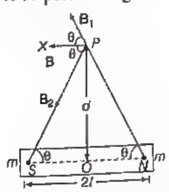
$$= \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3} \qquad [\because l << d]$$

$$= \frac{10^{-7} \times 2 \times 0.40}{(0.5)^3}$$

$$= 6.4 \times 10^{-7} \text{T}$$

When Point Lies on Equatorial Line of a Bar Magnet

In the given figure, the point P is shown on equatorial line of the same bar magnet, where $OP = d$. Magnetic field strength at P due to N-pole of magnet is



$$B_1 = \frac{\mu_0}{4\pi} \cdot \frac{m \times 1}{(NP)^2} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(d^2 + l^2)}, \text{ along } NP \text{ produced.}$$

Magnetic field strength at P due to S-pole of magnet is

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{m \times 1}{(SP)^2} = \frac{\mu_0}{4\pi} \cdot \frac{m}{(d^2 + l^2)}, \text{ along } PS \text{ produced.}$$

As $B_1 = B_2$ in magnitude, their components $B_1 \sin\theta$ along OP produced and $B_2 \sin\theta$ along PO will cancel out. However, components along PX parallel to NS will add. Therefore, magnetic field strength at P due to the bar magnet, $B = B_1 \cos\theta + B_2 \cos\theta = 2 B_1 \cos\theta$, along PX.

$$B = 2\frac{\mu_0}{4\pi} \cdot \frac{m}{(d^2 + l^2)} \times \frac{l}{\sqrt{d^2 + l^2}} \qquad \left[\because \cos\theta = \frac{l}{\sqrt{d^2 + l^2}}\right]$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{m \times 2l}{(d^2 + l^2)^{3/2}}$$

$$\Rightarrow \qquad \boxed{B = \frac{\mu_0}{4\pi} \cdot \frac{M}{(d^2 + l^2)^{3/2}}}$$

If the magnet is short, $l^2 << d^2$, such that l^2 is neglected.

$$\therefore \qquad B = \frac{\mu_0}{4\pi} \cdot \frac{M}{(d^2)^{3/2}}$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{M}{d^3}$$

$$\Rightarrow \qquad \boxed{B = \frac{\mu_0}{4\pi} \cdot \frac{M}{d^3}}$$

The direction of B is along PX, a line parallel to NS.

Note From the formulae of magnetic field due to a bar magnet at a point in axial position and at a point in equatorial position, it is clear that magnetic field due to a short bar magnet at any point on the axial line of magnet is twice the magnetic field at a point at the same distance on the equatorial line of the magnet.

EXAMPLE |8| Determine the magnitude of the equatorial fields due to a bar magnet of length 6 cm at a distance of 60 cm from its mid-point. The magnetic moment of the bar magnet is 0.60 A - m^2.

Sol. Given, magnetic length of bar magnet, $2l = 6$ cm

$$\Rightarrow \qquad l = 3 \text{ cm} = 3 \times 10^{-2} \text{ m}$$

Distance, $d = 60 \text{cm} = 0.6 \text{m}$

Magnetic moment, $M = 0.60 \text{ A-m}^2$

$$\therefore \text{ Magnetic field, } B = \frac{\mu_0}{4\pi} \times \frac{M}{(d^2 + l^2)^{3/2}}$$

$$= \frac{\mu_0 M}{4\pi d^3} \qquad [\because l << d]$$

$$= \frac{4\pi \times 10^{-7} \times 0.60}{4\pi \times (0.6)^3} = 27 \times 10^{-7} \text{T}$$

[redacted]

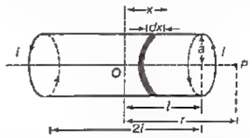
Bar Magnet as an Equivalent Solenoid

The magnetic field lines for a bar magnet and a current carrying solenoid resemble very closely. Therefore, a bar magnet can be thought as a large number of circulating currents in analogy with a solenoid.

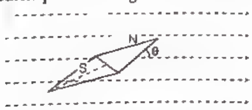
Cutting a bar magnet in half is like cutting a solenoid. We get two smaller solenoids with weaker magnetic properties. The field lines remain continuous, emerging from one face of the solenoid and entering into other face.

One can test this analogy by moving a small compass needle in the neighbourhood of a bar magnet and a current carrying finite solenoid and noting that the deflections of the needle are similar in both the cases.

To prove mathematically that magnetic field produced by a solenoid on any point on the axial line is same as that of a bar magnet. This analogy between bar magnet and solenoid can be shown by calculating the magnetic field at an axial point of solenoid which resembles to that of a bar magnet.



Let i be the current passing through a solenoid, a be the radius of solenoid, $2l$ be the length of solenoid and n be the number of turns per unit length of solenoid.



Let P be the point at distance r from centre at which magnetic field is to be calculated. Consider a small element of thickness dx of the solenoid at a distance (x) from the centre O.

Number of turns in the element $= n\,dx$

The magnitude of the field at point P due to the circular element is given by

$$dB = \frac{\mu_0 i a^2 (n\,dx)}{2[(r-x)^2 + a^2]^{3/2}} \quad \text{...(i)}$$

If P lies at a very large distance from O, i.e. $r >>> a$ and $r >>> x$, then $[(r-x)^2 + a^2]^{3/2} \approx r^3$

$$dB = \frac{\mu_0 i a^2 n\,dx}{2r^3} \quad \text{...(ii)}$$

Total magnetic field at point P due to current carrying solenoid.

$$B = \frac{\mu_0 n i a^2}{2r^3} \int_{-l}^{+l} dx$$

[$\because$ range of variation of x is from $-l$ to $+l$]

$$= \frac{\mu_0 n i a^2}{2r^3} [x]_{-l}^{+l} = \frac{\mu_0 n i a^2}{2r^3} (2l)$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{2n(2l)\, i\pi a^2}{r^3} \quad \text{...(iii)}$$

If M is the magnetic moment of the solenoid, then

M = Total number of turns × Current × Area of cross-section

$M = n(2l) \times i \times \pi a^2$

$$\Rightarrow \quad \boxed{B = \frac{\mu_0}{4\pi} \cdot \frac{2M}{r^3}}$$

This is the expression for magnetic field on the axial line of a short bar magnet.

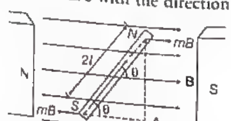
The magnetic moment of bar magnet is thus equal to the magnetic moment of an equivalent solenoid that produces the same magnetic field.

TORQUE ON A MAGNETIC DIPOLE IN A UNIFORM MAGNETIC FIELD

When a bar magnet is placed in a uniform magnetic field, torque acts on the magnet. Also, magnetic potential energy is associated with the magnet due to its orientation as discussed below.

In the figure below, a uniform magnetic field **B** is represented by equidistant parallel lines. NS is a bar magnet of length $2l$ and strength of each pole is m.

The magnet is held at $\angle\theta$ with the direction of **B**.



Bar magnet in a uniform magnetic field

Force on N-pole $= mB$, along **B**

Force on S-pole $= mB$, opposite to **B**

[redacted]

where, m = strength of each pole and

and B = strength of magnetic field.

These equal and unlike forces form a couple which tend to rotate the magnet clockwise, so as to align it along B.

Torque acting on the bar magnet,

τ = Force × Perpendicular distance

$\Rightarrow \quad \tau = mB \times NA$

In ΔNAS, $\sin\theta = \frac{NA}{NS} = \frac{NA}{2l}$

$\therefore \quad NA = 2l \sin\theta$

Now, $\quad \tau = mB \times 2l \sin\theta = B \times (m2l)\sin\theta$

$\Rightarrow \quad \boxed{\tau = MB\sin\theta}$

In vector form, $\boxed{\tau = \mathbf{M} \times \mathbf{B}}$

The direction of τ is perpendicular to the plane containing M and B and is given by right handed screw rule.

When $\quad B = 1$ and $\theta = 90°$,

then $\quad \tau = M \times 1 \sin 90° = M$

Hence, we may define **magnetic dipole moment** as the torque acting on a dipole held perpendicular to a uniform magnetic field of unit strength.

Unit of M is unit of τ divided by unit of B. Therefore, SI unit of M is joule per tesla ($J\,T^{-1}$).

EXAMPLE |9| A straight solenoid of length 50 cm has 1000 turns per metre and a mean cross-sectional area of 2×10^{-4} m^2. It is placed with its axis at 30°, with a uniform magnetic field of 0.32 T. Find the torque acting on the solenoid when a current of 2 A is passed through it.

Sol. Given, l = 50 cm

Number of turns per metre = 1000

$\therefore$ Total number of turns $(N) = 1000 \times \frac{1}{2} = 500$

Area, $A = 2\times10^{-4}\,m^2$

Current, I = 2 A

Magnetic field, B = 0.32T

$\therefore$ Torque, $\tau = MB\sin\theta = (NIA)B\sin\theta$

$= 500 \times 2 \times (2\times10^{-4}) \times 0.32 \times \frac{1}{2}$

$= 0.032$ N-m

EXAMPLE |10| A bar magnet when suspended horizontally and perpendicular to the earth's magnetic field experiences a torque of 3×10^{-4} N-m. What is the magnetic moment of the magnet? Horizontal component of earth's magnetic field at that place is 0.4×10^{-4} T.

Sol. Given, $\theta = 90°$, $\tau = 3\times10^{-4}$ N-m

and $\quad B = 0.4\times10^{-4}$ T

Since, torque $\tau = MB\sin\theta$

$$\therefore \quad M = \frac{\tau}{B\sin\theta} = \frac{3\times10^{-4}}{0.4\times10^{-4}\sin 90°}$$

$= 7.5\ JT^{-1}$

Oscillations of a Freely Suspended Magnet

The torque acting on the bar magnet makes it oscillate in the uniform magnetic field.

Since, in equilibrium, $\tau = MB\sin\theta$

Torque acting on a body, $\tau = I\alpha$

where, I = moment of inertia

and $\quad \alpha$ = angular acceleration.

Now, $\quad I\alpha = -MB\sin\theta \qquad [\because \tau = I\alpha]$

$$\Rightarrow \quad I\frac{d^2\theta}{dt^2} = -MB\sin\theta \qquad \left[\because \alpha = \frac{d^2\theta}{dt^2}\right]$$

In the above equation, negative sign with $MB\sin\theta$ indicates that restoring torque is in the opposite direction to the deflecting torque. For small values of θ in radians, $\sin\theta \approx \theta$.

$$\Rightarrow \quad I\frac{d^2\theta}{dt^2} = -MB\theta$$

$$\Rightarrow \quad \frac{d^2\theta}{dt^2} = \frac{-MB\theta}{I} \qquad \text{...(i)}$$

On comparing the Eq.(i) with equation of SHM, i.e. $\frac{d^2x}{dt^2} = -\omega^2 x$ or in angular terms, $\frac{d^2\theta}{dt^2} = -\omega^2\theta$

We can say that, the oscillations of a freely suspended magnet (magnetic dipole) in a uniform magnetic field are simple harmonic.

$$\therefore \quad \omega^2 = \frac{MB}{I} \text{ or } \omega = \sqrt{\frac{MB}{I}}$$

So, the time period of oscillations, $T = \frac{2\pi}{\omega}$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{I}{MB}}$$

or $$\boxed{B = \frac{4\pi^2 I}{MT^2}}$$

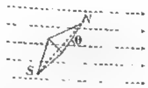
where, I = moment of inertia about the axis of the magnet

M = magnetic moment

and $\quad B$ = magnetic field intensity.

[redacted]

EXAMPLE |11| A magnetic needle is free to oscillate in a uniform magnetic field as shown in figure. The magnetic moment of magnetic needle is 7.2 A-m^2 and moment of inertia $I = 6.5 \times 10^{-6}$ kg-m^2. The number of oscillations performed in 5s is 10. Calculate the magnitude of magnetic field.



Sol. Here, $T = \dfrac{\text{Number of revolutions}}{\text{Time taken}} = \dfrac{5}{10} = 0.5$ s

$M = 7.2$ A-m^2, $I = 6.5 \times 10^{-6}$ kg-m^2

As, $T = 2\pi\sqrt{\dfrac{I}{MB}}$ or $T^2 = 4\pi^2 \dfrac{I}{MB}$

The magnitude of the magnetic field is

$$B = \frac{4\pi^2 I}{MT^2} = \frac{4 \times (3.14)^2 \times 6.5 \times 10^{-6}}{7.2 \times (0.5)^2}$$

$$= 1.42 \times 10^{-4} \text{ T}$$

POTENTIAL ENERGY OF A MAGNETIC DIPOLE IN A UNIFORM MAGNETIC FIELD

When a magnetic dipole of moment M is held at an angle θ with the direction of a uniform magnetic field B, the magnitude of the torque acting on the dipole is given by

$$\tau = MB\sin\theta$$

This torque tends to align the dipole in the direction of the field. Work has to be done in rotating the dipole against the action of the magnetic torque. This work done is stored as potential energy of the dipole.

Now, a small amount of work done in rotating the dipole through a small angle $d\theta$,

$$dW = \tau\, d\theta = MB\sin\theta \cdot d\theta$$

Total work done in rotating the dipole from $\theta = \theta_0$ to $\theta = \theta$,

$$W = \int_{\theta_0}^{\theta} dW = \int_{\theta_0}^{\theta} MB\sin\theta d\theta = MB[-\cos\theta]_{\theta_0}^{\theta}$$

$$\Rightarrow \quad W = -MB[\cos\theta - \cos\theta_0]$$

∴ Potential energy of the dipole,

$$U = W = -MB(\cos\theta - \cos\theta_0)$$

Let us assume that, $\theta_0 = 90°$

$$U = W = -MB(\cos\theta - \cos 90°)$$

Therefore, $\boxed{U = -MB\cos\theta}$

In vector notation, we may rewrite this equation as

$$\boxed{U = -\mathbf{M} \cdot \mathbf{B}}$$

Particular Cases

(i) When $\theta = 90°$,

$U = -MB\cos\theta$

$= -MB\cos 90° = 0$

i.e. when the dipole is perpendicular to magnetic field, its potential energy is zero.

(ii) When $\theta = 0°$, $U = -MB\cos\theta$

$= -MB\cos 0° = -MB$

i.e. when the magnetic dipole is aligned along the magnetic field, it is in **stable equilibrium** having minimum potential energy.

(iii) When $\theta = 180°$, $U = -MB\cos\theta$

$= -MB\cos 180° = MB$

which is maximum. This is the position of **unstable equilibrium.**

EXAMPLE |12| A circular coil of 100 turns and have an effective radius of 5 cm carries a current of 0.1 A. How much work is required to turn it in an external magnetic field of 1.5 Wb/m^2 through 180° about an axis perpendicular to the magnetic field? The plane of the coil is initially perpendicular to the magnetic field.

Sol. Given, number of turns, $N = 100$

Radius, $r = 5$ cm $= 0.05$ m

Current, $I = 0.1$ A

Magnetic field, $B = 1.5$ Wb/m^2

$\theta_1 = 0°, \theta_2 = 180°$

Area, $A = \pi r^2 = 3.14(0.05)^2$ m^2

∴ Required work done, $W = -MB(\cos\theta_2 - \cos\theta_1)$

$= -(NIA)B(\cos\theta_2 - \cos\theta_1)$

$= -100 \times 0.1 \times 3.14\,(0.05)^2 \times 1.5 \times (\cos 180° - \cos 0°)$

$= -10 \times 3.14(0.05)^2 \times 1.5(-1-1) = 0.24$ J

THE ELECTROSTATIC ANALOG

The magnetic dipole moment of a bar magnet is given by

$$M = m\,(2l)$$

where, m = strength of each pole

and $2l$ = length of the dipole.

The magnetic dipole is analogous to an electric dipole consisting of two equal charges of opposite signs ($\pm q$) separated by a certain distance (2a). It has an electric dipole moment, i.e. $\mathbf{p} = q\,(2\mathbf{a})$

The equations for magnetic field **B** due to a magnetic dipole can be obtained from the equations of electric field **E** due to an electric dipole by making the following changes,

$\mathbf{E} \to \mathbf{B}$, $\mathbf{p} \to \mathbf{M}$

[redacted]

$$\frac{1}{4\pi\varepsilon_0} \to \frac{\mu_0}{4\pi} \Rightarrow \frac{1}{\varepsilon_0} \to \mu_0$$

Thus, for any point on axial line of a bar magnet at a distance $d(d >>> l)$ from the centre of magnet.

$$B_A = \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3}$$

Similarly, for any point on equatorial line of a bar magnet at a distance, for $d >>> l$

$$B_E = \frac{\mu_0 M}{4\pi d^3}$$

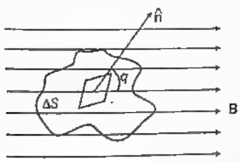
The table below shows the analogy between electric and magnetic dipoles.

The Dipole Analogy

	Electrostatics	Magnetism
Dipole moment	p	M
Equatorial field for a short dipole	$-p/4\pi\varepsilon_0 r^3$	$-\mu_0 M/4\pi r^3$
Axial field for a short dipole	$2p/4\pi\varepsilon_0 r^3$	$\mu_0 2M/4\pi r^3$
External field . Torque	$p \times E$	$M \times B$
External field : Energy	$-p \cdot E$	$-M \cdot B$

MAGNETISM AND GAUSS' LAW

The net magnetic flux (ϕ_B) through any closed surface is always zero.



This law suggests that the number of magnetic field lines leaving any closed surface is always equal to the number of magnetic field lines entering it. Suppose a closed surface S is held in a uniform magnetic field B. Consider a small vector area element ΔS of this surface. Magnetic flux through this area element is defined as, $\Delta\phi_B = B \cdot \Delta S$

Considering all small area elements of the surface, we obtain net magnetic flux through the surface as,

$$\phi_B = \sum_{all} \Delta\phi_B = \sum_{all} B \cdot \Delta S = 0$$

Comparing this with Gauss' law in electrostatics,

$$\phi_E = \oint_S E \cdot \Delta S = \frac{q}{\varepsilon_0}$$

The difference between the Gauss's law of magnetism and electrostatics is that isolated magnetic poles (also called monopoles) does not exist.

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. Two magnets have the same length and the same pole strength. But one of the magnets has a small hole at its centre. Then,
   (a) both have equal magnetic moment
   (b) one with hole has small magnetic moment
   (c) one with hole has large magnetic moment
   (d) one with hole loses magnetism through the hole

2. A large magnet is broken into two pieces so that their lengths are in the ratio 2 : 1. The pole strengths of the two pieces will have ratio
   (a) 2 : 1 (b) 1 : 2 (c) 4 : 1 (d) 1 : 1

3. The intensity of magnetic field at a point X on the axis of a small magnet is equal to the field intensity at another point Y on equatorial axis. The ratio of distance of X and Y from the centre of the magnet will be
   (a) $(2)^{-3}$ (b) $(2)^{-1/3}$ (c) 2^3 (d) $2^{1/3}$

4. Work done in rotating a bar magnet from 0 to angle 120° is
   (a) $\frac{1}{2}MB$ (b) $\frac{3}{2}MB$ (c) MB (d) $\frac{2}{3}MB$

5. Gauss's law for magnetism is
   (a) the net magnetic flux through any closed surface is $B \cdot \Delta S$
   (b) the net magnetic flux through any closed surface is $E \cdot \Delta S$
   (c) the net magnetic flux through any closed surface is zero
   (d) Both (a) and (c)

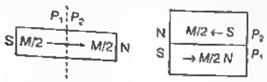
VERY SHORT ANSWER Type Questions

6. On what factors does the pole strength of a magnet depend?
7. What is Coulomb's law of magnetic force?
8. Define magnetic dipole moment. Also, write its SI unit.

[redacted]

9. A bar magnet is cut into two equal parts as shown in the Fig. (a). One part is now kept over the other such that, the P_2 is above P_1 as shown in the Fig. (b).

If M is the magnetic moment of the original magnet, what would be the magnetic moment of new combination of magnets so formed?



(a) (b)

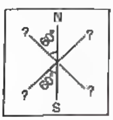
10. A coil of N turns and radius R carries a current I. It is unwound and rewound to make a square coil of side a having same number of turns N.

Keeping the current I same, find the ratio of the magnetic moments of the square coil and the circular coil. **Delhi 2013, Delhi 2013C**

11. Why do magnetic lines of force form continuous closed loops?

SHORT ANSWER Type Questions

12. Three identical bar magnets are rivetted together at centre in the same plane as shown in the figure. This system is placed at rest in a slowly varying magnetic field. It is found that the system of magnets does not show any motion. The North-South poles of one magnet is shown in the figure. Determine the poles of the remaining two. **NCERT Exemplar**



13. What happens to a bar magnet if it is cut into two pieces
(i) transverse to its length?
(ii) along its length?

14. State Gauss's law in magnetism and compare it with Gauss's law in electrostatics.

15. State whether the given statement is correct or incorrect and explain it.
"The magnetic field lines of a magnet form continuous closed loops unlike electric field lines."

16. A circular coil of closely wound N turns and radius r carries a current I. Write the expressions for the following:
(i) The magnetic field at its centre.
(ii) The magnetic moment of this coil. **All India 2012**

17. A short bar magnet placed with its axis making an angle θ with a uniform external field B, experiences a torque τ.
(i) What is the magnetic moment of the magnet?
(ii) Write the condition of stable equilibrium.

18. A small compass needle of magnetic moment M and moment of inertia I is free to oscillate in a magnetic field B. It is slightly disturbed from its equilibrium position and then released. Show that it executes simple harmonic motion. Hence, write the expression for its time period. **Delhi 2013, 2011C**

19. Two bar magnets having same geometry with magnetic moments M and $2M$ are placed in such a way that their similar poles are on the same side, then its time period of oscillation is T_1. Now, if the polarity of one of the magnets is reversed, then time period of oscillation is T_2, then find the relation between T_1 and T_2.

20. Suppose we want to verify the analogy between electrostatic and magnetostatic by an explicit experiment. Consider the motion of
(i) electric dipole **p** in an electrostatic field **E** and
(ii) magnetic dipole **M** in a magnetic field **B**.
Write down a set of conditions on **E**, **B**, **p**, **M**, so that the two motions are verified to be identical. (Assume identical initial conditions). **NCERT Exemplar**

21. Answer the following
(i) Is it possible to have a magnetic field configuration with three poles?
(ii) If magnetic monopoles existed, how would Gauss's law of magnetism be modified?

[redacted]

LONG ANSWER Type I Questions

22. An observer to the left of a solenoid of N turns each of cross-section area A observes that a steady current I in it flows in the clockwise direction. Depict the magnetic field lines due to the solenoid specifying its polarity and show that it acts as a bar magnet of magnetic moment $m = NIA$. **All India 2015**

23. A uniform conducting wire of length $12\,a$ and resistance R is wound up as a current carrying coil in the shape of
    (i) an equilateral triangle of side a,
    (ii) a square of sides a and
    (iii) a regular hexagon of side a. The coil is connected to a voltage source V_0. Find the magnetic moment of the coils in each case. **NCERT Exemplar**

24. An electron of mass m_e revolves around a nucleus of charge $+Ze$. Show that it behaves like a tiny magnetic dipole. Hence, prove that the magnetic moment associated with it is expressed as $\mu = -\frac{e}{2m_e}\mathbf{L}$, where $\mathbf{L}$ is the orbital angular momentum of the electron. Give the significance of negative sign. **Delhi 2017**

25. Verify the Gauss' law for magnetic field of a point dipole of dipole moment $\mathbf{M}$ at the origin for the surface which is a sphere of radius R. **NCERT Exemplar**

LONG ANSWER Type II Questions

26. Derive the expression for
    (i) magnetic field at a point lies on axial line of a bar magnet.
    (ii) magnetic field at a point lies on equatorial line of a bar magnet.
    Also, find the ratio of magnetic fields at the axial and equatorial points.

27. (i) Derive the expression for potential energy of a magnetic dipole in a magnetic field.
    (ii) Compare the magnetic fields of a bar magnet and a solenoid.

NUMERICAL PROBLEMS

28. A short bar magnet has a magnetic moment of 0.48 J/T. Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of the magnet on
    (i) the axis,
    (ii) the equatorial lines (normal bisector) of the magnet. **NCERT**

29. A short bar magnet of magnetic moment 5.25×10^{-2} J/T is placed with its axis perpendicular to the earth's field direction. At what distance from the centre of the magnet, the resultant field is inclined at 45° with the earth's field on (i) its normal bisector and (ii) its axis. Magnitude of the earth's field at the place is given to be 0.42 G. Ignore the length of the magnet in comparison to the distances involved. **NCERT**

30. A closely wound solenoid of 800 turns and area of cross-section $2.5 \times 10^{-4}\ m^2$ carries a current of 3.0 A. If it can be treated as a bar magnet, then find its magnetic moment. **(2 M)**

31. If the solenoid is treated as a magnet of moment (= 0.6 J/T) is free to turn about the vertical direction and a uniform horizontal magnetic field of 0.25 T is applied, what is the magnitude of torque on the solenoid when its axis makes an angle of 30° with the direction of applied field? **NCERT**

32. A closely wound solenoid of 2000 turns and area of cross-section $1.6 \times 10^{-4}\ m^2$, carrying a current of 4 A, is suspended through its centre allowing it to turn in a horizontal plane. If the solenoid is treated as magnet, then
    (i) What is the magnetic moment associated with the solenoid?
    (ii) What are the force and torque on the solenoid, if a uniform horizontal magnetic field of 7.5×10^{-2} T is set up at an angle of 30° with the axis of the solenoid? **NCERT**

33. A short magnet oscillates with a time period 0.1 s at a place, where horizontal magnetic field is 24 μT. A downward current of 18 A is established in a vertical wire 20 cm East of the magnet. What will be the new time period of the oscillator?

[redacted]

34. A circular coil of 16 turns and radius 10 cm carrying a current of 0.75 A, rests with its plane normal to an external field of magnitude 5.0×10^{-2} T. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of $2.0\ s^{-1}$. What is the moment of inertia of the coil about its axis of rotation? **NCERT**

35. If two magnets having magnetic moments M and $M\sqrt{3}$ are joined to form a cross (i.e. ×). The combination is suspended freely in a uniform magnetic field. In equilibrium position, the magnet having magnetic moment M makes an angle θ with the field. Calculate the value of θ.

36. A short bar magnet of magnetic moment $M = 0.32$ J/T is placed in a uniform magnetic field of 0.15 T. If the bar is free to rotate in the plane of the field, which orientation would correspond to its (i) stable and (ii) unstable equilibrium? What is the potential energy of the magnet in each case? **NCERT**

37. A bar magnet of magnetic moment 1.5 J/T lies aligned with the direction of a uniform magnetic field of 0.22 T.
   (i) What is the amount of work required by an external torque to turn the magnet, so as to align its magnetic moment (a) normal to the field direction, (b) opposite to the field direction?
   (ii) What is the torque on the magnet in cases (a) and (b)? **NCERT**

HINTS AND SOLUTIONS

1. (b) As, we know, magnetic dipole moment $m = m(2l)$, so hole reduces the effective length of the magnet and hence magnetic moment reduces.
2. (d) Pole strength does not depend on length. So, strength of the two pieces will remain same.
3. (d) If d_1 is distance of point X one axial line and d_2 is distance of point Y on equatorial line.

Then, $B_A = \frac{\mu_0}{4\pi}\frac{2m}{d_1^3}$, $B_E = \frac{\mu_0}{4\pi}\cdot\frac{m}{d_2^3}$

As, $B_A = B_E$

$\therefore \frac{\mu_0 2m}{4\pi d_1^3} = \frac{\mu_0 m}{4\pi d_2^3} \Rightarrow d_1^3 = 2d_2^3 \Rightarrow d_1/d_2 = 2^{1/3}$

4. (b) Work done in rotating a magnet (from angle 0 to 120°) is given by

$$W = \int_0^{120^\circ} \tau\, d\theta$$
$$= MB\int_0^{120^\circ} \sin\theta\, d\theta = MB(-\cos\theta)_0^{120^\circ}$$
$$= MB(-\cos 120^\circ + \cos 0^\circ) \Rightarrow MB\left(1+\frac{1}{2}\right) = \frac{3}{2}MB$$

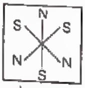
5. (c) Gauss's law for magnetism is the net magnetic flux through any closed surface is zero.
6. The pole strength of a magnet may depend on its cross-section, nature of material.
7. Coulomb's law of magnetic force is inversely proportional to the squared distance between the magnetic poles and directly proportional to the product of magnetic poles.
8. The magnetic moment of a magnet is a quantity that determines the torque, it will experience in an external magnetic field. Its SI unit is A-m^2.
9. When the bar magnet is cut into two equal parts, as shown in the Fig. (a), P_1 behaves as N and P_2 behaves as S and magnetic moment of each part of magnet becomes $M/2$. When pole P_2 is placed over pole P_1 as shown in Fig. (b), the net magnetic moment of the combination is zero, i.e. $\frac{M}{2} - \frac{M}{2} = 0$.
10. Ratio of the magnetic moments,

$$\frac{M_s}{M_c} = \frac{2INA_s}{I\,NA_c} = \frac{2\left(\frac{R}{2}\right)^2}{(R)^2} = \frac{1}{2}$$

11. Magnetic lines of force come out from North pole and enter into the South pole outside the magnet and travels from South pole to North pole inside the magnet. So, magnetic lines of force form closed loop, magnetic monopoles do not exist.

Note When South pole of the magnet is viewed with the frame of reference inside the magnet would appear as North pole and similarly, North pole as South pole. Therefore magnetic lines of force traversed from South pole to North pole inside the magnet.

12. The system of magnets will be in stable equilibrium, if the net force on the system is zero and net torque on it is also zero. It will be possible, if the poles of the remaining two magnets is as shown in the figure.



13. In both the cases (i) and (ii), we get two magnets, each with a North and South pole.
14. Refer to text on page 227.

[redacted]

15. The statement is correct. The number of magnetic field lines leaving a surface is balanced by the number of lines entering it. The net magnetic flux is zero.

16. (i) Magnetic field at centre due to circular current carrying coil, $B = \frac{\mu_0 NI}{2r}$

(ii) Magnetic moment, $M = NIA = NI(\pi r^2)$

$$M = \pi NIr^2$$

where, r is the radius of circular coil, μ_0 is permeability of free space and N is number of turns.

17. (i) Refer to text on pages 224 and 225.

(ii) Refer to text on page 226.

18. Refer to text on page 225.

19. Using the formula for time period for magnetic system

$$T = 2\pi\sqrt{\left(\frac{I}{MH}\right)} \Rightarrow T \propto \frac{1}{\sqrt{M}} \quad ...(i)$$

When similar poles placed at same side, then

$$M_1 = M + 2M = 3M$$

So, from Eq. (i), $T_1 \propto \frac{1}{\sqrt{3M}}$ ...(ii)

When the polarity of a magnet is reversed, then

$$M_2 = 2M - M = M$$

So, from Eq. (i),

$$T_2 \propto \frac{1}{\sqrt{M}} \quad ...(iii)$$

Now, on dividing Eq. (ii) by Eq. (iii), we get

$$\frac{T_1}{T_2} = \frac{\sqrt{M}}{\sqrt{3M}} = \frac{1}{\sqrt{3}} \Rightarrow T_2 = \sqrt{3}\,T_1$$

Hence, $T_1 < T_2$

20. Now, suppose that the angle between M and B is θ.

Torque on magnetic dipole moment M in magnetic field B,

$$\tau' = MB\sin\theta$$

Two motions will be identical, if

$$pE\sin\theta = MB\sin\theta$$

$\Rightarrow$ $$pE = MB \quad ...(i)$$

But $$E = cB$$

Putting this value in Eq. (i), we get

$$pcB = MB \Rightarrow p = \frac{M}{c}$$

21. (i) Yes

(ii) $\Sigma \mathbf{B} \cdot \Delta \mathbf{S} = mM_0$

22. Since, it is given that the current flows in the clockwise direction for an observer on the left side of the solenoid.

It means that the left face of the solenoid acts as South pole and right face acts as North pole. Inside a bar, the magnetic field lines are directed from South to North.

Therefore, the magnetic field lines are directed from left to right in the solenoid.

Magnetic moment of a single current carrying loop is given by, $m' = IA$.

So, magnetic moment of the whole solenoid is given by

$$m = Nm' = N(IA)$$

23. We know that magnetic moment of the coil $M = NIA$. Since, the same wire is used in three cases with same potentials, therefore, same current flows in three cases

Hints: The different shapes form figures of different area and hence, their magnetic moments vary.

(i) For an equilateral triangle of side a,

$N = 4$, as the total wire of length $= 12a$

Magnetic moment of the coil.

$N=4$

$$M = NIA = 4I\left(\frac{\sqrt{3}}{4}a^2\right)$$

$$\Rightarrow \quad M = Ia^2\sqrt{3}$$

(ii) For a square of side a, $A = a^2$

$N = 3$, as the total wire of length $= 12a$

Magnetic moment of the coil.

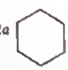
$N=3$

$$M = NIA = 3I(a^2) = 3I\,a^2$$

(iii) For a regular hexagon of sides a,

$N = 2$, as the total wire of length $= 12a$

Magnetic moment of the coils,



$N=2$

$$M = NIA = 2I\left(\frac{6\sqrt{3}}{4}a^2\right) = 3\sqrt{3}a^2 I$$

$\therefore$ M is in a geometric series.

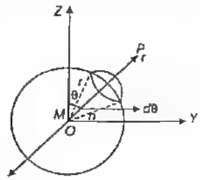
24. Refer to text on page 219.

In vector form, $\mu = \frac{-e\mathbf{L}}{2m_e}$

Here, negative sign indicates that μ directs away from $\mathbf{L}$.

[redacted]

25. Let us draw the figure for given situation,



We have to prove that $\oint \mathbf{B} \cdot d\mathbf{S} = 0$. This is called Gauss's law in magnetisation.

According to the question,

Magnetic moment of dipole at origin O is $\mathbf{M} = M\hat{\mathbf{k}}$

Let P be a point at distance r from O and OP makes an angle θ with Z-axis. Component of M along $OP = M\cos\theta$.

Now, the magnetic field induction at P due to dipole of moment $M\cos\theta$ is $\mathbf{B} = \frac{\mu_0}{4\pi}\cdot\frac{2M\cos\theta}{r^3}\hat{r}$

From the diagram, r is the radius of sphere with centre at O lying in YZ-plane. Take an elementary area dS of the surface at P. Then,

$$dS = r(r\sin\theta\, d\theta)\hat{r} = r^2\sin\theta\, d\theta\hat{r}$$

$$\therefore \oint \mathbf{B}\cdot d\mathbf{S} = \oint \frac{\mu_0}{4\pi}\cdot\frac{2M\cos\theta}{r^3}\hat{r}(r^2\sin\theta\, d\theta\hat{r})$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{M}{r}\int_0^{2\pi} 2\sin\theta\cos\theta\, d\theta$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{M}{r}\int_0^{2\pi}\sin 2\theta\, d\theta = \frac{\mu_0}{4\pi}\cdot\frac{M}{r}\left(\frac{-\cos 2\theta}{2}\right)_0^{2\pi}$$

$$= -\frac{\mu_0}{4\pi}\cdot\frac{M}{2r}[\cos 4\pi - \cos 0]$$

$$= \frac{\mu_0}{4\pi}\cdot\frac{M}{2r}[1-1] = 0$$

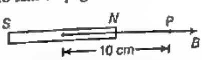
26. (i) Refer to text on pages 222 and 223.

(ii) Refer to text on page 223.

27. (i) Refer to text on page 226.

(ii) Refer to text on page 224.

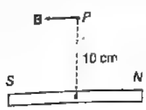
28. (i)



Refer to Example 7 on page 223.

The direction of magnetic field is from S to N-pole of magnet.

(ii)



Refer to Example 8 on page 223.

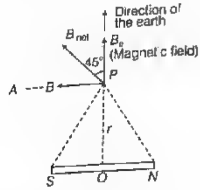
The direction of magnetic field is from N to S-pole of magnet

29. Given, magnetic moment,

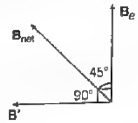
$$m = 5.25 \times 10^{-2}\ \text{J/T}$$

Let the resultant magnetic field be B_{net}.

It makes an angle of 45° with B_e.



According to the vector analysis,



$$\Rightarrow \quad \tan 45° = \frac{B\sin 90°}{B\cos 90° + B_e}$$

$$\Rightarrow \quad 1 = \frac{B}{B_e} \text{ or } B = B_e \qquad \text{...(i)}$$

From Eq. (i), we get

$$\Rightarrow 0.42\times10^{-4} = \frac{\mu_0}{4\pi}\cdot\frac{m}{r^3}$$

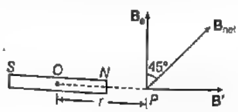
$$0.42\times10^{-4} = \frac{10^{-7}\times5.25\times10^{-2}}{r^3}$$

$$r^3 = \frac{5.25\times10^{-9}}{0.42\times10^{-4}} = 12.5\times10^{-5}$$

$$r = 0.05\ \text{m}$$

(ii) When Point Lies on Axial Line

Let the resultant magnetic field B_{net} makes an angle 45° from B_e. The magnetic field on the axial line of the magnet at a distance of r from the centre of magnet.

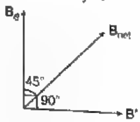


$$B' = \frac{\mu_0}{4\pi}\cdot\frac{2m}{r^3} \qquad \text{...(i)}$$

Direction of magnetic field is from S to N.

[redacted]

According to the vector analysis



$$\tan 45° = \frac{B' \sin 90°}{B' \cos 90° + B_e}$$

$$\Rightarrow \quad 1 = \frac{B'}{B_e} \Rightarrow B_e = B'$$

From Eq. (i), we get

$$0.42 \times 10^{-4} = \frac{\mu_0}{4\pi} \times \frac{2m}{r^3}$$

$$\Rightarrow \quad 0.42 \times 10^{-4} = \frac{10^{-7} \times 2 \times 5.25 \times 10^{-2}}{r^3}$$

$$\Rightarrow \quad r^3 = \frac{10^{-9} \times 2 \times 5.25}{0.42 \times 10^{-4}} = 25.0 \times 10^{-5}$$

$$\therefore \quad r = 0.063 \text{ m or } 6.3 \text{ cm}$$

30. Given, number of turns, $n = 800$

Area of cross section of solenoid, $A = 2.5 \times 10^{-4}$ m^2

Current through solenoid, $I = 3$ A

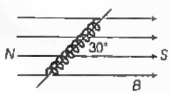
Magnetic moment of bar magnet,

$M = nIA = 800 \times 3 \times 2.5 \times 10^{-4}$

$= 0.6$ J/T along the axis of the solenoid

31. Given, magnetic field, $B = 0.25$ T

Angle between magnetic moment and the magnetic field, $\theta = 30°$



Magnetic moment, $M = 0.6$ J/T

Torque acting on the solenoid, when it is placed at an angle θ with the magnetic field.

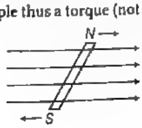
$\tau = MB \sin\theta = 0.6 \times 0.25 \sin 30°$

$= 0.6 \times 0.25 \times \frac{1}{2} = 0.075$ N-m

Thus, the magnitude of torque on the solenoid is 0.075 N-m.

32. (i) $M = 1.28$ J/T;

(ii) The force (net) on the solenoid is zero because two equal and opposite forces (on each of its poles) are acting but their lines of action are parallel, so they form a couple thus a torque (not force) is applied on it.



$\tau = 4.8 \times 10^{-2}$ N-m; refer to Q. 33 on page 240.

33. Initially,

$$T = 2\pi\sqrt{\frac{I}{mB'}} \text{ and finally, } T' = 2\pi\sqrt{\frac{I}{m(B + B')}}$$

where, B' = horizontal magnetic field $= 24\mu$T

and $\quad B$ = magnetic field due to downward conductor

$$= \frac{\mu_0}{4\pi} \cdot \frac{2i}{a} = 18\mu\text{T}$$

$$\therefore \quad \frac{T'}{T} = \sqrt{\frac{B'}{B + B'}}$$

$$\Rightarrow \quad \frac{T'}{0.1} = \sqrt{\frac{24}{18 + 24}}$$

$$\Rightarrow \quad T' = 0.076\text{s}$$

34. Given, number of turns of circular coil, $n = 16$

Radius of circular coil, $r = 10$ cm $= 0.1$ m

Current, $I = 0.75$ A, frequency, $f = 2\,\text{s}^{-1}$

Magnetic field, $B = 5.0 \times 10^{-2}$ T

Magnetic moment of the coil,

$M = nIA = 16 \times 0.75 \times \pi\,(0.1)^2$

$= 16 \times 0.75 \times 3.14 \times (0.1)^2$

$= 0.377$ J/T

Frequency of oscillation of the coil, $f = \frac{1}{2\pi}\sqrt{\frac{M \times B}{I}}$

where, I = moment of inertia of the coil.

Squaring on both sides, we get

$$f^2 = \frac{1}{4\pi^2} \cdot \frac{MB}{I} \Rightarrow I = \frac{MB}{4\pi^2 f^2}$$

$$= \frac{0.377 \times 5 \times 10^{-2}}{4 \times 3.14 \times 3.14 \times 2 \times 2}$$

$$= 1.2 \times 10^{-4} \text{ kg-m}^2$$

Thus, the moment of inertia of the coil

$= 1.2 \times 10^{-4}$ kg-m^2

35. If magnet of magnetic moment M makes an angle θ with the field, then other magnet of magnetic moment $M\sqrt{3}$ makes an angle $(90° - \theta)$ with the field.

In equilibrium, $\tau_1 = \tau_2$

$$\Rightarrow \quad MB\sin\theta = M\sqrt{3}\,B\cos\theta$$

$$\Rightarrow \quad \frac{\sin\theta}{\cos\theta} = \sqrt{3}$$

$$\Rightarrow \quad \tan\theta = \sqrt{3}$$

$$\Rightarrow \quad \theta = 60°$$

36. Given, magnetic moment of magnet, $M = 0.32$ J/T

Magnitude of magnetic field, $B = 0.15$ T

(i) For stable equilibrium, the angle between magnetic moment **M** and magnetic field **B** is $\theta = 0°$.

[$\because$ In this position, it will be in a direction parallel to the magnetic field, thus no torque will act on it.]

[redacted]

∵ The potential energy of the magnet,

$U = -\mathbf{M}\cdot\mathbf{B} = -MB\cos\theta$ [$\because \mathbf{M}\cdot\mathbf{B} = MB\cos\theta$]

$= -0.32\times 0.15\cos 0^\circ$

$= -4.8\times 10^{-2}$ J

Thus, for the stable equilibrium the potential energy is -4.8×10^{-2} J.

(ii) For unstable equilibrium, the angle between the magnetic moment and magnetic field is 180°.

($\because$ At $\theta = 180^\circ$, although torque is zero but if it is displaced by small angle $d\theta$, then resulting torque would not restore it to the original position).

Potential energy of the magnet,

$U = -MB\cos 180^\circ$

$= -0.32\times 0.15(-1)$

$= 4.8\times 10^{-2}$ J

Thus, for the unstable equilibrium, the potential energy is 4.8×10^{-2} J.

37. Given, magnetic moment of magnet, $M = 1.5$ J/T

Uniform magnetic field, $B = 0.22$ T

(i) (a) Angle, $\theta_1 = 0^\circ$ [$\because$ the magnet lies aligned in the direction of field]

and $\theta_2 = 90^\circ$ [$\because$ the magnet is to be aligned normal to the field direction]

Work done in rotating the magnet from angle θ_1 to θ_2,

$W = -MB(\cos\theta_2 - \cos\theta_1)$

$= -1.5\times 0.22(\cos 90^\circ - \cos 0^\circ)$

$= 0.33$ J

(b) Angle, $\theta_1 = 0^\circ$ and $\theta_2 = 180^\circ$

[$\because$ Magnet is to be aligned opposite to the direction of field]

Work done $= -MB(\cos\theta_2 - \cos\theta_1)$

$= -1.5\times 0.22(\cos 180^\circ - \cos 0^\circ) = 0.66$ J

(ii) Using the formula of torque,

$\tau = MB\sin\theta$

(a) When magnetic moment is normal to the field, $\theta = 90^\circ$

$\tau = 1.5\times 0.22\sin 90^\circ = 0.33$ N-m

(b) When magnetic moment is opposite to the field, $\theta = 180^\circ$

$\tau = 1.5\times 0.22\sin 180^\circ = 0$

|TOPIC 2|
Magnetic Properties of Materials

VARIOUS TERMS RELATED TO MAGNETISM

Various terms related to the magnetism are given below.

Magnetic Intensity (H)

The capability of magnetic field to magnetise the substance is measured in terms of magnetic intensity of the field. The magnitude of magnetic intensity may be defined as the number of ampère turns flowing round the unit length of toroid to produce the magnetic induction B_0, in the toroid. It is denoted by H.

$$\boxed{H = \frac{B_0}{\mu_0}} \quad \text{...(i)}$$

where, B_0 = magnetic field inside vacuum

and $\mu_0 = 4\pi\times 10^{-7}$ T-m A^{-1}

Its SI unit is Am^{-1}.

Magnetic intensity is also known as **magnetising force** and **magnetic field strength.**

Intensity of Magnetisation (I)

The intensity of magnetisation of a magnetised substance represents the degree to which the substance is magnetised. It is defined as the net magnetic moment M developed per unit volume V, when a magnetic specimen is subjected to magnetising field. It is denoted by I.

$$\boxed{I = \frac{M}{V}}$$

Its SI unit is Am^{-1}. Its dimension is [L^{-1}A] and it is a vector quantity.

Magnetic Induction (B)

It is defined as the number of magnetic lines of induction crossing per unit area normally through the magnetic substance. It is denoted by B. Magnetic induction B is the sum of the magnetic field B_0 and the magnetic field $\mu_0 I$ produced due to the magnetisation of the substance.

Thus, $B = B_0 + \mu_0 I = \mu_0 H + \mu_0 I$

$$\boxed{B = \mu_0(H + I)} \quad \text{...(ii)}$$

[redacted]

Magnetic induction is also known as **magnetic flux density** or simply magnetic field.

Its SI unit is T or Wbm^{-2}.

Magnetic Susceptibility (χ_m)

It is a measure of how easily a substance is magnetised in a magnetising field. The magnetic susceptibility of a magnetic substance is defined as the ratio of the intensity of magnetisation to the magnetic intensity. It is denoted by χ_m.

$$\boxed{\chi_m = \frac{I}{H}} \quad \text{...(iii)}$$

As units of H and I are same (Am^{-1}), therefore it has no unit.

Magnetic Permeability (μ)

It is a measure of conduction of magnetic field lines through a substance. The magnetic permeability of a magnetic substance is defined as the ratio of the magnetic induction to the magnetic intensity. It is denoted by μ.

$$\boxed{\mu = \frac{B}{H}}$$

Its SI unit is TmA^{-1}.

Relative Magnetic Permeability (μ_r)

It is the ratio of the magnetic permeability μ of the substance to the permeability of free space.

$$\boxed{\mu_r = \frac{\mu}{\mu_0}}$$

It is a dimensionless quantity and is equal to 1 for vacuum.

The relative magnetic permeability of a substance is defined as the ratio of magnetic flux density B in that substance and flux density B_0 in vacuum in the same field.

$$\boxed{\mu_r = \frac{B}{B_0}}$$

Relation between Relative Magnetic Permeability (μ_r) and Magnetic Susceptibility (χ_m)

When a substance is placed in a magnetising field, it becomes magnetised. The total magnetic flux density B within the substance is the flux density that would have been produced by the magnetising field in vacuum plus the flux density due to the magnetisation of the substance. If I be the intensity of magnetisation of the substance, then by definition, the magnetic intensity of the magnetising field is given by

$$H = \frac{B}{\mu_0} - I \quad \text{or} \quad B = \mu_0 (H + I)$$

But $\quad I = \chi_m H$

where, χ_m is the susceptibility of the substance.

$\therefore \quad B = \mu_0 H (1 + \chi_m)$

Again $B = \mu H$, where μ is the permeability of the substance.

$$\therefore \quad \mu = \mu_0 (1 + \chi_m) \quad \text{or} \quad \frac{\mu}{\mu_0} = 1 + \chi_m$$

$\frac{\mu}{\mu_0}$ is the relative permeability μ_r.

Thus, $\quad \boxed{\mu_r = 1 + \chi_m}$

The quantity $(1 + \chi_m) = \mu_r$ is the analog of dielectric constant in electrostatics and is known as relative magnetic permeability. It is a dimensionless quantity.

The value of magnetic susceptibility is small and positive for paramagnetic materials and small and negative for diamagnetic materials.

EXAMPLE |1| The magnetic field B and the magnetic intensity H in a material are found to be 1.6 T and 1000 Am^{-1}, respectively. Determine the relative permeability μ_r and the susceptibility χ_m of the material.

Sol. Magnetic permeability, $\mu = \frac{B}{H} = \frac{1.6}{1000}$

$$= 1.6 \times 10^{-3} \, TmA^{-1}$$

Since, relative magnetic permeability,

$$\mu_r = \frac{\mu}{\mu_0} = \frac{1.6 \times 10^{-3}}{4\pi \times 10^{-7}} = 0.127 \times 10^4$$

Therefore, susceptibility, $\chi_m = \mu_r - 1 = 1.27 \times 10^3 - 1$

$\Rightarrow \quad \chi_m = 1.27 \times 10^3$

EXAMPLE |2| A solenoid of 600 turns per metre is carrying a current of 4 A. Its core is made of iron with relative permeability of 5000. Calculate the magnitudes of magnetic intensity, intensity of magnetisation and magnetic field inside the core.

Sol. Given, current, $I = 4$ A

Number of turns per unit length, $n = 600$

Relative permeability, $\mu_r = 5000$

Since, magnetic intensity, $H = nI = 600 \times 4 = 2400 \, Am^{-1}$

Since, $\quad \mu_r = 1 + \chi_m$

$\Rightarrow \quad \chi_m = \mu_r - 1$

$$= 5000 - 1 = 4999 \approx 5000$$

[redacted]

Here, χ_m = magnetic susceptibility.

Intensity of magnetisation can be given as

$$I = \chi_m H = 5000 \times 2400$$
$$= 1.2 \times 10^7 \text{ Am}^{-1}$$

Therefore, magnetic field, $B = \mu_r \mu_0 H$

$$= 5000 \times (4\pi \times 10^{-7}) \times 2400$$
$$= 15 \text{ T}$$

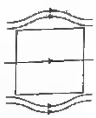
MAGNETIC PROPERTIES OF MATERIALS

Materials can be classified as diamagnetic, paramagnetic and ferromagnetic on the basis of susceptibility (χ_m).

Diamagnetic	Paramagnetic	Ferromagnetic
$-1 \le \chi_m < 0$	$0 < \chi_m < \varepsilon$	$\chi_m >> 1$
$0 \le \mu_r < 1$	$1 < \mu_r < 1 + \varepsilon$	$\mu_r >> 1$
$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$

Here, ε is a small positive number introduced to quantify paramagnetic materials.

Diamagnetism



Diamagnetic substances are those substances which have a tendency to move from stronger to the weaker part of the external magnetic field.

When a bar of diamagnetic material is placed in an external magnetic field, the field lines are repelled or expelled and the field inside the material is reduced.

Explanation of Diamagnetism

Diamagnetic substances are those substances in which resultant magnetic moment in an atom is zero.

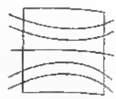
When magnetic field is applied, those electrons having orbital magnetic moment in the same direction slow down and those in opposite directions speed up. This happens due to induced current in accordance with Lenz's law. Thus, the substance develops a net magnetic moment in the direction opposite to that of the applied field and hence, repels. e.g. Bismuth, copper, lead, silicon, nitrogen (at STP), water and sodium chloride.

> **Meissner Effect**
>
> Superconductors exhibit perfect diamagnetism. A superconductor repels a magnet and (by Newton's third law) is repelled by the magnet. This phenomenon of perfect diamagnetism in superconductors is called the **Meissner effect**. Superconducting magnets have been used for running magnetically levitated superfast trains.

Paramagnetism

The substances which get weakly magnetised in the direction of external field, when placed in an external magnetic field are called paramagnetic substances. These substances have the tendency to move from a region of weak magnetic field to strong magnetic field, i.e. they get weakly attracted to a magnet.

Explanation of Paramagnetism



The atoms of a paramagnetic material possess a permanent magnetic dipole moment of their own. On account of the ceaseless random motion of the atoms, no net magnetisation is seen. But in the presence of an external field B_0, which is strong enough and at low temperatures, the individual atomic dipole moment can be made to align and point in the same direction as B_0.

The field lines get concentrated inside the material and the field inside is enhanced.

When placed in a non-uniform magnetic field, the paramagnetic material tends to move from weaker part of the field to the stronger part.

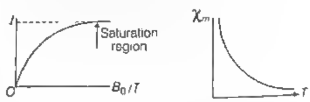
e.g. Aluminium, sodium, calcium, oxygen (at STP) and a copper chloride.

Curie's Law

Magnetisation of a paramagnetic material is inversely proportional to the absolute temperature (T).

$$I = C\frac{B_0}{T} \quad \text{or equivalently} \quad \chi_m = \frac{C\mu_0}{T}$$

where, C is called Curie's constant.



The variation of intensity of magnetisations with B_0/T is shown in the figure. At a particular stage, all the atomic dipoles present in the specimen align in the direction of the external field and this leads to saturation region.

Ferromagnetism

The substances which get strongly magnetised when placed in an external magnetic field are called ferromagnetic substances. They have strong tendency to move from a region of weak magnetic field to strong magnetic field, i.e. they get strongly attracted to a magnet.

[redacted]

Curie-Weiss Law

This describes the magnetic susceptibility χ_m of a ferromagnet in the paramagnetic region above the Curie point. It is expressed as

$$\chi_m = \frac{C}{T - T_C} \qquad [\because T > T_C]$$

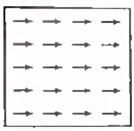
where, C is called Curie's constant, T is absolute temperature in kelvin and T_C is Curie temperature.

Explanation of Ferromagnetism

The atoms in a ferromagnetic material possess a dipole moment aligned in a common direction over a macroscopic volume called **domain**. Each domain has net magnetisation.

Randomly oriented domains

When we apply an external magnetic field B_0, the domains orient themselves in the direction of B_0 and simultaneously the domains grow in size.



Aligned domains

In some ferromagnetic materials, the magnetisation persists on removal of external magnetic field. Such materials are called **hard magnetic materials or hard ferromagnets**, e.g. alnico (an alloy of iron, aluminium, nickel, cobalt and copper) is one such material which forms permanent magnets to be used among other things as a compass needle. There are some ferromagnetic materials in which the magnetisation disappears on removal of external magnetic field, e.g. soft iron. Such materials are called **soft magnetic materials or soft ferromagnets**, e.g. Iron, cobalt, nickel, gadolinium, etc. The relative magnetic permeability of these substances is greater than 1000.

The ferromagnetic property depends on the temperature. At high temperature, a ferromagnet becomes a paramagnet. The transition of temperature from ferromagnetism to paramagnetism is called the **Curie temperature** (T_C). The susceptibility in the paramagnetic phase is described by

$$\chi_m = \frac{C}{T - T_C} \qquad [\because T > T_C]$$

Curie Temperature T_C of Some Ferromagnetic Materials

Material	T_C (K)
Cobalt	1394
Iron	1043
Ferric oxide	893
Nickel	631
Gadolinium	317

EXAMPLE |3| A solenoid having 5000 turns/m carries a current of 2A. An aluminium ring at temperature 300K inside the solenoid provides the core.

(a) If the magnetisation I is 2×10^{-2} A/m, find the susceptibility of aluminium at 300 K.

(b) If temperature of the aluminium ring is 320 K, what will be the magnetisation?

Sol. (a) Here, $H = nI = 5000 \times 2 = 10^4$ A/m

and $I = \chi H$

$$\therefore \quad \chi = \frac{I}{H} = \frac{2 \times 10^{-2}}{10^4} = 2 \times 10^{-6}$$

(b) According to Curie law,

$$\chi = \frac{C}{T}$$

$$\Rightarrow \quad \frac{\chi_2}{\chi_1} = \frac{T_2}{T_1}$$

$$\chi_2 = \frac{T_2}{T_1}\chi_1 = \frac{320}{300} \times 2 \times 10^{-6}$$

$$= 2.13 \times 10^{-6}$$

$\therefore$ Magnetisation at 320 K,

$$I = \chi_2 H = 2.13 \times 10^{-6} \times 10^4$$

$$= 2.13 \times 10^{-2} \text{ A/m}$$

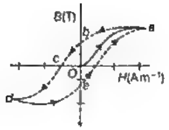
Hysteresis Curve

The hysteresis curve represents the relation between the magnetic induction B or intensity of magnetisation I of a ferromagnetic material with magnetic intensity H. The graph shows the behaviour of the material as we take it through one cycle of magnetisation shown in the figure.

Formation of Hysteresis Curve

An unmagnetised sample is placed in a solenoid and current through the solenoid is increased. The magnetic field B in the material rises and saturates as depicted in the curve Oa. Next, if H is decreased and reduces to zero.

[redacted]



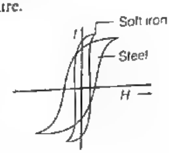
Then, at $H = 0, B \neq 0$ (curve ab)

The value of B at $H = 0$ is called retentivity.

Now, the current in the solenoid is reversed and slowly increased, we again obtain saturation in the reverse direction at d. The value of H at c is called coercivity.

Now, the current is reduced (curve de), increased, reversed (curve ea). The cycle repeats itself. For a given value of H, B is not unique, but depends on previous history of the sample. This phenomenon is called hysteresis.

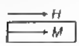
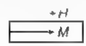
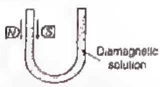
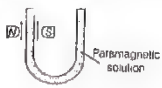
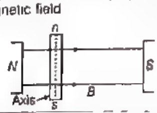
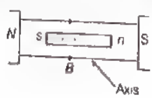
It is found that the area of hysteresis loop is proportional to the net energy absorbed per unit volume by the material, as it is taken over a complete cycle of magnetisation. The energy so, absorbed by the specimen appears in the form of heat energy. Hysteresis loop for soft iron is large and narrow, whereas the hysteresis loop for steel is short and wide as shown in the figure.



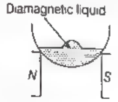
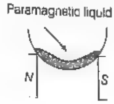
From the hysteresis loops of soft iron and steel, we say

(i) Retentivity of soft iron is greater than that of steel.
(ii) Soft iron is more strongly magnetised than steel.
(iii) Coercivity of soft iron is less than that of steel.
(iv) As area of hysteresis loop for soft iron is smaller than that for steel, therefore, hysteresis loss in case of soft iron is smaller than that in case of steel.

Comparative Study of Magnetic Materials

Diamagnetic Substances	Paramagnetic Substances	Ferromagnetic Substances
These substances when placed in a magnetic field, acquire feeble magnetism opposite to the direction of the magnetic field. 	These substances when placed in a magnetic field acquire feeble magnetism in the direction of the magnetic field 	These substances when placed in a magnetic field are strongly magnetised in the direction of the field 
These substances are feebly repelled by a magnet.	These substances are feebly attracted by a magnet	These substances are strongly attracted by a magnet
When a diamagnetic solution is poured into a U-tube and one arm is placed between the poles of strong magnet the level of solution in that arm is lowered 	The level of the paramagnetic solution in that arm rises 	No liquid is ferromagnetic.
If a rod of diamagnetic material is suspended freely between two magnetic poles, its axis becomes perpendicular to the magnetic field 	Paramagnetic rod becomes parallel to the magnetic field 	Ferromagnetic rod also becomes parallel to the magnetic field 
In non-uniform magnetic field, the diamagnetic substances are attracted towards the weaker fields, i.e. they move from stronger to weaker magnetic field.	In non-uniform magnetic field paramagnetic substances move from weaker to stronger part of the magnetic field slowly	In non-uniform magnetic field, ferromagnetic substances move from weaker to stronger magnetic field rapidly

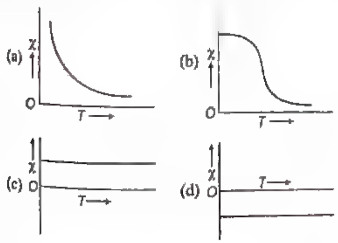
[redacted]

Diamagnetic Substances	Paramagnetic Substances	Ferromagnetic Substances
Their permeability is less than one ($\mu < 1$)	Their permeability is slightly greater than one ($\mu > 1$)	Their permeability is much greater than one ($\mu >> 1$)
Their susceptibility is small and negative. Their susceptibility is independent of temperature.	Their susceptibility is small and positive. Their susceptibility is inversely proportional to absolute temperature, which is Curie's law i.e. $\chi_m \propto \frac{1}{T}$	Their susceptibility is large and positive. They follow Curie-Weiss law, when heated above Curie's temperature (T_C). i.e. $\chi_m \propto \frac{1}{T - T_C}$ At Curie temperature, ferromagnetic substances change into paramagnetic substances.
Shape of diamagnetic liquid in a glass crucible and kept over two magnetic poles. 	Shape of paramagnetic liquid in a glass crucible and kept over two magnetic poles 	No liquid is ferromagnetic.
In these substances, the magnetic field lines are farther than in air	In these substances, the magnetic field lines are closer than in air	In these substances, the magnetic field lines are much closer than in air
The resultant magnetic moment of these substances is zero.	These substances have a permanent magnetic moment.	These substances also have a permanent magnetic moment

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. The variation of magnetic susceptibility (χ) with temperature for a diamagnetic substance is best represented by figure



2. The relative permeability of a substance X is slightly less than unity and that of substance Y is slightly more than unity, then
(a) X is paramagnetic and Y is ferromagnetic
(b) X is diamagnetic and Y is ferromagnetic
(c) X and Y both are paramagnetic
(d) X is diamagnetic and Y is paramagnetic

VERY SHORT ANSWER Type Question

3. In what way, the behaviour of a diamagnetic material is different from that of a paramagnetic, when kept in an external magnetic field? **All India 2016**

SHORT ANSWER Type Questions

4. From molecular view point, discuss the temperature dependence of susceptibility for diamagnetism, paramagnetism and ferromagnetism. **NCERT Exemplar**

5. Show diagrammatically the behaviour of magnetic field lines in the presence of
(i) paramagnetic and **All India 2014**
(ii) diamagnetic substances. How does one explain this distinguishing feature?

6. Out of the two magnetic materials, A has relative permeability slightly greater than unity while B has less than unity. Identify the nature of the materials A and B. Will their susceptibilities be positive or negative? **Delhi 2014**

7. A ball of superconducting material is dipped in liquid nitrogen and placed near a bar magnet.
(i) In which direction will it move?
(ii) What will be the direction of its magnetic moment? **NCERT Exemplar**

[redacted]

8. Explain quantitatively the order of magnitude difference between the diamagnetic susceptibility of N_2 ($\sim 5 \times 10^{-9}$) (at STP) and Cu ($\sim 10^{-5}$). **NCERT Exemplar**

9. (i) How does a diamagnetic material behave when it is cooled at very low temperature?
(ii) Why does a paramagnetic sample display greater magnetisation when cooled? Explain. **Delhi 2012**

10. The susceptibility of a magnetic material is 0.9853. Identify the type of magnetic material. Draw the modification of the field pattern on keeping a piece of this material in a uniform magnetic field. **CBSE 2018**

11. Out of the following, identify the materials which can be classified as
(i) paramagnetic
(ii) diamagnetic
(a) Aluminium (b) Bismuth
(c) Copper (d) Sodium

LONG ANSWER Type I Questions

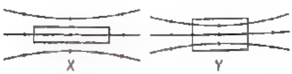
12. Three identical specimens of a magnetic material, nickel, antimony, aluminium are kept in a non-uniform magnetic field. Draw the modification in the field lines in each case. Justify your answer. **Delhi 2011**

13. Answer the following questions:
(i) Why does a paramagnetic sample display greater magnetisation (for the same magnetising field) when cooled?
(ii) If a toroid uses bismuth for its core, then will the field in the core be (slightly) greater or (slightly) less than when the core is empty?
(iii) Is the permeability of a ferromagnetic material independent of the magnetic field? If not, is it more for lower or higher fields?
(iv) Magnetic field lines are always nearly normal to the surface of a ferromagnet at every point. (This fact is analogous to the static electric field lines being normal to the surface of a conductor at every point.) Why?
(v) Would the maximum possible magnetisation of a paramagnetic sample be of the same order of magnitude as the magnetisation of a ferromagnet? **NCERT**

14. Answer the following questions.
(i) Explain qualitatively on the basis of domain picture the irreversibility in the magnetisation curve of a ferromagnet.
(ii) The hysteresis loop of a soft iron piece has a much smaller area than that of a carbon steel piece. If the material is to go through repeated cycles of magnetisation, which piece will dissipate greater heat energy?
(iii) A system displaying a hysteresis loop such as a ferromagnet, is a device for storing memory. Explain the meaning of this statement.
(iv) What kind of ferromagnetic material is used for coating magnetic tapes in a cassette player or for building 'memory stores' in a modern computer?
(v) A certain region of space is to be shielded from magnetic fields. Suggest a method. **NCERT**

15. A bar magnet of magnetic moment 6 J/T is aligned at 60° with a uniform external magnetic field of 0.44 T. Calculate (a) the work done in turning the magnet to align its magnetic moment (i) normal to the magnetic field, (ii) opposite to the magnetic field, and (b) the torque on the magnet in the final orientation in case (ii). **CBSE 2018**

16. When two materials are placed in an external magnetic field, the behaviour of magnetic field lines is as shown in the figure. Identify the magnetic nature of each of these two materials. **Delhi 2009C**



17. A sample of paramagnetic salt contains 2×10^{24} atomic dipoles, each of dipole moment 1.5×10^{-23} J/T. The sample is placed under a homogenous magnetic field of 0.84 T and cooled to a temperature of 4.2 K. The degree of magnetic saturation achieved is equal to 15%. What will be the total dipole moment of the sample for a magnetic field of 0.98 T and at a temperature of 2.8 K? **NCERT**

[redacted]

LONG ANSWER Type II Question

18. (i) Discuss briefly electron theory of magnetism for diamagnetic and paramagnetic materials.
(ii) Give two methods to destroy the magnetism of a magnet.

NUMERICAL PROBLEMS

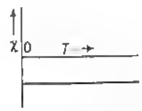
19. If the bar magnet in Q. 14 is turned around by 180°, where will the new null points be located? NCERT

20. A short bar magnet placed in a horizontal plane has its axis aligned along the magnetic North-South direction. Null points are found on the axis of the magnet at 14 cm from the centre of the magnet. The earth's magnetic field at the place is 0.36 gauss and the angle of dip is zero.

What is the total magnetic field on the normal bisector of the magnet at the same distance as the null point (i.e. 14 cm) from the centre of the magnet? (At null points, field due to a magnet is equal and opposite to the horizontal component of the earth's magnetic field.) NCERT

HINTS AND SOLUTIONS

1. (d) For diamagnetic substances, the magnetic susceptibility is negative, and it is independent of temperature. Therefore, choice (d) is correct in figure



2. (d) As $\mu_r < 1$ for substance X, it must be diamagnetic and $\mu_r > 1$ for substance Y, is must be paramagnetic.

3. When paramagnetic materials are placed in external magnetic field, these are feebly magnetised in the direction of the applied external magnetic field whereas in case of diamagnetic materials, these are feebly magnetised opposite to that of applied external magnetic field.

4. Susceptibility of magnetic material $\chi = \frac{I}{H}$, where I is the intensity of magnetisation induced in the material and H is the magnetising force

Diamagnetism is due to orbital motion of electrons in an atom developing magnetic moments opposite to applied field. Thus, the resultant magnetic moment of the diamagnetic material is zero and hence, the susceptibility χ of diamagnetic material is not much affected by temperature.

Paramagnetism and ferromagnetism is due to alignment of atomic magnetic moments in the direction of the applied field. As temperature is raised, the alignment is disturbed, resulting decrease in susceptibility of both with increase in temperature.

5. Refer to page 236 for diagram.

Magnetic permeability of paramagnetic substance is more than air, so it allows more lines to pass through it while permeability of diamagnetic substance is less than air, so it does not allow lines to pass through it. Thus, diamagnetic substances expel magnetic field lines, while paramagnetic substances attract them.

6. The nature of the material A is paramagnetic and its susceptibility χ_m is positive.

The nature of the material B is diamagnetic and its susceptibility χ_m is negative.

7. Both a superconducting material and nitrogen are diamagnetic in nature. When a ball of superconducting material is dipped in liquid nitrogen, it behaves as a diamagnetic material. When placed near a bar magnet, it will be feebly magnetised opposite to the direction of magnetising field.

Because of this, (i) it will be repelled (i.e. move away from magnet) (ii) the direction of magnetic moment will be opposite to the direction of magnetic field of bar magnet.

8. Here, $\chi_{m(N_2)} = 5 \times 10^{-9}$ and $\chi_{m(Cu)} = 10^{-5}$

$$\therefore \quad \frac{\chi_{m\,(N_2)}}{\chi_{m\,(Cu)}} = \frac{5 \times 10^{-9}}{10^{-5}} = 5 \times 10^{-4}$$

As, $\chi_m = \frac{I}{H} = \frac{M/V}{H} = \frac{M}{HV} = \frac{M\rho}{Hm}$

where, M = magnetic moment

V = volume, m = mass and ρ = density

$$\therefore \quad \chi_m \propto \rho \text{ for given value of } \frac{M}{Hm}$$

Thus, $\frac{\chi_{m\,(N_2)}}{\chi_{m\,(Cu)}} = \frac{\rho_{N_2}}{\rho_{Cu}} = \frac{28\text{ g}/22400\text{ cc}}{8\text{ g cc}} = 1.6 \times 10^{-4}$

9. (i) As, the resistance (electrical of metal decreases with decrease in temperature.

But for diamagnetic substances, the variation of susceptibility is very small $(0 < \chi_m < \varepsilon)$, i.e. diamagnetic materials are unaffected by the change in temperature (except bismuth).

(ii) Paramagnetic materials when cooled due to thermal agitation tendency alignment of magnetic dipoles decreases. Hence, they shows greater magnetisation.

10. Given, susceptibility, $\chi_m = 0.9853$

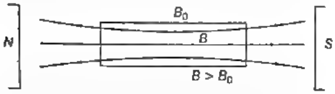
As the susceptibility of material is positive but small.

$\therefore$ The material is paramagnetic in nature. For paramagnetic material, magnetic lines of external

[redacted]

magnetic field will passes through the material without much deviation, when it is placed in between magnetic poles.

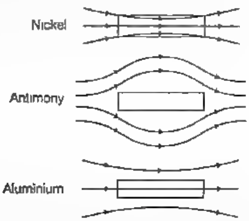
The modification of the field pattern is shown in the following figure.



11. (i) **Paramagnetic substance** Aluminium, sodium

(ii) **Diamagnetic substance** Bismuth, copper, the susceptibility of the diamagnetic materials is small and negative, i.e. $-1 < \chi_m < 0$, whereas for paramagnetic substance the susceptibility is small and positive, i.e. $0 < \chi_m < a$, where a is a small number

12.



The modification in the field lines shown in the figure are as such because

(i) nickel is a ferromagnetic substance.

(ii) antimony is a diamagnetic substance.

(iii) aluminium is a paramagnetic substance.

Refer to text on pages 236 and 237.

13. (i) A paramagnetic sample displays greater magnetisation when cooled because at the lower temperatures, the tendency to disrupt the alignment of magnetic dipoles decreases due to the reduced random thermal motion of atoms or molecules.

(ii) Bismuth is a diamagnetic element, so the magnetic field in the core will be slightly less than when the core is empty, because the diamagnetic substances are feebly magnetised in the opposite direction of magnetic field.

(iii) No, the permeability of a ferromagnetic material is not independent of the magnetic fields. By observing the hysteresis curve, the value of permeability is greater for lower fields

(iv) The magnetic field lines are always nearly normal to the surface of a ferromagnet at every point because the value of permeability for ferromagnetic substance is always greater than 1 ($\mu >> 1$). It is based on the conditions of B and H at the interface of two media in the hysteresis curve.

(v) Yes, the maximum possible magnetisation of a paramagnetic sample will be of the same order of magnitude as the magnetisation of a ferromagnet. Although, the condition of saturation for paramagnets, requires very high magnetising fields which cannot be achieved.

14. (i) To explain qualitatively the domain picture of the irreversibility in the magnetisation curve of a ferromagnet, we draw the hysteresis curve for ferromagnetic substance. We can observe that the magnetisation persists even when the external field is removed. This gives the idea of irreversibility of a ferromagnet.

(ii) As we know that, in hysteresis curve, the energy dissipated per cycle is directly proportional to the area of hysteresis loop. So, as according to the question, the area of hysteresis loop is more for carbon steel, thus carbon steel piece will dissipate greater heat energy.

(iii) The magnetisation of a ferromagnet depends not only on the magnetising field, but also on the history of magnetisation (i.e. how many times it was already magnetised in the past). Thus, the value of magnetisation of a specimen is a record of memory of the cycles of magnetisation, it had undergone. The system displaying such a hysteresis loop can thus act as a device for storing memory.

(iv) The ferromagnetic materials which are used for coating magnetic tapes in a cassette player or for building memory stores in the modern computer are ferrites. The most commonly ferrites used are $MnFe_2O_4$, $FeFe_2O_4$, $CoFe_2O_4$, $NiFe_2O_4$, etc

(v) To shield any space from magnetic field, surround the space with soft iron ring. As the magnetic field lines will be drawn into the ring, the enclosed region will become free of magnetic field.

15. (a) Given, magnetic moment, $M = 6$ J/T

Aligned angle, $\theta_1 = 60°$

External magnetic field, $B = 0.44$ T

(i) When the bar magnet is align normal to the magnetic field, i.e. $\theta_2 = 90°$

$\therefore$ Amount of work done in turning the magnet.

$$W = -MB(\cos\theta_2 - \cos\theta_1)$$
$$= -6 \times 0.44 (\cos 90° - \cos 60°)$$
$$= +6 \times 0.44 \times \frac{1}{2} \quad \left(\begin{array}{l}\because \cos 90° = 0 \\ \text{and } \cos 60° = 1/2\end{array}\right)$$
$$= 1.32 \text{ J}$$

(ii) When the bar magnet align opposite to the magnetic field, i.e. $\theta_2 = 180°$

$$\therefore W = -MB(\cos 180° - \cos 60°)$$
$$= -6 \times 0.44 \left(-1 - \frac{1}{2}\right) \quad (\because \cos 180° = -1)$$
$$= 6 \times 0.44 \times \frac{3}{2} = 3.96 \text{ J}$$

(b) We know that, torque,

$$\tau = \mathbf{M} \times \mathbf{B} = MB\sin\theta$$

For case (ii), $\theta = 180°$

[redacted]

$\therefore \quad \tau = MB\sin 180° \quad (\because \sin 180° = 0)$
$= 0$

$\therefore$ Amount of torque is zero for case (ii).

16. (i) Material X is paramagnetic substance. When a specimen of a paramagnetic substance is placed in a magnetising field, the lines of force prefer to pass through the specimen rather than through air. Thus, magnetic induction inside the sample is more than the magnetic intensity.

(ii) Material Y is ferromagnetic substance. These are the substances in which a strong magnetism is produced in the same direction as the applied magnetic field, these are strongly attracted by a magnet, exhibits highly concentrated lines of force.

17. According to Curie's law, $\chi_m = \frac{C}{T}$

As magnetic susceptibility, $\chi_m = \frac{I}{H}$

$\Rightarrow \quad I = \frac{M}{V}$ and $H = \frac{B}{\mu}$

$\Rightarrow \quad \frac{M/V}{B/\mu} = \frac{C}{T}$

$\Rightarrow \quad M = \frac{CV}{\mu}\left(\frac{B}{T}\right)$

For a given sample, $CV/\mu =$ constant

Thus, $M = \left(\frac{B}{T}\right)$

or $\frac{M_1}{M_2} = \frac{B_1/T_1}{B_2/T_2}$

Given, $B_1 = 0.84$ T, $B_2 = 0.98$ T
$T_1 = 4.2$ K, $T_2 = 2.8$ K

Thus, $\frac{M_1}{M_2} = \frac{0.84/4.2}{0.98/2.8} = \frac{4}{7}$

or $M_2 = \left(\frac{7}{4}\right) M_1$

Initial total magnetic moment of the sample,

$$M_1 = 15\% \text{ of } (2\times10^{24})(1.5\times10^{-23}) = 4.5 \text{ J/T}$$

Thus, $M_2 = \left(\frac{7}{4}\right) 4.5 = 7.9$ J/T

18. (i) Refer to text on page 236.

(ii) We can destroy the magnetism of a magnet
(a) by heating it.
(b) by applying magnetic field across it in reverse direction.

19. When a bar magnet is turned by 180°, then the null points are obtained on the equatorial line.

So, magnetic field on the equatorial line at a distance d' is given by

$$B' = \frac{\mu_0}{4\pi} \cdot \frac{M}{d'^3}$$

This magnetic field is equal to the horizontal component of the earth's magnetic field.

$$B' = \frac{\mu_0}{4\pi} \cdot \frac{M}{d'^3} = H \quad \ldots(i)$$

As we know that,

Magnetic field, $B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3} = H \quad \ldots(ii)$

From Eqs. (i) and (ii), we get

$$\frac{\mu_0}{4\pi} \cdot \frac{M}{d'^3} = \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3}$$

or $\frac{1}{d'^3} = \frac{2}{d^3}$

or $d'^3 = \frac{d^3}{2} = \frac{(14)^3}{2} \quad [\because d = 14 \text{ cm}]$

or $d' = \frac{14}{(2)^{1/3}} = 11.1$ cm

Thus, the null points are located on the equatorial line at a distance of 11.1 cm.

20. Distance of the null point from the centre of magnet,

$$d = 14 \text{ cm} = 0.14 \text{ m}$$

The earth's magnetic field, where the angle of dip is zero, is the horizontal component of the earth's magnetic field.

i.e. $H = 0.36$ gauss

Initially, the null points are on the axis of the magnet. We use the formula of magnetic field on axial line (consider that the magnet is short in length).

$$B_1 = \frac{\mu_0}{4\pi} \frac{2M}{d^3}$$

This magnetic field is equal to the horizontal component of the earth's magnetic field.

i.e. $B_1 = \frac{\mu_0}{4\pi} \cdot \frac{2M}{d^3} = H \quad \ldots(i)$

On the equatorial line of magnet at same distance d magnetic field due to the magnet,

$$B_2 = \frac{\mu_0}{4\pi} \cdot \frac{M}{d^3} = \frac{B_1}{2} = \frac{H}{2} \quad \ldots(ii)$$

The total magnetic field on equatorial line at this point (as given in question),

$$B = B_2 + B_1 = \frac{H}{2} + H$$
$$= \frac{3}{2}H = \frac{3}{2} \times 0.36$$
$$= 0.54 \text{ gauss}$$

The direction of magnetic field is in the direction of earth's field.

[redacted]

SUMMARY

- The phenomenon of attraction of small bits of iron, steel, cobalt, nickel, etc., towards the ore is called magnetism.
- Magnetic materials tend to point in the North-South direction. Like magnetic poles repel and unlike poles attract each other
- Cutting a bar magnet creates two smaller magnets. Therefore, magnetic poles cannot be separated, i.e. magnetic monopole does not exist
- **Force between two magnetic poles** is given by, $F = \frac{km_1m_2}{r^2}$

  where k is magnetic force constant and is given by

  $$k = \frac{\mu_0}{4\pi} = 10^{-7}$$
- The magnetic dipole moment of a magnetic dipole is given by

  $$\mathbf{M} = m \times 2l$$

  where, m is pole strength and $2l$ is dipole length directed from S to N. The SI unit of magnetic dipole moment is A-m^2 or JT^{-1}. It is a vector quantity and its direction is from South pole to North pole
- **Magnetic Dipole** is defined as two magnetic poles of equal and opposite strengths separated by a small distance. e.g. bar magnet, compass needle, etc
- **The Magnetic Field Lines** These are the imaginary lines which continuously represent the direction of magnetic field.
- **Magnetic Field Strength at a Point due to Bar Magnet** The force experienced by a hypothetical unit North pole placed at that point
  - (i) **When Point Lies on Axial Line of Bar Magnet** In this case, $B = \frac{\mu_0 2Md}{4\pi(d^2 - l^2)^2}$
  - (ii) **When Point Lies on Equatorial Line of a Bar Magnet** In this case, $B = \frac{\mu_0 M}{4\pi(d^2 + l^2)^{3/2}}$
- Torque on a bar magnet in a uniform magnetic field is

  $$\tau = MB\sin\theta = \mathbf{M} \times \mathbf{B}$$
- **Oscillation of a Freely Suspended Magnet** The oscillations of a freely suspended magnet (magnetic dipole) in a uniform magnetic field are SHM

  The time period of oscillation, $T = 2\pi\sqrt{\frac{I}{MB}}$
- **Potential energy** of a magnetic dipole in a magnetic field is given by $U = -MB\cos\theta = -\mathbf{M}\cdot\mathbf{B}$

  where, θ is the angle between **M** and **B**
- **Magnetism and Gauss' Law** The number of magnetic field lines leaving any closed surface is always equal to the number of magnetic field lines entering it
- **Magnetic Intensity** i.e. $H = \frac{B_0}{\mu_0}$
- **Intensity of Magnetisation** i.e. $I = \frac{M}{V}$
- **Magnetic Induction** (B) i.e. $B = \mu_0(H + I)$
- **Magnetic Susceptibility** i.e. $\chi_m = I/H$
- **Magnetic Permeability** i.e. $\mu = (B/H)$
- **Relative Magnetic Permeability** i.e. $\mu_r = (\mu/\mu_0)$
- **Relation between Relative Permeability** (μ_r) **and Magnetic Susceptibility** (χ_m) It is given by, $\mu_r = 1 + \chi_m$
- Magnetic materials are broadly classified as diamagnetic, paramagnetic and ferromagnetic. For diamagnetic materials, χ is negative and small and for paramagnetic materials, it is positive and small.
- The magnetic susceptibility of a ferromagnetic materials varies as

  $$\chi_m \propto \frac{1}{(T - T_c)} \text{ or } \chi_m = \frac{C}{(T - T_c)}$$

  where, C is a constant. It is known as **Curie-Weiss** law and T_c is called Curie temperature.

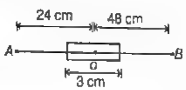
[redacted]

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. Cutting a bar magnet in half is like cutting a solenoid, such that we get two smaller solenoids with
   (a) weaker magnetic properties
   (b) strong magnetic properties
   (c) constant magnetic properties
   (d) Both (a) and (b)

2. A bar magnet of length 3 cm has points A and B along axis at a distance of 24 cm and 48 cm on the opposite ends. Ratio of magnetic fields at these points will be



   (a) 8 (b) 3 (c) 4 (d) $1/2\sqrt{2}$

3. A short bar magnet placed with its axis at 30° with an external field of 800 G experiences a torque of 0.016 Nm. The magnetic moment of the magnet is
   (a) 4 Am^2 (b) 0.5 Am^2
   (c) 2 Am^2 (d) 0.40 Am^2

4. A bar magnet has magnetic dipole moment M. Its initial position is parallel to the direction of uniform magnetic field B. In this position, the magnitudes of torque and force acting on it, respectively are **CBSE 2021 Term-I**
   (a) 0 and MB (b) MB and MB
   (c) 0 and 0 (d) $|M \times B|$ and 0

5. If a diamagnetic substance is brought near the North or the South-pole of a bar magnet, then it is
   (a) attracted by the both poles
   (b) repelled by both the poles
   (c) repelled by the North-pole and attracted by the South-pole
   (d) attracted by the North-pole and repelled by the South-pole

6. Ferromagnetism show their properties due to
   (a) filled inner subshells
   (b) vacant inner subshells
   (c) partially filled inner subshells
   (d) all the subshells equally filled

7. The relative permeability of a substance is 0.9999. The nature of substance will be
   (a) diamagnetic
   (b) paramagnetic
   (c) magnetic moment
   (d) intensity of magnetic field

8. Hysteresis loss is minimised by using
   (a) alloy of steel
   (b) shell type of core
   (c) thick wire which has low resistance
   (d) metal

ASSERTION AND REASON

Directions (Q. Nos. 9-19) *In the following questions; two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

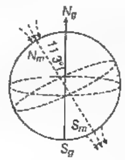
9. **Assertion** The true geographic North direction cannot be found by using a compass needle.
   Reason The magnetic meridian of the earth is along the axis of rotation of the earth.

10. **Assertion** The axis of the dipole does not coincide with the axis of rotation of the earth but is presently tilted by approximately 11.3° with respect to the later.

    Reason The magnetic poles are located where the magnetic field lines due to the dipole enter or leave the earth.

[redacted]

11. Consider the figure,
Assertion Unlike in the case of bar magnet, the field lines go into the earth at the North magnetic pole (N_m) and come out from the South magnetic pole (S_m).
Reason The magnetic North was the direction to which the North-pole of a magnetic needle pointed; the North-pole of a magnet was so named as it was the North seeking pole.



12. **Assertion** A magnetic needle, which is free to swing horizontally, would lie in the magnetic meridian and the North-pole of the needle would point towards the magnetic North-pole.
Reason The line joining the magnetic poles is tilted with respect to the geographic axis of the earth, the magnetic meridian at a point makes angle with the geographic meridian.

13. **Assertion** When magnetic field is applied to a diamagnetic substance, those electrons having orbital magnetic moment in the same direction slow down and those in the opposite direction speed up.
Reason This happens due to induced current in accordance with Lenz's law and the substance develops a net magnetic moment in direction opposite to that of the applied field and hence repulsion.

14. **Assertion** Susceptibility is defined or the ratio of intensity of magnetisation I to magnetic intensity H.
Reason Greater the value of susceptibility smaller value of intensity magnetisation I.

15. **Assertion** The pole of magnet cannot be separated by breaking into two pieces.
Reason The magnetic moment will be reduced to half when a magnet is broken into two equal pieces.

16. **Assertion** Substances which at room temperature retain their ferromagnetic property for a long period of time are called permanent magnets.
Reason Permanent magnet can be made by placing a ferromagnetic rod in a solenoid and passing current through it.

17. **Assertion** Ferromagnetic substances are those which gets strongly magnetised when placed in an external magnetic field.
Reason The individual atoms (or ions or molecules) in a ferromagnetic material possess a dipole moment as in a paramagnetic material.

18. **Assertion** A paramagnetic sample displays greater magnetisation (for the same magnetising field) when cooled.
Reason The magnetisation does not depends on temperature.

19. **Assertion** The poles of a bar magnet cannot be separated.
Reason Magnetic monopoles do not exist.

CASE BASED QUESTIONS

Directions (Q.No. 20) *This question is case study based question. Attempt any 4 sub-parts from this question. Each question carries 1 mark.*

20. **Dipole in Magnetic Fields**
To determine the magnitude of B accurately, a small compass needle of known magnetic moment m and moment of inertia I is allowed to oscillate in the magnetic field. This arrangement is shown in figure.

(i) The torque on the needle is
(a) $\tau = 2\mathbf{m} \times \mathbf{B}$ (b) $\tau = \mathbf{m} \times \mathbf{B}$
(c) $\tau = \mathbf{m} \times \mathbf{B}/2$ (d) $\tau = \mathbf{m} \times 2\mathbf{B}$

(ii) Which of the following represents a simple harmonic motion?
(a) $\frac{d^2\theta}{dt^2} = -\frac{mB}{I}\theta$
(b) $\frac{d\theta}{dt^2} = -\frac{mB}{I}\theta$
(c) $\frac{d^2\theta}{dt} = \frac{mB}{I}\theta$
(d) $\frac{d^2\theta}{dt^2} = \frac{mB}{I}\theta$

[redacted]

(iii) The time period of oscillation of the dipole is

(a) $2\pi\sqrt{\frac{2I}{mB}}$ (b) $2\pi\sqrt{\frac{I}{2mB}}$

(c) $4\pi\sqrt{\frac{I}{mB}}$ (d) $2\pi\sqrt{\frac{I}{mB}}$

(iv) The magnitude of the magnetic field if time period is T is

(a) $B=\frac{4\pi^2 I}{mT^2}$ (b) $B=\frac{2\pi^2 I}{mT^2}$

(c) $B=\frac{\pi^2 I}{2mT^2}$ (d) $B=\frac{3\pi^2 I}{2mT^2}$

(v) The magnetic potential energy U_m is given by

(a) $U_m=-\mathbf{m}\cdot\mathbf{B}$
(b) $U_m=\mathbf{m}\cdot\mathbf{B}$
(c) $U_m=2\mathbf{m}\cdot\mathbf{B}$
(d) $U_m=-2\mathbf{m}\cdot\mathbf{B}$

VERY SHORT ANSWER Type Questions

21. Name the physical quantity having unit J/T. CBSE Sample Paper

22. What is the basic difference between magnetic and electric field lines?

23. In a submarine, a compass becomes ineffective. Why?

24. Why is diamagnetism almost independent of temperature?

25. If a toroid uses bismuth for its core, will the field in the core be (slightly) greater or (slightly) less than when the core is empty?

26. One cannot write the proportionality $B=\mu H$ for the ferromagnets. Comment.

27. "Alkali halides are diamagnetic rather than paramagnetic." Explain why?

28. The magnetic susceptibility of magnesium at 300K is 1.2×10^5. At what temperature will its magnetic susceptibility become 1.44×10^5? [CBSE 2019]

29. The magnetic susceptibility of χ of a given material is -0.5. Identify the magnetic material. [CSBE 2019]

SHORT ANSWER Type Questions

30. What is the net magnetic moment of two identical magnets each of magnetic moment m_0 inclined at 60° with each other?

31. What are the magnetic field lines? State their properties. Why two such lines do not intersect each other?

32. A wire of length L is bent in the form of a circle of radius R and carries current I. What is its magnetic moment?

33. Derive an expression for the torque acting on a bar magnet placed in the uniform magnetic field.

34. Suppose you have two bars of identical dimensions, one made of paramagnetic substance and the another of diamagnetic substance. If you place these bars along a uniform magnetic field, show diagrammatically, what modifications in the field pattern would take place in each case?

LONG ANSWER Type I Questions

35. (a) State Gauss's law for magnetism. Explain its significance.
(b) Write the four important properties of the magnetic field lines due to a bar magnet. [CBSE 2019]

36. Write three points of differences between para-, dia- and ferro- magnetic materials, giving one example for each. [CBSE 2019]

37. Define magnetic susceptibility of a material. Name two elements, one having positive susceptibility and the other having negative susceptibility. What does negative susceptibility signify?

38. Draw a plot showing the variation of intensity of magnetisation with the applied magnetic field intensity for bismuth. Under what condition does a diamagnetic material exhibit perfect conductivity and perfect diamagnetism?

39. Explain the following:
(i) Diamagnetism is the result of induced magnetic dipole moments.
(ii) Hysteresis associated with a loss in electromagnetic energy.

[redacted]

40. Explain the phenomenon of hysteresis in magnetic materials. Draw a hysteresis loop showing remanence and coercive force.

LONG ANSWER Type II Question

41. (i) A bar magnet of magnetic moment M is aligned parallel to the direction of a uniform magnetic field B. What is the work done, to turn the magnets, so as to align its magnetic moment
(a) opposite to field direction and
(b) normal to field direction?
(ii) Steel is preferred for making permanent magnets, whereas soft iron is preferred for making electromagnets. Give one reason.

NUMERICAL PROBLEMS

42. Calculate the magnetic induction at a point 4 cm from the centre and along the equator of a bar magnet of length 6 cm and magnetic moment 0.26 A-m^2.

43. A bar magnet of length 0.1 m and a pole strength 10^{-4} A-m is placed in a magnetic field of 30 Wb/m^2 at an angle 30°. Determine the couple acting on it.

ANSWERS

1. (a) 2. (a) 3. (d) 4. (d) 5. (b)
6. (c) 7. (a) 8. (d)

9. (c) A compass is simply a needle shaped magnet that mounted so that it can rotate freely about a vertical axis When it is held in a horizontal plane, the North-pole end of the needle points, generally, towards the geomagnetic North-pole (really a South magnetic pole). Thus, true geographic North direction cannot be found by using a compass needle. Now, vertical plane passing through the magnetic axis of earth's magnet is called magnetic meridian.

10. (b)

11. (a) If one looks at the magnetic field lines of the earth one sees that unlike in the case of a bar magnet, the field lines go into the earth at the North magnetic pole (N_m) and come out from the South magnetic pole (S_m).
The convention arose because the magnetic North was the direction to which the North-pole of a magnetic needle pointed; the North-pole of a magnet was so named as it was the North seeking pole.
Thus, in reality, the North magnetic pole behaves like the South-pole of a bar magnet inside the earth and vice-versa.

12. (b) A magnetic needle which is free to swing horizontally would, lie in the magnetic meridian and the North-pole of the needle would point towards the magnetic North-pole.
The line joining the magnetic poles is tilted with respect to the geographic axis of the Earth, the magnetic meridian at a point makes angle with the geographic meridian.

13. (c)

14. (c) From the relation, susceptibility of the material is

$$\chi_m = \frac{I}{H} \Rightarrow I = \chi_m H$$

Thus, it is obvious that greater the value of susceptibility of a material greater will be the value of intensity of magnetisation i.e., more easily it can be magnetised

15. (b) 16. (b)
17. (c) 18. (b) 19. (a)

20. (i) (b) The torque on the needle is

$$\tau = \mathbf{m} \times \mathbf{B}$$

Magnitude of torque, $\tau = mB \sin\theta$

(ii) (a) Torque on the needle

$$\tau = mB\sin\theta$$

Here, τ is restoring torque and θ is the angle between $\mathbf{m}$ and $\mathbf{B}$.

Therefore, in equilibrium $I\dfrac{d^2\theta}{dt^2} = -mB\sin\theta$

Negative sign with $mB\sin\theta$ implies that restoring torque is in opposition to deflecting torque. For small values of θ in radians, we approximate $\sin\theta \approx \theta$ and we get

$$I\frac{d^2\theta}{dt^2} \approx -mB\theta \quad \text{or} \quad \frac{d^2\theta}{dt^2} = -\frac{mB}{I}\theta$$

This represents a simple harmonic motion.

(iii) (d) The square of the angular frequency is $\omega^2 = mB/I$ and the time period,

$$T = 2\pi\sqrt{\frac{I}{mB}} \quad \text{or} \quad B = \frac{4\pi^2 I}{mT^2}$$

(iv) (a) The magnitude of the magnetic field is $B = \dfrac{4\pi^2 I}{mT^2}$

(v) (a) An expression for magnetic potential energy can also be obtained on lines similar to electrostatic potential energy.
The magnetic potential energy U_m is given by

$$U_m = \int \tau(\theta)\, d\theta = \int mB\sin\theta\, d\theta$$
$$= (-mB\cos\theta) = -\mathbf{m}\cdot\mathbf{B}$$

[redacted]

21. The torque acting on a bar magnet placed in a uniform magnetic field is

$$\tau = \mathbf{M} \times \mathbf{B}$$

where, M is the magnetic dipole moment and B is the magnetic field

$$\Rightarrow \quad M = \frac{\tau}{B} = \text{Joule (J)/Tesla(T)}$$

$\therefore$ Magnetic dipole moment (M) has unit of J/T.

22. The magnetic field lines form continuous closed loops, whereas the electric field lines begin from a positive charge and end on the negative charge or escape to infinity.

23. The body of a submarine is made of steel and other magnetic substances which causes the compass needle to deviate from the magnetic meridian

24. Diamagnetism is independent of temperature because the value of susceptibility (a measure of relative amount of induced magnetism) is always negative.

25. Bismuth is a diamagnetic material. So, when it is kept in an external magnetic field the field lines are repelled and the field inside the material is reduced.

26. For ferromagnets, we cannot write $B = \mu H$ because the relation between B and H is not linear and it depends on the magnetic history of the sample. This phenomenon is called hysterisis.

27. The alkali halides are all diamagnetic because of the absence of unpaired electrons. So, they do not show paramagnetism.

28. The susceptibility of magnetic material is inversely proportional to temperature, i.e.

$$\chi_m \propto \frac{1}{T}$$

$$\therefore \quad \frac{\chi_m(T)}{\chi_m(300\,\text{K})} = \frac{300}{T} \Rightarrow T = \frac{300 \times 1.2 \times 10^5}{1.44 \times 10^5} = 250\,\text{K}$$

29. Substance having (small) negative value (-0.5) of magnetic susceptibility χ_m are diamagnetic.

30. $M = \sqrt{M_1^2 + M_2^2 + 2M_1M_2 \cos\theta}$

31. Refer to text on page 222.

32. As, $L = 2\pi R$

$$\Rightarrow \quad R = \frac{L}{2\pi}$$

$$\Rightarrow \quad M = IA = I \times \pi R^2$$

$$= I\pi \times \frac{L^2}{4\pi^2} = \frac{IL^2}{4\pi}$$

33. Refer to text on pages 224 and 225.

34. Refer to text on pages 236 and 237.

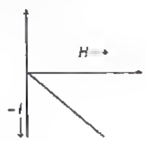
35. (a) Refer to text on page 227.
(Magnetism and Gauss' Law)
(b) Refer to text on page 222.
(Properties of Magnetic Field Lines)

36. **Difference between para-, dia- and ferro-magnetic materials**

Refer to text on pages 238 and 239.
(Comparative Study of Magnetic Materials)

37. Refer to text on page 235.

38. Here, intensity of magnetisation varies inversely with magnetic field strength i.e., $-I \propto H$ as shown in figure.



Refer to text on page 236.

39. (i) Refer to text on page 236.
(ii) Refer to text on page 238.

40. Refer to text on pages 237 and 238.

41. (i) $W = -MB(\cos\theta - \cos\theta_0)$
(ii) Refer to text on page 238.

42. Refer to Q. 29 on page 229.

43. Torque, $\tau = MB\sin\theta$

[redacted]

06

In the previous chapter, we have seen that a current carrying conductor when kept in a magnetic field, experiences force and torque. Now, in this chapter, we will study that the reverse phenomenon is also possible, i.e. if we rotate the conductor (coil) in a magnetic field, then the current will flow in it.

ELECTROMAGNETIC INDUCTION

Current can be induced in the coils when these coils are rotated in a magnetic field. This has led to the alternate ways of generating current. When electromagnetic induction was discovered, only source of emf available were those of chemical nature such as dry cells, but at present large-scale production and distribution of energy became possible because of this phenomenon of Electromagnetic Induction (EMI). Faraday and Henry independently discovered the principle of magnetically induced emfs and found methods to convert mechanical energy into electrical energy. EMI formed the principle of two important electrical devices namely, electric generator and transformer, which has revolutionised the life styles of mankind.

CHAPTER CHECKLIST
- Faraday's Laws and Motional Electromotive Force
- Self and Mutual Induction

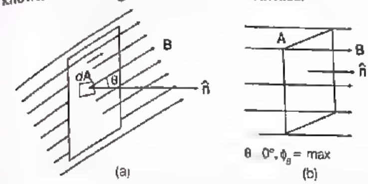
|TOPIC 1|
Faraday's Laws and Motional Electromotive Force

The phenomenon of generation of current or emf by changing the magnetic field is known as Electromagnetic Induction (EMI). The emf developed in the conductor by the process of EMI is known as **induced emf** and if the conductor is in the form of a closed loop, then the current flowing through the conductor is known as **induced current**. It is the reverse process of magnetic field production by electric current. The phenomenon of EMI was discovered by Michael Faraday in 1831, which is not merely of theoretical or academic interest but also of practical utility. We cannot imagine a world with no electricity, no electric lights, no trains, no telephones, no personal computers. Hence, today's civilisation owes a great deal to the discovery of EMI.

[redacted]

MAGNETIC FLUX

The total number of magnetic field lines crossing through any surface normally, when it is placed in a magnetic field is known as the magnetic flux of that surface.



Suppose a loop enclosing an area **A** is placed in a uniform magnetic field **B**. Then, the magnetic flux through the loop is given by

$$\boxed{\phi_B = \int \mathbf{B} \cdot d\mathbf{A}}$$

When the magnetic field is perpendicular to that plane of the loop, then magnetic flux will be

$$\phi_B = BA = \text{maximum value} \qquad ...(i)$$

This means that $B = \dfrac{\phi_B}{A}$, i.e. magnetic field strength B is the magnetic flux per unit area and is called **magnetic flux density** or **magnetic induction**.

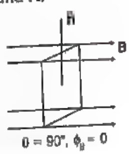
When the magnetic field B is not perpendicular to area A rather it is inclined at an angle θ with respect to the normal to the surface.

The magnetic flux becomes

$$\boxed{\begin{aligned}\phi_B &= \mathbf{B} \cdot \mathbf{A} = |\mathbf{B}|\,|\mathbf{A}| \cos\theta \\ &= BA\cos\theta\end{aligned}} \qquad ...(ii)$$

where, θ is the smaller angle between **B** and **A**.

If a plane is parallel to the magnetic field, then no field line will pass through it and the magnetic flux linked with that plane will be zero.



From Eq.(ii), it is clear that the flux can be varied by changing anyone or more of the terms **B**, **A** and θ.

The flux can also be altered by changing the shape of the coil (by stretching or by compressing) in a magnetic field or rotating a coil in a magnetic field, such that the angle θ between **B** and **A** changes.

The SI unit of magnetic flux (ϕ_B) is tesla-metre square, which is also called weber (abbreviated Wb).

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T-m}^2$$

1 weber is the amount of magnetic flux over an area of 1 m^2 held normal to a uniform magnetic field of 1 tesla (T).

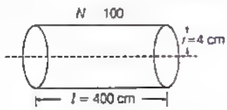
The CGS unit of magnetic flux (ϕ_B) is maxwell (Mx).

where, $\quad 1 \text{ weber} = 10^8 \text{ maxwell}$

Magnetic flux is a scalar quantity and its dimensional formula is $[ML^2T^{-2}A^{-1}]$.

EXAMPLE |1| A long solenoid of radius 4 cm, length 400 cm carries a current of 3 A. The total number of turns is 100. Assuming ideal solenoid, find the flux passing through a circular surface having centre on axis of solenoid of radius 3 cm and is perpendicular to the axis of solenoid (i) inside and (ii) at the end of solenoid.

Sol.



Number of turns per unit length is given by

$$n = \frac{N}{l} = \frac{100}{4} = 25 \text{ turns/m}$$

(i) Magnetic field of a solenoid at a point inside is

$$B = \mu_0 n i$$

Area of cross-section of the solenoid, $A = \pi r_1^2$ and $\theta = 0°$

Magnetic flux,

$$\begin{aligned}\phi_B &= BA\cos\theta \\ &= \mu_0 n i\, \pi r_1^2 \cos 0° \\ &= 4\pi \times 10^{-7} \times 25 \times 3 \times \pi \times (3 \times 10^{-2})^2 \\ &= 0.27 \times 10^{-6} \text{ Wb} \\ &= 0.27 \mu \text{ Wb}\end{aligned}$$

(ii) At the end, magnetic field of solenoid is

$$B = \frac{1}{2}\mu_0 n i$$

$$\therefore \quad \phi_B = \frac{0.27}{2} = 0.135\mu \text{ Wb}$$

EXPERIMENTS OF FARADAY AND HENRY

The discovery and understanding of electromagnetic induction are based on a long series of experiments carried out by Faraday and Henry.

It is the relative motion between the magnet and closed coil, which is responsible for generation or induction of electric current in the coil.

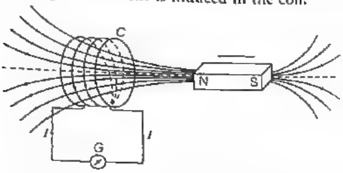
Whenever magnetic field linked with a closed coil changes, an emf is induced in the coil, which is called **induced emf**.

[redacted]

First Experiment
(Current Induced by a Magnet)

Consider a coil C of few turns of conducting material insulated from one another and is connected to a sensitive galvanometer G.

Whenever there is a relative motion between the coil and magnet, the galvanometer shows a deflection indicating that current is induced in the coil.

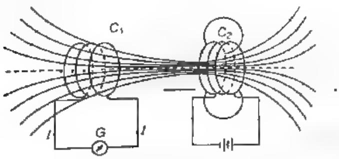


EMI with a stationary coil and moving magnet

Therefore, relative motion between the magnet and the coil generates electric current in the coil. So, the current generated is called **induced current.**

Second Experiment
(Current Induced by a Current)

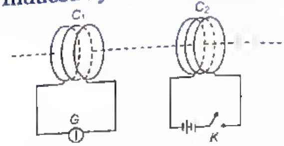
When the bar magnet is replaced by a second coil C_2 connected to a battery, the steady current in coil C_2 produces a steady magnetic field. As coil C_2 is moved towards coil C_1, the galvanometer shows a deflection. This indicates that electric current is induced in coil C_1.



EMI with one coil stationary and another moving

When coil C_2 is moved away, the galvanometer shows a deflection again but this time, in the opposite direction. The deflection lasts as long as coil C_2 is in the motion.

Third Experiment
(Current Induced by Changing Current)



EMI with changing current in one coil

The figure shows two coils C_1 and C_2 held stationary. Coil C_1 is connected to galvanometer G, while the second coil C_2 is connected to a battery through a tapping key K.

It is observed that the galvanometer shows a momentary deflection when the tapping key K is pressed. If the key is pressed continuously, there is no deflection in the galvanometer. When the key is released, a momentary deflection is observed again but in opposite direction. All experimental observations lead us to conclude that induced emf appears in a coil, whenever the amount of magnetic flux linked with the coil changes.

Note Presence of magnetic flux is not enough. The amount of magnetic flux linked with the coil must be change in order to produce an induced emf in the coil.

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

The two laws of electromagnetic induction given by Faraday are stated below

Faraday's First Law

Whenever the amount of magnetic flux linked with a circuit changes, an emf is induced in it. The SI unit of this induced emf is volt (V). The actual number of magnetic field lines passing through the circuit does not depend on the values of the induced emf. Induced current is determined by the rate at which the magnetic flux changes.

Faraday's Second Law

The magnitude of the induced emf in a circuit is equal to the rate of change of magnetic flux through the circuit. Mathematically, Faraday's second law can be expressed as,

Induced emf $\propto$ Rate of change of magnetic flux

[redacted]

i.e. $$e = \frac{-d\phi_B}{dt}$$

$$\left[\because \text{rate of change of magnetic flux} = \frac{\phi_2 - \phi_1}{t_2 - t_1}\right]$$

The negative sign in above relation indicates that the induced emf in the loop due to changing flux always opposes the change in the magnetic flux. In other words, the direction of induced emf is such that it always opposes the change in magnetic flux linked with the circuit. In the case of a closely wound coil of N turns, the change of flux associated with each turn is same. Therefore, the expression for the total induced emf is given by

$$\boxed{e = -N\frac{d\phi_B}{dt}}$$

The induced emf can be increased by increasing the number of turns N of a closed coil.

Induced Emf and Current

If N is the number of turns and R is the resistance of a coil, and the magnetic flux linked with its each turn changes by $d\phi$ in short time interval dt, then

Induced emf in the coil, $e = -N\frac{d\phi_B}{dt}$

Induced current flowing through the coil,

$$I = \frac{e}{R} = -\frac{N}{R}\cdot\frac{d\phi_B}{dt} \qquad \left[\because I = \frac{V}{R}\right]$$

Electric charge flows due to induced current,

$$q = Idt = \frac{N}{R}d\phi_B$$

EXAMPLE |2| **A magnetic field of flux density 10 T acts normally to the coil of 50 turns having 100 cm² area. Find emf induced, if the coil is removed from the magnetic field in 0.15 s.**

Sol. Given, $B = 10\text{ T}$, $N = 50$ turns,

$A = 100\text{ cm}^2 = 10^{-2}\text{ m}^2$, $dt = 0.15\text{ s}$

Magnetic flux linked with the coil initially,

$\phi_1 = NBA = 50 \times 10 \times 10^{-2} = 5\text{ Wb}$

But magnetic flux linked with the coil finally, i.e. (when removed from the magnetic field), $\phi_2 = 0$.

$\therefore$ Emf induced, $e = \frac{-d\phi}{dt} = -\left(\frac{\phi_2 - \phi_1}{dt}\right)$

$$= -\left(\frac{0-5}{0.15}\right)$$

$$= 33.33\text{ V}$$

EXAMPLE |3| **A square loop of side 10 cm and resistance 0.5 Ω is placed vertically in the East-West plane. A uniform magnetic field of 0.10 T is set up across the plane in the North-East direction. The magnetic field is decreased to zero in 0.70 s at a steady rate. Determine the magnitudes of induced emf and current during this time interval.**

Sol. Given, $B = 0.10\text{ T}$, $A = 100\text{ cm}^2 = 10^{-2}\text{ m}^2$, $\theta = 45°$

(as the angle made by area vector of the loop with magnetic field is 45°).

$R = 0.5\,\Omega$, $dt = 0.70\text{ s}$

Initial magnetic flux, $\phi_1 = BA\cos\theta$

$= 0.10 \times 10^{-2} \times \cos 45°$ $\left(\because \cos 45° = \frac{1}{\sqrt{2}}\right)$

$$= \frac{10^{-3}}{\sqrt{2}}\text{ Wb}$$

As, final magnetic flux, $\phi_2 = 0$

$\therefore$ Magnitude of induced emf is

$$e = -\frac{d\phi}{dt} = -\left(\frac{\phi_2 - \phi_1}{dt}\right) = -\left(\frac{0 - \frac{10^{-3}}{\sqrt{2}}}{0.70}\right)$$

$$= 10^{-3}\text{ V} = 1.0\text{ mV}$$

$\therefore$ Magnitude of induced current,

$$I = \frac{e}{R} = \frac{10^{-3}\text{ V}}{0.5\,\Omega}$$

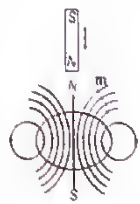
$$= 2 \times 10^{-3}\text{ A}$$

$$= 2\text{ mA}$$

LENZ'S LAW

According to this law, the polarity of emf induced is such that, it tends to produce a current which opposes the change in magnetic flux that produced it.

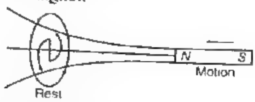
Illustration of Lenz's Law



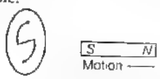
As the magnet is moved towards the loop, a particular amount of current is induced in the loop. The magnetic field is produced by the current, with magnetic dipole moment m oriented, so as to oppose the motion of magnet. Thus, the induced current must be counterclockwise as shown in the figure. When the North pole of a magnet moves towards a stationary loop, an induced current I flows in anti-clockwise sense as seen from the above, at which the magnet is located.

[redacted]

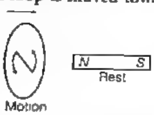
The anti-clockwise sense corresponds to the generation of North pole which opposes the motion of the approaching N-pole of the magnet.

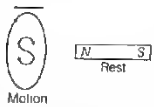


When the North pole of the magnet is moved away from the loop, the current I flows in the clockwise sense which corresponds to the generation of South pole as shown in figure. The induced South pole opposes the motion of the receding North pole.



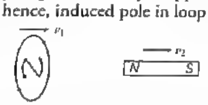
The directions of current induced in all above cases remain same, if instead of the loop, the magnet is kept stationary and loop is moved towards or away from it.



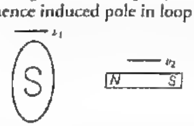


If both the loop and magnet are in relative motion w.r.t. each other, the induced pole on the loop facing magnet is according to Lenz's law.

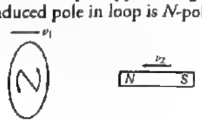
(i) When $v_1 > v_2$, i.e. the loop is approaching towards N-pole, hence, induced pole in loop is N-pole.



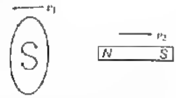
(ii) When $v_1 < v_2$, i.e. the loop is receding away from N-pole, hence induced pole in loop is S-pole.



(iii) When loop and magnet having opposite directions of velocities, then loop is approaching towards N-pole, hence, induced pole in loop is N-pole.



(iv) When loop and magnet having opposite directions of velocities and loop is receding away from N-pole, then induced pole in loop is S-pole.

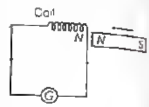


Thus, Lenz's law is used to find the direction of induced current in a closed circuit.

Note This topic has been frequently asked in previous years 2017, 2014, 2013, 2012, 2011, 2010

Lenz's Law and Conservation of Energy

Lenz's law is in accordance with the law of conservation of energy.

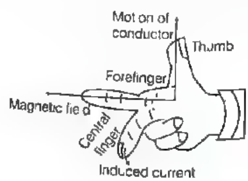


In the alongside circuit, when N-pole of magnet is moved towards the coil, the front face of the coil acquires North polarity. Thus, work has to be done against the force of repulsion in bringing the magnet closer to the coil.

When N pole of magnet is moved away, South pole develops on the front face of the coil. Therefore, work has to be done against the force of attraction in taking the magnet away from the coil. This mechanical work in moving the magnet w.r.t. the coil changes into electrical energy producing induced current. Hence, energy transformation takes place. When we do not move the magnet, work done is zero. Therefore, induced current is also zero. Hence, Lenz's law obeys the law of conservation of energy.

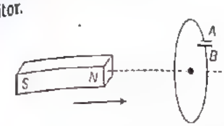
Fleming's Right Hand Rule

If we stretch the thumb, the forefinger and the central finger of right hand in such a way that all these three are mutually perpendicular to each other and if thumb represents the direction of motion of the conductor and the forefinger represents the direction of magnetic field, then central finger will represent the direction of induced current as shown below.



[redacted]

EXAMPLE |4| In the given figure, a bar magnet is quickly moved towards a conducting loop having a capacitor. Predict the polarity of the plates A and B of the capacitor.

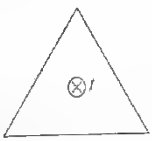


All India 2014

Hints: As, the magnet moves towards the coil, flux linked with the coil, increases, hence according to the Lenz's law, it will oppose the change

Sol. Here the North pole is approaching towards the magnet, so the induced current in the face of loop viewed from left side will flow in such a way that it will behave like North pole or South pole is developed in loop when viewed from right hand side of the loop. The flow of induced current is clockwise hence, A acquires positive polarity and B acquires negative polarity.

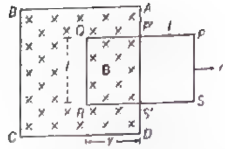
EXAMPLE |5| A current carrying straight wire passes inside a triangular coil as shown in figure. The current in the wire is perpendicular to paper inwards. Find the direction of the induced current in the loop, if current in the wire is increased.



Sol. Magnetic field lines around the current carrying wire are as shown in figure below. Since, the lines are tangential to the loop ($\theta = 90°$), the flux passing through the loop is zero, whether the current is increased or decreased. Hence, change in flux is zero. Therefore, induced current in the loop will be zero.

MOTIONAL ELECTROMOTIVE FORCE AND FARADAY'S LAW

Consider a uniform magnetic field B confined to the region $ABCD$ and a coil $PQRS$ is placed inside the magnetic field. At any time t, the part $P'Q = S'R = y$ of the coil is inside the magnetic field. Let l be the length of the arm of the coil.



Inducing current by changing the area of the rectangular loop

Area of the coil inside the magnetic field at time t,

$$\Delta S = QR \times RS' = ly$$

Magnetic flux linked with the coil at any time t,

$$\phi = B\Delta S = Bly$$

The rate of change of magnetic flux linked with the coil is given by

$$\frac{d\phi}{dt} = \frac{d}{dt}(Bly)$$

$$= Bl\frac{dy}{dt} = Blv \qquad \left[\because \frac{dy}{dt} = v\right]$$

where, v is the velocity with which the coil is pulled out of the magnetic field.

If e is the induced emf, then according to Faraday's law,

$$e = -\frac{d\phi}{dt} \text{ or } e = -Blv$$

From Fleming's right hand rule, the current due to induced emf will flow from the end R to Q, i.e. along $SRQP$ in the coil. This induced electromotive force (emf) Blv is called **motional emf.**

Note When a conducting rod of length l fixed at its one end moves on a circular path with angular velocity ω in a uniform magnetic field B normal to it, then induced emf produced in it is $e = \frac{1}{2}B\omega l^2$

EXAMPLE |6| A wire of length 0.3 m moves with a speed of 20 m/s perpendicular to the magnetic field of induction 1 Wb/m^2. Calculate the induced emf.

Sol. Given, velocity, $v = 20$ m/s

Length, $l = 0.3$ m

Angle, $\theta = 90°$

Magnetic field, $B = 1$ Wb/m^2

As, induced emf, $e = Blv$

$\therefore \quad e = 1 \times 0.3 \times 20 = 6$V

[redacted]

EXAMPLE |7| A wheel with 10 metallic spokes each 0.5 m long is rotated with a speed of 120 rev/min in a plane normal to the horizontal component of the earth's magnetic field H_E at a place. If $H_E = 0.4$ gauss at the place, what is the induced emf between the axle and the rim of the wheel? Take, 1 gauss $= 10^{-4}$ T.

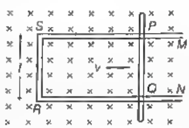
Sol. Induced emf $= \frac{1}{2} B\omega l^2 = (1/2)\,\omega BR^2$

$$\left[\begin{array}{l} \because \omega = 2\pi f = 2\pi \frac{120}{60} = 4\pi \\ \text{and } l = 2R \end{array}\right]$$

$$= (1/2) \times 4\pi \times 0.4 \times 10^{-4} \times (0.5)^2$$
$$= 6.28 \times 10^{-5} \text{ V}$$

The number of spokes is immaterial because the emf's across the spokes are in parallel.

ENERGY CONSIDERATION
(A QUANTITATIVE STUDY)



Let R be the resistance of movable arm PQ of the irregular conductor. We assume that the remaining arms QR, RS and SP have negligible resistances compared to R.

Thus, overall resistance of the rectangular loop is R and this does not change as PQ is moved.

Current I in the Loop is given by

$$I = \frac{e}{R} = \frac{Blv}{R} \quad \ldots\text{(i)}$$

Due to the presence of the magnetic field, there is a force on the arm PQ. This force is directed outwards in the direction opposite to the velocity of the rod.

The magnitude of this force is given by magnetic force

i.e. $\quad F = IlB \;$ or $\; \boxed{F = \frac{B^2 l^2 v}{R}}$

Alternatively, the arm PQ is being pushed with a constant speed v. Power required to do this is given by

$$P = Fv \text{ or } \boxed{P = \frac{B^2 l^2 v^2}{R}} \quad \ldots\text{(ii)}$$

The agent that does this work is mechanical energy.

This mechanical energy is dissipated as joule heat and is given by

$$P_J = I^2 R \text{ or } P_J = \left(\frac{Blv}{R}\right)^2 R \text{ or } \boxed{P_J = \frac{B^2 l^2 v^2}{R}}$$

This is identical to Eq (ii).

Thus, mechanical energy, which was required to move the arm PQ is converted into electrical energy and then to thermal energy.

Note The magnetic flux linked with a loop does not change when
(i) magnet and loop are moving with the same velocity
(ii) magnet is rotated around its axis without changing its distance from the loop
(iii) loop is moved in a uniform magnetic field and the whole of the loop remains in the field

Induced Quantities and Their Formulae

S.No.	Name of the quantity	Formula	Circuit open/ closed	Dependence upon resistance	SI unit
1.	Induced emf	$e = -\frac{d\phi}{dt}$	Open or closed	No	volt
2.	Induced current	$I = -\frac{d\phi}{Rdt}$	Closed	Yes	ampere
3.	Charge flown due to induced current	$dq = -\frac{d\phi}{R}$	Closed	Yes	coulomb
4.	Power required to pull a loop out of a magnetic field	$P = \frac{B^2 l^2 v^2}{R}$	Open or closed	Yes	watt

Induced Current in a Circuit

If R is the electrical resistance of the circuit, then induced current in the circuit is given by

$$I = \frac{e}{R}$$

If induced current is produced in a coil rotated in a uniform magnetic field, then

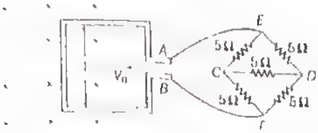
$$I = \frac{NBA\omega \sin\omega t}{R} = I_0 \sin\omega t$$

where, $I_0 = \frac{NBA\omega}{R}$ = peak value of induced current

N = number of turns in the coil,
B = magnetic field,
ω = angular velocity of rotation
and A = area of cross-section of the coil.

EXAMPLE |8| A square metal wire loop of side 20 cm and resistance 2 Ω is moved with a constant velocity v_0 in a uniform magnetic field of induction $B = 1 \text{ Wb/m}^2$ as shown in the figure. The magnetic field lines are perpendicular to the plane of the loop. The loop is connected to a network of resistance each of value 5 Ω. The resistances of the lead wires BF and AE are negligible. What should be the speed of the loop, so as to have a steady current of 2 mA in the loop? Give the direction of current in the loop.

[redacted]



Sol. From the figure, we see that, network $CEDF$ is balanced Wheatstone bridge, so no current will flow in branch CD.

So, the equivalent resistance of $CEDF$ network is

$$R_{eq} = \frac{10 \times 10}{10+10} = 5\,\Omega$$

Resistance of loop $= 2\,\Omega$

$$R_{total} = 2 + R_{eq} = 2 + 5 = 7\,\Omega$$

We know that, induced emf, $e = Bv_0 l$

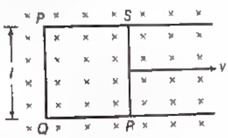
and induced current, $I = \frac{e}{R_{total}} = \frac{Bv_0 l}{7}$

$$\Rightarrow \qquad 2 \times 10^{-3} = \frac{1 \times v_0 \times 0.2}{7}$$

$$\Rightarrow \qquad v_0 = 7 \times 10^{-2}\ \text{m/s} = 7\ \text{cm/s}$$

As flux is decreasing, so induced current I will be clockwise.

EXAMPLE |9| Figure shows a rectangular conducting loop $PQRS$ in which arm RS of length l is movable. The loop is kept in a uniform magnetic field B directed downward perpendicular to the plane of the loop. The arm RS is moved with a uniform speed v.



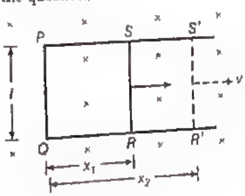
Deduce the expression for

(i) the emf induced across the arm RS

(ii) the external force required to move the arm and

(iii) the power dissipated as heat. **All India 2009**

Sol. According to the question,



(i) Let RS moves with speed v rightward and also RS is at distances x_1 and x_2 from PQ at instants t_1 at t_2, respectively.

$\therefore$ At t_1, flux linked with loop 1, i.e. $PQRS$, $\phi_1 = B(lx_1)$

Similarly, at instant t_2, flux linked with loop 2, i.e. $PQR'S'$, $\quad \phi_2 = B(lx_2)$

$\therefore$ Change in flux, $\Delta\phi = \phi_2 - \phi_1 = Bl(x_2 - x_1) = Bl\Delta x$

$$\Rightarrow \qquad \frac{\Delta\phi}{\Delta t} = Bl\frac{\Delta x}{\Delta t} = Blv \qquad \left[\because v = \frac{\Delta x}{\Delta t}\right]$$

By Faraday's law, magnitude of induced emf, $e = vBl$

(ii) If resistance of loop is R, then $I = \frac{vBl}{R}$

$$\therefore \text{ Magnetic force} = IBl \sin 90^\circ = \left(\frac{vBl}{R}\right)Bl$$

$$= \frac{vB^2l^2}{R} \qquad [\because \sin 90^\circ = 1]$$

$\therefore$ External force must be equal to magnetic force and in opposite directions

$$\therefore \text{ External force} = \frac{vB^2l^2}{R}$$

(iii) As, $\quad P = I^2R = \left(\frac{vBl}{R}\right)^2 \times R = \frac{v^2B^2l^2}{R^2} \times R$

$$\therefore \qquad P = \frac{v^2B^2l^2}{R}$$

EDDY CURRENTS

The currents induced in bulk pieces of conductors, when the magnetic flux linked with it changes, are known as eddy currents. These currents are always produced in a plane, perpendicular to the direction of magnetic field. They show both heating and magnetic effects.

The magnitude of eddy current is given by

$$I = \frac{\text{Induced emf}}{\text{Resistance}} = \frac{e}{R}$$

According to Faraday's law, $e = -\frac{d\phi}{dt}$, then $I = -\frac{d\phi/dt}{R}$

The direction of eddy currents can be given by Lenz's law or by Fleming's right hand rule. However, their flow patterns resemble swirling eddies in water. That is why, they are called eddy currents. These were discovered by **Foucault** in 1895 and hence, they are also named as **Foucault current**. e.g. When we move a metal plate out of a magnetic field, the relative motion of the field and the conductor again induces a current in the conductor. The conduction electrons build up the induced current whirl around within the plate as, if they were caught in an eddy of water. This is called the eddy current.

[redacted]

Undesirable Effects of Eddy Currents

Eddy currents are produced inside the iron cores of the rotating armatures of electric motors and dynamos and also in the cores of transformers, which experience flux changes, when they are in use. They cause unnecessary heating and wastage of power. The heat produced by eddy currents may even damage the insulation of coils. They are minimised by using laminations of metal to make a metal core. The laminations are separated by an insulating material. The plane of the laminations must be arranged parallel to the magnetic field, so that they cut across the eddy current paths. This arrangement reduces the strength of eddy currents.

Applications of Eddy Currents

Eddy currents are useful in many ways. Some of the important applications of eddy currents are as given below

(i) **Electromagnetic damping** In order to immediately bring the moving coil of a galvanometer to rest, we make the use of electromagnetic damping which uses eddy currents to bring the coil to rest. When the coil oscillates, the eddy currents generated in the core oppose the motion and bring the coil to rest.

(ii) **Induction furnace** In this, high temperature can be produced by using eddy currents. We generally use induction furnace in preparation of alloys by melting the constituents of metal. A coil is wound over the metal which needs to be melted and through the coil, we pass high frequency alternating current.

The eddy current generated in the metal produces high temperature to melt the metal.

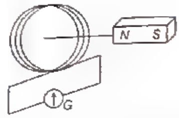
(iii) **Electric power meters** Old electric power meters (analog type) had a metallic disc. The disc rotates due to generation of eddy currents which are produced due to sinusoidally varying currents in the coil.

(iv) **Magnetic braking in electronic trains** Some electric powered trains make use of strong electromagnets which are situated above the rails. These electromagnets are used to produce eddy currents in the rails which oppose the motion of the train and thus stop it. In this case, as there is no mechanical linkage, the braking effect is smooth.

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. Current in the coil is larger



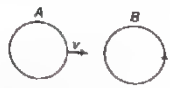
(a) when the magnet is pushed towards the coil faster
(b) when the magnet is pulled away the coil faster
(c) Both (a) and (b)
(d) Neither (a) nor (b)

2. The instantaneous magnetic flux linked with a coil is given by $\phi = (5t^3 - 100t + 300)$ Wb. The emf induced in the coil at time $t = 2$ s is

(a) -40 V (b) 40 V
(c) 140 V (d) 300 V

3. There are two coils A and B as shown in figure. A current starts flowing in B as shown, when A is moved towards B and stops when A stops moving. The current in A is counter clockwise. B is kept stationary when A moves. We can infer that NCERT Exemplar

(a) there is a constant current in the clockwise direction in A
(b) there is a varying current in A
(c) there is no current in A
(d) there is a constant current in the counter clockwise direction in A



4. A horizontal straight wire 20 m long extending from east to west is falling with a speed of 5.0 ms^{-1} at right angles to the horizontal component of the earth's magnetic field 0.30×10^{-4} Wbm^{-2}. The instantaneous value of the emf induced in the wire will be

(a) 6.0 mV (b) 3 mV
(c) 4.5 mV (d) 1.5 mV

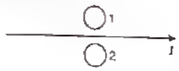
[redacted]

VERY SHORT ANSWER Type Questions

5. Two coils of wire A and B are placed mutually perpendicular. When a current induced is changed in any one coil, will the current induced in another coil?

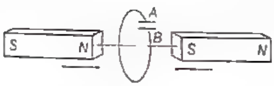
6. A long straight current carrying wire passes normally through the centre of circular loop. If the current through the wire increases, will there be an induced emf in the loop? Justify. **Delhi 2017**

7. What is the direction of induced currents in metal rings 1 and 2, when current I in the wire is increasing steadily?

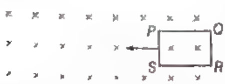


All India 2017

8. In the figure given, mark the polarity of plates A and B of a capacitor when the magnets are quickly moved towards the coil. **All India 2017 C**

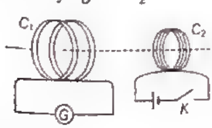


9. The closed loop $PQRS$ of wire is moved into a uniform magnetic field at right angles to the plane of the paper as shown in the figure. Predict the direction of the induced current in the loop. **Foreign 2012**



SHORT ANSWER Type Questions

10. A current is induced in coil C_1 due to the motion of current carrying coil C_2.



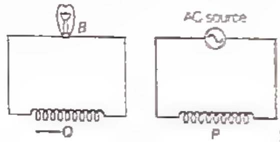
(i) Write any two ways by which a large deflection can be obtained in the galvanometer G.

(ii) Suggest an alternative device to demonstrate the induced current in place of a galvanometer. **Delhi 2011**

11. A coil Q is connected to low voltage bulb B and placed near another coil P as shown in the figure. Give reasons to explain the following observations

(i) The bulb B lights.

(ii) Bulb gets dimmer, if the coil Q is moved towards left.



Delhi 2010

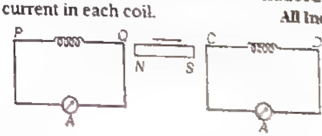
12. Two identical loops, one of copper and the other of aluminium are rotated with the same angular speed in the same magnetic field. Compare

(i) the induced emf and

(ii) the current produced in the two coils. Justify your answer. **All India 2010**

13. State Lenz's law. A metallic rod held horizontally along East-West direction, is allowed to fall under gravity. Will there be an emf induced at its ends? Justify your answer. **Delhi 2013**

14. A bar magnet is moved in the direction indicated by the arrow between two coils PQ and CD. Predict the directions of induced current in each coil. **All India 2012**



15. A rectangular loop of length l and breadth b is placed at distance of x from infinitely long wire carrying current i such that the direction of current is parallel to breadth. If the loop moves away from the current wire in a direction perpendicular to it with a velocity v, what will be the magnitude of emf in the loop?

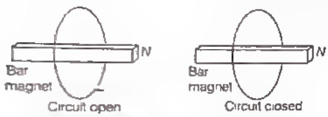
16. A metallic rod of length L is rotated with angular frequency of ω with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius L, about an axis passing through the centre and perpendicular to the plane of the ring. A constant and a uniform magnetic field B parallel to the axis is present

[redacted]

everywhere. Deduce the expression for the emf between the centre and the metallic ring.
Delhi 2012

17. Why is the coil of dead beat galvanometer wound on a metal frame?

18. Consider a magnet surrounded by a wire with an ON/OFF switch as shown in the figure. If the switch is thrown from the OFF position (open circuit) to the ON position (closed circuit), will a current flow in the circuit? Explain.
NCERT Exemplar



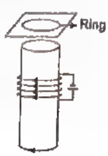
Hints : The magnetic flux linked with a uniform surface area A in a uniform magnetic field is given by $\phi = \mathbf{B} \cdot \mathbf{A} = BA \cos\theta$. So, flux linked will changes, only when either B or A or the angle between B and A changes

19. A wire in the form of tightly wound solenoid is connected to a DC source and carries a current I. If the coil is stretched, so that there are gaps between successive elements of the spiral coil, will the current increase or decrease? Explain.
NCERT Exemplar

Hints: Here, the application of Lenz's law is tested through this problem.

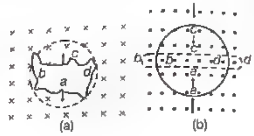
20. A solenoid is connected to a battery, so that a steady current flows through it. If an iron core is inserted into the solenoid, will the current increase or decrease? Explain.
NCERT Exemplar

21. (i) A metal ring is held horizontally and bar magnet is dropped through the ring with its length along the axis of the ring. What will be the acceleration of a falling magnet?
(ii) Consider a metal ring kept on top of a fixed solenoid (say on a cardboard) (see figure). The centre of the ring coincides with the axis of the solenoid. If the current is suddenly switched ON, the metal ring jumps up. Explain.
NCERT Exemplar



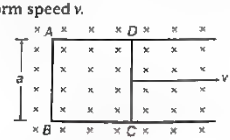
LONG ANSWER Type I Questions

22. Use Lenz's law to determine the direction of induced current in the situations described by figure.
(i) A wire of irregular shape turning into a circular shape.
(ii) A circular loop being deformed into a narrow straight wire.
NCERT



23. A metallic rod of length l is moved perpendicular to its length with velocity v in a magnetic field $\mathbf{B}$ acting perpendicular to the plane in which rod moves. Derive the expression for the induced emf. **All India 2017C**

24. Figure shows a rectangular conducting loop $ABCD$ in which arm CD of length a is movable. The loop is kept in a uniform magnetic field B directed downward perpendicular to the plane of the loop. The arm CD is moved with a uniform speed v.



Deduce an expression for
(i) the emf induced across the arm CD
(ii) the external force required to move the arm and
(iii) the power dissipated as heat.

25. (i) A rod of length l is moved horizontally with a uniform velocity v in a direction perpendicular to its length through a region in which a uniform magnetic field is acting vertically downward. Derive the expression for the emf induced across the ends of the rod.
(ii) How does one understand this motional emf by invoking the Lorentz force acting on the free charge carriers of the conductor? Explain. **All India 2014**

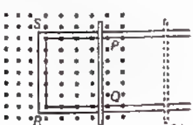
[redacted]

26. A metallic rod of length l is rotated with a frequency ν with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius r, about an axis passing through the centre and perpendicular to the plane of the ring.
A constant uniform magnetic field B parallel to the axis is present everywhere. Using Lorentz force, explain how emf is induced between the centre and the metallic ring and hence obtain the expression for it? **Delhi 2013**

LONG ANSWER Type II Questions

27. A metallic rod of length l and resistance R is rotated with a frequency ν, with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius l, about an axis passing through the centre and perpendicular to the plane of the ring. A constant and a uniform magnetic field B parallel to the axis is present everywhere.
   (i) Derive the expression for the induced emf and the current in the rod.
   (ii) Due to the presence of the current in the rod and of the magnetic field, find the expression for the magnitude and direction of the force acting on this rod.
   (iii) Hence, obtain the expression for the power required to rotate the rod. **All India 2014C**

28. State Faraday's law of electromagnetic induction. Figure shows a rectangular conductor $PQRS$ in which the conductor PQ is free to move in a uniform magnetic field B perpendicular to the plane of the paper. The field extends from $x = 0$ to $x = b$ and is zero for $x > b$. Assume that only the arm PQ possesses resistance r. When the arm PQ is pulled outward from $x = 0$ to $x = 2b$ and is then moved backward to $x = 0$ with constant speed v, obtain the expressions for the flux and the induced emf.
Sketch the variation of these quantities with distance $0 \le x \le 2b$. **All India 2010**



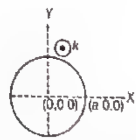
NUMERICAL PROBLEMS

29. A rectangular loop of area 20 cm × 30 cm is placed in magnetic field of 0.3 T with its plane
   (i) normal to the field
   (ii) inclined 30° to the field and
   (iii) parallel to the field.
   Find the flux linked with the coil in each case.

30. The magnetic flux through a coil perpendicular to the plane is given by $\phi = 5t^3 + 4t^2 + 2t$.
Calculate induced emf through the coil at $t = 2$s.

31. A circular coil of radius 10 cm, 500 turns and resistance 2 Ω is placed with its plane perpendicular to the horizontal component of the earth's magnetic field. It is rotated about its vertical diameter through 180° in 0.25 s. Estimate the magnitude of the emf and current induced in the coil. Horizontal component of the earth's magnetic field at the place is 3×10^{-5} T. **NCERT Intext**

32. A magnetic field in a certain region is given by $\mathbf{B} = B_0 \cos(\omega t)\hat{\mathbf{k}}$ and a coil of radius a with resistance R, is placed in the xy-plane with its centre at the origin in the magnetic field as shown in the figure. Find the magnitude and the direction of the current at $(a, 0, 0)$ at
$t = \frac{\pi}{2\omega}$, $t = \frac{\pi}{\omega}$ and $t = \frac{3\pi}{2\omega}$.



NCERT Exemplar

33. A wheel with 15 metallic spokes each 60 cm long, is rotated at 360 rev/min in a plane normal to the horizontal component of the earth's magnetic field. The angle of dip at that place is 60°. If the emf induced between rim of the wheel and the axle is 400 mV, calculate the horizontal component of the earth's magnetic field at the place.
How will the induced emf change, if the number of spokes is increased? **All India 2017C**

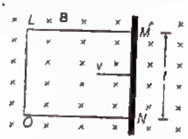
[redacted]

34. A horizontal straight wire 10 m long extending from East to West is falling with a speed of 5.0 m/s, at right angles to the horizontal component of the earth's magnetic field, 0.30×10^{-4} Wb/m^2.
(i) What is the instantaneous value of the emf induced in the wire?
(ii) What is the direction of the emf?
(iii) Which end of the wire is at the higher electrical potential? NCERT

35. A jet plane is travelling towards West at a speed of 1800 km/h. What is the voltage difference developed between the ends of the wing having a span of 25 m, if the earth's magnetic field at the location has a magnitude of 5×10^{-4} T and the dip angle is 30°? NCERT

36. A 1 m long conducting rod rotates with an angular frequency of 400 rad/s about an axis normal to the rod passing through its one end. The other end of the rod is in contact with a circular metallic ring. A constant magnetic field of 0.5 T parallel to the axis exists everywhere. Calculate the emf developed between the centre and the ring. NCERT

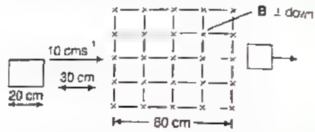
37. A rectangular conductor *LMNO* is placed in a uniform magnetic field of 0.5 T. The field is directed perpendicular to the plane of the conductor.



When the arm *MN* of length 20 cm is moved towards left with a velocity of 10 ms^{-1}, calculate the emf induced in the arm. Given, the resistance of the arm to be 5 Ω (assuming that other arms are of negligible resistance), find the value of the current in the arm. All India 2013

38. A rectangular loop of sides 8 cm and 2 cm with a small cut is moving out of a region of a uniform magnetic field of magnitude 0.3 T directed normal to the loop. What is the voltage developed across the cut, if velocity of loop is 1 cms^{-1} in a direction normal to the
(i) longer side?
(ii) shorter side of the loop?
For how long does the induced voltage last in each case? NCERT

39. A square loop of side 20 cm is initially kept 30 cm away from a region of a uniform magnetic field of 0.1 T as shown in the figure. It is then moved towards the right with a velocity of 10 cm s^{-1} till it goes out of the field. Plot a graph showing the variation of
(i) magnetic flux (ϕ) through the loop with time (t).
(ii) induced emf (ε) in the loop with time t.
(iii) induced current in the loop, if it has resistance of 0.1 Ω.



40. A square loop of side 12 cm with its sides parallel to X and Y-axes is moved with a velocity of 8 cm/s in the positive x-direction in an environment containing a magnetic field in the positive z-direction. The field is neither a uniform in space nor constant in time. It has a gradient of 10^{-3} T/cm along the negative x-direction (i.e. it increases by 10^{-3} T/cm as one moves in the negative x-direction) and it is decreasing in time at the rate of 10^{-3} T/s. Determine the direction and magnitude of the induced current in the loop, if its resistance is 4.50 mΩ. NCERT

41. A circular coil of radius 8.0 cm and 20 turns rotated about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of magnitude 3×10^{-2} T. Obtain the maximum and average emf induced in the coil. If the coil forms a closed loop of resistance 10 Ω, then calculate the maximum value of current in the coil. Calculate the average power loss due to joule heating. Where does this power come from? NCERT

[redacted]

HINTS AND SOLUTIONS

1. (c) Current will be larger, when the magnet is pushed faster towards the coil, also current is large when magnet is pulled faster away but now it is in opposite direction.

2. (b) Given, $\phi = (5t^3 - 100t + 300)$, $t = 2$ s

Induced electromotive force,

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(5t^3 - 100t + 300)$$

$$e = -5\times 3t^2 + 100 = -5\times 3(2)^2 + 100$$

$$= -5\times 12 + 100 = -60 + 100 = 40\text{ V}$$

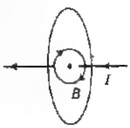
3. (d) When the A stops moving the current in B become zero, it possible only if the current in A is constant. If the current in A would be variable, there must be an induced emf (current) in B even if the A stops moving.

4. (b) Induced emf across the ends of wire

$$e = B_H lv = 0.30\times 10^{-4}\times 20\times 5 = 3\text{ mV}$$

5. No, this is because the magnetic field due to the current in coil (A or B) will be parallel to the plane of the other coil (B or A). Hence, the magnetic flux linked with the other coil will be zero and so no current will be induced in it.

6. The flux created by straight current carrying wire is depicted in the figure.



As, induced emf (e) $\propto$ rate of change of magnetic flux (ϕ_B)

and $\phi_B = \mathbf{B}\cdot\mathbf{A} = BA\cos\theta$

Here, $\mathbf{B}\perp\mathbf{A} \Rightarrow \phi_B = BA\cos 90° = 0$

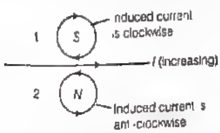
So, induced emf = 0

Hence, a change in current of wire will not create any emf in the loop.

7. Current in the wire is steadily increasing, so the induced current in rings 1 and 2 will flow in such a way that it opposes the increase of current.

So, it will flow in same direction. Now, from the figure, it is clear that the direction of induced current in

(i) ring 1 is clockwise, (ii) ring 2 is anti-clockwise.



8. From Lenz's law, the direction of induced current is in clockwise sense. This implies that plate A of the capacitor is at the higher potential than plate B, i.e. B will be negative plate, while A will be positive plate.

9. Since magnetic flux increases, when the loop moves into a uniform magnetic field. So, the induced current should oppose this increase. Thus, the flow will be from $QPSRQ$, i.e. anti-clockwise.

10. (i) Large deflection in the galvanometer can be obtained when change in magnetic flux is fast. So, according to the diagram given in question,
   (a) by moving quickly, the coil C_2 towards C_1 or by moving quickly the coil C_2 away from C_1.
   (b) by switching off and on the key.

   (ii) Alternating device in place of galvanometer can be LED or bulb.

11. (i) Due to varying current in P, the flux linked with P change and hence Q changes, which in turn induces the emf in Q and bulb B lights, where P and Q are coils.

   (ii) When Q is moved left or it goes away from P, the lesser flux charge takes place in Q. This leads to decrease the value of rate of change of magnetic flux and hence, lesser emf and bulb B gets dimmer.

12. (i) The induced emf in both the loops will be same as areas of the loop and time periods are same as they are identical and rotated with same angular speed.

   (ii) The current induces in Cu coil is more than Al coil as Cu coil has lesser resistance and $I \propto \frac{1}{R}$ (for the same voltage).

13. Lenz's law states that the polarity of induced emf is such that, it tends to produce a current which opposes the change in magnetic flux that produced it.

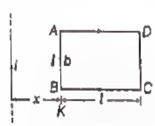
Yes, emf will be induced in the rod as there is change in magnetic flux.

14. From the figure, it is clear that North pole of the magnet is moving away from coil PQ, so the direction of current at end Q will flow in such a way that it will oppose the away moment of North pole, so it has to act as South pole. Hence, the direction of current will be clockwise.

Again, the South pole is approaching towards coil CD, so end C of the coil will act as South pole (to oppose the approaching South pole). Hence, the direction of current will be clockwise.

When a metallic rod held horizontally along East-West direction, is allowed to fall freely under gravity, i.e. fall from North to South direction, the intensity of magnetic lines of the earth's magnetic field changes through it, i.e. the magnetic flux changes and hence the induced emf in it. When we increase the number of turns, the induced emf will increase because induced emf is directly proportional to the number of turns.

15. Since, loop is moving away from the wire, so the direction of current in the loop will be as shown in the figure.



Net magnetic field on the loop due to wire,

[redacted]

$$B = \frac{\mu_0 I}{2\pi}\left[\frac{1}{x} - \frac{1}{l+x}\right] = \frac{\mu_0 Il}{2\pi x(l+x)}$$

So, the magnitude of the emf in the loop,

$$e = vBb = \frac{\mu_0 Il v b}{2\pi x(l+x)}$$

16. Angular velocity of rod, $\omega = \frac{2\pi}{T}$, where T = time period

$\therefore$ Charge in flux in one revolution $= BA = B(\pi L^2)$

According to Faraday's law of EMI, magnitude of induced emf,

$$e = \frac{\Delta\phi}{\Delta T} = \frac{B\pi L^2}{T} = \frac{B\pi L^2}{\left(\frac{2\pi}{\omega}\right)} \qquad \left[\because T = \frac{2\pi}{\omega}\right]$$

$$\therefore \qquad = \frac{1}{2}B\omega L^2$$

which is the required expression.

17. On switching ON, the current in a galvanometer, the coil of the galvanometer does not come to rest immediately. It oscillates about its equilibrium position but the coil of a dead beat galvanometer comes to rest immediately. It is due to the reason that the eddy currents are set up in the metallic frame, over which the coil is wound and the eddy currents oppose the oscillatory motion of the coil.

18. When the switch is thrown from the OFF position (open circuit) to the ON position (closed circuit), then neither B nor A and the angle between B and A does not change. Thus, no change in magnetic flux linked with coil occur, hence no electromotive force is produced and consequently, no current will flow in the circuit.

19. When the coil is stretched, so that there are gaps between successive elements of the spiral coil, i.e. the wires are pulled apart which lead to the flux leakage through the gaps. According to Lenz's law, the emf produced must oppose this decrease, which can be done by an increase in current. So, the current will increase.

20. When the iron core is inserted in the current carrying solenoid, the magnetic field increases due to the magnetisation of iron core and consequently, the flux increases. According to Lenz's law, the emf produced must oppose this increase in flux, which can be done by making decrease in current. So, the current will decrease.

21. (i) As the magnet falls, the magnetic flux linked with the ring increases. This induces emf in the ring which opposes the motion of the falling magnet, hence $a < g$.

(ii) When current is suddenly switched ON, magnetic flux linked with the solenoid and thus, with metal ring increases. Current is induced in the ring in anti-clockwise direction (as seen from top of the ring). Since, the direction of flow of current in the ring is opposite to the current in the solenoid, therefore they will repel each other and the ring jumps up.

22. (i) Here, the direction of magnetic field is perpendicularly inwards to the plane of paper. If a wire of irregular shape turns into a circular shape then its area increases (therefore the circular loop has greater area than the loop of irregular shape), so that the magnetic flux linked also increases. Now, the induced current is produced in a direction such that it decreases the magnetic field, i.e. the current will flow in such a direction, so that the wire forming the loop is pulled inward in all directions (to decrease the area), i.e. current is in anti-clockwise direction, i.e. along *adcba*.

(ii) When a circular loop deforms into a narrow straight wire, the magnetic flux linked with it also decreases. The current induced due to change in flux will flow in such a direction that it will oppose the decrease in magnetic flux, so it will flow anti-clockwise, i.e. along *adcba* due to which the magnetic field produced will be out of the plane of paper.

23. Refer to text on page 255.

24. Refer to text Example 9 on page 257.

25. (i) Refer to text on page 255.

(ii) During motion, free electrons are shifted at one end due to magnetic force. So, due to polarisation of rod electric field is produced which applies electric force on free electrons in an opposite direction.

At equilibrium of Lorentz force,

$$F_e + F_m = 0$$

where, F_e − force due to electric field $= qE$

F_m − force due to magnetic field $= q(v \times B)$

$\therefore \quad q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) = 0$

$\Rightarrow \quad \mathbf{E} = -\mathbf{v} \times \mathbf{B} = \mathbf{B} \times \mathbf{v}$

$\Rightarrow \quad |E| = Bv\sin\theta$

Case I If B, E and v are collinear, then charged particle is moving parallel or anti-parallel.

Case II If v, E and B are mutually perpendicular, i.e. $\theta = 90°$, then Lorentz force is zero which means particle will pass through the field without any change.

26. Suppose the length of the rod is greater than the radius of the circle and rod rotates anti-clockwise. Suppose the direction of the rod at any instant be along positive y-direction and the direction of the magnetic field is along positive z-direction.

Then, using Lorentz law, we get the following,

$$\mathbf{F} = -e(\mathbf{v} \times \mathbf{B}) \Rightarrow \mathbf{F} = -e(v\hat{\mathbf{j}} \times B\hat{\mathbf{k}})$$

$$\Rightarrow \quad \mathbf{F} = -evB\hat{\mathbf{i}} \qquad [\because \hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}]$$

Thus, the direction of force on the electrons is along X-axis. Thus, the electrons will move towards the centre, i.e. the fixed end of the rod. This movement of electrons will result in current having the direction opposite that of electrons and hence, it will produce emf in the rod

[redacted]

between the fixed end and the point touching the ring. Let θ be the angle between the rod and radius of the circle at any time t.

Then, area swept by the rod inside the circle $= \frac{1}{2}\pi r^2\theta$

Now, induced emf $= B \times \frac{d}{dt}\left(\frac{1}{2}\pi r^2\theta\right) = \frac{1}{2}\pi r^2 B\frac{d\theta}{dt}$

$$= \frac{1}{2}\pi r^2 B\omega = \frac{1}{2}\pi r^2 B(2\pi v) = \pi^2 r^2 Bv$$

27. (i) Refer to text on pages 255 and 256.
 (ii) Refer to text on page 256.
 (iii) Refer to text on page 256.

28. **For statement of Faraday's law of electromagnetic induction.** Refer to text on pages 252 and 253.
According to the given figure.

Case I When PQ moves forward.

(i) For $0 \le x < b$

Magnetic field B exists in the region.

$\therefore$ Area of loop $PQRS = lx$

$\therefore$ Magnetic flux linked with loop $PQRS$,

$$\phi = BA = Blx$$

$\Rightarrow$ $\phi = Blx$ ...(i)

(ii) For $2b \ge x \ge b$ $(b > x \ge 0)$

$B = 0$

$\therefore$ Flux linked with loop $PQRS$ is a uniform and given by

$\phi' = Blb$ ...(ii)

$\Rightarrow x = b$

Forward journey

Thus, for $b > x \ge 0$

Flux, $\phi = Blx \Rightarrow \phi \propto x$

For $2b \ge x \ge b$

Flux, $\phi = Bbl$ [constant]

Backward journey

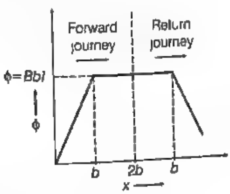
For $b \le x \le 2b$,

ϕ = constant $= Bbl$

For $0 \le x \le b$,

$\phi = Blx$ [decreasing]

Graphical representation



Case II For $b > x \ge 0$, $B = 0$

As, $\phi = Blx$

$\Rightarrow$ $\frac{d\phi}{dt} = Bl\frac{dx}{dt} = Bvl$ $\left[\because v = \frac{dx}{dt}\right]$

Induced emf, $e = -\frac{d\phi}{dt} = -vBl$

For $2b \ge x \ge b$,

As, $\phi' = Bbl \Rightarrow \frac{d\phi'}{dt} = 0 \Rightarrow e = 0$

Forward journey

For $b > x \ge 0 \Rightarrow e = -vBl$

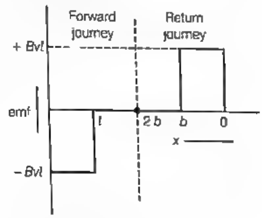
For $2b \ge x \ge b \Rightarrow e = 0$

Backward journey

For $b > x \ge 0 \Rightarrow e = vBl$

For $2b \ge x \ge b$, $e = 0$

Variation of induced emf



29. Here, $A = 20\text{ cm} \times 30\text{ cm} = 6 \times 10^{-2}\text{ m}^2$

$B = 0.3$ T

Let θ be the angle made by the field B with the normal to the plane of the coil.

(i) Here, $\theta = 90° - 90° = 0°$

So, flux, $\phi = BA\cos\theta$

$\phi = 0.3 \times 6 \times 10^{-2} \times \cos 0°$

$= 1.8 \times 10^{-2}$ Wb

(ii) Here, $\theta = 90° - 30° = 60°$

$\phi = 0.3 \times 6 \times 10^{-2} \times \cos 60°$

$\phi = 0.9 \times 10^{-2}$ Wb

(iii) Here, $\theta = 90°$

$\phi = 0.3 \times 6 \times 10^{-2} \times \cos 90°$

$\phi = 0$

30. As we know that, $e = \frac{d\phi}{dt}$

As, $\phi = 5t^3 + 4t^2 + 2t$

So, $e = 15t^2 + 8t + 2$

For $t = 2$ s, $e = 15 \times 2^2 + 8 \times 2 + 2$

$= 60 + 16 + 2 = 78$ V

31. Here, radius, $r = 10\text{ cm} = 10^{-1}$ m, $N = 500$ turns

Resistance, $R = 2\,\Omega$, $\theta_1 = 0°, \theta_2 = 180°$

$dt = 0.25$ s, $e = ?$, $I = ?$, $B = 3 \times 10^{-5}$ T,

$A = \pi r^2 = 3.14\,(10^{-1})^2 = 3.14 \times 10^{-2}\text{ m}^2$

[redacted]

$$\Rightarrow \quad e = -\frac{N(d\phi)}{dt} = -\frac{N(\phi_2 - \phi_1)}{dt}$$

$$= -\frac{NBA(\cos\theta_2 - \cos\theta_1)}{dt}$$

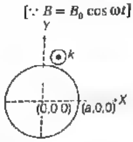
$$= -\frac{500 \times 3 \times 10^{-5} \times 3.14 \times 10^{-2}(\cos 180^\circ - \cos 0^\circ)}{0.25}$$

$$= \frac{2 \times 500 \times 3 \times 3.14 \times 10^{-7}}{0.25} = 3.8 \times 10^{-3}\text{ V}$$

and $I = \frac{e}{R} = \frac{3.8 \times 10^{-3}}{2} = 1.9 \times 10^{-3}$ A

32. At any instant, flux passing through the ring is given by $\phi = \mathbf{B} \cdot \mathbf{A} = BA\cos\theta = BA$ $[\because \theta = 0]$

or $\phi = B_0(\pi a^2)\cos\omega t$ $[\because B = B_0 \cos\omega t]$



By Faraday's law of electromagnetic induction, the magnitude of induced emf is given by

$$e = \frac{d\phi}{dt} = B_0(\pi a^2)\,\omega \sin\omega t$$

This causes flow of induced current, which is given by

$$I = B_0(\pi a^2)\,\omega \sin\omega t / R$$

Now, finding the values of current at different instants. So, we have current at

$$t = \frac{\pi}{2\omega} \Rightarrow I = \frac{B_0(\pi a^2)\omega}{R} \text{ along } \hat{j}$$

Because $\sin\omega t = \sin\left(\omega \frac{\pi}{2\omega}\right) = \sin\frac{\pi}{2} = 1$

At $\quad t = \frac{\pi}{\omega} \Rightarrow I = \frac{B_0(\pi a^2)\omega}{R} \times \sin\pi = 0$

Because $\sin\omega t = \sin\left(\omega \frac{\pi}{\omega}\right) = \sin\pi = 0$

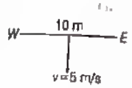
At $\quad t = \frac{3\pi}{2\omega} \Rightarrow I = \frac{B_0(\pi a^2)\omega}{R}$ along $-\hat{j}$

Because $\quad \sin\omega t = \sin\left(\omega \frac{3\pi}{2\omega}\right) = \sin\frac{3\pi}{2} = -1$

33. Refer to Example 7 on page 256, $B_H = 58 \times 10^{-3}$T

The number of spokes is immaterial because the emf's across the spokes are in parallel

34. Given, velocity of straight wire, $v = 5$ m/s



Horizontal component of the earth's magnetic field,

$$B = 0.30 \times 10^{-4}\text{ Wb/m}^2$$

Length of wire, $l = 10$ m

(i) The emf induced in the wire, $e = Blv \sin\theta$

Here, $\theta = 90^\circ$

$\therefore \sin\theta = 1$ [$\because$ wire is falling at right angle to the earth's horizontal magnetic field component]

$$e = 0.30 \times 10^{-4} \times 10 \times 5$$
$$= 1.5 \times 10^{-3}\text{ V}$$

(ii) According to the Fleming's right hand rule, if the force is downward, then the direction of induced emf will be from West to East.

(iii) As the direction of induced emf is from West to East, the West end of the wire is at higher potential.

35. Given, speed of jet plane,

$$v = 1800\text{ km/h} = 1800 \times \frac{5}{18}$$
$$= 500\text{ m/s}$$

and l = distance between the ends of the wings = 25 m

The magnitude of magnetic field,

$$B = 5 \times 10^{-4}\text{ T}$$

Angle of dip, $\delta = 30^\circ$

Use the formula of motional emf,

$e = B_V vl$ or $e = (B\sin\delta)\,vl$

[$\because$ vertical component of the earth's magnetic field, $B_V = B\sin\delta$]

$$\Rightarrow e = 5 \times 10^{-4} \sin 30^\circ \times 500 \times 25 = 3.1\text{ V}$$

Thus, the voltage difference developed between the ends is 3.1 V.

36. Refer to Example 7 on page 256, $e = 10$ V

37. Given, $B = 0.5$ T

$l = 20$ cm $= 0.2$ m

$v = 10\text{ ms}^{-1}$

emf induced $|e| = |Blv| = |-0.5 \times 0.2 \times 10| = 1$ V

Current in the arm, $I = \frac{e}{R} = \frac{1}{5} = 0.2$ A

38. Here, area, $A = 8 \times 2 = 16\text{ cm}^2 = 16 \times 10^{-4}\text{ m}^2$

$B = 0.3$T, $v = 1\text{ cm/s} = 10^{-2}$ m/s

Induced emf, $e = ?$

(i) When velocity is normal to longer side,

$$l = 8\text{ cm} = 8 \times 10^{-2}\text{ m}$$
$$e = Blv = 0.3 \times 8 \times 10^{-2} \times 10^{-2}$$
$$= 2.4 \times 10^{-4}\text{ V}$$

Induced emf lasts till the loop comes out of field.

$$\text{Time, } t = \frac{\text{Distance covered by coil in uniform magnetic field}}{\text{Velocity of the coil}}$$

$$= \frac{2 \times 10^{-2}}{10^{-2}} \Rightarrow t = 2\text{ s}$$

(ii) When velocity is normal to shorter side,

$$l = 2\text{ cm} = 2 \times 10^{-2}\text{ m}$$
$$e = Blv = 0.3 \times 2 \times 10^{-2} \times 10^{-2}$$
$$= 0.6 \times 10^{-4}\text{ V}$$

$\therefore$ The induced voltage lasts for time;

[redacted]

$$t = \frac{\text{Distance covered by coil in uniform magnetic field}}{\text{Velocity of the coil}}$$

$$= \frac{8\times10^{-2}}{10^{-2}} = 8\text{ s}$$

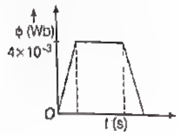
39. Given, $l = 20\text{cm} = 0.2\text{ m}$,

$B = 0.1\text{T}$, $v = 10\text{ cms}^{-1} = 0.1\text{ms}^{-1}$

(i) Magnetic flux through loop $\phi = B\cdot A = Blx$

$\phi_{max} = 0.1\times0.2\times0.2$

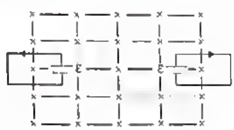
$= 0.004\text{ Wb} = 4\times10^{-3}\text{ Wb}$

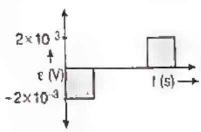


(ii) Induced emf, $\varepsilon = \frac{-d\phi}{dt} = -Blv$

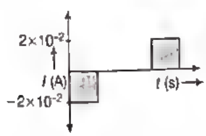
$\therefore |\varepsilon|_{max} = 0.1\times0.2\times0.1 = 0.002\text{ V}$

$= 2\times10^{-3}\text{ V}$





(iii) Induced current, $I = \frac{|\varepsilon|}{R} = \frac{2\times10^{-3}}{0.1} = 2\times10^{-2}\text{ A}$



40. Given, side of loop, $a = 12\text{ cm}$

$\therefore$ Area of loop, $A = a^2 = (12)^2 = 144\text{ cm}^2 = 144\times10^{-4}\text{m}^2$

Velocity, $v = 8\text{ cm/s} = 8\times10^{-2}\text{ m/s}$ [X-axis]

Rate of change of magnetic field with distance,

$$\frac{dB}{dx} = 10^{-3}\text{ T/cm} \quad \text{[negative } X\text{-axis]}$$

Rate of change of magnetic field with time,

$$\frac{dB}{dt} = 10^{-3}\text{ T/s}$$

Resistance of the loop, $R = 4.5\text{ m}\Omega = 4.5\times10^{-3}\ \Omega$

Rate of change of magnetic flux with respect to time,

$$\frac{d\phi}{dt} = \frac{d(BA)}{dt} = \left(\frac{dB}{dt}\right)A \quad [\because \phi = BA]$$

$$= 10^{-3}\times144\times10^{-4}$$

$$= 1.44\times10^{-5}\text{ Wb/s}$$

Rate of change of magnetic flux due to the motion of loop,

$$\frac{d\phi}{dt} = \frac{dB}{dx}\cdot A\cdot\frac{dx}{dt} = 10^{-3}\times144\times10^{-4}\times8 \quad \left[\because \frac{dx}{dt} = \text{velocity}\right]$$

$$= 11.52\times10^{-5}\text{ Wb/s}$$

Both of the effects cause a decrease in magnetic flux along the positive z-direction.

Total induced emf in the loop,

$$e = 1.44\times10^{-5} + 11.52\times10^{-5}$$

$$e = 12.96\times10^{-5}\text{ V}$$

Induced current in the loop $= \frac{e}{R} = \frac{12.96\times10^{-5}}{4.5\times10^{-3}}$

$$= 2.88\times10^{-2}\text{ A}$$

The direction of induced current is such as to increase the flux through the loop along positive z-direction, i.e. induced current will be anti-clockwise.

41. Given, radius of coil, $r = 8.0\text{cm} = 8\times10^{-2}\text{ m}$

$N = 20$ turns, $\omega = 50\text{ rad s}^{-1}$,

$B = 3\times10^{-2}\text{ T}$, $e_0 = ?$, $e_{av} = ?$

Resistance, $R = 10\ \Omega$, $P = ?$

As, $e_0 = NAB\omega = N(\pi r^2)B\omega \quad [\because A = \pi r^2]$

$$e_0 = 20\times\frac{22}{7}\times(8\times10^{-2})^2\times3\times10^{-2}\times50$$

$$e_0 = 0.603\text{ V}$$

Average value of emf induced over a full cycle,

$$e_{av} = 0$$

$$I_{max} = \frac{e_0}{R} = \frac{0.603}{10} = 0.0603\text{ A}$$

Average power dissipated,

$$P_{av} = \frac{e_0 I_0}{2} = \frac{0.603\times0.0603}{2}$$

$$P_{av} = 0.018\text{ W}$$

The induced current causes a torque opposing the rotation of the coil. An external agent (rotor) must supply torque (and do work) to counter this torque in order to keep the coil rotating uniformly. Thus, the source of power dissipated as heat in the coil is the external rotor.

[redacted]

|TOPIC 2|
Self and Mutual Induction

INDUCTANCE

Flux linkage of a closely wound coil is directly proportional to the current I,

i.e. $$\phi_B \propto I$$

If the geometry of the coil does not varying with time, then

$$\frac{d\phi_B}{dt} \propto \frac{dI}{dt}$$

For a closely wound coil of N turns, the same magnetic flux is linked with all turns. The flux ϕ_B through the coil changes, each turn contributes to the induced emf. Therefore, flux linked with the coil (flux linkage) is equal to $N\phi_B$. In this case,

Total flux, $$N\phi_B \propto I$$

The constant of proportionality in this relation is called **inductance**. Therefore, inductance is basically a measure of the ratio of the flux to the current.

The SI unit of inductance is the tesla-square metre per ampere (T-m^2/A). We call this as henry (H), named in the honour of American physicist Joseph Henry, who discoveres the law of induction and a contemporary of Faraday and its dimensions are $[ML^2T^{-2}A^{-2}]$.

Thus, $$1 \text{ H} = 1 \text{ T-m}^2/\text{A}$$

Inductance is a scalar quantity which plays same role in an electrical circuit as played by inertia in mechanics. It depends only on the geometry of the coil and intrinsic material properties.

SELF-INDUCTANCE

It is the property of a coil by virtue of which, the coil opposes any change in the strength of current flowing through it by inducing an emf in itself. This induced emf is also called back emf. When the current in a coil is switched ON, the opposes the growth of the current and when the current is switched OFF, the self-induction opposes the decay of the current. So, self-induction is also called the **inertia of electricity**.

Coefficient of Self-Induction

Let us consider a coil of N turns carrying a current I. Let ϕ_B be the magnetic flux linked with each turn of the coil. Then, the number of flux linked through the coil will be $N\phi_B$.

If no magnetic materials (iron, etc.) are present near the coil, then the number of flux linkages with the coil is proportional to the current I, i.e.

$$N\phi_B \propto I \text{ or } N\phi_B = LI$$

where, L is constant called the **coefficient of self-induction** or **self-inductance** of the coil.

By the above equation, we have

$$L = \frac{N\phi_B}{I} \quad \text{...(i)}$$

If $I = 1$, then $L = N\phi_B$.

Hence, the coefficient of self-induction of a coil is numerically equal to the number of magnetic flux linkages with the coil when unit current is flowing through the coil.

If on changing the current through the coil, the back emf induced in the coil be e, then by Faraday's law, we have

$$e = -N\frac{\Delta\phi_B}{\Delta t} = -\frac{\Delta(N\phi_B)}{\Delta t}$$

where, $\Delta(N\phi_B)/\Delta t$ is the rate of change of magnetic flux (due to change of current) in the coil. But $N\phi_B = LI$.

$$\therefore \quad e = -\frac{\Delta(LI)}{\Delta t} = -L\frac{\Delta I}{\Delta t}$$

where, $\Delta I/\Delta t$ is the rate of change of current in the coil. The negative sign indicates that the induced emf e is always in such a direction that it opposes the change of current in the coil. From the above formula, we have

$$\boxed{L = -\frac{e}{\Delta I/\Delta t}} \quad \text{...(ii)}$$

If $\Delta I/\Delta t = 1$, then $L = e$ (numerically).

Hence, the coefficient of self-induction of a coil is numerically equal to the emf induced in the coil when the rate of change of current in the coil is unity.

The SI unit of the coefficient of self-induction is henry (H) and its dimensions are $[ML^2T^{-2}A^{-2}]$. Thus, the self-inductance of a coil is 1 henry when an induced emf of 1volt is set up in the coil due to a current changing at the rate of 1 ampere per second in the coil, i.e.

$$1 \text{ henry} = \frac{1 \text{ volt}}{1 \text{ ampere/second}}$$

Thus, as before, $1 \text{ H} = 1 \text{ Vs A}^{-1} = 1 \text{Wb A}^{-1}$

But from Eq. (i), 1 henry $=$ 1 weber/ampere.

[redacted]

The smaller units for L are millihenry (mH) and microhenry (μH).

$$1\text{ mH} = 10^{-3}\text{ H},\ 1\,\mu\text{H} = 10^{-6}\text{ H}$$

Note When two coils of coefficient of self-induction L_1 and L_2 are

(i) connected in series, then $L = L_1 + L_2$

(ii) connected in parallel, then $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$

Self-Inductance of Long Solenoid

A long solenoid is one whose length is very large as compared to its area of cross-section. The magnetic field B at any point inside such a solenoid is practically constant and is given by

$$B = \frac{\mu_0 NI}{l} = \mu_0 nI \qquad \left[\because n = \frac{N}{l}\right] \quad \text{...(i)}$$

where, μ_0 = magnetic permeability of free space,
N = total number of turns in the solenoid,
l = length of the solenoid
and n = number of turns per unit length.

$\therefore$ Magnetic flux through each turn of the solenoid,

$$\phi = B \times \text{area of the each turn}$$

$$\phi = \left(\mu_0 \frac{N}{l} I\right) A$$

where, A = area of each turn of the solenoid.

Total magnetic flux linked with the solenoid

= Flux through each turn $\times$ Total number of turns

$$N\phi = \mu_0 \frac{N}{l} IA \times N \quad \text{...(ii)}$$

If L is coefficient of self-inductance of the solenoid, then

$$N\phi = LI \quad \text{...(iii)}$$

From Eqs. (ii) and (iii), we get

$$LI = \mu_0 \frac{N}{l} IA \times N \text{ or } \boxed{L = \frac{\mu_0 N^2 A}{l}}$$

If core of any other magnetic material μ is placed, then

$$\mu = \mu_0 \mu_r \quad [\mu_r = \text{relative magnetic permeability}]$$

$$\therefore \quad \boxed{L = \frac{\mu_0 \mu_r N^2 A}{l}}$$

Note This topic has been frequently asked in previous years 2015, 2014, 2013, 2012, 2011, 2010.

EXAMPLE |1| Current in a circuit falls steadily from 2.0 A to 0.0 A in 10 ms. If an average emf of 200 V is induced, calculate the self-inductance of the circuit.

Foreign 2011

Sol. Given, $\Delta I = -2\text{ A}$, $\Delta t = 10 \times 10^{-3}\text{ s}$

$e = 200\text{ V}$, $L = ?$

$\because$ Induced emf, $e = -L\frac{\Delta I}{\Delta t} \Rightarrow 200 = -L\left(\frac{-2}{10 \times 10^{-3}}\right)$

$\Rightarrow \quad 200 = L \times 2 \times 10^2$

$\therefore$ Self-induction, $L = 1$ H

EXAMPLE |2| What is the self-inductance of a solenoid of length 40 cm, area of cross-section 20 cm² and total number of turns is 800?

Sol. Given, $l = 40\text{ cm} = 0.4\text{ m}$

$A = 20\text{ cm}^2 = 20 \times 10^{-4}\text{ m}^2$

$N = 800$, $L = ?$

$$\therefore \quad L = \frac{\mu_0 N^2 A}{l} = \frac{4\pi \times 10^{-7} \times (800)^2 \times 20 \times 10^{-4}}{0.4}$$

$= 4.02 \times 10^{-5}$ H

EXAMPLE |3| In the circuit diagram shown in figure, $R = 10\,\Omega$, $L = 5$ H, $E = 20$ V, $I = 2$ A. This current is decreasing at a rate of -1.0 A/s. Find V_{ab} at this instant.

Sol. Potential difference across inductor is given as

$$V_L = L\frac{dI}{dt} = (5)(-1.0) = -5\text{ V}$$

Now, using Kirchhoff's second law, $V_a - IR - V_L - E = V_b$

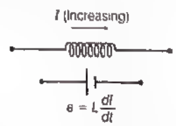
$\therefore \quad V_{ab} = V_a - V_b = E + IR + V_L$

$= 20 + (2)(10) - 5$

$= 35$ V

Energy Stored in an Inductor

The energy of a capacitor is stored in the electric field between its plates. Similarly, an inductor has the capability of storing energy in its magnetic field.



An increasing current in an inductor causes an emf between its terminals.

The work done per unit time is power,

$$\boxed{P = \frac{dW}{dt} = -eI = -LI\frac{dI}{dt}}$$

[redacted]

From $\quad dW = -dU$

or $\quad \dfrac{dW}{dt} = -\dfrac{dU}{dt}$, we have

$$\frac{dU}{dt} = LI\frac{dI}{dt}$$

or $\quad dU = LI\,dI$

The total energy U supplied while the current increases from zero to a final value I is

$$U = L\int_0^I I\,dI = \frac{1}{2}LI^2$$

$$\therefore \quad \boxed{W = U = \frac{1}{2}LI^2}$$

Energy stored per unit volume(V) in magnetic field is known as **energy density**.

$$\therefore \quad \boxed{\text{Energy density} = \frac{U}{V} = \frac{1}{2}\frac{B^2}{\mu_0}}$$

Thus, if $I = 1\,A$, then $2W = L$

Hence, the coefficient of self-inductance is equal to twice the work done in establishing a flow of one ampere current in the circuit.

EXAMPLE |4| Two coils having self-inductances, $L_1 = 5\,mH$ and $L_2 = 1\,mH$. The current in the coil is increasing at same constant rate at a certain instant and the power supplied to the coils is also same. Find the ratio of

(i) induced voltages (ii) currents

(iii) energy stored in two coils at that instant.

Sol. Given, $L_1 = 5\,mH$ and $L_2 = 1\,mH$

(i) As we know, induced voltage is given by $e = \dfrac{LdI}{dt}$

$$\Rightarrow \quad \frac{e_1}{e_2} = \frac{L_1(dI/dt)}{L_2(dI/dt)} = \frac{L_1}{L_2} = \frac{5}{1} = 5:1 \qquad \text{...(i)}$$

(ii) Power in the coil is given by

$$P = eI$$

Here, $P_1 = P_2 \Rightarrow e_1 I_1 = e_2 I_2 \Rightarrow \dfrac{I_1}{I_2} = \dfrac{e_2}{e_1}$

Using Eq. (i), we can write as, $\dfrac{I_1}{I_2} = \dfrac{1}{5} = 1:5$

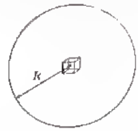
(iii) Energy stored in a coil is given by $U = \dfrac{1}{2}LI^2$

$$\therefore \quad \frac{U_1}{U_2} = \frac{(1/2)L_1I_1^2}{(1/2)L_2I_2^2} = \frac{L_1}{L_2}\left(\frac{e_2}{e_1}\right)^2$$

$$= \frac{5}{1}\left(\frac{1}{5}\right)^2 = 1:5$$

EXAMPLE |5| Suppose a cube of volume $2\,mm^3$ is placed at the centre of a circular loop of radius 5cm carrying current 2A. Find the magnetic energy stored inside the cube.

Sol.



Magnetic field at the centre of the circular loop is given by $B = \dfrac{\mu_0 i}{2R}$.

We know, energy density, $\mu = \dfrac{B^2}{2\mu_0}$ and energy stored in the cube will be given by

$$U = \mu V_0 = \frac{B^2}{2\mu_0}V_0$$

where, V_0 is the volume of the cube

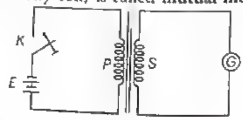
Substituting the values in the above equation.

$$= \frac{1}{2\mu_0}\times\left(\frac{\mu_0 I}{2R}\right)^2 V_0 = \frac{\mu_0 I^2 V_0}{8R^2}$$

$$= \frac{4\pi\times10^{-7}\times2^2\times2\times10^{-9}}{8\times(0.05)^2} = 16\pi\times10^{-14}\,J$$

MUTUAL INDUCTANCE

The phenomenon according to which an opposing emf is produced in a coil (i.e. primary coil) as a result of change in current or magnetic flux linked with a neighbouring coil (i.e. secondary coil) is called **mutual induction**.



Mutual induction

Coefficient of Mutual Induction

Let a current of I_1 ampere flows in the primary coil (P). Due to this current, the magnetic flux linked with each turn of the secondary coil (S) be ϕ_2. If N_2 is the number of turns in the secondary coil, then the number of flux linkages in the coil will be $N_2\phi_2$. This number of flux linkages is proportional to the current I_1 flowing in the primary coil, i.e.

$$N_2\phi_2 \propto I_1 \quad \text{or} \quad N_2\phi_2 = MI_1$$

[redacted]

where, M is a constant called the coefficient of mutual induction or mutual inductance of the two coils. From the above equation, we have

$$M = \frac{N_2\phi_2}{I_1}$$

If $I_1 = 1$, then $\quad M = N_2\phi_2$

Hence, the coefficient of mutual induction of two coils is equal to the number of magnetic flux linkages in one coil when a unit current flows in the other.

If on changing the current in the primary coil, the emf induced in the secondary coil is e_2, then according to Faraday's law, we have

$$e_2 = -N_2\frac{\Delta\phi_2}{\Delta t} = -\frac{\Delta(N_2\,\phi_2)}{\Delta t}$$

where, $\Delta\phi_2/\Delta t$ is the rate of change of magnetic flux in the secondary coil (due to change of current in the primary coil). But $N_2\phi_2 = MI_1$.

$$\therefore \quad e_2 = -\frac{\Delta(MI_1)}{\Delta t} = -M\frac{\Delta I_1}{\Delta t}$$

where, $\Delta I_1/\Delta t$ is the rate of change of current in the primary coil. The negative sign indicates that the direction of emf induced in the secondary coil is always such that it opposes any change in current in the primary coil.

From the above expression, we have

$$\boxed{\text{Mutual inductance, } M = -\frac{e_2}{\Delta I_1/\Delta t}}$$

If $\Delta I_1/\Delta t = 1$, then $M = e_2$ (numerically).

Hence, the coefficient of mutual induction of two coils is equal to the numerical value of the induced emf in one coil when the rate of change of current in other coil is unity.

The SI unit of the coefficient of mutual inductance is henry (H). Thus, the mutual inductance of two coils is 1 henry when an induced emf of 1 volt is set up in one of them due to a current changing at the rate of 1 ampere per second in the other.

i.e. $$1 \text{ henry} = \frac{1 \text{ volt}}{1 \text{ ampere/second}}$$

or $$1\,\text{H} = 1\,\text{V s A}^{-1} = 1\,\text{Wb A}^{-1}$$

The coefficient of mutual induction or mutual inductance of two coils depend on

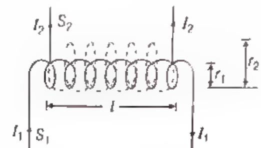
(i) geometry of two coils, i.e. size of coils, their shape, number of turns, nature of material on which two coils are wound.

(ii) distance between two coils.

(iii) relative placement of two coils (i.e. orientation of the two coils).

Mutual Inductance of Two Long Coaxial Solenoids

Consider two long solenoids S_1 and S_2 of same length l, such that solenoid S_2 surrounds solenoid S_1 completely.



Two long coaxial solenoids of same length l

Let n_1 be the number of turns per unit length of S_1, n_2 be the number of turns per unit length of S_2, I_1 be current passed through solenoid S_1 and ϕ_{21} be flux linked with S_2 due to current flowing through S_1.

$$\phi_{21} \propto I_1 \text{ or } \phi_{21} = M_{21}\,I_1$$

where, M_{21} is the coefficient of mutual induction of the two solenoids.

When current is passed through solenoid S_1, an emf is induced in solenoid S_2. Magnetic field produced inside solenoid S_1 on passing current through it.

$$B_1 = \mu_0 n_1 I_1$$

Magnetic flux linked with each turn of solenoid S_2 will be equal to B_1 times the area of cross-section of solenoid S_1.

Magnetic flux linked with each turn of the solenoid $S_2 = B_1 A$.

Therefore, total magnetic flux linked with the solenoid S_2 will be

$$\phi_{21} = B_1 A \times n_2 l$$
$$= \mu_0 n_1 I_1 \times A \times n_2 l$$
$$\phi_{21} = \mu_0 n_1 n_2 A I_1 l$$
$$M_{21} = \mu_0 n_1 n_2 A l \quad \text{...(i)}$$

Similarly, the mutual inductance between the two solenoids, when current is passed through solenoid S_2 and induced emf is produced in solenoid S_1 and is given by

$$M_{12} = \mu_0 n_1 n_2 A l$$
$$M_{12} = M_{21} = M \quad \text{(say)}$$

Hence, coefficient of mutual induction between two long solenoids.

$$M = \mu_0 n_1 n_2 A l$$

We can rewrite Eq. (i) as,

$$M = \mu_0\left(\frac{N_1}{l}\right)\left(\frac{N_2}{l}\right)\pi r_1^2 \times l$$
$$= \frac{\mu_0 N_1 N_2 A}{l}$$

[redacted]

If core of any other magnetic material μ is placed, then

$$\boxed{M = \frac{\mu_0 \mu_r N_1 N_2 A}{l}}$$

Note This topic has been frequently asked in previous years 2015, 2013, 2012, 2011, 2010.

EXAMPLE |6| There are two coils, which have mutual inductance of 10 H. When the circuit is closed, current in the primary coil is raised to 3 A within a time range of 1 millisecond. Calculate the emf induced in secondary coil.

Sol. Given, mutual inductance, $M = 10$ H

Change in current, $dI = 3$ A

Change in time, $dt = 1$ millisecond $= 10^{-3}$ s

emf induced, $e = ?$

emf induced in secondary coil is given by $e = \frac{M\,dI}{dt}$

$\therefore$ emf induced $= \frac{10 \times 3}{10^{-3}} = 3 \times 10^4$ V

EXAMPLE |7| A 1 m long solenoid with diameter 2 cm and 2000 turns has a secondary coil of 1000 turns wound closely near its mid-point. What will be the mutual inductance between the two coils?

Sol. Given, $l = 1\text{m}, r = \frac{2}{2}\text{cm} = 1\text{ cm} = 10^{-2}\text{m}$

$$N_1 = 2000, N_2 = 1000, A = \pi r^2 = \pi (10^{-2})^2 \text{ m}^2$$

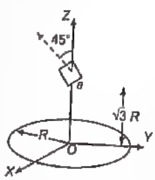
$$= \pi \times 10^{-4} \text{m}^2, M = ?$$

$\therefore$ Mutual inductance between the two coils is given by

$$M = \frac{\mu_0 N_1 N_2 A}{l} = \frac{4\pi \times 10^{-7} \times 2000 \times 1000 \times \pi \times 10^{-4}}{10^{-2}}$$

$$= 78.9 \times 10^{-3} \text{ H}$$

EXAMPLE |8| A circular wire loop of radius R is placed in the XY-plane centred at the origin O. A square loop of side a ($a << R$) having two turns is placed with its centre at $z = \sqrt{3}\,R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the Z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0\, a^2}{2^{p/2}\, R}$, then find the value of p.



Sol. If I current flows through the circular loop, then magnetic field at the location of square loop is

$$B = \frac{\mu_0 I R^2}{2(R^2 + Z^2)^{3/2}}$$

Substituting the value of $Z = \sqrt{3}R$, we have

$$B = \frac{\mu_0 I}{16R}$$

Now, total flux through the square loop is

$$\phi_T = NBS \cos \theta$$

$$= (2)\left(\frac{\mu_0 I}{16R}\right) a^2 \cos 45^\circ$$

Mutual inductance,

$$M = \frac{\phi_T}{I} = \frac{\mu_0 a^2}{2^{7/2} R}$$

$\therefore \quad p = 7$

AC GENERATOR

An AC generator produces electrical energy from mechanical work, just the opposite of what a motor does. In it, a shaft is rotated by some mechanical means, such as an engine or a turbine starts working and an emf is induced in the coil.

Principle

It is based on the phenomenon of electromagnetic induction which states that whenever magnetic flux linked with a conductor (or coil) changes, an emf is induced in the coil.

Main Parts of an AC Generator

Main parts of an AC generator are shown in the figure and discussed as given below

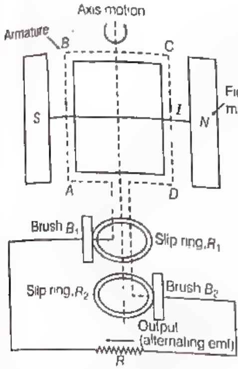
Armature A rectangular coil $ABCD$ consisting of a large number of turns of copper wire wound over a soft iron core is called armature. The soft iron core is used to increase the magnetic flux.

Field magnets Two pole pieces of a strong electromagnet.

Slip rings The ends of the coil $ABCD$ are connected to two hollow metallic rings R_1 and R_2.

Brushes B_1 and B_2 are two flexible metal plates or carbon rods. They are fixed and are kept in slight contact with R_1 and R_2.

[redacted]



AC generator

Theory and Working

As the armature of coil is rotated in the uniform magnetic field, angle θ between the field and normal to the coil changes continuously.

Therefore, magnetic flux linked with the coil changes and an emf is induced in the coil. According to Fleming's right hand rule, current induced in AB is from A to B and it is from C to D in CD. In the external circuit, current flows from B_2 to B_1.

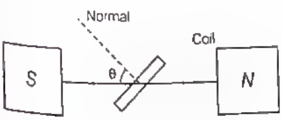
To calculate the magnitude of emf induced, suppose

A = area of each turn of the coil,

N = number of turns in the coil,

B = strength of magnetic field,

θ = angle which normal to the coil and B at any instant t.



Magnetic flux linked with the coil in this position,

$$\phi = N(\mathbf{B}\cdot\mathbf{A})$$
$$= NBA\cos\theta$$
$$= NBA\cos\omega t \qquad [\because \theta = \omega t]$$

where, ω is angular velocity of the coil and other symbols have usual meaning.

As, the coil rotates, angle θ changes. Therefore, magnetic flux ϕ linked with the coil changes and hence, an emf is induced in the coil. At this instant t, if e is the emf induced in the coil, then

$$e = -\frac{d\phi}{dt}$$
$$= -\frac{d}{dt}(NAB\cos\omega t)$$
$$= -NAB\frac{d}{dt}(\cos\omega t)$$
$$= -NAB(-\sin\omega t)\,\omega = NAB\,\omega\sin\omega t$$

where, $NBA\,\omega$ is the maximum value of the emf (also called peak value) which occurs when $\sin\omega t = \pm 1$.

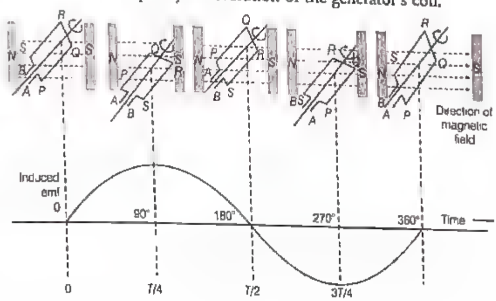
If $NBA\,\omega = e_0$, then $e = e_0\sin\omega t$

Since, the value of the sine function varies between +1 and −1. So, the polarity of emf changes with time. The emf has its extreme value when $\theta = 90°$ or $\theta = 270°$ as the change of flux is greatest at these points. The direction of the induced emf (and hence current) changes periodically as shown in the figure given below. Therefore, the current is called alternating current.

Since, $\omega = 2\pi\nu$

$\therefore \quad e = e_0\sin 2\pi\nu t$

where, ν is the frequency of revolution of the generator's coil.



In commercial generators, the mechanical energy required for rotation of armature is provided by water falling from height, e.g. dams. These are called **hydroelectric generators.**

If the steam at high pressure is used to produce the rotation of armature, these are called **thermal generators.** Instead of coal, if a nuclear fuel is used, we get **nuclear power generators.**

Note In India, the frequency of generation of AC is 50 Hz.

[redacted]

EXAMPLE |9| An AC generator consists of a coil of 1000 turns and cross-sectional area of 100 cm^2, rotating at an angular speed of 100 rpm in a uniform magnetic field of 3.6×10^{-2} T. Calculate the maximum emf produced in the coil.

Sol. Given, $N = 1000, A = 100\text{ cm}^2 = 10^{-2}\text{ m}^2$,

$$\nu = 100\text{ rpm} = \frac{100}{60}\text{ rps},$$

$$B = 3.6\times10^{-2}\text{T}, \ e_0 = ?$$

$\therefore$ Maximum emf produced in the coil is

$$e_0 = NBA\,\omega = NBA\,(2\pi\nu)$$

$$= 1000\times3.6\times10^{-2}\times10^{-2}\times2\times\frac{22}{7}\times\frac{100}{60}$$

$$= 3.77\text{ V}$$

TOPIC PRACTICE 2

OBJECTIVE Type Questions

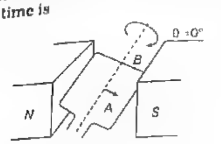
1. The self-inductance of a coil is 2 mH. The rate of flow of current in it is 10^3 A/s. The induced electromotive force in the coil is
   (a) 1 V (b) 2 V
   (c) 3 V (d) 4 V

2. The self inductance L of a solenoid of length l and area of cross-section A, with a fixed number of turns N increases as
   (a) l and A increase
   (b) l decreases and A increases
   (c) l increases and A decreases
   (d) both l and A decrease

3. If a medium of relative permeability μ_r had been present instead of air, the mutual inductance would be
   (a) $M = \mu_r \mu_0 n_1 n_2 \pi r_1 l$ (b) $M = \mu_0 n_1 n_2 \pi r_1^2 l$
   (c) $M = \mu_r n_1 n_2 \pi r_1^2 l$ (d) $M = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l$

4. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon
   (a) the rates at which currents are changing in the two coils
   (b) relative position and orientation of the two coils
   (c) the materials of the wires of the coils
   (d) the currents in the two coils

5. The effective area of the coil exposed to the magnetic field lines changes with time, the flux at any time is



   (a) $\phi_B = BA \cot \omega t$
   (b) $\phi_B = BA \cos \omega t$
   (c) $\phi_B = BA \tan \omega t$
   (d) $\phi_B = BA \sec \omega t$

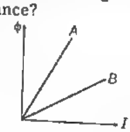
VERY SHORT ANSWER Type Questions

6. Can a straight wire act as an inductor?

7. Define the term self-inductance of a coil. Write its SI unit. **Delhi 2015**

8. How can the self-inductance of a given coil having N number of turns, area of cross-section A and length l be increased? **Foreign 2009; Delhi 2012**

9. A plot of magnetic flux (ϕ) *versus* current (I) is shown in the figure for two inductors A and B. Which of the two has larger value of self-inductance? **Delhi 2010**



10. Self-induction is called the inertia of electricity. Why?

11. Define mutual inductance. Write its SI unit. **Delhi 2016**

12. How does the mutual inductance of a pair of coils change when
    (i) distance between the coils is increased and
    (ii) number of turns in the coils is increased? **All India 2013**

SHORT ANSWER Type Questions

13. A source of emf e is used to establish a current I through a coil of self-inductance L. Show that the work done by the source to build up the current I is $\frac{1}{2}LI^2$. **Delhi 2010C**

[redacted]

14. Define mutual inductance between two long coaxial solenoids. Find out the expression for the mutual inductance of inner solenoid of length l having the radius r_1 and the number of turns n_1 per unit length due to the second outer solenoid of same length and n_2 number of turns per unit length. **Delhi 2012**

15. Two concentric circular coils, one of radius r and the other of radius R are placed coaxially with their centres coinciding. For $R >>> r$, obtain an expression for the mutual inductance of the arrangement. **Delhi 2011**

16. A small square loop of wire of side l is placed inside a large square loop of wire of side $L (L >> l)$. The loops are coplanar and their centres coincide. Give the dependence of mutual inductance.

LONG ANSWER Type I Questions

17. (i) Define self-inductance. Write its SI units.
    (ii) Derive the expression for self-inductance of a long solenoid of length l, cross-sectional area A having N number of turns. **Delhi 2009**

18. Starting from the expression for the energy, $W = \frac{1}{2}LI^2$, stored in a solenoid of self-inductance L to build up the current I, obtain the expression for the magnetic energy in terms of the magnetic field B, area A and length l of the solenoid having n number of turns per unit length. Hence, show that the energy density is given by $B^2/2\mu_0$. **Delhi 2013C**

19. The current through two inductors of self-inductance 12 mH and 30 mH is increasing with time at the same rate. Draw graphs showing the variation of the **All India 2017 C**
    (i) emf induced with the rate of change of current in each inductor.
    (ii) energy stored in each inductor with the current flowing through it.

    Compare the energy stored in the coils, if the power dissipated in the coils is the same.

20. (i) Define the term 'self-inductance' and write its SI unit.
    (ii) Obtain the expression for the mutual inductance of two long coaxial solenoids S_1 and S_2 wound one over the other, each of length L and radii r_1 and r_2; and n_1 and n_2 number of turns per unit length, when a current I is set up in the outer solenoid S_2. **Delhi 2017**

21. Define mutual inductance between a pair of coils. Derive an expression for the mutual inductance of two long coaxial solenoids of same length wound one over the other. **All India 2017**

22. (i) Draw a schematic sketch of an AC generator describing its basic elements. State briefly its working principle. Show a plot of variation of
    (a) magnetic flux and
    (b) alternating emf *versus* time generated by a loop of wire rotating in a magnetic field.
    (ii) Why is choke coil needed in the use of fluorescent tubes with AC mains? **Delhi 2014**

23. State the principle of an AC generator and explain its working with the help of a labelled diagram. Obtain the expression for the emf induced in a coil having N turns each of cross-sectional area A, rotating with a constant angular speed ω in a magnetic field (B), directed perpendicular to the axis of rotation. **CBSE 2018**

LONG ANSWER Type II Questions

24. (i) Describe a simple experiment (or activity) to show that the polarity of emf induced in a coil is always such that it tends to produce a current which opposes the change of magnetic flux that produces it.
    (ii) The current flowing through an inductor of self-inductance L is continuously increasing. Plot a graph showing the variation of
    (a) magnetic flux *versus* current.
    (b) induced emf *versus* dI/dt.
    (c) magnetic potential energy stored *versus* the current. **Delhi 2014**

25. (i) Define mutual inductance and write its SI units.
    (ii) Derive the expression for the mutual inductance of two long coaxial solenoids of same length wound one over the other.
    (iii) In an experiment, two coils C_1 and C_2 are placed close to each other. Find out the expression for the emf induced in the coil C_1 due to a change in the current through the coil C_2. **All India 2015**

[redacted]

26. (i) Explain the meaning of the term mutual inductance. Consider two concentric circular coils, one of the radius r_1 and the other of radius $r_2 (r_1 < r_2)$ placed coaxially with centres coinciding with each other. Obtain the expression for the mutual inductance of the arrangement.

(ii) A rectangular coil of area A, having number of turns N is rotated at f revolutions per second in a uniform magnetic field B, the field being perpendicular to the coil.

Prove that the maximum emf induced in the coil is $2\pi fNBA$. **All India 2016**

27. (i) Draw a labelled diagram of AC generator and state its working principle.

(ii) How is magnetic flux linked with the armature coil changes in a generator?.

(iii) Derive the expression for maximum value of the induced emf and state the induced emf and state the rule that gives the direction of the induced emf.

(iv) Show the variation of the emf generated *versus* time as the armature is rotated with respect to the direction of the magnetic fields. **Delhi 2014**

28. (i) State the principle on which AC generator works. Draw a labelled diagram and explain its working.

(ii) A conducting rod held horizontally along East-West direction is dropped from rest from a certain height near the Earth's surface. Why should there be an induced emf across the ends of the rod?

Draw a plot showing the instantaneous variation of emf as a function of time from the instant it begins to fall. **Foreign 2012**

29. State the working of AC generator with the help of a labelled diagram. The coil of an AC generator having N turns, each of area A, is rotated with a constant angular velocity ω. Deduce the expression for the alternating emf generated in the coil. What is the source of energy generation in this device? **Delhi 2010**

NUMERICAL PROBLEMS

30. A 200 turn coil of self-inductance 30 mH carries a current of 5 mA. Find the magnetic flux linked with each turn of the coil. **Delhi 2011**

31. Self-induction of an air core inductor increases from 0.01 mH to 10 mH on introducing an iron core into it. What is the relative permeability of the core used?

32. Current in a circuit falls from 5.0 A to 0.0 A in 0.1 s. If an average emf of 200 V is induced, give an estimate of the self-inductance of the circuit. **NCERT**

33. If a rate of change of current of 4 As^{-1} induces an emf of 20 mV in a solenoid, what is the self-inductance of the solenoid? **Delhi 2010**

34. A long solenoid with 15 turns per cm has a small loop of area 2.0 cm^2 placed inside, normal to the axis of solenoid. If the current carried by the solenoid changes steadily from 2 A to 4 A in 0.1 s, what is the induced voltage in the loop, while the current is changing? **NCERT**

35. A coil has a self-inductance of 10 mH. What is the maximum magnitude of the induced emf in the inductor, when a current $I = 0.1 \sin 200\,t$ ampere is sent through it.

36. A solenoid of radius 3 cm and length 1 m has 600 turns per metre. Calculate its self-inductance.

37. The current flowing in the two coils of self-inductance $L_1 = 16$ mH and $L_2 = 12$ mH are increasing at the same rate. If the power supplied to the two coils are equal, find the ratio of

(i) induced voltages (ii) the currents and

(iii) the energies stored in the coil at a given instant. **Foreign 2014**

38. A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil? **Delhi 2016**

39. Two coils have mutual inductance of 1.5 H. If current in primary coil is raised to 5 A in one millisecond after closing the circuit, what is the emf induced in the secondary coil?

40. There are two coils A and B separated by some distance. If a current of 2 A flows through A, a magnetic flux of 10^{-2} Wb passes through B (no current through B). If no current passes through A and a current of 1 A passes through B, what is the flux through A? **NCERT Exemplar**

[redacted]

41. The flux linked with a large circular coil of radius R is 0.5×10^{-3} Wb. When a current of 0.5 A flows through a small neighbouring coil of radius r, calculate the coefficient of mutual inductance for the given pair of coils. If the current through the small coil suddenly falls to zero, what would be its effect in the larger coil? **Delhi 2008**

42. An AC generator consists of coil of 100 turns and cross-sectional area of 3 m^2, rotating at a constant angular speed of 60 rad s^{-1} in a uniform magnetic field 0.04T. The resistance of the coil is 500 Ω. Calculate (i) maximum current drawn from the generator and (ii) minimum power dissipation of the coil.

HINTS AND SOLUTIONS

1. (b) Given, coefficient of self-inductance,
$$L = 2 \times 10^{-3} \text{ H}$$
Rate of flow of current, $di/dt = 10^3$ A/s
Induced electromoture force,
$$|e| = \frac{Ldi}{dt} = 2 \times 10^{-3} \text{ V} = 2 \text{ V}$$

2. (b) The self-inductance of a long solenoid of cross-sectional area A and length l, having n turns per unit length, filled the inside of the solenoid with a material of relative permeability (e.g., soft iron, which has a high value of relative permeability) is given by
$$L = \mu_r \mu_0 n^2 Al$$
where, $n = N/l$

3. (d) Air as the medium within the solenoids. Instead, if a medium of relative permeability μ_r had been present, the mutual inductance would be $M = \mu_r \mu_0 n_1 n_2 \pi r_1^2 l$.

It is also important to know that the mutual inductance of a pair of coils, solenoids etc., depends on their separation as well as their relative orientation.

4. (b) Mutual inductance of the pair of coils depends on distance between two coils and geometry of two coils.

5. (b) The effective area of the coil exposed to the magnetic field lines changes with time, the flux at any time t is
$$\phi_B = BA\cos\theta = BA\cos\omega t$$

6. A straight wire cannot act as an inductor as the magnetic flux linked with the wire of negligible area of cross-section is zero. The wire has to be in the form of a coil to serve as an inductor.

7. Self-inductance is the property of a coil by virtue of which, the coil opposes any change in the strength of current flowing through it by inducing an emf in itself. Its SI unit is henry (H).

8. The self-inductance can be increased with the help of electric fields. It does not depend on the current through circuit but depends upon the permeability of material from which the core is made up off.

9. Self inductance of the inductor, $L = \phi/I$.
The slope of I-ϕ graph gives self-inductance of the coil. Inductor A have got greater slope than inductor B, therefore self-inductance of A is greater than self-inductance of B.

10. Self-induction of coil is the property by virtue of which it tends to maintain the magnetic flux linked with it and opposes any change in the flux by inducing current in it. This property of a coil is analogous to mechanical inertia, i.e. why self-induction is called the inertia of electricity.

11. The phenomenon according to which an opposing emf is produced in a coil as a result of change in current of magnetic flux linked with a neighbouring coil is called mutual induction.
Its SI unit is henry.

12. (i) As $\phi = MI$, with the increase in the distance between the coils, the magnetic flux linked with the secondary coil decreases and hence, the mutual inductance of the two coils will decrease with the increase of separation between them.
(ii) Mutual inductance of two coils can be found out by $M = \mu_0 N_1 N_2 Al$, i.e. $M \propto N_1 N_2$, so with the increase in number of turns, mutual inductance increases.

13. Refer to text on pages 269 and 270.

14. Refer to text on pages 271 and 272.

15. The magnetic field produced by current carrying large coil C_1 in the vicinity of small coil C_2 is given by
$$B_1 = \frac{\mu_0 I_1}{2R}$$
The magnetic flux linked with shorter coil C_2 is given by
$$\phi_2 = B_1 A_2 = \frac{\mu_0 I_1}{2R} \pi r^2$$
$$\text{Mutual inductance, } M = \frac{\phi_2}{I_1} = \frac{\mu_0 \pi r^2}{2R} \text{ henry}$$

16. Magnetic field produced by current carrying square loop of wire of side L is given by
$$B_1 = \frac{2\sqrt{2}\mu_0 I_1}{2L}$$
The magnetic flux linked with shorter square of side l is given by
$$\phi_2 = B_1 A_2 = \frac{2\sqrt{2}\mu_0 I_1}{2L} l^2$$
$$\therefore \text{Mutual inductance, } M = \frac{\phi_2}{I_1} = \frac{2\sqrt{2}\mu_0 l^2}{2L} \text{ henry}$$
$$\Rightarrow \quad M \propto \frac{l^2}{L}$$

[redacted]

17. Refer to text on pages 268 and 269.

18. Energy stored in the magnetic field,

$$W = \frac{1}{2}LI^2 = \frac{1}{2}\cdot\frac{\mu_0 N^2 A}{l}\cdot\frac{B^2 l^2}{\mu_0^2 N^2}$$

$$= \frac{1}{2\mu_0}B^2\,Al \qquad \left[\because L = \frac{\mu_0 N^2 A}{l},\ B = \frac{\mu_0 NI}{l}\right]$$

Energy density,

$$u_B = \frac{\text{Energy}}{\text{Volume}} = \frac{\frac{1}{2\mu_0}B^2 AL}{AL} = \frac{B^2}{2\mu_0} \qquad [\because \text{volume} = Al]$$

19. Given, $L_1 = 12$ mH, $L_2 = 30$ mH

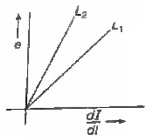
Also, $\frac{dI_1}{dt} = \frac{dI_2}{dt}$

(i) Induced emf in the inductors, $|e| = L\frac{dI}{dt}$

As, $\frac{dI_1}{dt} = \frac{dI_2}{dt}$

$\Rightarrow e \propto L$

Thus, graph of e *versus* $\frac{dI}{dt}$ for two inductors is as shown in figure.



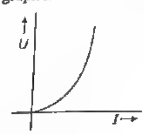
$\because L_2 > L_1$

$\therefore$ For the given $\frac{dI}{dt}$, $e_2 > e_1$.

(ii) Energy stored in inductor, $U = \frac{1}{2}LI^2$

For a given L, $U \propto I^2$

Thus, U *versus* I graph is curved as shown in the figure.



Given, $P_1 = P_2$

$\Rightarrow e_1 I_1 = e_2 I_2 \Rightarrow \frac{I_1}{I_2} = \frac{e_2}{e_1}$

$$\therefore \frac{U_1}{U_2} = \frac{\frac{1}{2}(L_1 I_1^2)}{\frac{1}{2}(L_2 I_2^2)} = \frac{L_1}{L_2}\left(\frac{I_1}{I_2}\right)^2 = \frac{L_1}{L_2}\left(\frac{e_2}{e_1}\right)^2$$

$$= \frac{12}{30}\left[\frac{L_2\,(dI/dt)}{L_1\,(dI/dt)}\right]^2$$

$$= \left(\frac{12}{30}\right)\times\left(\frac{30}{12}\right)^2 = \frac{30}{12} = \frac{5}{2}$$

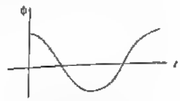
20. (i) Refer to text on page 268

(ii) Refer to text on pages 271 and 272.

21. Refer to text on pages 271 and 272.

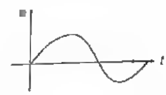
22. (i) Refer to text on pages 272 and 273.

(a) Variation of magnetic flux with time



$\phi = BNA\cos\omega t$

(b) Variation of alternating emf with time

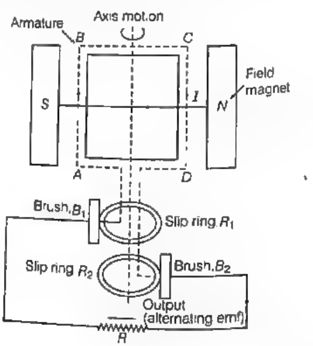


$e = BNA\,\omega\cos\omega t$

(ii) The choke coil is used to reduce the current. Therefore, it is required in the use of fluorescent tubes with AC mains.

23. **Principle**

An AC generator is based on the phenomenon of electromagnetic induction which states that whenever magnetic flux linked with a conductor (or coil) changes, an emf is induced in the coil.



AC generator

[redacted]

Working

As the armature of coil is rotated in the uniform magnetic field, angle θ between the field and normal to the coil changes continuously.

Therefore, magnetic flux linked with the coil changes and an emf is induced in the coil. According to Fleming's right hand rule, current induced in AB is from A to B and it is from C to D in CD. In the external circuit current flows from B_2 to B_1.

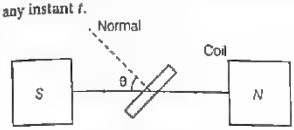
To calculate the magnitude of emf induced, suppose

A = area of each turn of the coil,

N = number of turns in the coil,

B = strength of magnetic field

and θ = angle which normal to the coil makes with B at any instant t.



Magnetic flux linked with the coil in this position,

$$\phi = N(\mathbf{B}\cdot\mathbf{A})$$

or $$\phi = NBA\cos\theta$$

$$\phi = NBA\cos\omega t$$

where, ω is angular velocity of the coil and symbols have usual meaning.

As we know, due to the rotation of the coil, an emf is being induced.

Thus, at this instant t, if e is the emf induced in the coil, then

$$e = -\frac{d\phi}{dt}$$

$$e = -\frac{d}{dt}(NAB\cos\omega t)$$

$$e = -NAB\frac{d}{dt}(\cos\omega t)$$

or $$e = -NAB(-\sin\omega t) = NAB\sin\omega t$$

where, $NBA\omega$ is the maximum value of the emf (also called peak value) which occurs when $\sin\omega t = \pm 1$

If $NBA\omega = e_0$, then $e = e_0 \sin\omega t$

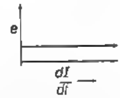
24. (i) According to Lenz's law, the polarity of the induced emf is such that it opposes the change in magnetic flux responsible for its production.

Refer to text on pages 253 and 254.

(ii) (a) **Magnetic flux *versus* current**

(b) **Induced emf *versus* $\frac{dI}{dt}$**

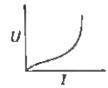
$$\Rightarrow \qquad e = -L\frac{dI}{dt}$$



$\frac{dI}{dt}$ is positive, and e is negative and constant.

(c) **Magnetic potential energy stored *versus* current**

$$U = \frac{1}{2}LI^2$$



$$\Rightarrow \qquad U \propto I^2$$

25. For parts (i) and (ii) refer to text on pages 270, 271 and 272.

(iii) Suppose that a current I is flowing through the coil C_2 at any instant. Flux linked with the coil C_1 is given by

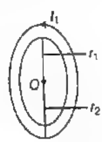
$$\phi \propto I \Rightarrow \phi = MI$$

where, M is the coefficient of mutual induction. If e is the induced emf produced in the coil C_1, then

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(MI) = -M\frac{dI}{dt}$$

26. (i) Whenever the current passing through a coil or circuit changes, the magnetic flux linked with a neighbouring coil or circuit will also change. Hence, an emf will be induced in the neighbouring coil or circuit. This phenomenon is called 'mutual induction'.

According to question, let the current in big coil of radius r_2 be I_1, so magnetic field at point O due to this coil will be $\frac{\mu_0 I_1}{2r_2}$.



Change in magnetic flux in the coil of radius r_1 is

$$\phi = BA = \frac{\mu_0 I_1}{2r_2} \times \pi r_1^2$$

Mutual inductance,

$$M = \frac{\phi}{I_1} = \frac{\mu_0 I_1 \pi r_1^2}{2r_2 \times I_1} = \frac{\mu_0 \pi r_1^2}{2r_2}$$

This is the required expression.

[redacted]

(ii) According to question, if the coil rotates with an angular velocity of ω and N turns through an angle θ in time t, thus $\theta = \omega t$.

$\therefore \quad \phi = BA\cos\theta = BA\cos\omega t$

As the coil rotates, the magnetic flux linked with it changes. An induced emf is set up in the coil which is given by

$$e = \frac{-d\phi}{dt} = \frac{-d}{dt}(BA\cos\omega t)$$
$$= BA\omega\sin\omega t$$

For N number of turns, $e = NBA\omega\sin\omega t$

For maximum value of emf ωt must be equals to 90°.

So, maximum emf induced is $= NBA\omega$.

i.e. $\quad e = NBA2\pi f \quad [\because \omega = 2\pi f]$

27. Refer to text on pages 272 and 273.

28. (i) Refer to the text on pages 272 and 273.

(ii) As the earth's magnetic field lines are cut by the falling rod, the change in magnetic flux takes place. This change in flux induces an emf across the ends of the rod.

Since, the rod is falling under gravity.

$$v = gt \quad [\because u = 0]$$

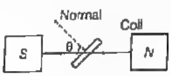
Induced emf,

$$\varepsilon = Blv \Rightarrow \varepsilon = Blgt$$

$\therefore \quad \varepsilon \propto t$

29. Refer to text on pages 272 and 273.

Direction of induced emf can be determined using Fleming's right hand rule given below. If we stretch the thumb and the first two Fingers of our right hand in mutually perpendicular directions and if the forefinger points in the direction of the magnetic field, thumb in the direction of motion of the conductor, then the central finger points in the direction of current induced in the conductor.



30. Let ϕ be the magnetic flux linked with each of the N turns of the coil.

Then, $N\phi \propto I$

$\Rightarrow \quad N\phi = LI$ or $\phi = \dfrac{LI}{N}$

$\Rightarrow \quad \phi = \dfrac{30\times10^{-3}\times5\times10^{-3}}{200} = 7.5\times10^{-7}\,\text{Wb}$

31. Here, $L_0 = 0.01$ mH $= 10^{-5}$ H

$L = 10$ mH $= 10^{-2}$ H

$\mu_r = ?, \mu_r = \dfrac{L}{L_0} = \dfrac{10^{-2}}{10^{-5}} = 10^3$

or $\quad \mu_r = 1000$

32. 4 H; refer to Example 1 on page 269.

33. Here, $\dfrac{dI}{dt} = 4\ \text{As}^{-1}$

$\Rightarrow \quad |e| = 20\text{ mV} \Rightarrow |e| = 20\times10^{-3}$ V

$\Rightarrow \quad |e| = L\dfrac{dI}{dt}$

$\therefore \quad L = \dfrac{|e|}{dI/dt}$

$\Rightarrow \quad L = \dfrac{20\times10^{-3}}{4} = 5\times10^{-3}\text{ H} = 5\text{ mH}$

34. Here, number of turns per unit length,

$$n = \frac{N}{l} = 15\text{ turns/cm} = 1500\text{ turns/m}$$
$$A = 2.0\text{ cm}^2 = 2\times10^{-4}\text{ m}^2$$

$\therefore \quad \dfrac{dI}{dt} = \dfrac{4-2}{0.1}$ or $\dfrac{dI}{dt} = 20\,\text{As}^{-1}$

$\therefore \quad |e| = \dfrac{d\phi}{dt} = \dfrac{d}{dt}(BA) \quad \left[\because B = \dfrac{\mu_0 NI}{l}\right]$

$$= \frac{Ad}{dt}\left(\mu_0\frac{NI}{l}\right) = A\mu_0\left(\frac{N}{l}\right)\frac{dI}{dt}$$
$$= (2\times10^{-4})\times4\pi\times10^{-7}\times1500\times20\text{ V}$$
$$= 7.5\times10^{-6}\text{ V}$$

35. Here, $L = 10$ mH $= 10^{-2}$ H, $I = 0.1\sin 200t$

$\because \quad \dfrac{dI}{dt} = 0.1\cos 200t\times200 = 20\cos 200t$

$\Rightarrow \quad \left(\dfrac{dI}{dt}\right)_{max} = 20\times1 = 20\text{As}^{-1}$

As, $\quad |e| = L\,(dI/dt) \Rightarrow e_{max} = L\,(dI/dt)_{max}$

$\Rightarrow \quad e_{max} = 10^{-2}\times20 = 0.2$ V

36. From the relation of self-inductance, we have

$$L = \frac{\mu_0 N^2 A}{l} = \frac{4\pi\times10^{-7}\times(600)^2\times\pi\times(3\times10^{-2})^2}{1}$$
$$= 4\pi^2\times36\times10^4\times9\times10^{-11}$$
$$= 1.28\times10^{-3}\text{ H}$$

37. Refer to the Example 4 on page 270.

(i) $\dfrac{e_1}{e_2} = \dfrac{4}{3}$ (ii) $\dfrac{I_1}{I_2} = \dfrac{3}{4}$ (iii) $\dfrac{E_1}{E_2} = \dfrac{3}{4}$

38. emf induced in the secondary coil is given by

$$e = \frac{-MdI}{dt}$$

$\Rightarrow \quad \dfrac{d\phi}{dt} = \dfrac{-MdI}{dt}$

$\Rightarrow \quad d\phi = -MdI$

or, $\quad d\phi = -1.5\times20 = -30\,\text{Wb}$

[redacted]

39. Given, $M = 1.5$ H, $\Delta i_1 = 5$A, $\Delta t = 10^{-3}$s

We know that, $M = -\dfrac{e_2}{\Delta i_1 / \Delta t}$

$\Rightarrow$ $$e_2 = M \times \frac{\Delta i_1}{\Delta t} = 1.5 \times \frac{5}{10^{-3}} = 7.5\ 10^{3}\text{ V}$$

40. Applying the mutual inductance of coil A with respect to coil B,

$$M_{21} = \frac{N_2 \phi_2}{I_1}$$

Therefore, we have

Mutual inductance $= \dfrac{10^{-2}}{2} = 5$ mH

Again, applying this formula for other case,

$$N_1 \phi_1 = M_{12} I_2 = 5\text{ mH} \times 1\text{ A} = 5\text{ mWb}$$

41. Flux linked with larger coil of radius,

$$\phi = 0.5 \times 10^{-3}\text{ Wb and } I = 0.5\text{ A}$$

Flows through neighbouring coil of radius r.

Total flux (ϕ) linked with one coil = MI

[$\because$ I current flows in neighbour coil]

$\therefore$ $0.5 \times 10^{-3} = M \times 0.5$

$$M = 10^{-3}\text{ H}$$

Mutual inductance of two coils, $M = 10^{-3}$ H.

With the fall of current in small coil to zero, the magnetic flux linked with long coil decreases to zero quickly which in turn produces large induced emf in it.

42. Here, total number of turns,

$$N = 100, A = 3\text{ m}^2, \omega = 60\text{ rad s}^{-1}, B = 0.04\text{ T}$$

(i) Maximum emf produced in the coil,

$$e_0 = NBA\omega = 100 \times 0.04 \times 3 \times 60$$

$$e_0 = 720\text{ V}$$

Since, resistance of the coil is 500 Ω, the maximum current drawn from the generator is

$$I_0 = \frac{e_0}{R} = \frac{720}{500} = 1.44\text{ A}$$

(ii) Maximum power dissipation in the coil,

$$P = e_0 I_0 = 720 \times 1.44 = 1036.8\text{ W}$$

[redacted]

SUMMARY

- **Magnetic Flux** The total number of magnetic field lines crossing through any surface normally when it is placed in a magnetic field is known as the magnetic flux through that surface
- **Faraday's Law of EMI** Faraday gave two laws of EMI
  - (i) **First Law** An emf is induced in a circuit when the magnetic flux linked with circuit charges
  - (ii) **Second Law** The magnitude of induced emf in a circuit is equal to the rate of change of magnetic flux through the circuit
- **Induced emf and Current**
  Induced emf, $e = -N\frac{d\phi_B}{dt}$
  Induced current, $I = \frac{e}{R} = \frac{-N}{R}\frac{d\phi_B}{dt}$
  Here, R = resistance of the circuit and N = number of turns
- **Lenz's Law** According to this law, the polarity of induced emf is such that it tends to produce a current which appose the change in magnetic flux produced it
- **Fleming's Right Hand Rule** If we stretch the thumb, the forefinger, the central finger of our right hand in such a way that all three are mutually perpendicular to each other, then if the thumb represents the direction of force, forefinger represent the direction of magnetic field, then the central finger will represent the direction of induced current
- **Induced Current in a Circuit** If induced current is produced in a coil rotated in a uniform magnetic field, then $I = I_0 \sin \omega t$
- **Motional emf and Faraday's Law** If e is the induced emf, then according to Faraday's law,
  $$e = (-d\phi / dt) = -Blv$$
- **Energy Consideration** Power required to move a conductor in a uniform magnetic field perpendicular is, $P = \frac{B^2 l^2 v^2}{R}$
  Here, R = resistance of the circuit through which current is flowing
  B = a uniform magnetic field, l = length of the conductor, v = speed.
- **Eddy Currents** The current induced in the bulk of conductors when the magnetic flux linked with the condutor charges are known as eddy currents
- **Undesirable Effects of Eddy Currents** Eddy Currents cause unnecessary heating and wastage of power
  The heat produced by eddy currents may even damage the insulation of coils
- **Inductance** It is the ratio of the flux to the current. It depends on the geometry of the coil and intrinsic material properties
- **Self-Inductance** It in the property of a coil by virtue of which it opposes any changes in the strength of current flowing through it by inducing an emf in itself
- When current in a coil changes, it induces a back emf in the same coil. The self-induced emf is given by, $e = -L\frac{dI}{dt}$, where L is the self-inductance of the coil
- If coil of N turns and area A is rotated with ν revolutions per second in a uniform magnetic field B, then the induced emf is $e = e_0 \sin \omega t = NAB\omega \sin \omega t = NBA(2\pi\nu)\sin(2\pi\nu)t$.
- **Self-Inductance of a Long Solenoid** Self-inductance of a long solenoid is given by
  $$L = \frac{\mu_0 N^2 A}{l}$$
- **Mutual Inductance** The phenomenon according to which an opposing emf is produced as a result of change in current or magnetic flux linked with a neighbouring coil
- **Mutual Inductance of Two Long Coaxial Solenoids** Mutual inductance of two long coaxial solenoids is given by
  $$M = \frac{\mu_0 N_1 N_2 A}{l}$$
- **AC Generator** A generator produced electrical energy from mechanical work just opposite of what a motor does.

[redacted]

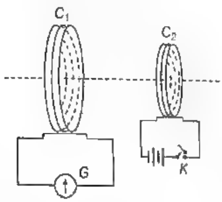
CHAPTER PRACTICE

OBJECTIVE Type Questions

1. A square of side L metres lies in the xy-plane in a region, where the magnetic field is given by $\mathbf{B} = B_0(2\hat{i} + 3\hat{j} + 4\hat{k})$ T, where B_0 is constant. The magnitude of flux passing through the square is **NCERT Exemplar**

   (a) $2B_0L^2$ Wb (b) $3B_0L^2$ Wb
   (c) $4B_0L^2$ Wb (d) $\sqrt{29}B_0L^2$ Wb

2. What will happen with the galvanometer when the tapping key K is pressed?



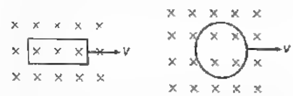
   (a) A momentary deflection
   (b) A long time deflection
   (c) No deflection
   (d) None of the above

3. The magnitude of the induced emf in a circuit is equal to the time rate of change of magnetic flux through the circuit, is statement of
   (a) Fleming's right hand rule
   (b) Fleming's left hand rule
   (c) Felming's third law
   (d) Faraday's law of electromagnetic induction

4. The direction of induced current is decided by
   (a) Lenz's law
   (b) Fleming's left hand rule
   (c) Biot-Savart's law
   (d) Ampere's law

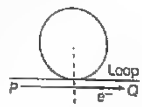
5. A 50 turns circular coil has a radius of 3 cm, it is kept in a magnetic field acting normal to the area of the coil. The magnetic field B increased from 0.10 T to 0.35 T in 2 ms^{-1}. The average induced emf in the coil is
   (a) 1.77 V (b) 17.7 V (c) 177 V (d) 0.177 V

6. A rectangular loop and a circular loop are moving out of a uniform magnetic field region in the given figure to a field free region with a constant velocity v. In which loop do you expect the induced emf to be constant during the passage out of the field region?



   (a) Rectangular loop (b) Circular loop
   (c) Both (a) and (b) (d) Neither (a) nor (b)

7. An electron moves along the line PQ which lies in the same plane as a circular loop of conducting wire as shown in figure. What will be the direction of the induced current in the loop?



   (a) Anti-clockwise
   (b) Clockwise
   (c) Alternating
   (d) Non-current will be induced

8. A constant current is flowing through a solenoid. An iron rod is inserted in the solenoid along its axis. Which of the following quantities will not increase? **CBSE 2021 (Term-I)**
   (a) The magnetic field at the centre
   (b) The magnetic flux linked with the solenoid
   (c) The rate of heating
   (d) The self-inductance of the solenoid

9. A coil of area 100 cm^2 is kept at an angle of 30° with a magnetic field of 10^{-1}T. The magnetic field is reduced to zero in 10^{-4}s. The induced emf in the coil is **CBSE 2021 (Term-I)**
   (a) $5\sqrt{3}$ V (b) $50\sqrt{3}$ V
   (c) 5.0 V (d) 50.0 V

[redacted]

10. The self-induced emf in a coil of 0.4 H self-inductance when current in it is changing at the rate of 50 As^{-1}, is

(a) 8×10^{-4} V (b) 8×10^{-3} V

(c) 20 V (d) 500 V

ASSERTION AND REASON

Directions (Q. Nos. 11-16) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

11. **Assertion** Faraday's law are consequence of conservation of energy.

Reason In purely resistive AC circuit, the current lags behind the emf in phase.

12. **Assertion** Lenz's law violates the principle of conservation of energy.

Reason Induced emf always opposes the change in magnetic flux responsible for its production.

13. **Assertion** In equation $\mathbf{F}=q(\mathbf{E}+\mathbf{v}\times\mathbf{B})$ when $v=0$, any force on the charge must arise from the electric field term E alone.

Reason To explain, the existence of induced emf or induced current in static conductor kept in time-varying magnetic field, we must assume that a time-varying magnetic field generates an electric field.

14. **Assertion** Eddy currents are undesirable.

Reason Eddy currents heat up the core and dissipate electrical energy in the form of heat.

15. **Assertion** Eddy current is produced in any metallic conductor when magnetic flux is changed around it.

Reason Electric potential determines the flow of charge.

16. **Assertion** If the inner solenoid was much shorter than (and placed well inside) the outer solenoid, then we could still have calculated the flux linkage $N_1\phi_1$.

Reason The inner solenoid is effectively immersed in a uniform magnetic field due to the outer solenoid.

CASE BASED QUESTIONS

Directions (Q.Nos. 17-18) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

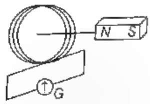
17. Faraday's Laws

According the Faraday's first law, whenever the amount of magnetic flux linked with a circuit changes, an emf is induced in it. Induced current is determined by the rate at which the magnetic flux changes.

Mathematically, the magnitude of the induced emf in a circuit is equal to the rate of change of magnetic flux through the circuit.

Induced emf $\propto$ Rate of change of magnetic flux

(i) On the basis of Faraday's law, current in the coil is larger



(a) when the magnet is pushed towards the coil faster

(b) when the magnet is pulled away the coil faster

(c) Both (a) and (b)

(d) Neither (a) nor (b)

(ii) The flux linked with a circuit is given by $\phi=t^3+3t-7$. The graph between time (X-axis) and induced emf (Y-axis) will be a

(a) straight line through the origin

(b) straight line with positive intercept

(c) straight line with negative intercept

(d) parabola not through the origin

(iii) Wire loop is rotated in a magnetic field. The frequency of change of direction of the induced emf is

(a) once per revolution

(b) twice per revolution

(c) four times per revolution

(d) six times per revolution

(iv) The instantaneous magnetic flux linked with a coil is given by $\phi=(5t^3-100t+300)$ Wb. The emf induced in the coil at time $t=2$ s is

(a) -40 V (b) 40 V

(c) 140 V (d) 300 V

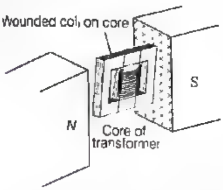
[redacted]

(v) A copper disc of radius 0.1 m is rotated about its centre with 20 rev/s in a uniform magnetic field of 0.1 T with its plane perpendicular to the field. The emf induced across the radius of the disc is

(a) $\frac{\pi}{20}$ V (b) $\frac{\pi}{10}$ V
(c) 20π mV (d) None of these

18. Eddy Current

Coil is wound over metallic core is helpful in reducing eddy currents in the metallic cores of transformers, electric motors, induction furnaces and other such devices (as shown below). Eddy current are undesirable since they heat up the core and dissipate electrical energy in the form of heat. These currents are minimised by using laminations of metal to make a metal core.



(i) How are eddy currents minimised to make a metal core of transformer on which coils are wound?
(a) By using laminations of metal
(b) By using solid metallic core
(c) Both (a) and (b)
(d) Neither (a) nor (b)

(ii) The plane of the laminations must be arranged parallel to the magnetic field, so that they cut across the
(a) keep on sliding
(b) keep on rotating
(c) cut across the induced eddy currents
(d) Both (a) and (b)

(iii) Induction furnace is used to produce
(a) low temperature to melt the metal
(b) high temperature to melt the metal
(c) constant low temperature 20°C
(d) high pressure

(iv) Induction furnace can be utilised to prepare
(a) alloys, by melting the constituent metals
(b) metal, by mixing electrons, protons, neutrons
(c) Both (a) and (b)
(d) Neither (a) nor (b)

(v) When a high frequency alternating current is passed through a coil which surrounds the metal to be melted. Then,
(a) the metal freezes
(b) coil rotates with frequency ω
(c) the metal melts
(d) None of the above

VERY SHORT ANSWER Type Questions

19. On what factors does the magnitude of induced emf in a coil depend?

20. If a coil is removed from a magnetic field
(i) slowly and
(ii) rapidly, then in which case, more work will be done?

21. Why is a core of transformer laminated?

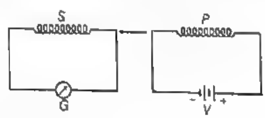
22. Give any two useful applications of eddy currents.

23. How can self-inductance of a given coil having N number of turns be changed, if N is doubled keeping other factors constant?

24. If two coils are very tightly wound over one another, will their mutual inductance increase or decrease as compared to the case when the coils are placed some distance apart?

SHORT ANSWER Type Questions

25. (i) When primary coil P is moved towards secondary coil S (as shown in the figure below), the galvanometer shows momentary deflection. What can be done to have larger deflection in the galvanometer with the same battery?

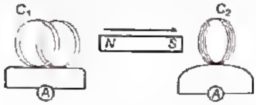


(ii) State the related law. **Delhi 2010**

26. Four shapes made of wires are situated in a magnetic field B, perpendicular to the plane of the paper, directed downwards. If B starts reducing, what will be the directions of the induced currents in these shapes?

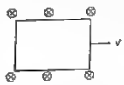
[redacted]

27. A magnet is quickly moved in the direction indicated by an arrow between two coils C_1 and C_2 as shown in the figure.



What will be the direction of induced current in each coil as seen from the magnet? Justify your answer.

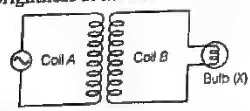
28. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B, constant in time and space, pointing perpendicular to the plane of the loop exists everywhere. What will be the current induced?



29. Two concentric circular coils C_1 and C_2, radius r_1 and r_2 ($r_1 << r_2$) respectively are kept coaxially. If current is passed through C_2, then find an expression for mutual inductance between the two coils.

LONG ANSWER Type I Questions

30. The figure given below shows an arrangement by which current flows through the bulb X connected with coil B, when AC is passed through coil A.
   (i) Name the phenomenon involved.
   (ii) If a copper sheet is inserted in the gap between the coils, explain how the brightness of the bulb would change?



31. State the law that gives the polarity of the induced emf. Give its illustration.

LONG ANSWER Type II Questions

32. State and explain Faraday's law of electromagnetic induction.
A cylindrical bar magnet is placed along the axis of a circular coil. Will there be a current induced in the coil, if magnet is rotated about its axis?

33. State Lenz's law. Give one example to illustrate this law. Lenz's law is a consequence of principle of energy conservation. Justify this statement.

34. How is the mutual inductance of a pair of coils affected, when
   (i) separation between the coils is increased?
   (ii) the number of turns of each coil is increased?
   (iii) a thin iron sheet is placed between the two coils, other factors remaining the same?
   Explain your answer in each case.

NUMERICAL PROBLEMS

35. A jet plane is travelling towards West at a speed of 450 m/s. If the horizontal component of the earth's magnetic field at that place is 4×10^{-4} T and dip angle is 30°, calculate the emf induced between the ends of wings having a span of 30 m.

36. A long solenoid of 10 turns/cm has a small loop of area 1 cm^2 placed inside with the normal of the loop parallel to the axis. Calculate the voltage across the small loop, if the current in the solenoid is changed from 1 A to 2 A in 0.1 s, during the duration of this charge.

37. A solenoid of length 80 cm, area of cross-section 1 m^2 with 20 turns per cm completely surrounds another coaxial solenoid of the same length, area of cross-section 25 cm^2 with 20 turns per cm. Calculate the mutual inductance of the system.

ANSWERS

1. (c)	2. (a)	3. (d)	4. (a)	5. (b)
6. (a)	7. (d)	8. (c)	9. (a)	10. (c)

11. (c) According to Faraday's law of the conservation mechanical energy into electrical energy is in accordance with the law of conservation of energy. It is also clearly known that in pure resistance, the emf is in phase with the current.

12. (d) Lenz's law is based on conservation of energy and induced emf always opposes the cause of it, i.e. change in magnetic flux.

13. (a) Increasing B causes induced current I due to induced electric field E along I.

Note This induced electric field is non-conservative, makes closed loop electric field lines.

So, time varying magnetic field generates electric field

14. (a)

[redacted]

15. (b)

16. (a) If the inner solenoid was much shorter than (and placed well inside) the outer solenoid, then we could still have calculated the flux linkage $N_1\phi_1$ because the inner solenoid is effectively immersed in a uniform magnetic field due to the outer solenoid.

17. (i) (c) Current will be larger, when the magnet is pushed faster towards the coil, also current is large when magnet is pulled faster away but now it is in opposite direction.

(ii) (d) $\phi = t^3 + 3t - 7$

$\therefore$ Induced emf, $e = -\dfrac{d\phi}{dt} = -(3t^2 + 3) = -3t^2 - 3$

At $t = 0; e = -3\text{ V}$

Therefore, shape of graph will be a parabola not through origin. $(\because e \propto t^2)$

(iii) (b) If a wire loop is rotated in a magnetic field, the frequency of change in the direction of the induced emf is twice per revolution.

(iv) (b) Given, $\phi = (5t^3 - 100t + 300)$, $t = 2\text{ s}$

Induced electromotive force,

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}(5t^3 - 100t + 300)$$

$$e = -5\times 3t^2 + 100 = -5\times 3(2)^2 + 100$$

$$= -5\times 12 + 100 = -60 + 100 = 40\text{ V}$$

(v) (c) From Faraday's law of electromagnetic induction,

$$e = -\frac{d\phi}{dt} = -BAN \quad (\because dt = 1\text{s})$$

Given, $B = 0.1$ T, $N = 20$, $A = \pi r^2 = \pi(0.1)^2$

$\therefore \quad e = 0.1 \times 20 \times \pi(0.1)^2 = 20\pi$ mV

18. (i) (a) Eddy current are minimised by using laminations of metal to make a metal core.

(ii) (c) The laminations are separated by an insulating material like lacquer. The plane of the laminations must be arranged parallel to the magnetic field, so that they cut across the eddy current paths. This arrangement reduces the strength of the eddy current. Since, the dissipation of the strength of electric current, heat loss is substantially reduced.

(iii) (b) Induction furnace can be used to produce high temperatures and can be utilised to prepare alloys, by melting the constituent metals. A high frequency alternating current is passed through a coil which surrounds the metals to be melted. The eddy currents generated in the metals produce high temperatures sufficient to melt it.

(iv) (a) Since, induction furnaces uses the concept of eddy currents. Thus they are used to prepare alloys, by melting the constituent metals.

(v) (c) Eddy current generated in the metal produce high temperature sufficient to melt it.

19. Number of turns in coils, and rate of change of magnetic flux.

20. More work will be done in (ii) Case.

21. To prevent it from eddy current being produced in the core

22. In electric power meters and in induction furnace.

23. Doubled.

24. Increases

25. Refer to text on page 252.

26. Refer to text on page 253.

27. Refer to Q. 10 on page 259.

28. Current induced is zero.

29. Refer to Q. 15 on page 275.

30. Refer to Q. 11 on page 259.

31. **Lenz's law** Refer to text on pages 253 and 254.

32. Refer to text on pages 252 and 253.

33. Refer to text on pages 253 and 254.

34. Refer to text on page 271. Also, refer Q. 12 on page 274.

35. $e = 3$ kV.
Refer to Q. 35 on page 262.

36. Voltage $= 12.57 \times 10^{-7}$ V
Refer to Q. 34 on page 276.

37. Mutual inductance = 3.92 H
Refer to Example 7 on page 272.

[redacted]

07

Most of the electric power generated and used in the world is in the form of alternating current. This is because

(i) alternating voltages can be easily and efficiently converted from one value to the other by means of transformers.

(ii) the alternating current energy can be transmitted and distributed over long distances economically without much loss of energy.

ALTERNATING CURRENT

In this chapter, we will study about some alternating current system that transfers energy efficiently and we will also discuss some of the devices that make use of alternating current.

CHAPTER CHECKLIST

- Introduction to Alternating Current
- AC Circuits
- AC Devices

|TOPIC 1|
Introduction to Alternating Current

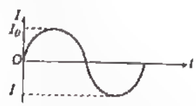
Alternating Current (AC)

If the direction of current changes alternatively (periodically) and its magnitude changes continuously with respect to time, then the current is called alternating current. It is sinusoidal (i.e. represented by sine or cosine angles) in nature.

Alternating current can be defined as the current whose magnitude and direction changes with time and attains the same magnitude and direction after a definite time interval It changes continuously between zero and a maximum value and flows in one direction in the first half cycle and in the opposite direction in the next half cycle.

The instantaneous value of AC is given by

$$I = I_0 \sin\omega t \qquad \left[\because \omega = \frac{2\pi}{T} = 2\pi\nu\right]$$



Current vs time graph of an AC

where, I = current at any instant t, I_0 = maximum/peak value of AC, ν = frequency and ω = angular frequency.

Note Current whose direction does not change with time through a load is known as direct current (DC)

[redacted]

Advantages of AC over DC

(i) AC generation is easy and economical.
(ii) It can be easily converted into DC with the help of rectifier.
(iii) In AC, energy loss is minimum, so it can be transmitted over large distances.

Disadvantages of AC over DC

(i) AC shock is attractive while DC shock is repulsive so, 220V AC is more dangerous than 220V DC.
(ii) AC cannot be used in electroplating process because here constant current with constant polarity is needed which is given by DC.

Alternating emf or Voltage

It can be defined as the voltage whose magnitude and direction changes with time and attains the same magnitude and direction after a definite time interval. The instantaneous value of alternating emf or voltage is given by

$$E = E_0 \sin \omega t$$

where, E = voltage at any time t, E_0 = maximum/peak value of alternating voltage and ω = angular frequency.

Note Alternating current, alternating emf, flux etc. all are sinusoidal waves.

MEAN OR AVERAGE VALUE OF AC

It is defined as the value of AC (Alternating Current) which would send same amount of charge through a circuit in half cycle (i.e. $T/2$) that is sent by steady current in the same time. It is denoted by I_m or I_{av}.

Let the instantaneous value of alternating current is represented by

$$I = I_0 \sin \omega t \quad \text{... (i)}$$

The AC changes continuously with time. Suppose current is kept constant for small time (dt). Then, small amount of charge (dq) in small time (dt) is given by

$$dq = I dt = I_0 \sin \omega t \, dt \quad \text{[from Eq. (i)]}$$

To calculate total charge send by AC over half cycle is given by

$$\int dq = \int_0^{T/2} I_0 \sin \omega t \, dt$$

or

$$q_s = I_0 \int_0^{T/2} \sin \omega t \, dt$$

Here, q_s is steady charge over half cycle.

$$\Rightarrow \quad q_s = I_0 \left[\frac{-\cos \omega t}{\omega}\right]_0^{T/2} = \frac{-I_0}{\omega}[\cos \omega t]_0^{T/2}$$

$$= \frac{-I_0}{\omega}\left[\cos \frac{\omega T}{2} - \cos 0°\right]$$

$$= \frac{-I_0}{\omega}\left[\cos \frac{\omega T}{2} - 1\right] = \frac{-I_0}{\omega}\left[\cos \frac{2\pi}{2} - 1\right]$$

$$\left[\because \omega = \frac{2\pi}{T} \Rightarrow \omega T = 2\pi\right]$$

$$= \frac{-I_0}{\omega}[\cos \pi - 1] = \frac{-I_0}{\omega}[-1-1] \quad [\because \cos \pi = -1]$$

$$\Rightarrow \quad q_s = \frac{2I_0}{\omega}$$

Also, the charge sent by AC in positive half cycle is

$$q_{AC} = I_m \times \frac{T}{2}$$

where, I_m is mean value of AC over half cycle.

According to the definition,

$$q_s = q_{AC} \quad \text{[over any half cycle]}$$

$$\Rightarrow \quad \frac{2I_0}{\omega} = I_m \times \frac{T}{2}$$

$$\Rightarrow \quad I_m = \frac{4I_0}{\omega T} = \frac{4I_0}{2\pi} \quad [\because \omega T = 2\pi]$$

$$\Rightarrow \quad I_m = \frac{2I_0}{\pi} = 0.637 I_0$$

$$\therefore \quad \boxed{I_m = 0.637 I_0}$$

Mean value of AC (I_m) is 63.7% of the peak value of AC (I_0) over positive half cycle. For negative half cycle, the mean value of AC will be $-2I_0/\pi$. Therefore, in a complete cycle, the mean value of AC will be zero.

In the same way, mean value of alternating emf (E_m) is

$$\boxed{E_m = \frac{2E_0}{\pi} = 0.637 E_0}$$

Root Mean Square (RMS) Value of AC

It is defined as that value of Alternating Current (AC) over a complete cycle which would generate same amount of heat in a given resistor that is generated by steady current in the same resistor and in the same time during a complete cycle.

It is also called virtual value or effective value of AC.

It is represented by I_{rms} or I_{eff} or I_V. Suppose I is the current which flows in the resistor having resistance (R) in time (T) produces heat (H).

Instantaneous value of AC,

$$I = I_0 \sin \omega t$$

[redacted]

If dH is small amount of heat produced in time dt in resistor R, then

$$dH = I^2 R\, dt \qquad [\because H = I^2RT] \quad \ldots(i)$$

In complete cycle $(0 \to T)$, the total heat produced is H. After integrating Eq. (i), we get

$$\int dH = \int_0^T I^2 R\, dt \Rightarrow H = \int_0^T I^2 R\, dt$$

Put the value of I in the above equation, we get

$$H = \int_0^T (I_0 \sin \omega t)^2 R\, dt$$

$$= I_0^2 R \int_0^T \sin^2 \omega t\, dt = I_0^2 R \int_0^T \left[\frac{1-\cos 2\omega t}{2}\right] dt$$

$$\left[\because \sin^2 \omega t = \frac{1-\cos 2\omega t}{2}\right]$$

$$= \frac{I_0^2 R}{2} \int_0^T (1-\cos 2\omega t)\, dt$$

$$= \frac{I_0^2 R}{2}\left[\int_0^T dt - \int_0^T \cos 2\omega t\, dt\right]$$

$$= \frac{I_0^2 R}{2}\left[|t|_0^T - \left|\frac{\sin 2\omega t}{2\omega}\right|_0^T\right]$$

$$= \frac{I_0^2 R}{2}\left[(T-0) - \frac{1}{2\omega}|\sin 2\omega T - \sin 0°|\right]$$

$$= \frac{I_0^2 R}{2}\left[T - \frac{1}{2\omega}|\sin 2 \times 2\pi - 0|\right] [\because \omega T = 2\pi]$$

$$= \frac{I_0^2 R}{2}\left[T - \frac{1}{2\omega}|0-0|\right] \qquad [\because \sin 4\pi = 0]$$

$$= \frac{I_0^2 RT}{2} \qquad \ldots(ii)$$

If I_{rms} is rms value of alternating current and H is the heat produced by rms current (I_{rms}), then

$$H = I_{rms}^2 RT \qquad \ldots(iii)$$

On comparing Eqs. (ii) and (iii), we get

$$I_{rms}^2 RT = \frac{I_0^2 RT}{2} \Rightarrow I_{rms}^2 = \frac{I_0^2}{2}$$

$$\Rightarrow \boxed{I_{rms} = \sqrt{\frac{I_0^2}{2}} = \frac{I_0}{\sqrt{2}} = 0.707 I_0}$$

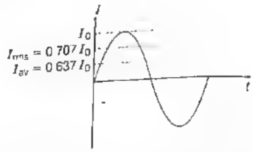
$$\Rightarrow \boxed{I_{rms} = 70.7\% \text{ of } I_0}$$

From the above equation, we conclude that rms value of current is 70.7% of the peak value of current.

In the same way, the rms value of alternating emf (E_{rms} or E_{eff} or E_V) is

$$E_{rms} = \frac{E_0}{\sqrt{2}} = 0.707 E_0 = 70.7\% \text{ of } E_0$$

The different values I_0, I_{av} and I_{rms} are shown in figure given below.



RMS and Average value of current on the same graph

AC Ammeter and Voltmeter

AC ammeter and voltmeter always measure the virtual value of AC or alternating emf.

They are also called hot wire instruments because deflection in the needle depends upon the heat produced in any coil.

$$H \propto I_{rms}^2$$

If we connect ordinary DC ammeter or voltmeter to AC circuit, they read zero because average value of alternating current/voltage over a full cycle is zero.

EXAMPLE |1| The instantaneous current from an AC source is given by $I = 5 \sin 314t$. What is the rms value of the current?

Sol. Given, $I = 5 \sin 314t$ ...(i)

We know that, $I = I_0 \sin \omega t$ ...(ii)

On comparing Eqs. (i) and (ii), we have

$I_0 = 5$ A and $\omega = 314$

$\therefore \quad I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}} = 3.54$ A

EXAMPLE |2| Calculate the instantaneous voltage for AC supply of 220 V and 50Hz.

Sol. Given, $E_V = 220$ V, $\nu = 50$ Hz and $E = ?$

Since, we know that for calculating the peak value of alternating voltage E_0, we can use the relation

$$E_0 = \sqrt{2} E_V = 1.414 \times 220 = 311 \text{ V}$$

Therefore, instantaneous voltage, $E = E_0 \sin \omega t$

$$E = E_0 \sin(2\pi\nu)t$$
$$= 311 \sin(2\pi \times 50)t$$
$$= 311 \sin 100\pi t$$

[redacted]

EXAMPLE |3| In an AC circuit, the rms voltage is $100\sqrt{2}$ V. Determine the peak value of voltage and its mean value during a positive half cycle.

Sol. Given, $E_V = 100\sqrt{2}$ V

Peak value of voltage, $E_0 = ?$

Mean value of voltage, $E_m = ?$

$$\because \quad E_0 = \sqrt{2}E_V = \sqrt{2}(100\sqrt{2}) = 200\text{V}$$

$\therefore$ During positive half cycle $(0 \to T/2)$,

$$E_m = \frac{2E_0}{\pi} = \frac{2 \times 200}{3.14} = 127.4 \text{ V}$$

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. The peak voltage in a 220 V, AC source is
 (a) 220 V (b) about 160 V
 (c) about 310 V (d) 440 V

2. If the rms current in a 50 Hz AC circuit is 5 A, the value of the current 1/300 s after its value becomes zero is **NCERT Exemplar**
 (a) $5\sqrt{2}$ A (b) $5\sqrt{3/2}$ A
 (c) $5/6$ A (d) $5/\sqrt{2}$ A

3. If the reading of AC mains voltage by a voltmeter is 200 V, then the root mean square value of this voltage will be
 (a) $200\sqrt{2}$ V (b) $100\sqrt{2}$ V
 (c) 200 V (d) $400/\pi$ V

4. The reading of an ammeter in an alternating circuit is 4 A. The peak (maximum) value of current in the circuit is
 (a) 4 A (b) 8 A
 (c) $4\sqrt{2}$ A (d) $\frac{2}{\sqrt{2}}$ A

5. When a voltage measuring device is connected to AC mains, the meter shows the steady input voltage of 220 V. This means **NCERT Exemplar**
 (a) input voltage cannot be AC voltage, but a DC voltage
 (b) maximum input voltage is 220 V
 (c) the meter reads not V but $< V^2 >$ and is calibrated to read $\sqrt{< V^2 >}$
 (d) the pointer of the meter is stuck by some mechanical defect

VERY SHORT ANSWER Type Questions

6. Define the term rms value of the current. How is it related to the peak value? **All India 2010C**

7. The peak value of emf in AC is E_0. Write its (i) rms (ii) average value over a complete cycle. **Foreign 2011**

8. In many European homes and offices, the rms voltage available from a wall socket is 240 V. What is the maximum voltage in this case?

9. An AC current $I = I_0 \sin \omega t$ produces certain heat H in a resistor R over a time $T = 2\pi/\omega$. Write the value of the DC current that would produce the same heat in the same resistor in the same time. **All India 2009C**

10. An alternating current is given by $I = I_1 \cos\omega t + I_2 \sin\omega t$. Determine the rms value of current through the circuit.

11. The current through an AC circuit is $I_t = I_0(t/\tau)$ for sometime. Determine the rms current through the circuit over time interval $t = 0$ to $t = \tau$.

12. Can the instantaneous power output of an AC source ever be negative? Can the average power output be negative? **NCERT Exemplar**

SHORT ANSWER Type Questions

13. Establish an expression for the average voltage of AC voltage $V = V_0 \sin \omega t$ over the time interval $t = 0$ and $t = \pi/\omega$.

14. Which of the following 120 V AC devices cost more to operate (i) one that draws an rms current of 10 A or (ii) one that draws a peak current of 12 A? Explain the reason for your answer.

15. Show that heat produced in a cycle of AC is same as the heat produced by DC with $I = I_{rms}$.

16. Both alternating current and direct current are measured in amperes. But how is the ampere defined for an alternating current? **NCERT Exemplar**

NUMERICAL PROBLEMS

17. The instantaneous value of current in an AC circuit is $I = 2 \sin (100\pi t + \pi/3)$ A. At what first time, the current will be maximum?

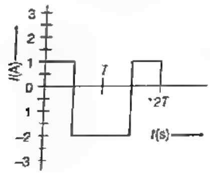
[redacted]

18. An alternating current in a circuit is given by $I = 20\sin(100\pi t + 0.05\pi)$ A. What is the rms value of current?

19. (i) The peak voltage of an AC supply is 300 V. What is its rms voltage?
(ii) The rms value of current in an AC circuit is 10 A. What is the peak current? **NCERT**

20. A light bulb is rated 100 W for 220 V AC supply of 50 Hz. Calculate
(i) resistance of the bulb.
(ii) the rms current through the bulb.
All India 2012

21. The alternating current in a circuit is described by the graph shown in the figure. Find the rms current in this graph. **NCERT Exemplar**



HINTS AND SOLUTIONS

1. (c) Given, $E_V = 220$ V

Peak voltage, $E_0 = \sqrt{2}E_V = \sqrt{2}\times 220 = 310$ V

Thus, option (c) is correct.

2. (b) Given, $\nu = 50$ Hz, $I_{rms} = 5$A

$$t = \frac{1}{300}\text{ s}$$

We have to find $I(t)$

$$I_0 = \text{Peak value}$$
$$= \sqrt{2}\,I_{rms} = \sqrt{2}\times 5$$
$$= 5\sqrt{2}\ \text{A}$$
$$I = I_0 \sin\omega t = 5\sqrt{2}\sin 2\pi\nu t$$
$$= 5\sqrt{2}\sin 2\pi\times 50\times\frac{1}{300}$$
$$= 5\sqrt{2}\sin\frac{\pi}{3}$$
$$= 5\sqrt{2}\times\frac{\sqrt{3}}{2} = 5\sqrt{3/2}\ \text{A}$$

3. (c) $E_V = 200\text{ V} = E_{rms}$

Root mean square value of this voltage is the effective value of voltage i.e., equal to the voltage indicated in voltmeter.

4. (c) Given, $I_{rms} = 4$ A

The peak value of current, $I_0 = I_{rms}\sqrt{2} = 4\sqrt{2}$ A

5. (c) The voltmeter connected to AC mains reads mean value ($<V^2>$) and is calibrated in such a way that it gives value of $<V^2>$, which is multiplied by form factor to give rms value.

6. It is defined as the value of Alternating Current (AC) over a complete cycle which would generate same amount of heat in a given resistor that is generated by steady current in the same resistor and in the same time during a complete cycle. It is also called virtual value or effective value of AC

Let the peak value of the current be I_0.

$$\therefore \quad I_{rms} = \frac{I_0}{\sqrt{2}}$$
$$\Rightarrow \quad I_{rms} = \frac{I_0}{\sqrt{2}}$$

where, I_0 = peak value of AC.

7. E_0 = peak value of emf in a complete cycle,

(i) rms value $[E_{rms}] = \dfrac{E_0}{\sqrt{2}}$

(ii) average value $[E]$ = zero

8. As we know, $E_{rms} = \dfrac{E_0}{\sqrt{2}}$

$\therefore E_0$ (Maximum voltage)

$$= E_{rms}\times 1.414 = 339.36\text{ V}$$

9. An AC current $I = I_0 \sin\omega t$ produces certain heat H in a resistor R over a time $T = 2\times 3.14/\omega$, is given by

$H = \left(\dfrac{I_0}{\sqrt{2}}\right)^2 RT$ and the same amount of heat produced by DC current in same time is given by $H = I^2RT$.

As, these heats are equal, then $I^2RT = \left(\dfrac{I_0}{\sqrt{2}}\right)^2 RT$

So, $I = \dfrac{I_0}{\sqrt{2}}$, where I stands for DC and I_0 is the peak value of AC current.

10. As, $I_{rms_1} = \dfrac{I_1}{\sqrt{2}}$

and $I_{rms_2} = \dfrac{I_2}{\sqrt{2}}$

Hence, the resultant of these two currents,

$$I_{rms} = \sqrt{\left(\frac{I_1}{\sqrt{2}}\right)^2 + \left(\frac{I_2}{\sqrt{2}}\right)^2} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$$

[redacted]

11. The mean square current is

$$\overline{I^2} = \frac{1}{\tau}\int_0^{\tau} I_0^2 (t/\tau)^2 dt$$

$$= \frac{I_0^2}{\tau^3}\int_0^{\tau} t^2 dt = \frac{I_0^2}{\tau^3}\left[\frac{t^3}{3}\right]_0^{\tau} = \frac{I_0^2}{3} \quad [\because t = \tau]$$

$$\therefore I_{rms} = \sqrt{\overline{I^2}} = \sqrt{\left(\frac{I_0^2}{3}\right)} = \frac{I_0}{\sqrt{3}}$$

12. Yes, the instantaneous power can be negative as, $P_{instantaneous} = I_{in} \times V_{in} = I_0 \sin \omega t \times V_0 \cos \omega t$
No, because it is average, so it will be positive.

13. Average voltage, V_m or $V_{av} = \frac{2V_0}{\pi} = 0.637 V_0$

Refer to the text on page 290.

14. Alternating currents and voltage are generaly measured in the terms of their rms values.
Since, the electric cost is calculated on the power used, i.e. $P \propto I_{rms}^2$

$\therefore$ For $I_{rms} = 10$ A

$$P_1 \propto (10)^2 \quad \text{...(i)}$$

and for $I_0 = 12$ A,

$$\Rightarrow I_{rms} = \frac{12}{1.414} = 8.48 \text{ A} \quad \left[\because I_{rms} = \frac{I_0}{\sqrt{2}}\right]$$

$$\Rightarrow P_2 \propto (8.48)^2 \quad \text{...(ii)}$$

So, from Eqs. (i) and (ii), the device that draws a rms current of 10 A costs more to operate.

15. For an AC, $I_t = I_0 \sin\omega t$
Heat produced in a resistance in small time dt,

$$dU = I_t^2 R\, dt = (I_0 \sin\omega t)^2 R\, dt$$

$\therefore$ Heat produced during a full cycle of AC,

$$U = \int dU = I_0^2 R \int_0^T \sin^2 \omega t\, dt$$

$$= \frac{I_0^2}{2} R[T] \quad [\because \omega T = 2\pi]$$

$$\Rightarrow U = I_{rms}^2 RT \quad \left[\because I_{rms} = \frac{I_0}{\sqrt{2}}\right]$$

Thus, we see that AC produces same heating effect as DC of value $I = I_{rms}$.

16. As we know that, the AC current changes its direction with time. So, AC ampere must be defined in terms of some property which is independent of direction of current. Thus, Joule's heating effect is the property, which defines rms value of AC.

17. Here, $I = 2\sin(100\,\pi t + \pi/3)$ A ...(i)
Since, the relation between current and time gives us

$$\frac{2\pi t}{T} = 100\,\pi t$$

$$\therefore T = \frac{2\pi}{100\pi} = \frac{1}{50}\text{ s} \quad \text{...(ii)}$$

$$\Rightarrow t = \frac{T}{4} = \frac{1}{50 \times 4} = \frac{1}{200}\text{ s}$$

18. $I_{rms} = 10\sqrt{2}$ A
Refer to the Example 1 on page 290.

19. (i) $E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{300}{\sqrt{2}} = 212.1$ V $\quad [\because E_0 = \text{peak voltage}]$

(ii) $I_{rms} = 10$ A

$$\Rightarrow I_{rms} = \frac{I_0}{\sqrt{2}} \quad [\because I_0 = \text{peak current}]$$

$$\Rightarrow I_0 = \sqrt{2} I_{rms} = 10 \times \sqrt{2} = 14.14 \text{ A}$$

20. (i) Power, $P = EI \Rightarrow P = E \times \frac{E}{R}$ $\quad \left[\because I = \frac{E}{R}\right]$

$$\Rightarrow R = \frac{E^2}{P} = \frac{(220)^2}{100}$$

$$= \frac{48400}{100} = 484\ \Omega$$

(ii) The peak voltage of the source is $E_{rms} = \frac{E_0}{\sqrt{2}}$

$$\Rightarrow E_0 = E_{rms} \times \sqrt{2}$$

$$= 220\sqrt{2} = 311.13 \text{ V}$$

$$\therefore I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{E_0}{R\sqrt{2}} = \frac{311.13}{484\sqrt{2}}$$

$$= \frac{311.13}{684.479} = 0.45 \text{ A}$$

21. From the graph, $I_1 = 1$ A, $I_2 = -2$A and $I_3 = 1$A

$$I_{rms} = \sqrt{\frac{I_1^2 + I_2^2 + I_3^2}{3}} = \sqrt{\frac{1^2 + (-2)^2 + 1^2}{3}}$$

$$= \sqrt{\frac{6}{3}} = \sqrt{2} = 1.414 \text{ A}$$

[redacted]

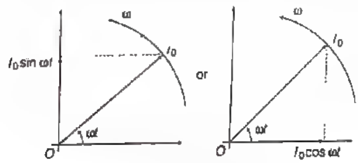
|TOPIC 2|
AC Circuits

Phasor Diagrams

The study of AC circuit is much simplified, if we represent alternating current, alternating emf as rotating vector, with the angle between the vectors equal to the phase difference between the current and the emf. These rotating vectors representing current and alternating emf are called **phasors**.

A diagram representing alternating current and alternating emf (of same frequency) as rotating vectors (phasors) with the phase angle between them is called **phasor diagram**.

The length of the vector represents the maximum or peak value, i.e. I_0 and E_0. The projection of the vector on fixed axis gives the instantaneous value of alternating current and alternating emf. In sine form, ($I = I_0 \sin \omega t$ and $E = E_0 \sin \omega t$), projection is taken on Y-axis. In cosine form, ($I = I_0 \cos \omega t$ and $E = E_0 \cos \omega t$), projection is taken on X-axis.



Phasor diagram

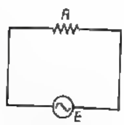
DIFFERENT TYPES OF AC CIRCUIT

In this section, we will derive voltage current relations for individual as well as combined circuit elements carrying a sinusoidal current. Here, we will only consider resistors, inductors and capacitors.

AC through Resistor

Suppose a resistor of resistance R is connected to an AC source of emf with instantaneous value (E), which is given by

$$E = E_0 \sin \omega t \qquad \text{...(i)}$$



An AC voltage applied to a resistor

Let E be the potential drop across resistance (R), then

$$E = IR \qquad \text{...(ii)}$$

∵ Instantaneous emf = Instantaneous value of potential drop

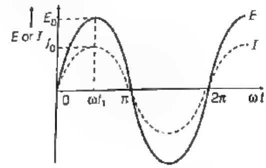
From Eqs. (i) and (ii), we have

$$IR = E = E_0 \sin \omega t$$

$$\Rightarrow \quad I = \frac{E}{R} = \frac{E_0 \sin \omega t}{R}$$

$$\Rightarrow \quad \boxed{I = I_0 \sin \omega t} \qquad \left[\because I_0 = \frac{E_0}{R}\right] \qquad \text{...(iii)}$$

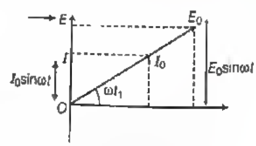
Comparing $I_0 = E_0/R$ with Ohm's law, we find that resistors work equally well for both AC and DC voltages. From Eqs. (i) and (iii), we get that for resistor there is zero phase difference between instantaneous alternating current and instantaneous alternating emf (i.e. they are in same phase).



Graph of E and I versus ωt

Phasor Diagram

Here, peak values E_0 and I_0 are represented by vectors rotating with angular velocity ω with respect to horizontal reference. Their projections on vertical axis give their instantaneous values.



Phasor diagram for a purely resistive circuit

EXAMPLE |1| A resistance of 20 Ω is connected to a source of alternating current rated 110 V, 50 Hz. Find the

(i) rms current.

(ii) maximum instantaneous current in the resistor.

(iii) time taken by the current to change from its maximum value to the rms value.

[redacted]

Sol. Given, resistance, $R = 20\,\Omega$

The rms value of voltage, $E_{rms} = 110\,V$

Frequency, $\nu = 50$ Hz

(i) $I_{rms} = \frac{E_{rms}}{R} = \frac{110}{20} = 5.5$ A

(ii) $I_0 = \sqrt{2}I_{rms} = 1.414 \times 5.5 = 7.8$ A

(iii) Let the AC be represented by $I = I_0 \cos \omega t$

At $t = 0, I = I_0 \cos 0 = I_0(\text{max})$

At $t = t$, let $I = I_V = \frac{I_0}{\sqrt{2}} = I_0 \cos \omega t$

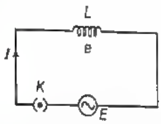
$\because \cos \omega t = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4} \Rightarrow \omega t = \frac{\pi}{4} \Rightarrow 2\pi\nu t = \frac{\pi}{4}$

$\therefore \quad t = \frac{1}{8\nu} = \frac{1}{8 \times 50} = 2.5 \times 10^{-3}$ s

AC through Inductor

Suppose an inductor with self-inductance (L) is connected to an AC source with instantaneous emf (E), which is given by

$$E = E_0 \sin \omega t \quad \ldots(\text{i})$$



An AC source connected to an inductor

When key K is closed, then current I begins to grow because magnetic flux linked with it changes and induced emf produces which opposes the applied emf.

According to Lenz's law,

$$e = -L\frac{dI}{dt} \quad \ldots(\text{i})$$

where, e is induced emf and $\frac{dI}{dt}$ is the rate of change of current.

To maintain the flow of current in the circuit, applied voltage must be equal and opposite to the induced emf i.e.

$E = -e$

$\therefore \quad E = -\left(-\frac{LdI}{dt}\right) = \frac{LdI}{dt}$ or $dI = \frac{E}{L}dt$ [from Eq. (i)]

Integrating the above equation on both sides, we get

$\int dI = \int \frac{E}{L}dt \Rightarrow I = \int \frac{E_0 \sin \omega t}{L}dt \quad [\because E = E_0 \sin \omega t]$

$\Rightarrow \quad I = \frac{E_0}{L}\left[\frac{-\cos \omega t}{\omega}\right]$

$\Rightarrow \quad I = -\frac{E_0}{\omega L}\sin(\pi/2 - \omega t) \quad \left[\because \sin\left(\frac{\pi}{2} - \omega t\right) = \cos \omega t\right]$

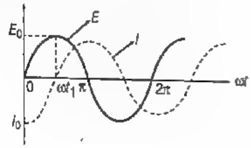
$\Rightarrow \quad I = \frac{E_0}{\omega L}\sin(\omega t - \pi/2) \quad \ldots(\text{ii})$

If $\sin(\omega t - \pi/2) = \text{maximum} = 1$, then $I = I_0$

where, peak value of current, $I_0 = \frac{E_0}{\omega L}$

$\therefore \quad \boxed{I = I_0 \sin(\omega t - \pi/2)} \quad \ldots(\text{iii})$

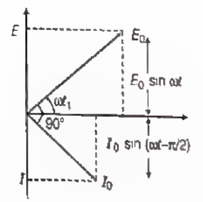
From Eqs. (i) and (iii), it is clear that in a pure inductor, the current lags behind the voltage by a phase angle of $\pi/2$ radians (90°) or the voltage leads the current by a phase angle of $\pi/2$ radians (90°).



Graph of E and I versus ωt

Phasor Diagram

The phasor representing peak emf E_0 makes an angle ωt_1 in anti-clockwise direction from horizontal axis. As current lags behind the voltage by 90°, so the phasor representing I_0 is turned 90° clockwise with the direction of E_0.



Phasor digram for purely inductive circuit

Inductive Reactance (X_L)

The opposing nature of inductor to the flow of alternating current is called inductive reactance.

As, $\quad I = \frac{E_0}{\omega L}\sin(\omega t - \pi/2)$ or $I_0 = E_0/\omega L$

Comparing the above with Ohm's law, $I_0 = \frac{E_0}{R}$. The quantity ωL is analogous to the resistance and is denoted by X_L.

So, $\quad \boxed{X_L = \omega L}$

where, X_L is called inductive reactance.

[redacted]

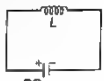
If f is the frequency of AC source, then

$$X_L = \omega L = 2\pi\nu L \quad ...(i) \quad \left[\because \omega = \frac{2\pi}{T} = 2\pi\nu\right]$$

The dimension of inductive reactance is the same as that of resistance and its SI unit is ohm (Ω). The inductive reactance limits the current in a purely inductive circuit in the same way as the resistance limits the current in a purely resistive circuit. It is also directly proportional to the inductance and to the frequency of the AC current. Thus, if the frequency of AC increases, its inductive reactance also increases.

If inductor is connected to DC source

$$\nu = 0 \qquad \left[\because \nu = \frac{1}{T}\right]$$



DC source

Here, ν is frequency. So, from Eq. (i)

$X_L = 0$.

Therefore, inductor passes DC and blocks AC of very high frequency.

EXAMPLE |2| Alternating emf of $E = 220\sin 100\pi t$ is applied to a circuit containing an inductance of $(1/\pi)$ H. Write equation for instantaneous current through the circuit. What will be the reading of AC galvanometer connected in the circuit?

Sol. Given, $E = 220\sin 100\pi t$

$$E_0 = 220\text{ V}, \omega = 100\pi, L = (1/\pi)\text{ H}$$

Since, inductive reactance, $X_L = \omega L$

$$X_L = 100\pi \times \frac{1}{\pi} = 100\,\Omega$$

$$\therefore \quad I_0 = \frac{E_0}{X_L} = \frac{220}{100} = 2.2\text{ A}$$

As current lags behind the emf by a phase angle of $\frac{\pi}{2}$.

$$\therefore \quad I = I_0 \sin(\omega t - \pi/2) = 2.2 \sin(100\pi t - \pi/2)$$

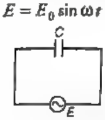
$\therefore$ Reading of AC galvanometer, $I_V = \frac{I_0}{\sqrt{2}} = \frac{2.2}{\sqrt{2}}$

$$= \frac{2.2}{\sqrt{2}} = 1.55\text{ A}$$

AC through Capacitor

Let us consider a capacitor with capacitance C be connected to an AC source with an emf having instantaneous value,

$$E = E_0 \sin \omega t \quad ...(i)$$



An AC source connected to a capacitor

Due to this emf, charge will be produced and it will charge the plates of capacitor with positive and negative charges. If potential difference across the plates of capacitor is V, then

$$V = \frac{q}{C} \text{ or } q = CV$$

The instantaneous value of current in the circuit,

$$I = \frac{dq}{dt} = \frac{d}{dt}(CE) \qquad [\because V = E]$$

$$= \frac{d}{dt}(CE_0 \sin \omega t) \qquad [\because E = E_0 \sin \omega t]$$

$$= CE_0 \cos \omega t \times \omega$$

$$= \frac{E_0}{1/\omega C} \cos \omega t$$

$$\Rightarrow \quad I = \frac{E_0}{1/\omega C} \sin(\omega t + \pi/2) \quad ...(ii)$$

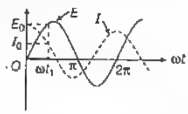
$$[\because \cos \omega t = \sin(\pi/2 + \omega t)]$$

I will be maximum when $\sin(\omega t + \pi/2) = 1$, so that $I = I_0$

where, peak value of current is, $I_0 = \frac{E_0}{1/\omega C}$

$$\therefore \quad \boxed{I = I_0 \sin(\omega t + \pi/2)} \quad ...(iii)$$

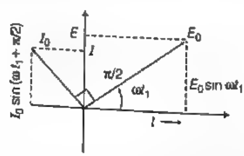
From Eqs. (i) and (iii), it is clear that in a perfect capacitor, the current leads the voltage by a phase angle of $\pi/2$ radians (90°) or the voltage lags behind the current by a phase angle of $\pi/2$ radians (90°).



Graph of E and I versus ωt

Phasor Diagram

The phasor representing peak emf E_0 makes an angle ωt_1 in anti-clockwise direction with respect to horizontal axis. As current leads the voltage by 90°, the phasor representing I_0 is turned 90° anti-clockwise with the phasor representing E_0. The projections of these phasors on the vertical axis give instantaneous values of E and I.



Phasor digram for purely capacitive circuit

[redacted]

Capacitive Reactance (X_C)

The instantaneous value of alternating current through a capacitor is given by

$$I = \frac{E_0}{1/\omega C}\sin(\omega t + \pi/2) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

Comparing the above with Ohm's law we get, $I_0 = \dfrac{E_0}{1/\omega C}$

$$\therefore \quad \boxed{X_C = \frac{1}{\omega C}}$$

where, X_C is called capacitive reactance.

The opposing nature of capacitor to the flow of alternating current is called capacitive reactance.

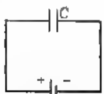
If ν is the frequency of the alternating current, then

$$\boxed{X_C = \frac{1}{2\pi\nu C}} \qquad \left[\because \omega = \frac{2\pi}{T} = 2\pi\nu\right]$$

The dimension of capacitive reactance is same as that of resistance and its SI unit is ohm (Ω). The capacitive reactance limits the amplitude of the current in a purely capacitive circuit in the same way as the resistance limits the current in a purely resistive circuit. It is inversely proportional to the capacitance and frequency of the current.

Thus, if frequency of AC increases, then its capacitive reactance decreases.

When capacitor is connected to DC source,



DC source

$$X_C = \frac{1}{\omega C} = \frac{1}{0} = \infty$$

$[\because$ for DC, $\omega = 2\pi\nu = 0$, as $\nu = 0]$

Thus, capacitor blocks DC and acts as open circuit while it passes AC of high frequency.

EXAMPLE |3| **A capacitor of 10μF is connected to an AC source of emf $E = 220\sin 100\pi t$. Write the equation of instantaneous current through the circuit. What will be the reading of AC ammeter connected in the circuit?**

Sol. Given, capacitance, $C = 10\mu F = 10 \times 10^{-6}$ F,

emf, $E = 220\sin 100\pi t = E_0 \sin\omega t$

$\therefore \quad E_0 = 220$ V, $\omega = 2\pi\nu = 100\pi \Rightarrow \nu = 50$ Hz

Since, capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 50 \times 10^{-5}} = 318.5\,\Omega$$

$$I_0 = \frac{E_0}{X_C} = \frac{220}{318.5} = 0.691 \text{ A}$$

So, reading of AC ammeter,

$$I_V = \frac{I_0}{\sqrt{2}} = \frac{0.691}{1.414} = 0.489 \text{ A}$$

AC THROUGH *L-C-R* CIRCUIT

Suppose that an inductor (L), a capacitor (C) and a resistor (R) are connected in series to an AC source. I is the current passing through this circuit. As R, L and C are in series, therefore at any instant through the three elements, AC has the same amplitude and phase. Let it be represented by

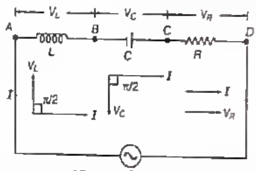
$$I = I_0 \sin\omega t$$

However, voltage across each element bears a different phase relationship with the current.

$V_L = I_0 X_L$ $\quad$ [V_L is maximum voltage across L]

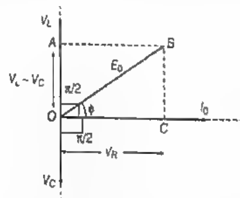
$V_C = I_0 X_C$ $\quad$ [V_C is maximum voltage across C]

$V_R = I_0 R$ $\quad$ [V_R is maximum voltage across R]



An AC source connected to L-C-R circuit

Inside the above figure for a L-C-R circuit, phasor diagrams of each L, C and R are given. To form phasor diagram for series L-C-R circuit, combine all these phasor diagrams.



Phasor diagram for a series L-C-R circuit

Since, voltage (V_L) is in upward direction and voltage (V_C) in downward direction, so net voltage upto point A is $V_L - V_C$ (assuming $V_L > V_C$) and net maximum voltage is V_0.

From phasor diagram,

$$OB = \sqrt{(OC)^2 + (CB)^2} = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\Rightarrow \quad E_0 = \sqrt{(I_0 R)^2 + (I_0 X_L - I_0 X_C)^2} \quad [\because OB = E_0]$$

$$\Rightarrow \quad E_0 = I_0\sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore \quad Z = \frac{E_0}{I_0} = \sqrt{R^2 + (X_L - X_C)^2}$$

Here, Z is called impedance.

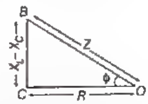
[redacted]

Impedance

It is the total resistance of a circuit applied in the path of alternating current. It is given by

$$\boxed{Z=\sqrt{R^2+(X_L-X_C)^2}} \quad \ldots\text{(i)}$$

From phasor diagram, it is clear that voltage leads the current by an angle ϕ.



Impedance diagram of L-C-R circuit

$\therefore$ From ΔOCB,

$$\tan\phi=\frac{CB}{OC}=\frac{V_L-V_C}{V_R}=\frac{I_0X_L-I_0X_C}{I_0R}$$

$$\Rightarrow \quad \tan\phi=\frac{X_L-X_C}{R} \quad \ldots\text{(ii)}$$

So, the alternating emf in the series L-C-R circuit would be represented by $E=E_0\sin(\omega t+\phi)$.

Eqs. (i) and (ii) are graphically shown in the above shown graph. This is called impedance diagram, which is a right angled triangle with Z as its hypotenuse.

The amplitude and phase of current for an L-C-R series circuit is obtained by using the technique of phasors. But this method of analysing AC circuits have certain disadvantages. Firstly, the phasor diagram does not signify anything about initial condition. One can take any arbitrary value of t and draw different phasors which shows the relative angle between different phasors. The solution so obtained is called the steady state solution.

Special Cases

(i) When $X_L=X_C$, then $Z=R$ and $\tan\phi=0$ $[\because \phi=0°]$

Hence, voltage and current are in the same phase. Therefore, the AC circuit is non-inductive.

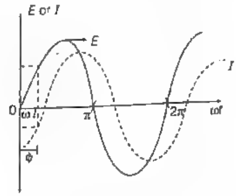
(ii) When $X_L>X_C$, then $\tan\phi$ is positive.

Hence, voltage leads the current by a phase angle ϕ. Therefore, the AC circuit is inductance dominated circuit.

(iii) When $X_C>X_L$, then $\tan\phi$ is negative.

Hence, voltage lags behind the current by a phase angle ϕ. Therefore the AC circuit is capacitance dominated circuit.

A graph (given below) is showing variation of E and I with ωt for the case, $X_L>X_C$.



Graph of E and I *versus* ωt for series L-C-R circuit when $X_C<X_L$

EXAMPLE |4| A capacitor of 100μF and a coil of resistance $50\,\Omega$ and inductance 0.5 H are connected in series with a 110 V -50 Hz source. Calculate the rms value of current in the circuit.

Sol. Given, capacitance, $C=100\,\mu F=100\times10^{-6}\,F=10^{-4}\,F$

Resistance, $R=50\,\Omega$

Inductance, $L=0.5\,H$

Rms value of voltage, $E_V=110\,V$

Frequency, $\nu=50\,Hz$

Since, capacitive reactance,

$$X_C=\frac{1}{\omega C}=\frac{1}{2\pi\nu C}=\frac{1}{2\times3.14\times50\times10^{-4}}$$

$$X_C=31.85\,\Omega$$

and inductive reactance,

$$X_L=\omega L=2\pi\nu L=2\times3.14\times50\times0.5=157\,\Omega$$

$$\because \text{Impedance, } Z=\sqrt{R^2+(X_L-X_C)^2}$$

$$=\sqrt{(50)^2+(157-31.85)^2}=134.77\,\Omega$$

$$\Rightarrow \quad I_V=\frac{E_V}{Z}=\frac{110}{134.77}=0.816\,A$$

EXAMPLE |5| A coil of 0.01H inductance and 1Ω resistance is connected to 200 V, 50 Hz AC supply. Find the impedance of the circuit and time lag between maximum alternating voltage and current. **NCERT Exemplar**

Sol. Given, inductance, $L=0.01\,H$

Resistance, $R=1\,\Omega$

Voltage, $V=200\,V$

Frequency, $\nu=50\,Hz$

Impedance of the circuit, $Z=\sqrt{R^2+X_L^2}$

$$=\sqrt{R^2+(2\pi\nu L)^2}=\sqrt{1^2+(2\times3.14\times50\times0.01)^2}$$

$$=\sqrt{10.86}=3.3\,\Omega$$

[redacted]

$$\tan\phi = \frac{\omega L}{R} = \frac{2\pi\nu L}{R} = \frac{2\times 3.14\times 50\times 0.01}{1} = 3.14$$

$\Rightarrow \quad \phi = \tan^{-1}(3.14) = 72°$

Phase difference, $\phi = \frac{72\times\pi}{180}$ rad

Time lag between maximum alternating voltage and current,

$$\Delta t = \frac{\phi}{\omega} = \frac{72\pi}{180\times 2\pi\times 50} = \frac{1}{250}\text{ s}$$

Resonance

In a series *L-C-R* circuit, when phase (ϕ) between current and voltage is zero, then the circuit is said to be a **resonant circuit.**

As applied frequency increases, then

$X_L = \omega L$, X_L increases and $X_C = \frac{1}{\omega C}$, X_C decreases.

At some angular frequency (ω_r), $X_L = X_C$

where, $\quad X_L = \omega_r L,\ X_C = \frac{1}{\omega_r C}$

The frequency at which X_C and X_L become equal, is called **resonant frequency.**

$$\Rightarrow \quad \omega_r L = \frac{1}{\omega_r C} \text{ or } \omega_r^2 = \frac{1}{LC} \text{ or } (2\pi\nu_r)^2 = \frac{1}{LC}$$

[$\because \omega_r = 2\pi\nu_r$, where ν_r is resonating frequency]

$$2\pi\nu_r = \frac{1}{\sqrt{LC}}$$

$$\therefore \quad \boxed{\nu_r = \frac{1}{2\pi\sqrt{LC}}}$$

At resonating frequency,

$$Z = R = \text{Minimum}$$

$$\therefore \quad I = \frac{E}{Z} = \text{Maximum}$$

Since, Z is minimum, therefore I will be maximum.

EXAMPLE |6| A 2 μF capacitor, 100 Ω resistor and 8 H inductor are connected in series with an AC source. What should be the frequency of source for which the current drawn in the circuit is maximum? If peak value of emf of source is 200 V, find the maximum current, inductive reactance, capacitive reactance, total impedance, peak value of current in the circuit. What is the phase relation between the voltages across inductor and resistor? Also, give the phase relation between voltages across inductor and capacitor.

Sol. Given, capacitance, $C = 2\,\mu F = 2\times 10^{-6}$ F

Resistance, $R = 100\,\Omega$

Inductance, $L = 8$ H

Peak value of voltage, $E_0 = 200$ V

When frequency of AC source is equal to resonant frequency,

then current drawn in the circuit is maximum.

$$\therefore \quad \nu = \nu_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\times 3.14\times\sqrt{8\times 2\times 10^{-6}}}$$

$$= \frac{1000}{8\times 3.14} = 39.8\text{ Hz}$$

Peak value of current, $I_0 = \frac{E_0}{R} = \frac{200}{100} = 2$ A

$\because \quad X_C = X_L = \omega L = 2\pi\nu L$

$= 2\times 3.14\times 39.8\times 8 = 2000\,\Omega \Rightarrow Z = R = 100\,\Omega$

The voltages across inductor and resistor differ in phase by 90° and the voltages across inductor and capacitor differ in phase by 180°.

Quality Factor (*Q*-Factor)

It is the measure of sharpness of the resonance of an *L-C-R* circuit. It is defined as the ratio of voltage developed across the inductance or capacitance at resonance to the impressed voltage, which is the voltage applied across *R*.

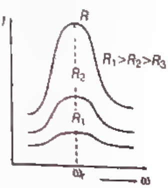
$$Q\text{-factor} = \frac{\text{Voltage across } L \text{ (or } C\text{)}}{\text{Voltage across } R}$$

$$Q\text{-factor} = \frac{V_L \text{ or } V_C}{V_R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$\boxed{Q\text{-factor} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}}$$

Q is just a number having no dimensions, it can also be called voltage multiplication factor of the circuit.

The electronic circuit with high Q values would respond to a very narrow range of frequencies and *vice-versa*. Higher the value of Q, the narrower and sharper is the resonance.



I versus ω graph of an *L-C-R* circuit

Q-factor can also be defined as the ratio of the resonant frequency to the difference in two frequencies taken on both sides of the resonant frequency such that at each frequency, the current amplitude becomes $\frac{1}{\sqrt{2}}$ times the value at resonant frequency.

[redacted]

Mathematically,

$$Q\text{-factor or } Q = \frac{\omega_r}{\omega_1 - \omega_2}$$

where, ω_1 and ω_2 are frequencies when current decreases to 0.707 $(1/\sqrt{2})$ times the peak value of current.

We can also write, $\omega_1 = \omega_r + \Delta\omega$

$\omega_2 = \omega_r - \Delta\omega$

The difference $\omega_1 - \omega_2 = 2\Delta\omega$ is often called the bandwidth of the circuit.

Thus, from the above, Q-factor can also be defined as the ratio of resonant angular frequency to bandwidth of the circuit.

The smaller the bandwidth $(\Delta\omega)$, the sharper and narrower is the resonance.

Significance of Q-Factor

- Q-factor denotes the sharpness of tuning.
- High Q-factor indicates lower rate of energy loss.
- Higher value of Q-factor indicates sharper peak in the current.
- For $R = 0$, Q-factor infinity

AVERAGE POWER ASSOCIATED IN AC CIRCUIT

Power is defined as the rate of doing work.

$$P = \frac{dW}{dt} \quad \ldots(i)$$

or

Power is defined as the product of voltage and current.

In AC circuit, both emf and current change continuously with respect to time. So in it we have to calculate average power in complete cycle $(0 \to T)$.

Instantaneous power, $P = EI$ ...(ii)

$[\because E = E_0 \sin\omega t,\ I = I_0 \sin(\omega t + \phi)]$

Here, E and I are instantaneous voltage and current, respectively. If the instantaneous power remains constant for a small time dt, then small amount of work done in maintaining the current for a small time dt is

$$\frac{dW}{dt} = EI$$

$$\Rightarrow \quad dW = EI\,dt \quad \ldots(iii)$$

Integrating Eq. (iii) on both sides, we get

$$\int dW = \int_0^T EI\,dt$$

Total work done or energy spent in maintaining current over one full cycle,

$$W = \int_0^T E_0 \sin\omega t \cdot I_0 \sin(\omega t + \phi)\,dt$$

$$= E_0 I_0 \int_0^T \sin\omega t(\sin\omega t\cos\phi + \cos\omega t\sin\phi)\,dt$$

$$= E_0 I_0 \left[\cos\phi \int_0^T \sin^2\omega t\,dt + \sin\phi \int_0^T \sin\omega t\cos\omega t\,dt\right]$$

$$= E_0 I_0 \left[\cos\phi \int_0^T \left(\frac{1-\cos 2\omega t}{2}\right)dt + \frac{\sin\phi}{2}\int_0^T 2\sin\omega t\cos\omega t\,dt\right]$$

$$= \frac{E_0 I_0}{2}\left[\cos\phi\left(\int_0^T dt - \int_0^T \cos 2\omega t\,dt\right) + \sin\phi \int_0^T \sin 2\omega t\,dt\right]$$

$$= \frac{E_0 I_0}{2}\left[\left(\cos\phi[t]_0^T - \int_0^T \cos 2\omega t\,dt\right) + \sin\phi\int_0^T \sin 2\omega t\,dt\right]$$

But $\int_0^T \cos 2\omega t\,dt = 0$ or and $\int_0^T \sin 2\omega t\,dt = 0$

$$\therefore \quad W = \frac{E_0 I_0 T}{2}\cos\phi$$

Average power associated in AC circuit,

$$P_{av} = \frac{W}{T} = \frac{E_0 I_0 T\cos\phi}{2T} = \frac{E_0 I_0}{2}\cos\phi$$

$$\boxed{P_{av} = \frac{E_0}{\sqrt{2}} \cdot \frac{I_0}{\sqrt{2}}\cos\phi}$$

or $\quad P_{av} = E_{rms} I_{rms}\cos\phi$

$= E_V I_V \cos\phi$

Here, $\cos\phi$ is power factor, which is defined as the cosine of the angle of lag or lead.

If P_{av} is true power or average power, then power factor is given by,

$$\cos\phi = \frac{P_{av}}{E_{rms} I_{rms}} = \frac{\text{True power}}{\text{Apparent power}} \quad \cos\phi = \frac{R}{Z}$$

Here, ϕ is the phase difference between I_{rms} and E_{rms}.

Special Cases

(i) **AC circuit containing R**

When $\phi = 0°$, then $P_{av} = E_V I_V \cos 0°$

$P_{av} = E_V I_V$

So, average power in R is maximum.

[redacted]

(ii) AC circuit containing L

When $\phi = \frac{\pi}{2}$, then $P_{av} = E_V I_V \cos\frac{\pi}{2}$

$$P_{av} = 0$$

So, average power in L is zero.

(iii) AC circuit containing C

When $\phi = \frac{\pi}{2}$, then $P_{av} = E_V I_V \cos\frac{\pi}{2}$

$$P_{av} = 0$$

So, average power in C is zero.

(iv) AC circuit containing L and R

When $\tan\phi = \frac{\omega L}{R} \Rightarrow \cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$,

then $P_{av} = E_V I_V \cdot \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$

(v) AC circuit containing C and R

When $\tan\phi = \frac{1/\omega C}{R} \Rightarrow \cos\phi = \frac{R}{\sqrt{R^2 + 1/\omega^2 C^2}}$

then $P_{av} = E_V I_V \cdot \frac{R}{\sqrt{R^2 + 1/\omega^2 C^2}}$

(vi) AC circuit containing L, C and R

When $\tan\phi = \frac{\omega L - 1/\omega L}{R}$

$$\Rightarrow \quad \cos\phi = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

then $P_{av} = E_V I_V \cdot \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

EXAMPLE |7| A sinusoidal voltage of peak value 283V and frequency 50 Hz is applied to a series L-C-R circuit in which $R = 3\,\Omega$, $L = 25.48$ mH and $C = 796\mu$F. Find

(i) the impedance of the circuit.

(ii) phase difference between the voltage across the source and current.

(iii) the power dissipated in the circuit.

(iv) the power factor. **NCERT**

Sol. Given, $E_0 = 283\text{V}, \nu = 50\text{Hz}, R = 3\Omega$,

$$L = 25.48\,\text{mH} = 25.48 \times 10^{-3}\text{H}$$

and $C = 796\,\mu\text{F} = 796 \times 10^{-6}\text{F}$

(i) Since, inductive reactance, $X_L = \omega L$

$$\Rightarrow X_L = 2\pi\nu L$$
$$= 2 \times 3.14 \times 50 \times 25.48 \times 10^{-3} = 8\Omega$$

Since, capacitive reactance,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C} = \frac{1}{2 \times 3.14 \times 50 \times 796 \times 10^{-6}}$$

$$X_C = 4\Omega$$

$\therefore$ Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{3^2 + (8-4)^2} = 5\Omega$$

(ii) Phase difference,

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{8-4}{3}\right) = 53.1°$$

It means that the current in the circuit lags behind the voltage by 53.1°

(iii) Power dissipated in the circuit, $P = I_V^2 R$

$$\because \quad I_V = \frac{I_0}{\sqrt{2}Z} = \frac{283}{1.414 \times 5} = 40\text{A}$$

$$\therefore \quad P = I_V^2 R = (40)^2 \times 3 = 4800\,\text{W}$$

(iv) Power factor, $\cos\phi = \cos 53.1° = 0.60°$

EXAMPLE |8| Suppose the frequency of the source in the above example can be varied.

(i) What is the frequency of the source at which resonance occurs ?

(ii) Calculate the impedance, the current and power dissipated of resonant condition. **NCERT**

Sol. (i) Resonant frequency, $\nu = \frac{1}{2\pi\sqrt{LC}}$

$$= \frac{1}{2 \times 3.14 \times \sqrt{25.48 \times 10^{-3} \times 796 \times 10^{-6}}}$$
$$= 35.4\,\text{Hz}$$

(ii) At resonance, $Z = R = 3\Omega$

$$\Rightarrow \quad I_V = \frac{E_V}{Z} = \frac{283}{\sqrt{2} \times 3} = 66.7\,\text{A} \quad \left[\because E_V = \frac{E_0}{\sqrt{2}}\right]$$

$\therefore$ Power dissipated, $P = I_V^2 R$

$$= (66.7)^2 \times 3 = 13350\,\text{V}$$

WATTLESS CURRENT

The current which consumes no power for its maintenance in the circuit is called wattless current or idle current.

or

If the resistance in an AC circuit is zero, although current flows in the circuit, then the average power remains zero, i.e. there is no energy dissipation in the circuit, such a circuit is called wattless circuit and the current flowing is called wattless current.

If the circuit contains either inductance or capacitance only, then phase difference between current and voltage is 90°, i.e. $\phi = 90°$. The average power in such a circuit is

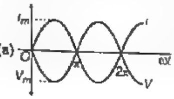
$$P_{av} = V_{rms} \times I_{rms} \times \cos\phi = V_{rms} \times I_{rms} \times \cos 90° = 0$$

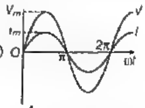
[redacted]

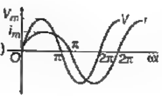
TOPIC PRACTICE 2

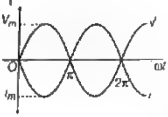
OBJECTIVE Type Questions

1. Which of the following graphs shows, in a pure resistor, the voltage and current are in phase?

(a) 

(b) 

(c) 

(d) 

2. Voltage and current in an AC circuit are given by

$V = 5\sin(100\pi t - \pi/6)$

and $I = 4\sin(100\pi t + \pi/6)$

(a) voltage leads the current by 30°
(b) current leads the voltage by 30°
(c) current leads the voltage by 60°
(d) voltage leads the current by 60°

3. A resistance of 20 Ω is connected to a source of an alternating potential, $V = 220\sin(100\pi t)$. The time taken by current to change from its peak value to rms value is

(a) 0.2 s (b) 0.25 s
(c) 25×10^{-3} s (d) 2.5×10^{-1} s

4. The inductive reactance is directly proportional to the

(a) inductance
(b) frequency of the current
(c) Both (a) and (b)
(d) amplitude of current

5. A pure inductor of 25.0 mH is connected to a source of 220 V. Find the inductive reactance if the frequency of the source is 50 Hz.

(a) 785 Ω (b) 6.50 Ω
(c) 7.85 Ω (d) 8.75 Ω

6. Current I across the capacitor in a purely capacitive AC circuit is

(a) $i_m \sin(\omega t + \pi/4)$
(b) $i_m \sin(\omega t + \pi/2)$
(c) $I_m \cos(\omega t + \pi/4)$
(d) $i_m \cos(\omega t + \pi/2)$

7. The amplitude of the oscillating current in the a pure capacitive AC circuit is, if $V = V_m \sin\omega t$ and capacitance = C.

(a) ωCV_m (b) $2\omega CV_m$ (c) $\frac{\omega CV_m}{4}$ (d) $\frac{3\,\omega CV_m}{2}$

8. A 15.0 μF capacitor is connected to a 220 V, 50 Hz source. The capacitive reactance is

(a) 220 Ω (b) 215 Ω (c) 212 Ω (d) 204 Ω

9. L, C and R represents self inductance, capacitance and resistance respectively. Which of the following dimensional formula is not of frequency?

(a) $\frac{1}{RC}$ (b) $\frac{R}{L}$ (c) $\frac{1}{\sqrt{LC}}$ (d) $\frac{C}{L}$

10. To reduce the resonant frequency in an L-C-R series circuit with a generator **NCERT Exemplar**

(a) the generator frequency should be reduced
(b) another capacitor should be added in parallel to the first
(c) the iron core of the inductor should be removed
(d) dielectric in the capacitor should be removed

11. In a series L-C-R circuit, the capacitance C is changed to $4C$. To keep the resonant frequency same, the inductance must be changed by

(a) $2L$ (b) $L/2$
(c) $4L$ (d) $L/4$

12. Which of the following combinations should be selected for better tuning of an L-C-R circuit used for communication? **NCERT Exemplar**

(a) $R = 20\,\Omega$, $L = 1.5$ H, $C = 35\,\mu$F
(b) $R = 25\,\Omega$, $L = 2.5$ H, $C = 45\,\mu$F
(c) $R = 15\,\Omega$, $L = 3.5$ H, $C = 30\,\mu$F
(d) $R = 25\,\Omega$, $L = 1.5$ H, $C = 45\,\mu$F

VERY SHORT ANSWER Type Questions

13. An electric lamp is connected in series with a capacitor and an AC source is glowing with a certain brightness. How does the brightness of the lamp change on increasing the capacitance?

14. Explain the statement that a capacitor is a conductor at very high frequencies. Compare this behaviour with that of a capacitor in a DC circuit after the steady state. **NCERT**

15. How does the sign of the phase angle ϕ, by which the supply voltage leads the current in an L-C-R series circuit, change as the supply frequency is gradually increased from very low to very high values. **NCERT Exemplar**

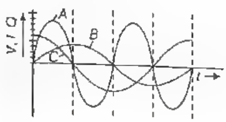
[redacted]

16. Define 'quality factor' of resonance in series L-C-R circuit. What is its SI unit? **Delhi 2016**

17. How can you improve the quality factor of a series resonance circuit?

18. Mention the significance of quality factor. **Foreign 2012**

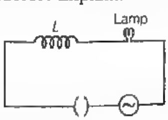
19. A device X is connected to an AC source $V = V_0 \sin \omega t$. The variation of voltage, current and power in one complete cycle is shown in the following figure.
   (i) Which curve shows power consumption over a full cycle?
   (ii) Identify the device X.



SHORT ANSWER Type Questions

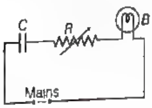
20. Explain, why the reactance offered by an inductor increases with increasing frequency of an alternating voltage? **NCERT Exemplar**

21. (i) When an AC source is connected to an ideal inductor, show that the average power supplied by the source over a complete cycle is zero.
   (ii) A lamp is connected in series with an inductor and an AC source. What happens to the brightness of the lamp when the key is plugged in and an iron rod is inserted inside the inductor? Explain.



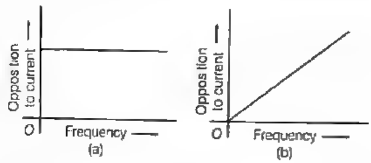
All India 2016

22. A capacitor C, a variable resistance R and a bulb B are connected in series to the AC mains in circuit as shown in the figure. The bulb glows with some brightness. How will the glow of the bulb change, if (i) a dielectric slab is introduced between the plates of the capacitor, keeping resistance R to be same;



   (ii) the resistance R is increased keeping the same capacitance? **Delhi 2014**

23. (i) The graphs (a) and (b) represent the variation of the opposition offered by the circuit element to the flow of alternating current with frequency of the applied emf. Identify the circuit elements corresponding to each graph.



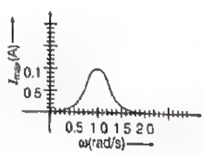
   (ii) Write the expression for the impedance offered by the series combination of the above two elements connected across the AC source. Which will be ahead in phase in this circuit, voltage or current? **All India 2011**

24. (i) Draw a graph showing variation of amplitude of circuit current with changing frequency of applied voltage in a series L-C-R circuit for two different values of resistance R_1 and $R_2 (R_1 > R_2)$.
   (ii) Define the term 'Sharpness of Resonance'. Under what condition, does a circuit become more selective? **Foreign 2016**

25. Prove that an ideal capacitor in an AC circuit does not dissipate power. **All India 2017 C**

26. In the analogy between series L-C-R circuit and a mass on a spring, the mass is analogous to the inductance and the spring constant is analogous to the inverse of the capacitance. Explain giving reason.

27. In series L-C-R circuit, the plot of I_{max} *versus* ω is shown in the figure. Find the bandwidth and mark in the figure.



NCERT Exemplar

[redacted]

LONG ANSWER Type I Questions

28. An inductor L of inductance X_L is connected in series with a bulb B and an AC source. How would brightness of the bulb change when

(i) number of turns in the inductor is reduced?

(ii) an iron rod is inserted in the inductor?

(iii) a capacitor of reactance $X_C = X_L$ is inserted in series in the circuit? Justify your answer in each case. **All India 2015**

29. (i) When an AC source is connected to an ideal capacitor. Show that the average power supplied by the source over a complete cycle is zero.

(ii) A lamp is connected in series with a capacitor. Predict your observations when the system is connected first across a DC and then an AC source. What happens in each case, if the capacitance of the capacitor is reduced? **Delhi 2013 C**

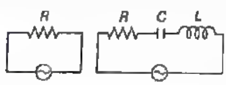
30. Answer the following questions.

(i) What is the minimum value of the power factor of a circuit? Under what circumstances can it occur?

(ii) State the maximum value of the power factor? Under what circumstances can this occur?

31. An AC voltage $V = V_m \sin \omega t$ is applied across an inductor of inductance L. Find the instantaneous power P_i supplied to the inductor. Show graphically the variation of P_i with ωt.

32. Study the circuits (a) and (b) shown in the figure and answer the following questions:



(a) (b)

(i) Under which conditions would the rms currents in the two circuits be the same?

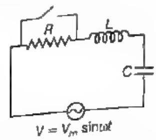
(i) Can the rms current in circuit (b) be larger than that in (a)? **NCERT Exemplar**

33. In the L-C-R circuit, shown in the figure, the AC driving voltage is $V = V_m \sin \omega t$.

(i) Write down the equation of motion for $q(t)$.

(ii) At $t = t_0$, the voltage source stops and R is short circuited. Now, write down how much energy is stored in each of L and C.

(iii) Describe subsequent motion of charges. **NCERT Exemplar**



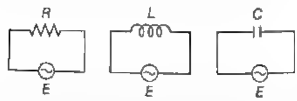
LONG ANSWER Type II Questions

34. An AC source of voltage $V = V_0 \sin \omega t$ is connected to a series combination of L, C and R. Use the phasor diagram to obtain expressions for impedance of the circuit and phase angle between voltage and current. Find the condition when current will be in phase with the voltage. What is the circuit in the condition called? **Delhi 2015**

35. (i) What do you understand by sharpness of resonance in a series L-C-R circuit? Derive an expression for Q-factor of the circuit.

(ii) Three electrical circuits having AC sources of variable frequency are shown in the figures. Initially, the current flowing in each of these is same. If the frequency of the applied AC source is increased, how will the current flowing in these circuits be affected?

Give the reason for your answer. **Delhi 2011**



36. Derive an expression for the impedance of a series L-C-R circuit connected to an AC supply of variable frequency. Plot a graph showing variation of current with the frequency of the applied voltage. Explain briefly how the phenomenon of resonance in the circuit can be used in the tuning mechanism of a radio or a TV set? **Delhi 2011**

37. (i) Show that a series L-C-R circuit at resonance behaves as a purely resistive circuit. Compare the phase relation between current and voltage in series L-C-R circuit for

(a) $X_L > X_C$

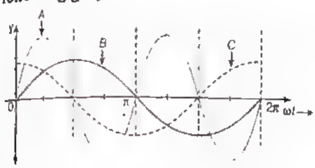
(b) $X_L = X_C$ using phasor diagrams.

(ii) What is an acceptor circuit and where it is used?

[redacted]

38. In a series, L-C-R circuit connected to an AC source of variable frequency and voltage $V = V_m \sin \omega t$, draw a plot showing the variation of current I with angular frequency ω, for two different values of resistance R_1 and $R_2 (R_1 > R_2)$. Write the condition under which the phenomenon of resonance occurs. For which value of the resistance out of the two curves, a sharper resonance is produced? Define Q-factor of the circuit and give its significance. **All India 2013**

39. A device X is connected to an AC source, $V = V_0 \sin \omega t$. The variation of voltage, current and power in one cycle is shown in the following graph.



(i) Identify the device X.
(ii) Which of the curves A, B and C represent the voltage, current and the power consumed in the circuit? Justify the answer.
(iii) How does its impedance vary with frequency of the AC source? Show graphically.
(iv) Obtain an expression for the current in the circuit and its phase relation with AC voltage. **All India 2017**

40. (i) A voltage $V = V_0 \sin \omega t$ applied to a series L-C-R circuit derives a current $I = I_0 \sin \omega t$ in the circuit. Deduce the expression for the average power dissipated in the circuit.
(ii) For circuits used for transporting electric power, a low power factor implies large power loss in transmission. Explain.
(iii) Define the term wattless current. **Delhi 2012**

41. A device X is connected across an AC source of voltage $V = V_0 \sin \omega t$. The current through X is given as $I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$.
(a) Identify the device X and write the expression for its reactance.
(b) Draw graphs showing variation of voltage and current with time over one cycle of AC, for X.
(c) How does the reactance of the device X vary with frequency of the AC? Show this variation graphically.
(d) Draw the phasor diagram for the device X. **CBSE 2018**

NUMERICAL PROBLEMS

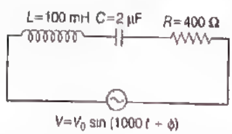
42. An alternating voltage given by $E = 140 \sin 314t$ is connected across a pure resistor of 50 Ω. Find
(i) the frequency of the source.
(ii) the rms current through the resistor. **All India 2012**

43. A coil of inductance 0.5 H and resistance 100 Ω is connected to a 240 V, 50 Hz AC supply.
(i) What is the maximum current in the coil?
(ii) What is the time lag between the voltage maximum and current maximum? **NCERT**

44. A 100 μF capacitor in series with a 40 Ω resistance is connected to a 110 V, 60 Hz supply.
(i) What is the maximum current in the circuit?
(ii) What is the time lag between the current maximum and the voltage maximum? **NCERT**

45. A resistor of 400 Ω, an inductor of $5/\pi$ H and a capacitor of $\frac{50}{\pi}$ μF are connected in series across a source of alternating voltage of $140 \sin 100\pi t$ V. Find the voltage (rms) across the resistor, the inductor and the capacitor. Is the algebraic sum of these voltage more than the source voltage? If yes, resolve the paradox. **Foreign 2010**

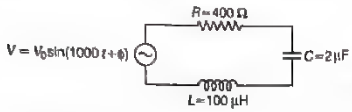
46. (i) Find the value of the phase difference between the current and the voltage in the series L-C-R circuit shown below. Which one leads in phase, current or voltage?



(ii) Without making any other change, find the value of the additional capacitor C', to be connected in parallel with the capacitor C, in order to make the power factor of the circuit unity. **Delhi 2017**

[redacted]

47. Determine the value of phase difference between the current and the voltage in the given series L-C-R circuit. **Delhi 2015**



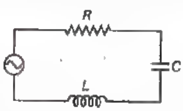
48. A 10 V, 650 Hz source is connected to a series combination of $R=100\ \Omega$, $C=10\mu F$ and $L=0.15$ H. Find out the time in which resistance will get heated by 10 °C, if thermal capacity of the material = 2J/°C.

49. Calculate the quality factor of a series L-C-R circuit with $L=2.0$ H, $C=2\mu F$ and $R=10\ \Omega$. **Foreign 2012**

50. Resonance frequency of a circuit is ν. If the capacitance is made 4 times the initial value, find the change in the resonance frequency.

51. A 2μF capacitor, 100 Ω resistor and 8 H inductor are connected in series with an AC source.
   (i) What should be the frequency of the source such that current drawn in the circuit is maximum? What is this frequency called?
   (ii) If the peak value of emf of the source is 200 V, find the maximum current. **Foreign 2016**

52. The figure shows a series L-C-R circuit with $L=10.0$ H, $C=40\ \mu F$, $R=60\ \Omega$ connected to variable frequency 240 V source. Calculate



   (i) the angular frequency of the source which drives the circuit at resonance.
   (ii) the current at the resonating frequency.
   (iii) the rms potential drop across the inductor at resonance. **Delhi 2012**

53. Obtain the resonant frequency (ω_r) of a series L-C-R circuit with $L=2.0$ H, $C=32\mu F$ and $R=10\ \Omega$. What is the Q-value of this circuit? **NCERT**

54. An inductor of 200 mH, capacitor of 400 μF and a resistor of 10 Ω are connected in series to AC source of 50V of variable frequency. Calculate
   (i) the angular frequency at which maximum power dissipation occurs on the circuit and the corresponding value of effective current, and
   (ii) the value of Q-factor on the circuit. **All India 2017C**

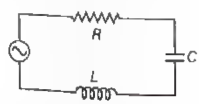
55. Obtain the resonant frequency and Q-factor of a series L-C-R circuit with $L=3.0$ H, $C=27\mu F$ and $R=7.4\ \Omega$. It is designed to improve the sharpness of resonance of the circuit by reducing its full width at half maximum by a factor of 2. Suggest a suitable way. **NCERT**

56. A 100 Ω resistor is connected to a 220 V, 50 Hz supply.
   (i) What is the rms value of current in the circuit?
   (ii) What is the net power consumed over a full cycle? **NCERT**

57. A 44 mH inductor is connected to 220 V, 50 Hz AC supply. Determine the rms value of the current in the circuit. What is the net power absorbed over a complete cycle? Explain. **NCERT**

58. A 60 μF capacitor is connected to a 110 V, 60 Hz AC supply. Determine the rms value of current in the circuit. What is the net power absorbed over a complete cycle? Explain. **NCERT**

59. A series L-C-R circuit connected to a variable frequency 230 V source has $L=5.0$ H, $C=80\ \mu F$, $R=40\ \Omega$, as shown in the figure.



   (i) Determine the source frequency which drives the circuit in resonance.
   (ii) Obtain the impedance of the circuit and amplitude of current at the resonant frequency.
   (iii) Determine the rms potential drop across the three elements of the circuit. Show that the potential drop across the L-C combination is zero at the resonating frequency. **NCERT**

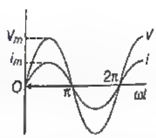
[redacted]

60. A circuit containing 80 mH inductor and a 60μF capacitor in series is connected to a 230 V, 50 Hz supply. The resistance in the circuit is negligible.
 (i) Obtain the current amplitude and rms value.
 (ii) Obtain the rms value of potential drop across each element.
 (iii) What is the average power transferred to inductor?
 (iv) What is the average power transferred to capacitor?
 (v) What is the total average power absorbed by the circuit? NCERT

61. A series L-C-R circuit with $L = 0.12$ H, $C = 480$ nF, $R = 23\ \Omega$ is connected to a 230 V variable frequency supply.
 (i) What is the source frequency for which current amplitude is maximum? Obtain this maximum value.
 (ii) What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of maximum power.
 (iii) For which frequency of the source is the power transferred to the circuit half the power at resonant? What is the current amplitude at these frequencies?
 (iv) What is the Q-factor of the given circuit? NCERT

HINTS AND SOLUTIONS

1. (b)



In a pure resistor, the voltage and current are in phase. The minima zero and maxima occur at the same respective times.

2. (c) Phase difference $\Delta\phi = \phi_2 - \phi_1 = \pi/6 - (-\pi/6) = \pi/3$

So, current leads the voltage by $\pi/3$.

3. (d) Current in at peak value so its equation is

$$i = i_0 \sin(100\pi t + \pi/2)$$

Peak value to rms value means current becomes $1/\sqrt{2}$ times.

So, from $\quad i = i_0 \sin(100\pi t + \pi/2)$

$$\frac{i_0}{\sqrt{2}} = i_0 \sin(100\pi t + \pi/2)$$

$$\sin 3\pi/4 = \sin(100\pi t + \pi/2)$$

$$\Rightarrow \quad t = \frac{1}{400}\ \text{s}$$

Time taken by current to change from its peak value to rms value,

i.e., $\quad t = \frac{1}{400}\ \text{s} = 2.5 \times 10^{-3}\ \text{s}$

4. (c) Inductive reactance, $X_L = \omega L = 2\pi f L$

5. (c) The inductive reactance,

$$X_L = 2\pi\nu L = 2 \times 3.14 \times 50 \times 25 \times 10^{-3} = 7.85\ \Omega$$

6. (b) Current I across the capacitor is $i_m \sin(\omega t + \pi/2)$.

7. (a) The amplitude of the oscillating current is

$$I_m = V_m / X_c = \omega C V_m$$

8. (c) The capacitive reactance is

$$X_C = \frac{1}{2\pi\nu C} = \frac{1}{2\pi\ (50\ \text{Hz})\ (15.0 \times 10^{-6}\ \text{F})} = 212\ \Omega$$

9. (d) $\frac{C}{L}$ is not the dimensional formula of frequency because

$\frac{C}{L} = \frac{[M^{-1}L^{-2}T^4A^2]}{[ML^2T^{-2}A^{-2}]}$ but dimensional formula of frequency is $[T^{-1}]$.

10. (b) We know that resonant frequency in an L-C-R circuit is given by

$$\nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

Now to reduce ν_0 either we can increase L or we can increase C.

To increase capacitance, we must connect another capacitor parallel to the first

11. (d) The resonant frequency, $f = \frac{1}{\sqrt{4\pi^2 LC}}$

Again, $\quad f = \frac{1}{\sqrt{4\pi^2 (L/4) \times 4C}}$

$$\Rightarrow \quad f = \frac{1}{\sqrt{4\pi^2 LC}}$$

If the value of L is changed to $L/4$, then the resonant frequency will remain unchanged.

12. (c) Quality factor (Q) of an L-C-R circuit is given by

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

where R is resistance, L is inductance and C is capacitance of the circuit. To make Q high,

R should be low, L should be high and C should be low. These conditions are best satisfied by the values given in option (c).

[redacted]

13. Capacitive reactance is given by,

$$X_C = \frac{1}{\omega C} \Rightarrow X_C \propto \frac{1}{C}$$

This means, with the increase in the capacitance, the capacitive reactance decreases. So, if an electric lamp is connected in a series with a capacitor and an AC source is glowing with certain brightness, then with the increase in the capacitance, the brightness of the lamp increases.

14. By comparison, at very high frequency, the resistance due to capacitor is negligible and hence it works like a pure conductor of negligible capacitive reactance.

In DC circuits, $\omega = 0$ (at steady state)

$$\Rightarrow \quad X_C = \frac{1}{\omega C} = \infty$$

So, it behaves like an open circuit.

15. The phase angle (ϕ) by which voltage leads the current in L-C-R series circuit is given by

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{2\pi\nu L - \dfrac{1}{2\pi\nu C}}{R}$$

If $\tan\phi < 0$ (for $\nu < \nu_0$), then circuit is capacitive

If $\tan\phi > 0$ (for $\nu > \nu_0$), then circuit is inductive.

At resonance, $\tan\phi = 0$ $\quad \left[\text{for } \nu = \nu_0 = \dfrac{1}{2\pi\sqrt{LC}}\right]$

16. The quality factor (Q) of resonance in series L-C-R circuit is defined as the ratio of voltage drop across inductor (or capacitor) to the applied voltage,

i.e. $Q = \dfrac{V_L \text{ or } V_C}{V_R} = \dfrac{\omega_0 L}{R} = \dfrac{1}{\omega_0 CR}$

It is an indicator of sharpness of the resonance. Quality factor has no unit.

17. To improve quality factor, ohmic resistance should be made as small as possible.

18. Refer to text on pages 299 and 300

19. (i) Curve A shows power consumption over a full cycle.

(ii) Device X is a capacitor. As in a perfect capacitor, the current (curve C) leads the voltage (curve B) by a phase angle of $\dfrac{\pi}{2}$.

20. Refer to text on pages 295 and 296.

21. (i) As, $P_{av} = V_{rms} I_{rms} \cos\phi$

In ideal inductor, current I_{rms} lags behind applied voltage V_{rms} by $\pi/2$.

$\therefore \phi = \pi/2$

Thus, $P_{av} = V_{rms} I_{rms} \cos \pi/2$

$= V_{rms} I_{rms} \times 0$

$= 0$

(ii) Brightness of the lamp decreases. It is because when iron rod is inserted inside the inductor, its inductance L increases, thereby increasing its inductive reactance X_L and hence impedance Z of the circuit. As $I_{rms} = \dfrac{V_{rms}}{Z}$, so this decreases the current I_{rms} in the circuit and hence the brightness of lamp.

22. (i) As the dielectric slab is introduced between the plates of the capacitor, its capacitance will increase. Hence, the potential drop across the capacitor will decrease, i.e. $V = \dfrac{Q}{C}$

As a result, the potential drop across the bulb will increase as they are connected in series. Thus, its brightness will increase.

(ii) As the resistance R is increased, the potential drop across the resistor will increase. As a result, the potential drop across the bulb will decrease as they are connected in series. Thus, its brightness will decrease.

23. (i) From graph (a), it is clear that resistance (opposition to current) is not changing with frequency, i.e. resistance does not depend on frequency of applied voltage, so the circuit element here is pure resistive (R). From graph (b), it is clear that resistance increases linearly with frequency, so the circuit element here is inductive in nature.

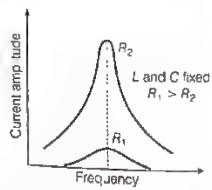
Inductive resistance, $X_L = 2\pi\nu L \Rightarrow X_L \propto \nu$

(ii) Impedance offered by the series combination of resistance (R) and inductor (L).

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi\nu L)^2}$$

In L-R circuit, the applied voltage leads the current by phase ϕ, where $\tan\phi = \dfrac{X_L}{R}$

24. (i) Graph showing the variation of amplitude of circuit current with changing frequency is given below.



(ii) **Sharpness of resonance** Refer to text on pages 299 and 300.

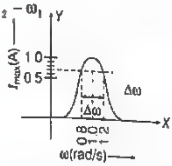
Circuit become more selective if the resonance is more sharp, maximum current is more, the circuit is close to resonance for smaller range of ($2\Delta\omega$) of frequencies. Thus, the tuning of the circuit will be good.

25. Refer to text on page 301.

26. Refer to text on page 302.

[redacted]

27. Consider the diagram,
Bandwidth $= \omega_2 - \omega_1$



where, ω_1 and ω_2 correspond to frequencies at which magnitude of current is $\frac{1}{\sqrt{2}}$ times of maximum value.

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \approx 0.7 \text{ A}$$

Clearly, from the diagram, the corresponding frequencies are 0.8 rad/s and 1.2 rad/s.

$$\Delta\omega = \text{Bandwidth} = 1.2 - 0.8 = 0.4 \text{ rad/s}$$

28. (i) We know that if the number of turns in the inductor decreases, then inductance L decreases. So, the net resistance of the circuit decreases. Hence, the current through the circuit increases, increasing the brightness of the bulb.

(ii) As the current increases and brightness of bulb increases, because L increases.

(iii) If the capacitor of reactance $X_C = X_L$ is connected in series with the circuit, then

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$$\Rightarrow \quad Z = R \qquad [\because X_L = X_C]$$

This is a case of resonance. In this case, the maximum current will flow through the circuit. Hence, the brightness of the bulb will increase.

29. (i) Refer to text on pages 296 and 297.

(ii) When DC source is connected, the condenser is charged but no current flows in the circuit. Therefore, the lamp does not glow. No change occurs even when capacitance of capacitor is reduced.

When AC source is connected, the capacitor offers capacitive reactance $X_C = \frac{1}{\omega C}$. The current flows in the circuit and the lamp glows. On reducing C, X_C increases. Therefore, glow of the bulb reduces.

30. Refer to the text on pages 300 and 301.

31. In an inductor, the current lags the voltage by 90°. If the source voltage is sinusoidal, then the current is also sinusoidal, but shifted in phase. The instantaneous power defined as the product of the instantaneous voltage and current can also be seen to be sinusoidal in time. However, in contrast to the resistive load, the instantaneous power in the inductor goes negative for part of the cycle of the source driving it.

As, $V(t) = V_m \sin \omega t$

$\therefore \quad I(t) = -I_m \cos \omega t$

Instantaneous power, $P_i = V(t) \cdot I(t)$

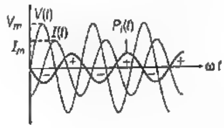
$$= V_m \sin \omega t \times (-I_m \cos \omega t)$$

$$= -\frac{V_m I_m}{2} \times 2 \sin \omega t \cos \omega t$$

$$= -\frac{V_m I_m}{2} [\sin 2\omega t + \sin 0]$$

$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

The variation of P_i with ωt is as given in the figure.



The instantaneous power alternates positive and negative at twice the frequency of source supplying it.

32. Let $(I_{rms})_a$ = rms current in circuit (a)

$(I_{rms})_b$ = rms current in circuit (b)

$$(I_{rms})_a = \frac{V_{rms}}{R} = \frac{V}{R}$$

$$(I_{rms})_b = \frac{V_{rms}}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

(i) When $(I_{rms})_a = (I_{rms})_b$

$$R = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow \quad X_L = X_C \text{ in resonance condition}$$

(ii) As, $Z \geq R$

$$\Rightarrow \quad \frac{(I_{rms})_a}{(I_{rms})_b} = \frac{\sqrt{R^2 + (X_L - X_C)^2}}{R} = \frac{Z}{R} \geq 1$$

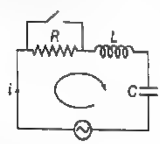
$$\Rightarrow \quad (I_{rms})_a \geq (I_{rms})_b$$

No, the rms current in circuit (b) cannot be larger than that in (a).

33. (i) Consider the R-L-C circuit as shown in the figure.

Given, $\quad V = V_m \sin \omega t$

Let current at any instant be i.



Note We have to apply KVL, write the equations in the form of current and charge, double differentiate the equation with respect to time and find the required relations.

Applying KVL in the given circuit,

$$iR + L\frac{di}{dt} + \frac{q}{C} - V_m \sin \omega t = 0 \qquad \text{...(i)}$$

Now, we can write, $\quad i = \frac{dq}{dt} \Rightarrow \frac{di}{dt} = \frac{d^2 q}{dt^2}$

[redacted]

From Eq. (i), we get

$$\frac{dq}{dt}R + L\frac{d^2q}{dt^2} + \frac{q}{C} = V_m \sin \omega t$$

$$\Rightarrow \quad L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V_m \sin \omega t$$

This is the required equation of variation motion of charge.

(ii) Let $q = q_m \sin(\omega t + \phi) = -q_m \cos(\omega t + \phi)$

$i = i_m \sin(\omega t + \phi) = q_m \omega \sin(\omega t + \phi)$

$$i_m = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

and $\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$

When R is short circuited at $t = t_0$, energy is stored in L and C.

$$U_L = \frac{1}{2}Li^2 = \frac{1}{2}L\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \sin^2(\omega t + \phi)$$

and $U_C = \frac{1}{2} \times \frac{q^2}{C} = \frac{1}{2C} \times \left(\frac{i_m}{\omega}\right)^2 \cos^2(\omega t_0 + \phi)$

$$= \frac{i_m^2}{2C\omega^2}\cos^2(\omega t_0 + \phi) \qquad [\because i_m = q_m\omega]$$

$$= \frac{1}{2C}\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \frac{\cos^2(\omega t_0 + \phi)}{\omega^2}$$

$$= \frac{1}{2C\omega^2}\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \cos^2(\omega t_0 + \phi)$$

(iii) When R is short circuited, it becomes an L-C oscillator. The capacitor will go on discharging and all energy will go to L.

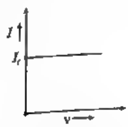
Hence, there is an oscillation of energy from electrostatic to magnetic and again to electrostatic.

34. Refer to text on page 298.

35. (i) Refer to text on page 299.

(ii) Let initially, I_i be current flowing in all the three circuits. If frequency of applied AC source is increased, then the change in current will occur in following manner.

(a) **AC circuit containing resistance only** where, ν_i = initial frequency of AC source.



Frequency of AC source

There is no effect on current with the increase in frequency.

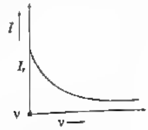
(b) **AC circuit containing inductance only** with the increase of frequency of AC source, inductive reactance increases as,

$$I = \frac{V_{rms}}{X_L} = \frac{V_{rms}}{2\pi\nu L}$$

$$\Rightarrow \quad X_L = 2\pi\nu L$$

For given circuit, $I \propto \frac{1}{\nu}$

Current decreases with the increase in frequency.



Frequency of AC source

(c) **AC circuit containing capacitor only**

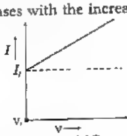
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\nu C}$$

$$\text{Current, } I = \frac{V_{rms}}{X_C} = \frac{V_{rms}}{\left(\frac{1}{2\pi\nu C}\right)}$$

$$I = 2\pi\nu C V_{rms}$$

For given circuit, $I \propto \nu$

Current increases with the increase in frequency.



Frequency of AC source

36. Refer to text on page 298.

For graph showing variation of current with frequency Refer to text on page 299.

The receiving antenna picks up the frequencies transmitted by different stations and a number of voltage appears in L-C-R circuit corresponding to different frequencies. But maximum current flows in circuit for that AC voltage which have got the frequency is equal to resonant frequency of circuit

i.e. $\nu = \frac{1}{2\pi\sqrt{LC}}$

For higher quality factor resonance, the signal received from other stations becomes weak due to sharpness of resonance. Thus, signal of desired frequency or program is tunned in.

37. (i) Refer to text on pages 297, 298 and 299.

(ii) Acceptor circuit is series L-C-R circuit.

38. For graph refer to text on page 299 and for conditions and Q-factor refer to text on pages 299 and 300.

[redacted]

39. (i) Device X is a capacitor.

As, the current is leading voltage by $\frac{\pi}{2}$ radians.

(ii) As, $E(t) = E_0 \sin \omega t$

Current, $I(t) = I_0 \cos \omega t$

As, in the case of capacitor,

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \quad \text{[current is leading voltage]}$$

Average power, $P = E(t)I(t) = E_0 I_0 \cos \phi / 2$

where, ϕ = phase difference

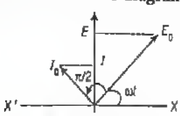
Hence, curve A represents power, curve B represents voltage and curve C represents current.

(iii) As, X_C = capacitive reactance $= \frac{1}{C\omega}$

where, ω is angular frequency.

So, reactance or impedance decreases with increase in frequency. Graph of X_C *versus* ω is shown below,

Phasor diagram



Current leads emf by π/2 radians

(iv) Refer to text on page 296.

40. (i) Refer to text on page 300.

(ii) Average power delivered by an AC circuit is

$P_{av} = V_{rms} I_{rms} \cos \phi$

where, $\cos \phi$ is known as the power factor for the circuit.

If $\cos \phi$ is minimum, the power delivered is minimum and hence, power dissipated will be maximum for the circuit.

(iii) Refer to text on page 301.

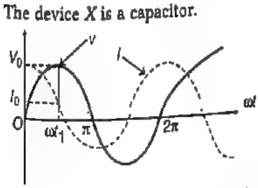
41. (a) Given, $V = V_0 \sin \omega t$

$$I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

As it is clear that, the current leads the voltage by a phase angle $\frac{\pi}{2}$.

$\therefore$ The device X is a capacitor.

(b)



(c) The reactance of the capacitance is given as

$$X_C = \frac{1}{\omega C}$$

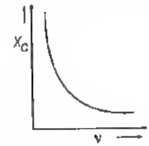
where, ω = angular frequency

and C = capacitance of capacitor.

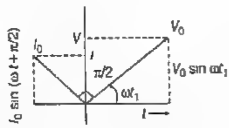
$$\therefore \quad X_C = \frac{1}{2\pi v C}$$

where, v = frequency of AC or $X_C \propto \frac{1}{v}$

$\therefore$ The graphical representation between reactance of capacitance and frequency is given as



(d) **Phasor diagram**



42. (i) As given, $E = 140 \sin 314t$

On comparing with $E = E_0 \sin \omega t$, we have

$\omega = 314$, $E_0 = 140$ V

$\because \quad \omega = 2\pi v$

$$\Rightarrow \quad v = \frac{\omega}{2\pi} = \frac{314}{2 \times 3.14} = 50 \text{ Hz}$$

(ii) $E_0 = 140$ V

$$E_{rms} = \frac{E_0}{\sqrt{2}} = \frac{140}{\sqrt{2}} = 99.29 \text{ V}$$

$$\therefore I_{rms} = \frac{E_{rms}}{R} = \frac{99.29}{50} = 1.98 \text{ A}$$

43. Given, $L = 0.5$ H, $R = 100\ \Omega$,

$v = 50$ Hz, $V_{rms} = 240$ V

$$\text{(i) } I_0 = \frac{V_0}{\sqrt{R^2 + \omega^2 L^2}} = \frac{\sqrt{2} \times 240}{\sqrt{(100)^2 + (100 \times \pi \times 0.5)^2}}$$

$[\because \omega = 2\pi v = 100\pi]$

$= 1.82$ A

(ii) 3.19×10^{-3} s; refer to Example 5 of on pages 298 and 299.

44. (i) Impedance, $Z = \sqrt{R^2 + X_C^2}$

$$= \sqrt{R^2 + \left(\frac{1}{2\pi v C}\right)^2}$$

$$= \sqrt{(40)^2 + \left(\frac{1}{2 \times 3.14 \times 60 \times 100 \times 10^{-6}}\right)^2} = 48\,\Omega$$

As, $I_V = \frac{E_V}{Z} \Rightarrow I_V = \frac{110 \text{ V}}{48\,\Omega}$

and $I_0 = \sqrt{2}\, I_V = 1.414 \times \frac{110}{48} = 3.24$ A

[redacted]

(ii) As, $\tan\phi = \frac{X_C}{R} = \frac{1}{2\pi\nu CR}$

$$= \frac{1}{2\times 3.14\times 60\times 10^{-4}\times 40} = 0.6628$$

$$\Rightarrow \quad \phi = \tan^{-1}(0.6628) = 33.5° = \frac{33.5\pi}{180}\text{ rad}$$

$\therefore$ Time lag, $t = \frac{\phi}{\omega} = \frac{33.5\pi}{180}\times\frac{1}{2\pi\times 60}$

$$= 1.55\times 10^{-3}\text{ s}$$

45. Given, applied voltage, $V = 140\sin 100\pi t$ V

$$C = \frac{50}{\pi}\mu\text{F} = \frac{50}{\pi}\times 10^{-6}\text{ F},$$

$$L = \frac{5}{\pi}\text{ H}, R = 400\ \Omega$$

Comparing with $V = V_0 \sin\omega t$, we get

$$V_0 = 140\text{ V and } \omega = 100\pi$$

Inductive reactance, $X_L = \omega L = 100\pi\times\frac{5}{\pi} = 500\ \Omega$

Capacitive reactance, $X_C = \frac{1}{\omega C} = \frac{1}{100\pi\times\frac{50}{\pi}\times 10^{-6}}$

$$= 200\ \Omega$$

Impedance of the circuit, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$= \sqrt{(400)^2 + (500-200)^2}$$

$$= \sqrt{1600+900} = 500\ \Omega$$

Maximum current in the circuit,

$$I_0 = \frac{V_0}{Z} = \frac{140}{500}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{140}{500\times\sqrt{2}} = 0.2\text{ A}$$

V_{rms} across resistor, $V_R = I_{rms}R$

$$= 0.2\times 400 = 80\text{ V}$$

V_{rms} across inductor, $V_L = I_{rms}X_L$

$$= 0.2\times 500 = 100\text{ V}$$

V_{rms} across capacitor, $V_C = I_{rms}X_C$

$$= 0.2\times 200 = 40\text{ V}$$

Now, $\quad V \neq V_R + V_L + V_C$

Because V_C, V_L and V_R are not in same phase, instead

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{80^2 + (100-40)^2} = 100\text{ V}$$

which is same as that of applied rms voltage.

46. Refer to Example 7 on page 301.

$$\phi = 135°$$

Since, $\quad \omega L < \frac{1}{\omega C}$ or $X_L < X_C$

Therefore, current is leading in phase by a phase angle 135°.

(ii) For unit power factor, $\cos\phi = 1$

$$\Rightarrow \quad \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C'}\right)^2}} = 1$$

where, C' is the total capacitance

$$\Rightarrow \quad R^2 + \left(\omega L - \frac{1}{\omega C'}\right)^2 = R^2$$

$$\Rightarrow \quad \omega L = \frac{1}{\omega C'}$$

$$\Rightarrow \quad \omega L = 100 = \frac{1}{\omega C} = \frac{1}{1000\ C'}$$

$$\Rightarrow \quad C' = \frac{1}{10^5} = 10^{-5}\text{ F} = 10\ \mu\text{F}$$

Additional capacitance C' required in parallel

$$= C' - C = 10\ \mu\text{F} - 2\ \mu\text{F} = 8\ \mu\text{F}$$

47. Refer to Example 7 on page 301.

Phase difference, $\quad \phi = \frac{-\pi}{4}$

48. Here, $E_V = 10$ V, $\nu = 650$ Hz, $R = 100\ \Omega$,

$C = 10\ \mu\text{F} = 10\times 10^{-6}$ F, $L = 0.15$ H,

$\Delta\theta = 10°\text{C}$, $ms = 2\text{ J/°C}$

As, $\quad X_L = 2\pi\nu L = 2\times\frac{22}{7}\times 650\times 0.15 = 612.86\ \Omega$

and $X_C = \frac{1}{2\pi\nu C} = \frac{1}{2\times\frac{22}{7}\times 650\times 10\times 10^{-6}} = 24.48\ \Omega$

$$\Rightarrow \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$= \sqrt{(100)^2 + (612.86 - 24.48)^2} = 596.82\ \Omega$$

$$\Rightarrow \quad I_V = \frac{E_V}{Z} = \frac{10}{596.82} = 0.0168$$

As, $\quad I_V^2 Rt = (ms)\,\Delta\theta$

$$\therefore \quad t = \frac{(ms)\,\Delta\theta}{I_V^2 R} = \frac{2\times 10}{(0.0168)^2\times 100} = 708.6\text{ s}$$

49. Given, $L = 2.0$ H, $C = 2\ \mu\text{F} = 2\times 10^{-6}$ F,

$R = 10\ \Omega$

Now, Q-factor $= \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{2\times 10^{-6}}} = \frac{1}{10\times 10^{-3}}$

$$= \frac{1}{10^{-2}} = 100$$

50. As, resonance frequency, $\nu = \frac{1}{2\pi\sqrt{LC}}$

i.e. $\quad \nu \propto \frac{1}{\sqrt{C}}$

$$\therefore \quad \nu' \propto \frac{1}{\sqrt{C'}} = \frac{1}{\sqrt{4C}} = \frac{1}{2\sqrt{C}} = \frac{1}{2}\nu$$

[redacted]

51. (i) Refer to Example 6 on page 317.

$\nu = 39.80$ Hz

(ii) $\therefore I_0 = \frac{E_0}{R} = \frac{200}{100} = 2\text{A}$

52. Given, $L = 10$ H, $C = 40\,\mu$F, $R = 60\,\Omega$, $V_{rms} = 240$ V

(i) Refer to the Q. 54 on page 320.

$\omega_r = 50$ rad/s

(ii) Current at resonating frequency,

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{R} \qquad [\because \text{ at resonance, } Z = R]$$

$$= \frac{240}{60} = 4\text{ A}$$

(iii) Inductive reactance, $X_L = \omega L$

At resonance, $X_L = \omega_r L = 50 \times 10 = 500\,\Omega$

Potential drop across inductor,

$$V_{rms} = I_{rms} \times X_L = 4 \times 500 - 2000\text{ V}$$

53. Given, $L = 20$ H, $C = 32 \times 10^{-6}$ F and $R = 10\,\Omega$

$$\therefore \quad \omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{2.0 \times 32 \times 10^{-6}}}$$

$$= \frac{10^3}{8} = 125\text{ rad/s}$$

and $$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{10}\sqrt{\frac{2}{32 \times 10^{-6}}}$$

$$= \frac{1}{10 \times 4 \times 10^{-3}} = 25$$

54. Refer to Q. 51 and Q. 52 on page 306.

[Ans. 0.112×10^3 rad/s, 5A; 2.23]

55. 111.1 rad/s and 45.04; Refer to Q. 52 on page 306.

Now, to reduce the full width of half maximum by a factor of 2 without changing ω_r, we have to take

$$R' = \frac{R}{2} = \frac{7.4}{2} = 3.7\,\Omega$$

56. Here, $R = 100\,\Omega$, $E_V = 220$ V, $\nu = 50$ Hz

(i) $I_V = \frac{E_V}{R} = \frac{220}{100} = 2.2$ A

(ii) Net power consumed,

$$P_{av} = E_V I_V = 220 \times 2.2 = 484\text{ W}$$

57. Given, inductance, $L = 44$ mH $= 44 \times 10^{-3}$ H, $V_{rms} = 220$ V

Frequency of inductor, $\nu = 50$ Hz

Inductive reactance, $X_L = 2\pi\nu L$

$= 2 \times 3.14 \times 50 \times 44 \times 10^{-3} = 13.82\,\Omega$

The rms value of current in the circuit,

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{220}{13.82} = 15.9\text{ A}$$

Power absorbed, $P = V_{rms} I_{rms} \cos\phi$

For pure inductive circuit, $\phi = 90°$

$\therefore \quad P = 0$

Thus, power spent in one half cycle is retrieved in the other half cycle.

58. Given, $C = 60\,\mu\text{F} = 60 \times 10^{-6}$ F, $V_{rms} = 110$ V

and $\nu = 60$ Hz

$$\therefore \quad I_{rms} = \frac{V_{rms}}{X_C} = \frac{V_{rms}}{\frac{1}{2\pi\nu C}}$$

$$\Rightarrow \quad I_{rms} = V_{rms} 2\pi\nu C$$

$$= 110 \times 2 \times 3.14 \times 60 \times 60 \times 10^{-6} = 2.5\text{ A}$$

Power absorbed, $P = V_{rms} I_{rms} \cos\phi$

For pure capacitive circuit, $\phi = 90°$

$\therefore \quad P = 0$

Thus, power spent in one half cycle is retrieved in the other half cycle.

59. Given, $L = 5$ H, $C = 80\,\mu\text{F} = 80 \times 10^{-6}$ F,

$R - 40\,\Omega$, $V_{rms} = 230$ V

(i) Resonance angular frequency,

$$\omega_r = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5 \times 80 \times 10^{-6}}} = 50\text{ rad/s}$$

(ii) Impedance, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

At resonance, $\omega L = \frac{1}{\omega C}$

$\therefore \quad Z_r = R = 40\,\Omega$

Amplitude of current at resonance frequency,

$$I_0 = \frac{V_0}{Z_r} = \frac{\sqrt{2} \times 230}{40} = 813\text{ A}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{8.13}{\sqrt{2}} = 5.75\text{ A}$$

(iii) Potential difference across L,

$$V_L = I_{rms} \times (\omega_r \times L)$$

$$= 5.75 \times 50 \times 5 = 1437.5\text{ V}$$

Potential difference across C,

$$V_C = I_{rms} \times \frac{1}{\omega_r C} = \frac{5.75}{50 \times 80 \times 10^{-6}} = 1437.5\text{ V}$$

$\therefore$ Potential difference across L and C combination,

$$V_{LC} = I_{rms}\left(\omega_r L - \frac{1}{\omega_r C}\right) = 0$$

$\therefore$ Potential difference across R,

$$V_R = I_{rms} R = 5.75 \times 40 = 230\text{ V}$$

[redacted]

60. Given, $L = 80\text{ mH} = 80\times10^{-3}\text{H}, R = 0, \nu = 50\text{ Hz}$

$$C = 60\mu F = 60\times10^{-6}\text{ F},$$

$$\omega = 2\pi\nu = 100\pi \text{ rad/s}$$

$$V_{rms} = 230\text{ V},$$

and $V_0 = \sqrt{2}V_{rms} = \sqrt{2}\times 230\text{ V}$

(i) $I_0 = ?$ and $I_{rms} = ?$

$$\Rightarrow I_0 = \frac{V_0}{Z} = \frac{V_0}{\left(\omega L - \frac{1}{\omega C}\right)}$$

$$= \frac{230\sqrt{2}}{\left(100\pi\times80\times10^{-3} - \frac{1}{100\pi\times60\times10^{-6}}\right)}$$

$$= \frac{230\sqrt{2}}{\left(8\pi - \frac{1000}{6\pi}\right)} = -11.63\text{ A}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{-11.63}{\sqrt{2}} = -8.23\text{ A}$$

Hence, negative sign indicates that emf lags behind the current by 90°.

(ii) For L, $V_L = I_{rms}\omega L$

$$= 8.23\times100\pi\times80\times10^{-3}$$

$$= 206.84\text{ V}$$

For C, $V_C = I_{rms}\frac{1}{\omega C} = 8.23\times\frac{1}{100\pi\times60\times10^{-6}}$

$$= 436.84\text{ V}$$

Since, voltage across L and C are 180° out of phase, therefore they are subtracted.

Thus, applied rms voltage $= 436.84 - 206.84$

$$= 230.0\text{ V}$$

(iii) Average power transferred per cycle by source to inductor is always zero because of phase difference of $\pi/2$ between voltage and current through L.

(iv) Average power transferred per cycle by the source to capacitor is always zero because of phase difference of $\pi/2$ between voltage and current through C.

(v) $\therefore$ Total average power absorbed by the circuit is also zero.

61. Given, $L = 0.12\text{ H}, C = 480\text{ nF} = 480\times10^{-9}\text{ F}$

$$R = 23\ \Omega, V_{rms} = 230\text{ V},\quad V_0 = 230\sqrt{2}\text{ V}$$

(i) $I_0 = \dfrac{V_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

I_0 would be maximum, if

$$\omega_r = \omega = \frac{1}{\sqrt{LC}}$$

$$= \frac{1}{\sqrt{0.12\times480\times10^{-9}}}$$

$$= 4166.7\text{ rad/s}$$

$\therefore$ Source frequency, $\nu_r = \dfrac{\omega_r}{2\pi}$

$$= \frac{4166.7}{2\pi} = 663.14\text{ Hz}$$

$$\Rightarrow \quad I_0 = \frac{V_0}{R} = \frac{\sqrt{2}\times230}{23}$$

$$= 14.14\text{ A}$$

(ii) Average power absorbed by the circuit is maximum, if

$$I = I_0 \text{ at } \omega = \omega_r$$

$$P_{av} = \frac{1}{2}I_0^2R = \frac{1}{2}(14.14)^2\times23$$

$$= 2299.3\text{ W} \approx 2300\text{ W}$$

(iii) Power transferred to circuit is half the power of resonant frequency, when

$$\Delta\omega = \frac{R}{2L} = \frac{23}{2\times0.12} = 95.83\text{ rad/s}$$

$$\Delta\nu = \frac{\Delta\omega}{2\pi} = \frac{95.83}{2\pi} = 15.2\text{ Hz}$$

$\therefore$ Frequency when power transferred is half

$$= \nu_r \pm \Delta\nu = 663.14 \pm 15.2$$

$$= 678.34 \text{ and } 647.94\text{ Hz}$$

$\therefore$ Current amplitude at these frequencies

$$= \frac{I_0}{\sqrt{2}} = \frac{14.14}{1.414} = 10\text{ A}$$

(iv) $Q = \dfrac{\omega_r L}{R} = \dfrac{4166.7\times0.12}{23} = 21.74$

[redacted]

TOPIC 3 AC Devices

CHOKE COIL

Choke coil is an electrical device used for controlling current in an AC circuit without wasting electrical energy in the form of heat.

(i) To reduce low frequency alternating currents, choke coils with laminated soft iron cores are used. These are called *af* choke coils.

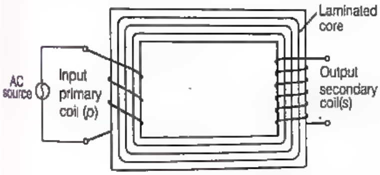
(ii) To reduce high frequency alternating currents, choke coils with air cores are used. These are called *rf* choke coils.

TRANSFORMER

It is a device, which is used to increase or decrease the alternating voltage.

The transformers are of the following types

1. Step-up transformer
2. Step-down transformer



Step-down transformer

Principle

Transformer is based upon the principle of mutual induction.

Construction

It consists of two coils, primary coil (p) and secondary coil (s), insulated from each other wounded on soft iron core. Often the primary coil is the input coil and secondary coil is the output coil. These soft iron cores are laminated to minimise eddy current loss.

Working and Theory

The value of the emf induced in secondary coil due to alternating voltage applied to primary coil depends on the number of turns in the secondary coil. We consider an ideal transformer in which the primary coil has negligible resistance and all the flux in the core links both primary and secondary windings. Let ϕ be the flux in each turn in the core at time t due to current in the primary when a voltage V_p is applied to it.

Then, the induced emf or voltage E_s, in the secondary with N_s turns is

$$E_s = -N_s \frac{d\phi}{dt} \quad \ldots(i)$$

The alternating flux ϕ also induces an emf, called back emf in the primary. This is

$$E_p = -N_p \frac{d\phi}{dt} \quad \ldots(ii)$$

But $E_p = V_p$. If this was not, so the primary current would be infinite, since the primary has zero resistance (as considered). If the secondary is an open circuit or the current taken from it is small, then to a good approximation.

$$E_s = V_s$$

where, V_s is the voltage across the secondary.

Therefore, Eqs. (i) and (ii) can be written as

$$V_s = -N_s \frac{d\phi}{dt} \quad \ldots(iii)$$

and

$$V_p = -N_p \frac{d\phi}{dt} \quad \ldots(iv)$$

From Eqs. (iii) and (iv), we have

$$\frac{V_s}{V_p} = \frac{N_s}{N_p} \quad \ldots(v)$$

The above relation has been obtained using three assumptions.

(i) The primary resistance and current are small.

(ii) The same flux links both the primary and the secondary as very little flux escapes from the core.

(iii) The secondary current is small.

If the transformer is assumed to be 100% efficient (no energy losses), the power input is equal to the power output. Since $P = IV$, we get

$$I_p V_p = I_s V_s \quad \ldots(vi)$$

Although, some energy is always lost, still this is a good approximation, since a well designed transformer may have an efficiency of more than 95%.

[redacted]

Combining Eqs. (v) and (vi), we have

$$\boxed{\frac{I_p}{I_s}=\frac{V_s}{V_p}=\frac{N_s}{N_p}} \quad \text{...(vii)}$$

Since, I and V both oscillate with the same frequency as the AC source, Eq. (vii) also gives the ratio of the amplitudes or rms values of corresponding quantities.

Now, we can observe how a transformer affects the voltage and current, we have

$$V_s=\left(\frac{N_s}{N_p}\right)V_p \text{ and } I_s=\left(\frac{N_p}{N_s}\right)I_p \quad \text{...(viii)}$$

That is, if the secondary coil has a greater number of turns than the primary (i.e. $N_s > N_p$), the voltage is stepped up ($V_s > V_p$). This type of arrangement is called a **step-up transformer**. However, in this arrangement, there is less current in the secondary than in the primary (i.e. $N_p/N_s < 1$ and $I_s < I_p$).

If the secondary coil has less number of turns than the primary (i.e. $N_s < N_p$), we have a **step down transformer.**

In this case, $V_s < V_p$ and $I_s > I_p$. That is, the voltage is stepped-down, (or reduced) and the current is increased. The equations obtained above apply to ideal transformers (without any energy losses).

Energy Loss in Transformers

In actual transformers, small energy losses do occur due to the following reasons.

(i) **Flux leakage** There is always some leakage of flux that is not all of the flux due to primary passes through the secondary. This is due to poor design of the core or the air gaps in the core. It can be reduced by winding the primary and secondary coils one over the other.

(ii) **Resistance of the windings** The wire used for the windings has some resistance and so, energy is also lost due to heat produced in the wire (I^2R). In high current, low voltage windings, energy losses are minimised by using thick wire.

(iii) **Eddy currents** The alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect is reduced by having a laminated core.

(iv) **Hysteresis** The magnetisation of the core is repeatedly reversed by an alternating magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a magnetic material which has a low hysteresis loss.

Uses of Transformers

Transformers are used in almost all AC operations. Some of the following are given below.

(i) In the induction furnaces.

(ii) In voltage regulators for TV, computer, refrigerator, etc.

(iii) A step-down transformer is used for the purpose of weldings.

EXAMPLE |1| How much current is drawn by the primary coil of a transformer which steps down 220 V to 22 V to operate device with an impedance of 220 Ω?

Sol. Given, $E_p = 220\text{ V}$, $E_s = 22\text{ V}$ and $R_s = 220\,\Omega$

Since, $I_s=\frac{E_s}{R_s}=\frac{22}{220}=0.1\text{A}$

In an ideal transformer, $\frac{I_p}{I_s}=\frac{E_s}{E_p}$

$\therefore \quad I_p=\frac{E_s}{E_p}\times I_s$

$=\frac{22\times 0.1}{220}=10^{-2}\text{ A}$

EXAMPLE |2| A step down transformer converts a voltage of 2200 V into 220 V in the transmission line. Number of turns in primary coil is 5000. Efficiency of transformer is 90% and its output power is 8 kW. Determine

(i) number of turns in the secondary coil.

(ii) input power.

Sol. Given, $E_p = 2200\text{ V}$, $E_s = 220\text{ V}$, $N_p = 5000$

Efficiency, $\eta = 90\%$

Output power, $P_o = 8\text{ kW}$

Since, efficiency,

$$\eta=\frac{\text{Output power}}{\text{Input power}}=\frac{P_o}{P_i}$$

$\Rightarrow \quad P_i=\frac{P_o}{\eta}=\frac{8}{90/100}=8.9\text{ kW}$

Also, $\frac{N_s}{N_p}=\frac{E_s}{E_p}\Rightarrow N_s=500$

[redacted]

TOPIC PRACTICE 3

OBJECTIVE Type Questions

1. A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230 V?
(a) 600 (b) 550 (c) 400 (d) 375

2. The output of a step-down transformer is measured to be 24 V when connected to a 12 W light bulb. The value of the peak current is **NCERT Exemplar**
(a) $1/\sqrt{2}$ A (b) $\sqrt{2}$ A
(c) 2 A (d) $2\sqrt{2}$ A

3. What is not possible in a transformer?
(a) Eddy current (b) Direct current
(c) Alternating current (d) Induced current

4. The large scale transmission and distribution of electrical energy over long distances is done with the use of
(a) dynamo (b) transformers
(c) generator (d) capacitor

5. A 60 W load is connected to the secondary of a transformer whose primary draws line voltage of 220 V. If a current of 0.54 A flows in the load, then what is the current in the primary coil?
(a) 2.7 A (b) 0.27 A
(c) 1.65 A (d) 2.85 A

VERY SHORT ANSWER Type Questions

6. Can we control direct current without much loss of energy? Can a choke coil do so?

7. Write the name of quantities which do not change during transformer operation.

8. Mention the two characteristic properties of the material suitable for making core of a transformer. **All India 2012**

9. A transformer is used to step-down AC voltage. What device do you use to step-down DC voltage?

10. A transformer has 150 turns in its primary and 1000 in secondary. If the primary is connected to 440 V DC supply, what will be the induced voltage in the secondary side?

11. What would happen if the primary winding of a transformer is connected to a battery?

SHORT ANSWER Type Questions

12. A 100% efficient transformer has n_1 turns in its primary and n_2 turns in its secondary. If the power input to the transformer is W (watt), what is the power output?

13. Answer the following questions.
(i) A choke coil in series with a lamp is connected to a DC line. The lamp is seen to shine brightly Insertion of an iron core in the choke causes no change in the lamp's brightness. Predict the corresponding observations, if the connection is to an AC line.
(ii) Why is choke coil needed in the use of fluorescent tubes with AC mains? Why we cannot use an ordinary resistor instead of the choke coil? **NCERT**

14. When a DC voltage is applied to a transformer, the primary coil sometimes will overheat and eventually burn. Explain, why?

LONG ANSWER Type I Questions

15. Write the function of a transformer. State its principle of working with the help of a diagram. Mention various energy losses in this device. **Delhi 2016**

16. Transformer A has a primary voltage E_p and a secondary voltage E_s. Transformer B has twice the number of turns on both its primary and secondary coils compared with transformer A. If the primary voltage on transformer B is $2E_p$, what is its secondary voltage? Explain briefly.

17. At a hydroelectric power plant, the water pressure head is at height of 300 m and the water flow available is 100 m^3/s. If the turbine generator efficiency is 60%, estimate the electric power available from the plant. (Take, $g = 9.8\ m/s^2$)

18. 1 MW power is to be delivered from a power station to a town 10 km away. One uses a pair of Cu wires of radius 0.5 cm for this purpose. Calculate the fraction of ohmic losses to power transmitted, if

[redacted]

(i) power is transmitted at 220 V. Comment on the feasibility of doing this.

(ii) a step-up transformer is used to boost the voltage at 11000 V, power transmitted, then a step-up transformer is used to bring voltage is 220 V. (Take, $\rho_{Cu} = 1.7 \times 10^{-8}$ SI unit)

NCERT Exemplar

LONG ANSWER Type II Questions

19. (i) Draw a labelled diagram of a step-down transformer. State the principle of its working.

(ii) Express the turn ratio in terms of voltages.

(iii) Find the ratio of primary and secondary currents in terms of turn ratio in an ideal transformer.

(iv) Define choke coil. All India 2016

20. Draw a schematic diagram of a step-up transformer. Using its working principle, deduce the expression for the secondary to the primary voltage in terms of number of turns in the two coils? In an ideal transformer, how is this ratio related to the currents in the two coils? How is this transformer used in large scale transmission and distribution of electrical energy over large distances?

NUMERICAL PROBLEMS

21. How much current is drawn by the primary of a transformer connected to 220 V supply when it delivers power to a 110 V-550 W refrigerator? All India 2016

22. A power transmission line feeds input power at 2200 V to a step-down transformer with its primary windings having 3000 turns. Find the number of turns in the secondary winding to get the power output at 220 V. Delhi 2017

23. 1 kW power is supplied to a 200 turns primary of the transformer at 500 mA. The secondary gives 220 V. Find the number of turns in the primary.

24. The primary coil of an ideal step up transformer has 100 turns and transformation ratio is also 100. The input voltage and power are respectively 220 V and 1100 W. Calculate

(i) the number of turns in secondary.

(ii) the current in primary.

(iii) the voltage across secondary.

(iv) the current in secondary.

(v) the power in secondary. Delhi 2016

25. A 60 W load is connected to the secondary of transformer whose primary draws line voltage. If current of 0.54 A flows in the load, what is the current in the primary coil? Comment on the type of transformer being used. NCERT Exemplar

26. A step-up transformer is operated on a 2.5 kV line. It supplies a load with 20 A. The ratio of the primary winding to the secondary is 10 : 1. If the transformer is 90% efficient, calculate

(i) the power output (ii) the voltage and

(iii) the current in the secondary.

27. A step-down transformer is used at 220 V to provide a current of 0.5 A to a 15 W bulb. If the secondary has 20 turns, find the number of turns in the primary coil and the current that flows in the primary coil.

28. A step-up transformer operates on a 220 V line and supplies a load of 2 A. The ratio of the primary to the secondary windings is 1:5. Determine the secondary voltage, primary current and power output. Assume efficiency to be 100%.

29. A small town with a demand of 800 kW of electric power at 220 V is situated 15 km away from an electric plant generating power at 440 V. The resistance of the two wires line carrying power is 0.5 Ω/km. The town gets power from the line through a 4000-220 V step-down transformer at a sub-station in the town.

(i) Estimate the line power of loss in the form of heat.

(ii) How much power of the plant supply, assuming there is negligible power loss due to leakage?

(iii) Characterise the step-up transformer at the plant. NCERT

30. Do the same question as above with the replacement of the earlier transformer by a 40000-220 V step-down transformer (neglect, as before, leakage losses though this may not be a good assumption any longer because of the very high voltage transmission involved). Hence, explain why high voltage transmission is preferred? NCERT

[redacted]

HINTS AND SOLUTIONS

1. (c) Here, $\varepsilon_p = 2300$ V, $N_p = 4000$, $\varepsilon_s = 230$ V

Let N_s be the required number of turns in the secondary

As, $\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p}$, $N_s = N_p\left(\frac{\varepsilon_s}{\varepsilon_p}\right)$

$$= 4000\left(\frac{230\text{ V}}{2300\text{ V}}\right) = 400$$

2. (a) Secondary voltage, $V_S = 24$V

Power associated with secondary,

$$P_S = 12\text{ W}$$

$$I_S = \frac{P_S}{V_S} = \frac{12}{24}$$

$$= \frac{1}{2}\text{ A} = 0.5\text{ A}$$

Peak value of the current in the secondary,

$$I_0 = I_S\sqrt{2}$$

$$= (0.5)(1.414) = 0.707 = \frac{1}{\sqrt{2}}\text{ A}$$

3. (b) Transformer is used to convert the value of AC voltage. It works on the principle of electromagnetic induction. So, direct current is not possible in it.

4. (b) Large scale distribution and transmission of electrical energy over long distances is done with the help of transformer

5. (b) $P = 60$ W, $\varepsilon_p = 220$ V, $i_s = 0.54$ A

As, $P = \varepsilon_s i$

$\Rightarrow$ $\varepsilon_s = \frac{60\text{ W}}{0.54\text{ A}} = 110$ V

As, $\varepsilon_p i_p = \varepsilon_s i_s$

$$i_p = \left(\frac{\varepsilon_s}{\varepsilon_p}\right) i_s = \left(\frac{110\text{ V}}{2200\text{ V}}\right)(0.54\text{ A}) = 0.27\text{ A}$$

6. No, there is no such device that can control DC without any energy loss. Even a choke coil cannot control DC.

7. Power and frequency.

8. (i) Low retentivity or coercivity.
(ii) Low hysteresis loss or high permeability and susceptibility.

9. An ohmic resistance can be used to step-down DC voltage, such as in potential dividing arrangement.

10. Zero, as transformer works only in AC and in case of DC supply, there is no induced emf in secondary because there is no change in flux through the transformer circuit.

11. Transformer works only in AC. When primary is connected to DC, there is no induced emf in secondary coil as there is no change in flux leakage.

12. For 100% efficient transformer, $P_i = P_o$

$\therefore$ The power output is W (watt).

13. (i) A choke has no impedance, if it is connected to DC line. Therefore, lamp shines brightly and has no effect on inserting iron core in the choke.
But choke offers impedance, if it is connected to AC line. So the bulb lights dimly. When an iron core is inserted in the choke, the impedance to AC increases. Hence, the brightness of the bulb decreases.
(ii) We use the choke coil instead of resistance because the power loss across resistor is maximum, while the power loss across choke is zero.
For resistor, $\phi = 0°$,
$$P = I_{rms}V_{rms}\cos 0°$$
$$= I_{rms}\cdot V_{rms} = \text{maximum}$$
For inductor, (choke coil)
$$\phi = 90°,$$
$$P = I_{rms}V_{rms}\cos 90° = 0$$

14. If in a case, the transformer primary winding would be connected to a DC supply, the inductive reactance of the winding would be zero as DC has no frequency. So, the effective impedance of the winding will therefore be very low and equal only to the resistance of the copper used. Thus, winding will draw a very high current from the DC supply causing it to overheat and eventually burn out, because as we know $I = V/R$.

15. Refer to text on pages 315 and 316.

16. Given, $N_{pB} = 2N_{pA}$, $N_{sB} = 2N_{sA}$, $E_{pB} = 2E_{pA}$

As we know, $\frac{N_s}{N_p} = \frac{E_s}{E_p}$

For transformer B, $\frac{N_{sB}}{N_{pB}} = \frac{2N_{sA}}{2N_{pA}} = \frac{E_{sB}}{E_{pB}}$

$$\Rightarrow \frac{E_{sB}}{E_{pB}} = \frac{E_{sB}}{2E_{pA}} = \frac{E_{sA}}{E_{pA}} \Rightarrow E_{sB} = 2E_{sA}$$

$\therefore$ Secondary voltage on transformer B is equal to the twice of secondary voltage on transformer A.

17. Given, height, $h = 300$ m, $V = \frac{\text{volume}}{\text{second}} = 100\text{ m}^3/\text{s}$,

$$\eta = 60\%, g = 9.8\text{ m/s}^2$$

Electric power = ?

Hydroelectric power $= \frac{\text{Work}}{\text{Time}} = \frac{\text{Force} \times \text{Displacement}}{\text{Time}}$

$$= \text{Force} \times \text{Velocity}$$
$$= \text{Pressure} \times \text{Area} \times \text{Velocity}$$
$$= \text{Pressure} \times \text{Volume} = p \times V$$

$\therefore$ Power available $= \frac{60}{100}pV = \frac{3}{5} \times h \times \rho \times g \times V$

[redacted]

$= \frac{3}{5} \times 300 \times 10^3 \times 9.8 \times 100$

[$\because$ density of water $= 10^3 \text{kg/m}^3$]

$= 1.764 \times 10^8 = 176.4$ MW

18. (i) The town is 10 km away, length of pair of Cu-wires used, $l = 20$ km $= 20000$ m.

Resistance of Cu wires,

$$R = \rho\frac{l}{A} = \rho\frac{l}{\pi(r)^2} = \frac{1.7\times 10^{-8} \times 20000}{3.14\,(0.5\times 10^{-2})^2} = 4\,\Omega$$

I at 220 V, $VI = 10^6$ W; $I = \frac{10^6}{220} = 0.45 \times 10^4$ A

$RI^2 =$ power loss $= 4\times (0.45)^2 \times 10^8 > 10^6$ W

Therefore, this method cannot be used for transmission.

(ii) When power, $P = 10^6$ W is transmitted at 11000 V.

$V'I' = 10^6$ W $= 11000\,I'$

Current drawn, $I' = \frac{1}{1.1} \times 10^2$

Power loss $= RI^2 = \frac{1}{121} \times 4 \times 10^6 = 3.3 \times 10^4$ W

$\therefore$ Fraction of power loss $= \frac{3.3\times 10^4}{10^6} = 3.3\%$

19. Refer to text on pages 315 and 316.

20. Refer to text on pages 315 and 316.

21. (iv) $P_{in} = P_{out} = 550 \text{ W} \Rightarrow \varepsilon_p I_p = 550$

$$220 \times I_p = 550 \Rightarrow I_p = \frac{550}{220} = \frac{5}{2} = 2.5 \text{ A}$$

22. Given, input voltage, $V_p = 2200$ V

Number of turns, $n_1 = 3000$

Output voltage, $V_s = 220$ V

As, $\frac{V_s}{V_p} = \frac{n_2}{n_1}$

$\Rightarrow \frac{220}{2200} = \frac{n_2}{3000}$

$\Rightarrow n_2 = \frac{220}{2200} \times 3000$

$\therefore$ Number of turns in the secondary winding,

$n_2 = 300$ turns.

23. $N_p = 22$; refer to Q. 22 on page 318.

24. Here, $N_p = 100$, $\frac{N_s}{N_p} = 100$

$\varepsilon_s = \varepsilon_p = 220$ V, $P_i = 1100$ W

(i) $N_p = 100$

$\therefore N_s = 10000$

(ii) $I_p = \frac{P_i}{\varepsilon_p} = \frac{1100}{220} = 5$ A

(iii) $\varepsilon_s = \frac{N_s}{N_p} \times \varepsilon_p = 100 \times 220 = 22000$ V

(iv) $I_s = \frac{P_o}{\varepsilon_s} = \frac{1100}{22000} = \frac{1}{20}$ A [$\because P_o = P_i$]

(v) $P_s = P_o = P_i = 1100$ W

25. Here, power, $P_L = 60$ W

Current, $I_L = 0.54$ A

Voltage, $V_L = \frac{P_L}{I_L} = \frac{60}{0.54}$

$= 111.11 \text{ V} = 111$ V

On average, the input current is half a load current.

$$I_p = \frac{I_L}{2} = \frac{0.54}{2} = 0.27 \text{ A}$$

The transformer is step-down.

26. Given, input voltage, $V_p = 2.5 \times 10^3$ V

Input current, $I_p = 20$ A

Also, $\frac{N_p}{N_s} = \frac{10}{1} \Rightarrow \frac{N_s}{N_p} = \frac{1}{10}$

Percentage efficiency $= \frac{\text{Output power}}{\text{Input power}} \times 100$

$\Rightarrow \frac{90}{100} = \frac{\text{Output power}}{V_p I_p}$

(i) Output power $= \frac{90}{100} \times (V_p I_p)$

$= \frac{90}{100} \times (2.5 \times 10^3 \text{ V}) \times (20 \text{ A})$

$= 4.5 \times 10^4$ W

(ii) $\because \frac{V_s}{V_p} = \frac{N_s}{N_p}$

$\Rightarrow V_s = \frac{N_s}{N_p} \times V_p$

Voltage, $V_s = \frac{1}{10} \times 2.5 \times 10^3 \text{ V} = 250$ V

(iii) $V_s I_s = 4.5 \times 10^4$ W

Current, $I_s = \frac{4.5\times 10^4}{V_s} = \frac{4.5 \times 10^4}{250} = 180$ A

27. Approx 147 turns, 0.0682 A; refer to Q. 25 on page 318.

28. 1000 V, 10 A, 2000 W; refer to Q. 26 on page 318.

29. Generating power of electric plant = 800 kW at $V = 220$ V, resistance/length $= 0.5\Omega$ / km

Distance = 15 km, generating voltage = 440 V,

Primary voltage, $V_p = 4000$ V

Secondary voltage, $V_s = 220$ V

(i) Power $= I_p \cdot V_p$

$\Rightarrow 800 \times 1000 = I_p \times 4000$

[redacted]

$\Rightarrow \quad I_p = 200$ A

Line power loss in form of heat

$$= (I_p)^2 \times R \text{ (2 lines)}$$
$$= (I_p)^2 \times 0.5 \times 15 \times 2$$
$$= (200)^2 \times 0.5 \times 15 \times 2$$
$$= 60 \times 10^4 \text{ W} = 600 \text{ kW}$$

(ii) If there is no power loss due to leakage, then the power supply by plant $= 800 + 600 = 1400$ kW

(iii) Voltage drop across the line $= I_p \cdot R$ (2 lines)

$$= 200 \times 0.5 \times 15 \times 2 = 3000 \text{ V}$$

Voltage from transmission $= 3000 + 4000 = 7000$ V

As, it is given that the power is generated at 440 V. So, the step-up transformer needed at the plant is 440V-7000 V.

30. Given, primary voltage, $V_p = 40000$ V

Let the current in primary be I_p.

$\therefore \quad V_p \cdot I_p = P$

$800 \times 1000 = 40000 \times I_p$

$I_p = 20$ A

(i) Line power loss $= I_p^2 \times R$ (2 lines)

$$= (20)^2 \times 2 \times 0.5 \times 15$$
$$= 6000 \text{ W} = 6 \text{ kW}$$

(ii) Power supply by plant $= 800 + 6 = 806$ kW

Voltage drop on line $= I_p \cdot R$ (2 lines)

$$= 20 \times 2 \times 0.5 \times 15$$
$$= 300 \text{ V}$$

Voltage for transmission $= 40000 + 300 = 40300$ V

Step-up transformer needed at the plant

$$= 440 \text{ V-}40300 \text{ V}$$

Power loss at higher voltage

$$= \frac{6}{800} \times 100 = 0.74\%$$

Power loss at lower voltage

$$= \frac{600}{1400} \times 100 = 42.8\%$$

Hence, the power loss is minimum at higher voltage. So, the high voltage transmission is preferred.

[redacted]

SUMMARY

- **Alternating Current** If the direction of current changes alternatively and its magnitude change continuously with respect to time is called Alternating current. It is sinusoidal in nature.
- The instantaneous value of AC is given by $I = I_0 \sin \omega t$ and instantaneous value of alternating emf is given by $E = E_0 \sin \omega t$.
- Mean value of AC is that value which send same change through a circuit in half cycle which is sent by steady current in same time.

  $\therefore \quad I_m = 0.637 I_0$ and $E_m = 0.637 E_0$

  where, I_0 and E_0 are the peak values of current and voltage respectively
- **Root mean square** (RMS) value of AC is that value over a complete cycle that generates same amount of heat in the given resistor that is generated by steady current in the same resistor.

  $$\therefore \quad I_{rms} = \frac{I_0}{\sqrt{2}} \text{ and } E_{rms} = \frac{I_0}{\sqrt{2}}$$
- A diagram representing alternating current and alternating emf (of same frequency) as rotating vectors (phasors) with the phase angle between them is called **phasor diagram**
- **AC through Resistor Only** In this case, there is zero phase difference between instantaneous alternating current and instantaneous alternating emf. So, they are in same phase.
- **AC through Capacitor Only** In this case, the current leads the voltage by a phase angle of $\frac{\pi}{2}$ or the voltage lags behind the current by the phase angle of $\frac{\pi}{2}$.

  Capacitive reactance, $X_C = \frac{1}{2\pi fC}$
- **AC through Inductor Only** In this case, the current lags behind the voltage by phase angle of $\frac{\pi}{2}$ or the voltage leads the current by phase angle of $\frac{\pi}{2}$. Inductive reactance, $X_L = 2\pi fL$.
- **AC through *L-C-R* Series Circuit** In this case, Impedance

  $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

  and $$\phi = \frac{X_L - X_C}{R}$$
- When $X_L > X_C$, then the AC circuit is inductance dominated circuit.
- When $X_C > X_L$, then the AC circuit is capacitance dominated circuit.
- In series *L-C-R* circuit, if phase (ϕ) between current and voltage is zero, then the circuit is said to be **resonant circuit**. Resonating frequency is given by,

  $$\nu_r = \frac{1}{2\pi\sqrt{LC}}$$
- **Quality Factor** (*Q*-Factor) determines the sharpness of the resonance.

  $$Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}}$$
- **Average Power Associated in AC Circuit,**

  $$P_{av} = I_{rms} E_{rms} \cos\theta$$

  $$\Rightarrow \quad P_{av} = \frac{E_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos\theta$$
- **Wattless Current** is the current which consumes no power for its maintenance in the circuit
- **Transformer** is used to increase or decrease the altrenating voltage.

  It is of the two types

  (i) **Step-up Transformer** $N_s > N_p$

  $V_s > V_p$

  $I_s < I_p$

  (ii) **Step-down Transformer** $N_s < N_p$

  $V_s < V_p$

  $I_s > I_p$

[redacted]

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. If an AC main supply is given to be 220 V. What would be the average emf during a positive half-cycle?
(a) 198 V (b) 386 V
(c) 256 V (d) None of these

2. If an alternating voltage is represented as $E = 141 \sin (628\, t)$, then the rms value of the voltage and the frequency are respectively
(a) 141 V, 628 Hz
(b) 100 V, 50 Hz
(c) 100 V, 100 Hz
(d) 141 V, 100 Hz

3. In a purely inductive AC circuit, $L = 30.0$ mH and the rms voltage is 150 V, frequency $\nu = 50$ Hz. The inductive reactance is
(a) $15.9\,\Omega$ (b) $9.42\,\Omega$ (c) $10\,\Omega$ (d) $8.85\,\Omega$

4. In an AC circuit, the power factor
(a) is zero when the circuit contains an ideal resistance only
(b) is unity when the circuit contains an ideal resistance only
(c) is unity when the circuit contains a capacitance only
(d) is unity when the circuit contains an ideal inductance only

5. If in an alternating circuit, the voltage is V and current is I, then the value of power dissipated in the circuit is
(a) VI
(b) $VI/2$
(c) $VI/\sqrt{2}$
(d) depends upon the angle between V and I

6. In an AC circuit, the instantaneous values of emf and current are $e = 200 \sin (314)\, t$ V and $I = \sin(314t + \pi/3)$ A. The average power consumed is
(a) 200 W (b) 100 W
(c) 50 W (d) 25 W

7. A coil of resistance $50\,\Omega$ and inductance 10 H is connected with a battery of 50 V. The energy stored in the coil is
(a) 125 J (b) 62.5 J (c) 250 J (d) 500 J

8. The value of power factor is maximum in an alternating circuit, when circuit consists
(a) only inductive (b) only capacitive
(c) only L - C (d) only resistive

9. In R-L C series circuit with $C = 1.00$ nF two values of R are
(i) $R = 100\,\Omega$ and
(ii) $R = 200\,\Omega$. For the source applied with $V_m = 100$ V. Resonant frequency is
(a) 1×10^3 rad/s (b) 1×10^6 rad/s
(c) 1.56×10^6 rad/s (d) 1.75×10^3 rad/s

10. The L-C-R circuit is connected to source of an alternating current. At the resonance, the phase difference between current flowing in the circuit and potential difference will be
(a) zero (b) $\pi/4$ (c) $\pi/2$ (d) π

11. The phenomenon of resonance is common among systems that have a tendency
(a) to oscillate at a particular frequency
(b) to get maximum amplitude
(c) Both (a) and (b)
(d) Neither (a) nor (b)

12. The value of emf in the secondary coil of transformer depends on
(a) the number of turns (b) material used
(c) voltage (d) induced flux

ASSERTION AND REASON

Directions (Q. Nos. 13-21) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

[redacted]

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false
(d) Assertion is false but Reason is true.

13. Assertion Today, most of the electrical devices use/require AC voltage.

Reason Most of the electrical energy sold by power companies is transmitted and distributed as alternating current.

14. Assertion Phasors V and I for the case of a resistor are in the same direction.

Reason The phase angle between the voltage and the current is zero.

15. Assertion When the capacitor is connected to an AC source, it limits or regulates the current, but does not completely prevent the flow of charge.

Reason The capacitor is alternately charged and discharged as the current reverses each half-cycle.

16. Assertion Capacitor serves as a barrier for DC and offers an easy path to AC.

Reason Capacitor reactance is inversely proportional to frequency.

17. Assertion If $X_C > X_L$, ϕ is positive and the circuit is predominantly capacitive. The current in the circuit leads the source voltage.

Reason If $X_C < X_L$, ϕ is negative and the circuit is predominantly inductive, the current in the circuit lags the source voltage.

18. Assertion In a series R-L-C circuit, the voltages across resistor, inductor and capacitor are 8V, 16V and 10V, respectively. The resultant emf in the circuit is 10 V.

Reason Resultant emf of the circuit is given by the relation.

$$E = \sqrt{V_R^2 + (V_L - V_C)^2}$$

19. Assertion Resonance phenomenon is exhibited by a circuit only if both L and C are present in the circuit.

Reason Voltage across L and C cancel each other and the current amplitude is V_m/R, the total source voltage appearing across R causes resonance.

20. Assertion In series L-C-R circuit resonance can take place.

Reason Resonance takes place if inductance and capacitive reactances are equal and opposite.

21. Assertion The wire used for the windings of transformer has some resistance.

Reason Energy is lost due to heat produced in the wire (I^2R).

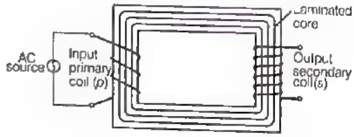
CASE BASED QUESTIONS

Directions (Q.Nos. 22-23) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

22. The Transformer

Transformer is a device, which is used to increase or decrease the alternating voltage. The transformers are of the following types

1. Step-up transformer 2. Step-down transformer



Transformer is based upon the principle of mutual induction. It consists of two coils, primary coil (p) and secondary coil (s), insulated from each other wounded on soft iron core. Often the primary coil is the input coil and secondary coil is the output coil. These soft iron cores are laminated to minimise eddy current loss.

(i) What is not possible in a transfomer?
(a) Eddy current
(b) Direct current
(c) Alternating current
(d) Induced current

(ii) Which quantities do not change during transformer operation?
(a) Power (b) Frequency
(c) Voltage (d) Both (a) and (b)

(iii) A transformer has 150 turns in its primary and 1000 in secondary. If the primary is connected to 440 V DC supply, what will be the induced voltage in the secondary side?
(a) 10 V (b) 3 V (c) 5 V (d) Zero

[redacted]

(iv) The ratio of secondary to primary turns in an ideal transformer is 4 : 5. If power input is P, then the ratio of power output to power input is

(a) 4 : 9 (b) 9 : 4
(c) 5 : 4 (d) 1 : 1

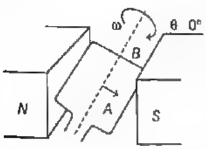
(v) A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230 V?

(a) 600 (b) 550
(c) 400 (d) 375

23. AC Generator

An AC generator produces electrical energy from mechanical work, just the opposite of what a motor does. In it, a shaft is rotated by some mechanical means, such as an engine or a turbine starts working and an emf is induced in the coil.

It is based on the phenomenon of electromagnetic induction which states that whenever magnetic flux linked with a conductor (or coil) changes, an emf is induced in the coil.



(i) Which method is used to induce an emf or current in a loop in AC generator?

(a) A change in the loop's orientation
(b) A change in its effective area
(c) Both (a) and (b)
(d) Neither (a) nor (b)

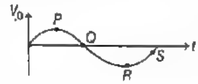
(ii) When the coil is rotated with a constant angular speed ω, the angle θ between the magnetic field vector B and the area vector A of the coil at any instant t, is

(a) $\theta = AB$ (b) $\theta = At$
(c) $\theta = \omega t$ (d) $\theta = Bt$

(iii) The change of flux is greatest at θ is equal to (given, $\phi_B = NBA\cos\omega t$)

(a) 90°, 270° (b) 90°, 45°
(c) 60°, 90° (d) 180°, 90°

(iv) The graph below shows the voltage output plotted against time. Which point on the graph shows that the coil is in a vertical position?



(a) P (b) Q (c) R (d) S

(v) An AC generator consists of a coil of 1000 turns and cross-sectional area of 100 cm^2, rotating at an angular speed of 100 rpm in a uniform magnetic field of 3.6×10^{-2} T. The maximum emf produced in the coil is

(a) 1.77 V (b) 2.77 V
(c) 3.77 V (d) 4.77 V

VERY SHORT ANSWER Type Questions

24. Prove mathematically that the average value of alternating current over one complete cycle is zero.

25. Draw the graph showing the variation of reactance of (i) a capacitor (ii) an inductor with the angular frequency of an AC circuit.

26. Distinguish between resistance, reactance and impedance for AC circuit.

27. The total impedance of a circuit decreases, when a capacitor is added in series with L and R. Explain why?

SHORT ANSWER Type Questions

28. Show mathematically that an ideal inductor does not consume any power in an AC circuit.

29. Discuss the use of transformer for long distance transmission of electrical energy.

30. What are the factors which reduces the efficiency of the transformer?

31. What is iron loss in a transformer and how it can be reduced?

LONG ANSWER Type I Questions

32. Draw a circuit diagram showing a series L-C-R circuit and derive an equation for its resonant frequency.

33. Explain the principle, construction and working of a step-down transformer. Can it be used with a DC circuit?

[redacted]

34. Find the expression for the true power and apparent power in an AC circuit. Determine the condition so that current in the circuit may be wattless.

35. An AC source of voltage, $V = V_m \sin \omega t$, is applied across a series L-C-R circuit. Draw the phasor diagram for this circuit when the,
   (i) capacitive impedance exceeds the inductive impedance.
   (ii) inductive impedance exceeds the capacitive impedance.

36. Answer the following questions.
   (i) In any AC circuit, is the applied instantaneous voltage equal to the algebraic sum of instantaneous voltage across the series elements of the circuit? Is the same true for rms voltage?
   (ii) A capacitor is used in the primary circuit of an induction coil. Why?
   (iii) An applied voltage signal consists of superposition of a DC voltage and an AC voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the DC signal will appear across C and the AC signal across L. NCERT

LONG ANSWER Type II Question

37. (i) Obtain the expression for the average power consumed in a series L-C-R circuit connected to AC source for which the phase difference between the voltage and the current in the circuit is ϕ.
   (ii) Define the quality factor in an AC circuit. Why should the quality factor have high value in receiving circuits? Name the factors on which it depends.

NUMERICAL PROBLEMS

38. The electric mains in a house are marked 220 V, 50 Hz. Write down the equation for instantaneous voltage.

39. What is the power dissipation in an AC circuit in which voltage and current are given by $E = 300\sin(\omega t + \pi/2)$ and $I = 5\sin\omega t$?

40. How much current is drawn by the primary coil of a transformer, which step-down 220 V-44 V to operate a device with an impedance of 440 Ω?

ANSWERS

1. (d)	2. (c)	3. (b)	4. (b)	5. (d)
6. (c)	7. (a)	8. (d)	9. (a)	10. (a)
11. (a)	12. (a)	13. (a)	14. (a)	15. (a)
16. (a)	17. (b)			

18. (a) The resultant emf in the L-C-R circuit is given by

$$E = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\Rightarrow \quad E = \sqrt{(8)^2 + (16-10)^2}$$

$$\Rightarrow \quad E = \sqrt{64 + 36} \Rightarrow E = 10\text{ V}$$

19. (a) It is important to note that resonance phenomenon is exhibited by a circuit only if both L and C are present in the circuit. Only then do the voltage across L and C cancel each other (both being out of phase) and the current amplitude is V_m/R, the total source voltage appearing across R. This means that we cannot have resonance in a R-L or R-C circuit.

20. (a)

21. (b)

22. (i) (b) Transformer is used to convert the value of AC voltage. It works on the principle of electromagnetic induction, so direct current is not possible in it.
   (ii) (d) The power and frequency do not change in a transformer operation. It changes voltage in a circuit.
   (iii) (d) As transformer works only in AC and in case of DC supply, there is no induced emf or voltage in secondary because there is no change in flux through the transformer circuit.
   (iv) (d) In an ideal transformer, there is no energy loss and flux is completely confined with the magnetic core, i.e. perfectly coupled.

   So, $\dfrac{P_{out}}{P_{in}} = 1$

   (v) (c) Here, $\varepsilon_p = 2300$ V, $N_p = 4000$, $\varepsilon_s = 230$ V
   Let N_s be the required number of turns in the secondary.

   As, $\dfrac{\varepsilon_s}{\varepsilon_p} = \dfrac{N_s}{N_p}$

$$\Rightarrow \quad N_s = N_p\left(\frac{\varepsilon_s}{\varepsilon_p}\right) = 4000\left(\frac{230\text{ V}}{2300\text{ V}}\right) = 400$$

23. (i) (c) One method to induce an emf or current in a loop is through a change in the loop's orientation or a change in its effective area.
   As the coil rotates in a magnetic field **B**, the effective area of the loop (the face perpendicular to the field) is $A\cos\theta$, where θ is the angle between **A** and **B**.

[redacted]

(ii) (c) When the coil is rotated with a constant angular speed ω, the angle θ between the magnetic field vector **B** and the area vector **A** of the coil at any instant t is $\theta = \omega t$ (assuming $\theta = 0^\circ$ at $t = 0$).

(iii) (a) $\frac{d\phi_B}{dt} = -NBA\omega \sin \omega t$, change of flux is greatest for $\omega t = \theta = 90^\circ, 270^\circ$, $\theta = 90^\circ, 270^\circ$.

(iv) (b) When the coil passes through its vertical position, its side is moving parallel to the magnetic flux between the magnetic poles, so no change of flux occurs. Hence, no emf is induced in it and output voltage is zero, i.e. at point Q.

(v) (c) Given, $N = 1000, A = 100 \text{ cm}^2 - 10^{-2} \text{ m}^2$,

$$v = 100 \text{ rpm} = \frac{100}{60} \text{ rps}$$

and $B = 3.6 \times 10^{-2}$ T

$\therefore$ Maximum emf produced in the coil is

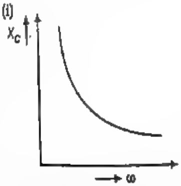
$$e_0 = NBA\,\omega = NBA\,(2\pi v)$$

$$= 1000 \times 3.6 \times 10^{-2} \times 10^{-2} \times 2 \times \frac{22}{7} \times \frac{100}{60}$$

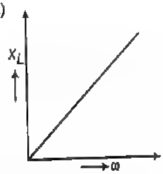
$$= 3.77 \text{ V}$$

24. Refer to text on page 300.

25. (i)



(ii)



26. The basic function of the three is the same i.e. to oppose the flow of current through the circuit. But the difference lie in their expressions. As,

Resistance, $R = V / I$

Capacitive reactance, $X_C = \frac{1}{\omega C}$

Inductive reactance, $X_L = \omega L$

Impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

27. Impedance of an L-R circuit is

$$Z = \sqrt{R^2 + X_L^2}$$

But with the introduction of a capacitor in series with a L and R, the new impedance will be

$$Z' = \sqrt{R^2 + (X_T - X_C)^2}$$

Hence, the total impedance decreases.

28. Refer to text on pages 295 and 296.

29. Refer to text on pages 316.

30. Refer to text on page 316.

31. Refer to text on page 316.

32. Refer to text on pages 297 and 298.

33. Refer to text on pages 315 and 316.

34. Refer to text on pages 300 and 301.

35. Refer to text on pages 297 and 298.

36. (i) Yes, it is true for instantaneous voltage.
No, it is not true for rms voltage because voltages across various elements may not be in same phase.

(ii) Because when the circuit is broken, then the large amount of induced voltage is used up in charging the capacitor. Thus, sparking is avoided.

(iii) $X_L = \omega L = 2\pi v L$

and $X_C = \frac{1}{\omega C} = \frac{1}{2\pi v C}$

Case I For DC, If $v = 0$, then $X_C = \infty$
Thus, capacitor blocks DC

Case II For AC of higher frequency, X_L is also higher, thus the inductor blocks the current. Hence, AC signal appears across L.

37. (i) Refer to text on page 300.
(ii) Refer to text on pages 299 and 300.

38. Refer to text on page 301. **[Ans. $220\sqrt{2} \sin 100\pi t$]**

39. 720 W, $P_{av} = E_0 / \sqrt{2} \times I_0 / \sqrt{2} \cos \theta$, [here, $\theta = 0^\circ$]

40. Refer to Example 1 on page 316. **[Ans. 0.02A]**

[redacted]

08

We have learnt that an electric current produces magnetic field and a varying magnetic field gives rise to an electric field. This brought together the phenomena of electricity and magnetism into a coherent and unified theory. After this discovery, Maxwell predicted variation of electric and magnetic field vectors perpendicular to each other leads to electromagnetic disturbance in space. He also concluded that, electromagnetic waves could travel with the speed of light. This led him to conclude that the light itself is an electromagnetic wave.

ELECTROMAGNETIC WAVES

CHAPTER CHECKLIST
- Displacement Current
- Maxwell's Equations
- Electromagnetic Waves
- Electromagnetic Spectrum

DISPLACEMENT CURRENT

Ampere's circuital law states that, the line integral of magnetic field B around any closed path is equal to μ_0 times the total current threading the closed path,

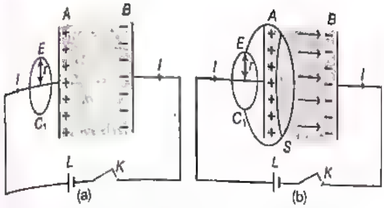
$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I \quad \ldots \text{(i)}$$

where, I is the net current threading the surface bounded by a closed path C.

Origin of Displacement Current

According to Maxwell, the Eq. (i) is logically inconsistent. With the help of following observations, it explained the same. He considered a parallel plate capacitor having plates A and B connected to a battery L, through a tapping key K. After pressing the key K, the conduction current flows through the connecting wires and the capacitor starts storing charge. As the charge on the capacitor grows, the conduction current in the wire decreases. When the capacitor is fully charged, the conduction current stops flowing in the wire. But during the charging of capacitor, there is no conduction current between the plates of capacitor. Let at an instant during charging, I be the conduction current in the wires. This current will produce magnetic field around the wires which can be detected by using a compass needle.

[redacted]



Circuit diagrams showing the inconsistency of Ampere's circuital law

After this, the magnetic field was found out at point E, which is at a perpendicular distance r from connecting wire, in a region outside the parallel plate capacitor. For this, a plane circular loop C_1 of radius r is considered. Its centre lies on wire and its plane is perpendicular to the direction of current carrying wire [see Fig. (a)]. The magnitude of magnetic field is same at all points on the loop and is acting tangentially along the circumference of the loop. If B is the magnitude of magnetic field at E, then by using Ampere's circuital law for loop C_1, we get

$$\oint_{C_1} \mathbf{B} \cdot d\mathbf{l} = \oint_{C_1} B\,dl \cos 0 = B \times 2\pi r = \mu_0 I$$

$$\Rightarrow \qquad B = \frac{\mu_0 I}{2\pi r} \qquad \text{...(ii)}$$

Now, a different surface, i.e. a tiffin box surface is considered. This surface is without lid with its circular rim, which has the same boundary as that of loop C_1 [see Fig. (b)].

On applying Ampere's circuital law to loop C_1 of this tiffin surface, we get

$$\oint \mathbf{B} \cdot d\mathbf{l} = B \cdot 2\pi r = \mu_0 \times 0 = 0 \qquad \text{...(iii)}$$

From Eqs. (ii) and (iii), it has been noticed that there is a magnetic field at E calculated through one way and no magnetic field at E, calculated through another way. As this contradiction arises from the use of Ampere's circuital law, hence, Ampere's circuital law is logically inconsistent.

Basic Idea of Displacement Current

Since, Ampere's circuital law for conduction current during charging of a capacitor was found inconsistent. Maxwell argued that the above inconsistency of Ampere's circuital law is because of some missing term. That term must be related to a changing electric field which passes through surface S between the plates of capacitor during charging. So, Maxwell introduced this missing term, i.e. displacement current, in order to make Ampere's circuital law logically consistent. Displacement current is that current which comes into play in the region in which the electric field and the electric flux is changing with time.

i.e. Displacement current, $\boxed{I_d = \varepsilon_0 \frac{d\phi_E}{dt}}$

Ampere's circuital law $\left(\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I\right)$ was modified to

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 (I_c + I_d)}$$

where, I_c = conduction current and I_d = displacement current.

It is called **modified Ampere's circuital law** or **Ampere Maxwell's-circuital law.**

Therefore, modified Ampere's circuital law may also be expressed as

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 \left(I_c + \varepsilon_0 \frac{d\phi_E}{dt}\right)}$$

The inferences can be drawn from the above discussion as given below

(i) The conduction and displacement currents are individually discontinuous, but the currents together possess the property of continuity through any closed electric circuit.

(ii) The displacement current is precisely equal to the conduction current, when the two present in different parts of the circuit.

(iii) The displacement current arises due to rate of change of electric flux (or electric field) between the two plates of the capacitor.

(iv) Just as the conduction current, the displacement current is also the source of varying magnetic field.

EXAMPLE |1| **In an electric circuit, there is a capacitor of reactance 100 Ω connected across the source of 220 V. Find the displacement current.**

Sol. Since, displacement current = conduction current.

Therefore, $I_d = \frac{V}{X_C} = \frac{220}{100} = 2.2\,\text{A}$

EXAMPLE |2| **In which way, you can establish an instantaneous displacement current of 1.0 A in the space between the parallel plates of 1 μF capacitor?**

Sol. ∴ Displacement current,

$$I_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d}{dt}(EA) \qquad [\because \phi_E = EA]$$

where, E is electric field and A is the area of cross-section.

$$= \varepsilon_0 A \frac{d}{dt}\left(\frac{V}{d}\right)$$

[redacted]

$$\Rightarrow \quad I_d = \frac{\varepsilon_0 A}{d} \times \frac{dV}{dt} = \frac{CdV}{dt} \quad \left[\because C = \frac{\varepsilon_0 A}{d}\right]$$

$$\Rightarrow \quad \frac{dV}{dt} = \frac{I_d}{C} = \frac{1.0}{10^{-6}} = 10^6\ \text{Vs}^{-1}$$

Thus, an instantaneous displacement current of 1 A can be set up by changing the potential difference across the parallel plates of capacitor at the rate of $10^6\ \text{Vs}^{-1}$.

MAXWELL'S EQUATIONS

Maxwell's equations are the basic laws of electricity and magnetism. These equations give complete description of all electromagnetic interactions. Maxwell on the basis of his equations, predicted the existence of electromagnetic waves.

There are four Maxwell's equations which are explained as given below

Gauss's Law of Electrostatics

This law states that, the total electric flux through any closed surface is always equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by that surface. It is given by

$$\boxed{\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}}$$

This equation is called Maxwell's first equation.

Gauss's Law in Magnetostatics

This law states that, the net magnetic flux through any closed surface is always zero. It is given by

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{S} = 0}$$

This equation is called Maxwell's second equation.

Faraday's Law of Electromagnetic Induction

This law states that, the induced emf produced in a circuit is numerically equal to rate of change of magnetic flux through it. It is given by

$$\boxed{\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\phi_B}{dt}}$$

This equation is called Maxwell's third equation.

Ampere-Maxwell's Circuital Law

This law states that, the line integral of the magnetic field along a closed path is equal to μ_0 times the total current (i.e. sum of conduction current and displacement current) threading the surface bounded by that closed path. It is given by

$$\boxed{\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0\left(I_c + \varepsilon_0 \frac{d\phi_E}{dt}\right)}$$

This equation is called Maxwell's fourth equation.

ELECTROMAGNETIC WAVES

These waves are produced due to the change in electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ sinusoidally and propagating through space such that, the two fields are perpendicular to each other and perpendicular to the direction of wave propagation.

Source of Electromagnetic Waves

An oscillating charge is an example of accelerating charge. It produces an oscillating electric field in space, which produces an oscillating magnetic field, which in turn produces an oscillating electric fields and so on. The oscillating electric and magnetic fields regenerate each other as a wave which propagates through space.

The frequency of EM wave is equal to the frequency of oscillation of charge, i.e.

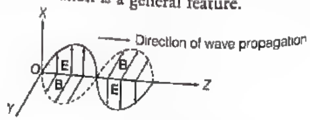
$$\nu = \frac{1}{2\pi\sqrt{LC}}$$

Electromagnetic waves are also produced when fast moving electrons are suddenly stopped by metal target of high atomic number. These electromagnetic waves are called X-rays.

Transverse Nature of Electromagnetic Waves

It can be shown from Maxwell's equations that electric and magnetic fields in an electromagnetic wave are perpendicular to each other and to the direction of wave propagation.

It was seen in the discussion of the displacement current also. If we would consider a parallel plate capacitor [refer to figure on page 330], the E inside the parallel plate capacitor was directed perpendicular to the plates. Also, the B which give rise to the displacement current was parallel to the capacitor . Thus, E and B were perpendicular in that case. But, this observation is a general feature.



A plane EM wave travelling along Z-axis

[redacted]

In the above figure, we see that permanent curve shows electric field E which is along x-direction and dotted curve shows magnetic field B which is along y-direction and the wave propagates along z-direction. Both E and B vary sinusodially and become maximum at same position and time.

Since, in electromagnetic wave, E and B are mutually perpendicular to each other, so they are transverse in nature.

The EM wave propagating in the positive z-direction may be represented by the following equations

Here, $E = E_x = E_0 \sin(kx - \omega t)$

$B = B_y = B_0 \sin(kz - \omega t)$

where, $k = 2\pi / \lambda$, [λ = wavelength]

$\omega = 2\pi\nu$, [ν − frequency]

E_0 = amplitude of varying electric field

and B_0 = amplitude of varying magnetic field.

Important Characteristics of Electromagnetic Waves

Important characteristics of EM waves are listed below

(i) The electromagnetic waves are produced by accelerated charge.

(ii) These waves do not require any material medium for propagation.

(iii) These waves travel in free space with the speed of light (3×10^8 ms^{-1}), given by the relation $c = 1/\sqrt{\mu_0 \varepsilon_0}$. It means that light waves are electromagnetic in nature.

(iv) Speed of electromagnetic wave in a medium is given by, $v = 1/\sqrt{\mu\varepsilon}$, where ε and μ are the permittivity and magnetic permeability of a material medium, respectively. This means, the speed of EM wave in a medium depends on electric and magnetic properties of a medium.

(v) The direction of variations of electric and magnetic fields are perpendicular to each other and also perpendicular to the direction of wave propagation. Thus, electromagnetic waves are transverse in nature.

(vi) In free space, the magnitudes of electric and magnetic fields in electromagnetic waves are related by $E_0/B_0 = c$.

(vii) The energy in electromagnetic waves is divided, on an average, equally between electric and magnetic fields.

$$U_e = U_m$$

where, U_e = energy of electric field

and U_m = energy of magnetic field.

(viii) The energy density (energy per unit volume) in an electric field E in vacuum is $\frac{1}{2}\varepsilon_0 E^2$ and that in magnetic field B is $\frac{B^2}{2\mu_0}$.

∴ Energy associated with an electromagnetic wave is given by

$$\boxed{U = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\cdot\frac{B^2}{\mu_0}}$$

Also, average energy density,

$$u_{av} = \frac{1}{4}\varepsilon_0 E_0^2 + \frac{1}{4}\frac{B_0^2}{\mu_0}$$

also $$u_{av} = \frac{1}{2}\varepsilon_0 E_0^2 = \frac{B_0^2}{2\mu_0}$$

(ix) Electromagnetic waves, being uncharged, are not deflected by electric and magnetic fields.

(x) The electromagnetic wave like other waves carries energy and momentum. Since, it has momentum, an electromagnetic wave also exerts pressure called **radiation pressure.**

If wave is incident on a completely absorbing surface, then momentum delivered is given by

momentum, $p = \frac{U}{c}$

Note Light carries energy from the sun to the earth, thus making life possible on the earth.

(xi) Electromagnetic waves are polarised and can be easily seen in the response of a portable AM radio to a broadcasting station. If an AM radio has a telescopic antenna, it responds to the electric part of the signal. When the antenna is turned horizontal, the signal will be greatly diminished.

EXAMPLE |3| An electromagnetic wave is travelling in vacuum with a speed of 3×10^8 m / s. Find its velocity in a medium having relative electric and magnetic permeability 2 and 1, respectively.

Sol. Given, velocity of electromagnetic wave in vacuum,

$$c = 3 \times 10^8 \text{ m/s}$$

Relative electric permeability, $\varepsilon_r = 2$

and magnetic permeability, $\mu_r = 1$

Since, velocity of electromagnetic wave in a medium can be calculated by

[redacted]

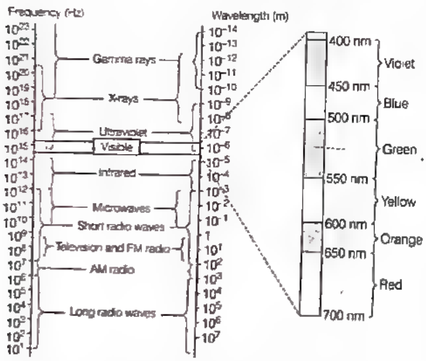
$$v = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \mu_0 \mu_r}} = \frac{1}{\sqrt{\varepsilon_0 \mu_0} \times \sqrt{\mu_r \varepsilon_r}}$$

where, $\frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c \Rightarrow v = \frac{c}{\sqrt{\mu_r \varepsilon_r}}$ ...(i)

Therefore, $v = \frac{3 \times 10^8}{\sqrt{2 \times 1}} \Rightarrow v = \frac{3}{\sqrt{2}} \times 10^8$ m/s

ELECTROMAGNETIC SPECTRUM

The orderly arrangement of EM waves in increasing or decreasing order of wavelength λ or frequency ν is called electromagnetic spectrum. The range varies from 10^{-12}m to 10^4 m, i.e. from γ-rays to radio waves.



Electromagnetic spectrum with common names for various parts of it

The wavelength ranges, frequency ranges and use of various regions of electromagnetic spectrum are summarised below

Radio waves

These are produced due to oscillating charge particles. The frequency varies from 500 kHz to 1000 MHz.

Uses of radio waves are given below

(i) These are used in AM (Amplitude Modulation) from 530 kHz to 1710 kHz. These are also used in ground wave propagation.

(ii) These are used in TV waves ranging from 54 MHz to 890 MHz.

(iii) These are used in FM (Frequency Modulation) ranging from 88 MHz to 108 MHz.

(iv) UHF (Ultra High Frequency) waves are used in cellular phones.

Microwaves

These waves are called short wavelength radio waves which are produced by vacuum tubes. Their frequency lies in the range of 1 GHz to 300 GHz (gigahertz).

Uses of microwaves are given below

(i) These are used in RADAR systems for aircraft navigation.

(ii) These are used in microwave oven for cooking purpose.

(iii) These are used in study of atomic and molecular structures.

(iv) These are used to measure the speed of vehicle, speed of cricket ball, etc.

Infrared Waves

These waves were discovered by Herschell. These waves are also called heat waves. These waves are produced from the heat radiating bodies and molecules.

They have high penetration power. Its frequency range is from 3×10^{11} Hz to 4×10^{14} Hz.

Uses of infrared waves are given below

(i) These are used in physical therapy.

(ii) These are used in satellite for army purpose.

(iii) These are used in weather forecasting.

(iv) These are used for producing dehydrated fruits.

(v) These are used in solar water heater, solar cells and cooker.

Visible Rays

It is that part of spectrum which is visible by human eye and its frequency range is from 4×10^{14} Hz to 7×10^{14} Hz.

Uses of visible rays are given below

Visible rays are used by the optical organs of humans and animals for three primary purposes given below:

(i) To see things, avoid bumping from them and escape danger.

(ii) To find stuff to eat.

(iii) To find other living things with which to consort so as to prolong the species.

Ultraviolet Rays

These rays were discovered by Ritter in 1801. These rays are produced by special lamps and very hot bodies. The sun is an important source of UV-rays but fortunately absorbed by ozone layer at an altitude of about 40-50km. Its frequency range is from 10^{14} Hz to 10^{16} Hz.

[redacted]

Uses of ultraviolet rays are given below

(i) These are used in burglar alarm.
(ii) These are used in checking mineral sample.
(iii) These are used to study molecular structure.
(iv) To kill germs in minerals.
(v) To sterilise surgical instruments.
(vi) These rays can be focussed into very narrow beams for high precision applications such as LASIK eye surgery.

X-Rays

These rays were discovered by German professor Roentgen. Its frequency range is from 3×10^{16} Hz to 3×10^{21} Hz.

Uses of X-rays are given below

(i) These are used in surgery to detect the fracture, diseased organs, stones in the body, etc.
(ii) These are used in engineering to detect fault, crack on bridges, testing of welds.
(iii) These are used at metro station to detect metal or explosive material.
(iv) These are used in scientific research.

Gamma (γ) Rays

These rays were discovered by Rutherford. They travel with the speed of light and having high penetration power. The frequency ranges from 3×10^{18} Hz to 5×10^{22} Hz.

Uses of gamma (γ) rays are given below

(i) These are used to produce nuclear reaction.
(ii) These are used in radio therapy for the treatment of tumour and cancer.
(iii) These are used in food industry to kill pathogenic microorganism.
(iv) These are used to provide valuable information about the structure of atomic nucleus.

Different Types of Electromagnetic Waves

Type	Wavelength range	Production	Detection
Radio wave	> 0.1 m	Rapid acceleration and decelerations of electrons in aerials	Receiver's aerials
Microwave	0.1 m to 1 mm	Klystron valve or magnetron valve	Point contact diodes
Infrared wave	1 mm to 700 nm	Vibration of atoms and molecules	Thermopiles bolometer, Infrared photographic film
Light	700 nm to 400 nm	Electrons in atoms emit light when they move from higher energy level to a lower energy level	The eye, Photocells, Photographic film
Ultraviolet rays	400 nm to 1 nm	Inner shell electrons in atoms moving from higher energy level to a lower energy level	Photocells, Photographic film
X-rays	1 nm to 10^{-3} nm	X-ray tubes or inner shell electrons	Photographic film, Geiger tubes, ionisation chamber
Gamma rays	$< 10^{-3}$ nm	Radioactive decay of the nucleus	Photographic film, ionisation chamber

Note This EM spectrum and its properties have been frequently asked in previous years 2014, 2013, 2012, 2011, 2010

CHAPTER PRACTICE
(SOLVED)

OBJECTIVE Type Questions

1. Which statement represents the symmetrical counterpart of Faraday's law and a consequence of the displacement current being a source of a magnetic field?

(a) An electric field changing with time gives rise to a magnetic field
(b) A magnetic field changing with time gives rise to an electric field
(c) An emf changing with time gives rise to an electric field
(d) An displacement current, changing with time gives rise to an electric field

2. A linearly polarised electromagnetic wave given as $\mathbf{E} = E_0 \hat{\mathbf{i}} \cos(kz - \omega t)$ is incident normally on a perfectly reflecting infinite wall at $z = a$. Assuming that the material of the wall is optically inactive, the reflected wave will be given as

(a) $\mathbf{E}_r = E_0 \hat{\mathbf{i}}(kz - \omega t)$
(b) $\mathbf{E}_r = E_0 \hat{\mathbf{i}} \cos(kz + \omega t)$
(c) $\mathbf{E}_r = -E_0 \hat{\mathbf{i}} \cos(kz + \omega t)$
(d) $\mathbf{E}_r = E_0 \hat{\mathbf{i}} \sin(kz - \omega t)$

3. Radiations of intensity 0.5 Wm^{-2} are striking a metal plate. The pressure on the plate is

(a) 0.166×10^{-8} Nm^{-2}
(b) 0.332×10^{-8} Nm^{-2}
(c) 0.111×10^{-8} Nm^{-2}
(d) 0.083×10^{-8} Nm^{-2}

[redacted]

4. Total energy density of electromagnetic waves in vacuum is given by the relation
(a) $\frac{1}{2}\cdot\frac{E^2}{\varepsilon_0}+\frac{B^2}{2\mu_0}$ (b) $\frac{1}{2}\varepsilon_0 E^2+\frac{1}{2}\mu_0 B^2$
(c) $\frac{E^2+B^2}{c}$ (d) $\frac{1}{2}\varepsilon_0 E^2+\frac{B^2}{2\mu_0}$

5. The speed of electromagnetic wave in vacuum depends upon the source of radiation
(a) increases as we move from γ-rays to radio waves
(b) decreases as we move from γ-rays to radio waves
(c) is same for all of them
(d) None of the above

6. One requires 11 eV of energy to dissociate a carbon monoxide molecule into carbon and oxygen atoms. The minimum frequency of the appropriate electromagnetic radiation to achieve the dissociation lies in **NCERT Exemplar**
(a) visible region (b) infrared region
(c) ultraviolet region (d) microwave region

VERY SHORT ANSWER Type Questions

7. A capacitor has been charged by a DC source. What are the magnitude of conduction and displacement current when it is fully charged? **Delhi 2013**

8. The charge on a parallel plate capacitor varies as $q = q_0 \cos 2\pi \nu t$. The plates are very large and close together (area = A and separation = d). Neglecting the edge effects, find the displacement current through the capacitor. **NCERT Exemplar**

9. A variable frequency AC source is connected to a capacitor. How will the displacement current change with decrease in frequency? **NCERT Exemplar**

10. The charging current for a capacitor is 0.25 A. What is the displacement current across its plates? **Foreign 2016**

11. What are the directions of electric and magnetic field vectors relative to each other and relative to the direction of propagation of electromagnetic waves? **All India 2012**

12. A charged particle oscillates about its mean position with frequency 10^9 Hz. What is the frequency of electromagnetic wave produced by the oscillators? **NCERT**

13. In which directions do the electric and magnetic field vectors oscillate in an electromagnetic wave propagating along the X-axis? **All India 2017**

14. How is the speed of electromagnetic waves in vacuum determined by the electric and magnetic fields? **Delhi 2017**

15. Do electromagnetic waves carry energy and momentum? **All India 2017**

16. An electromagnetic wave exerts pressure on the surface on which it is incident. Justify. **Delhi 2014**

17. To which part of the electromagnetic spectrum does a wave of frequency 5×10^{19} Hz belong? **All India 2014**

18. Name the type of electromagnetic wave used in food industry to will pathogenic microorganism. Also write its frequency range.

19. What physical quantity is the same for X-rays of wavelength 10^{-10} m, red light of wavelength 6800 Å and radio waves of wavelength 500 m? **NCERT**

20. Why are microwaves considered suitable for radar systems used in aircraft navigation? **Delhi 2016**

21. Name the electromagnetic waves which
(i) maintain the earth's warmth and
(ii) are used in aircraft navigation. **Foreign 2012**

22. Name the electromagnetic waves used in LASIK eye surgery and why?

23. Name the electromagnetic radiations used for (a) water purification, and (b) eye surgery. **CBSE 2018**

24. Why does microwave oven heats up a food item containing water molecules most efficiently? **NCERT Exemplar**

SHORT ANSWER Type Questions

25. When an ideal capacitor is charged by a DC battery, no current flows. However, when an AC source is used, the current flows continuously. How does one explain this, based on the concept of displacement current? **Delhi 2012**

26. A capacitor made of two parallel plates each of the plate A and separation d, is being charged by an external AC source. Show that the displacement current inside the capacitor is the same as the current charging the capacitor. **All India 2013**

[redacted]

27. (i) An electromagnetic wave is travelling in a medium with a velocity $\mathbf{v} = v\hat{\mathbf{i}}$. Draw a sketch showing the propagation of the electromagnetic wave indicating the direction of the oscillating electric and magnetic fields.
(ii) How are the magnitudes of the electric and magnetic fields related to velocity of the electromagnetic wave?
Delhi 2013, All India 2008C

28. Even though an electric field E exerts a force qE on a charged particle yet electric field of an electromagnetic wave does not contribute to the radiation pressure (but transfers energy). Explain. **NCERT Exemplar**

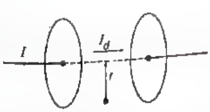
29. Show that the radiation pressure exerted by an EM wave of intensity I on a surface kept in vacuum is $\frac{I}{c}$. **NCERT Exemplar**

30. Poynting vectors is defined as a vector whose magnitude is equal to the wave intensity and whose direction is along the direction of wave propagation. Mathematically, it is given by $\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B})$. Show the nature of $\mathbf{S}$ *versus* t graph. **NCERT Exemplar**

31. Identify the electromagnetic waves whose wavelengths vary as
(i) $10^{-12}\text{ m} < \lambda < 10^{-8}\text{ m}$ (ii) $10^{-3}\text{ m} < \lambda < 10^{-1}\text{ m}$
Write one use for each. **All India 2017**

32. (a) Why are infrared waves often called heat waves? Explain.
(b) What do you understand by the statement, "electromagnetic waves transport momentum"? **CBSE 2018**

33. Show that the magnetic field B at a point in between the plates of a parallel plate capacitor during charging is $\frac{\mu_0 \varepsilon_0 r}{2} \cdot \frac{dE}{dt}$ (symbols having usual meaning). **NCERT Exemplar**



34. A capacitor of capacitance C is being charged by connecting it across a DC source along with an ammeter. Will the ammeter show a momentary deflection during the process of charging? If so, how would you explain this momentary deflection and the resulting continuity of current in the circuit? Write the expression for the current inside the capacitor. **All India 2012**

LONG ANSWER Type I Questions

35. Write the expression for the generalised form of Ampere's circuital law.
Discuss its significance and describe briefly how the concept of displacement current is explained through charging/discharging of a capacitor in an electric circuit. **Delhi 2015**

36. Show that average value of radiant flux density S over a single period T is given by $S = \frac{1}{2c\mu_0}E_0^2$. **NCERT Exemplar**

37. How are electromagnetic waves produced by oscillating charges?
Draw a sketch of linearly polarised electromagnetic waves propagating in the z-direction. Indicate the directions of the oscillating electric and magnetic fields. **Delhi 2016**

38. (i) Describe briefly how electromagnetic waves are produced by oscillating charges?
(ii) Give one use of each of the following
(a) Microwaves **All India 2011C**
(b) X-rays
(c) Infrared rays
(d) Gamma rays

39. Name the electromagnetic waves, in the wavelength range 10 nm to 10^{-3} nm. How are these waves generated? Write their two uses. **All India 2017 C**

40. Answer the following questions.
(i) Name the electromagnetic waves which are used for the treatment of certain forms of cancer. Write their frequency range.
(ii) Thin ozone layer on top of stratosphere is crucial for human survival. Why?
(iii) Why is the amount of the momentum transferred by the electromagnetic waves incident on the surface so small? **Delhi 2014**

41. Answer the following questions.
(i) Name the electromagnetic waves which are produced during radioactive decay of a nucleus. Write their frequency range.
(ii) Welders wear special glass goggles while working. Why? Explain.

[redacted]

(iii) Why are infrared waves often called as heat waves? Give their one application. Delhi 2014

42. Name the parts of the electromagnetic spectrum which is
(i) suitable for RADAR systems in aircraft navigations.
(ii) used to treat muscular strain.
(iii) used as a diagnostic tool in medicine.
Write in brief, how these waves can be produced? All India 2015

43. Name the following constituent radiations of electromagnetic spectrum which
(i) produce intense heating effect.
(ii) is absorbed by the ozone layer in the atmosphere.
(iii) is used for studying crystal structure. Write one more application for each of these radiations.

44. Identify the part of the electromagnetic spectrum which is
(i) suitable for radar system used in aircraft navigation.
(ii) produced by bombarding a metal target by high speed electrons. All India 2016

45. (i) Which segment of electromagnetic waves has highest frequency? How are these waves produced? Give one use of these waves.
(ii) Which EM waves lie near the high frequency end of visible part of EM spectrum? Give its one use. In what way, this component of light has harmful effects on humans? Foreign 2016

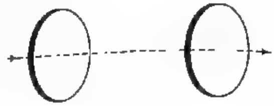
NUMERICAL PROBLEMS

46. The current in a circuit containing a capacitor is 0.15 A. What is the displacement current and where does it exist?

47. You are given a 2 μF parallel plate capacitor. How would you establish an instantaneous displacement current of 1 mA in the space between its plates? NCERT Exemplar

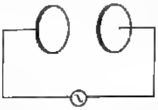
48. Calculate the displacement current between the square plates of side 1 cm of a capacitor, if electric field between the plates is changing at the rate of 3×10^6 V m^{-1} s^{-1}.

49. Figure shows a capacitor made of two circular plates each of radius 12 cm and separated by 5.0 cm. The capacitor is being charged by an external source (not shown in the figure). The charging current is constant and equal to 0.15 A.
(i) Calculate the capacitance and the rate of change of potential difference between the plates.
(ii) Obtain the displacement current across the plates.
(iii) Is Kirchhoff's first rule (junction rule) valid at each plate of the capacitor? Explain.



NCERT

50. A parallel plate capacitor (shown in the figure) made of circular plates each of radius $R = 6.0$ cm has a capacitance $C = 100$ pF. The capacitor is connected to a 230 V AC supply with angular frequency of 300 rad/s.



(i) What is the rms value of the conduction current?
(ii) Is the conduction current equal to the displacement current?
(iii) Determine the amplitude of B at a point 3.0 cm from the axis between the plates. NCERT

51. (i) A plane electromagnetic wave travels in vacuum along z-direction. What can you say about the directions of electric and magnetic field vectors?
(ii) If the frequency of the wave is 30 MHz. What is its wavelength? NCERT; Delhi 2012

52. A radio can tune into any station in the 7.5 MHz to 12 MHz band. What is its corresponding wavelength? NCERT

53. The magnetic field of a beam emerging from a filter facing a floodlight is given by
$$B = 12\times10^{-8}\sin(1.20\times10^7 z - 3.60\times10^{15} t)\,T.$$
What is the average intensity of the beam? NCERT Exemplar

[redacted]

54. About 5% of the power of a 100 W light bulb is connected to visible radiation. What is the average intensity of visible radiation at
 (i) distance of 1 m from the bulb
 (ii) distance of 10 m? Assume that the radiation is emitted isotropically and neglect reflection. NCERT

55. The amplitude of the magnetic field part of a harmonic electromagnetic wave in vacuum is $B_0 = 510$ nT. What is the amplitude of the electric field part of the wave? NCERT

56. In a plane, electric field oscillates sinusoidally at a frequency of 2×10^{10} Hz and amplitude 48 V/m.
 (i) What is the wavelength of the wave?
 (ii) What is the amplitude of the oscillating magnetic field?
 (iii) Show that the average energy density of the E field equals the average energy density of the B field. NCERT

57. Suppose that the electric field amplitude of an electromagnetic wave is $E_0 = 120$ N/C and that its frequency is $\nu = 50.0$ MHz. (i) Determine B_0, ω, k and λ (ii) Find expressions for E and B. NCERT

58. Suppose that the electric field part of an electromagnetic wave in vacuum is
$$E = [3.1\cos(1.8\,y + (5.4\times10^6 t))]\ \hat{\mathbf{i}}$$
 (i) What is the direction of propagation?
 (ii) What is the wavelength λ?
 (iii) What is the frequency ν?
 (iv) What is the amplitude of the magnetic field part of the wave?
 (v) Write an expression for the magnetic field part of the wave. NCERT

59. Use the formula, $\lambda_m T = 0.29$ cm-K to obtain the characteristic temperature ranges for different parts of the electromagnetic spectrum. What do the numbers that you obtain, tell you? NCERT

60. Given below are some famous numbers associated with electromagnetic radiations in different contexts in Physics. State the part of the electromagnetic spectrum to which each belongs.
 (i) 21 cm (wavelength emitted by atomic hydrogen in interstellar space).
 (ii) 1057 MHz (frequency of radiation arising from two close energy levels in hydrogen, known as Lamb shift).
 (iii) 2.7 K (temperature associated with the isotropic radiation filling all space thought to be a relic of the big-bang origin of the universe).
 (iv) 5890 Å-5896 Å (double lines of sodium).
 (v) 14.4 keV (energy of a particular transition in ^{57}Fe nucleus associated with a famous high resolution spectroscopic method (Mössbauer spectroscopy). NCERT

61. The terminology of different parts of the electromagnetic spectrum is given in the text. Use the formula $E = h\nu$ (for energy of a quantum of radiation: photon) and obtain the photon energy in units of eV for different parts of the electromagnetic spectrum. In what way are the different scales of photon energies that you obtain related to the sources of electromagnetic radiation? NCERT

HINTS AND SOLUTIONS

1. (a) The fact that an electric field changing with time gives rise to a magnetic field, is the symmetrical counterpart and is a consequence of the displacement current being a source of a magnetic field.

2. (b) When a wave is reflected from denser medium, then the type of wave doesn't change but only its phase changes by 180° or π radian.
Thus, for the reflected wave $\hat{\mathbf{z}} = -\hat{\mathbf{z}}, \hat{\mathbf{i}} = -\hat{\mathbf{i}}$ and additional phase of π in the incident wave.
Given, here the incident electromagnetic wave is,
$$\mathbf{E} = E_0\hat{\mathbf{i}}\cos(kz - \omega t)$$
The reflected electromagnetic wave is given by
$$\mathbf{E}_r = E_0(-\hat{\mathbf{i}})\cos[k(-z) - \omega t + \pi]$$
$$= -E_0\hat{\mathbf{i}}\cos[-(kz + \omega t) + \pi]$$
$$= E_0\hat{\mathbf{i}}\cos[-(k_z + \omega t) = E_0\hat{\mathbf{i}}\cos(kz + \omega t)]$$

3. (a) Intensity or power per unit area of the radiations
$$I = pc \Rightarrow p = \frac{I}{c} = \frac{0.5}{3\times10^8} = 0.166\times10^{-8}\ \text{Nm}^{-2}$$

4. (d) The energy in EM waves is divided equally between the electric and magnetic fields.
The total energy per unit volume is $u = u_e + u_m$
$$= \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2}\frac{B^2}{\mu_0}$$

[redacted]

5. (c) Speed of electromagnetic waves in vacuum $= \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

6. (c) Given, energy required to dissociate a carbon monoxide molecule into carbon and oxygen atoms $E = 11$ eV

We know that, $E = h\nu$, where $h = 6.62 \times 10^{-34}$ J-s

ν = frequency

$\Rightarrow 11 \text{ eV} = h\nu$

$$\Rightarrow \quad \nu = \frac{11 \times 1.6 \times 10^{-19}}{h} \text{ J}$$

$$= \frac{11 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} \text{ J} = 2.65 \times 10^{15} \text{ Hz}$$

This frequency radiation belongs to ultraviolet region.

7. Electric flux through plates of capacitor, $\phi_E = \dfrac{q}{\varepsilon_0}$.

where, charge, q = constant (as the capacitor is fully charged)

Displacement current, $I_d = \varepsilon_0 \dfrac{d\phi_E}{dt} = \varepsilon_0 \dfrac{d\left(\dfrac{q}{\varepsilon_0}\right)}{dt} = 0$

Conduction current, $I_c = C\dfrac{dV}{dt} = 0$ (as voltage becomes constant when the capacitor becomes fully charged).

8. The displacement current through the capacitor is given by

$$I_d = I_c = \frac{dq}{dt} \quad \text{...(i)}$$

Given, $q = q_0 \cos 2\pi\nu t$

Differentiating w.r.t. t on both sides, we get

$$\frac{dq}{dt} = q_0(-\sin 2\pi\nu t)(2\pi\nu)$$

Putting the value of $\dfrac{dq}{dt}$ in Eq. (i), we get

$$I_d = I_c = -(q_0 \sin 2\pi\nu t) \times 2\pi\nu = -2\pi\nu q_0 \sin 2\pi\nu t$$

9. Capacitive reactance, $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi\nu C}$

$$\therefore \quad X_C \propto \frac{1}{\nu}$$

As frequency decreases, X_C increases. As the conduction current is inversely proportional to X_C $\left[\because I \propto \dfrac{1}{X_C}\right]$.

So, displacement current also decreases because the conduction current is equal to the displacement current.

10. The displacement current is equal to 0.25 A, as the charging current is 0.25 A.

11. Direction of electric field E, direction of magnetic field B and direction of propagation of wave are mutually perpendicular to one another.

12. The frequency of electromagnetic wave produced by the oscillators is same as that of oscillating charged particle about its equilibrium position, i.e. 10^9 Hz.

13. E and B are perpendicular to direction of propagation of light. Also, direction of propagation is parallel to $\mathbf{E} \times \mathbf{B}$. Hence, E is along $\hat{j}$ or $+Y$-axis and B is along $\hat{k}$ or $+Z$-axis.

14. To determine speed of light in vacuum, we use the formula, $c = \dfrac{E_0}{B_0} = \dfrac{E_{rms}}{B_{rms}}$

where, E_0 and B_0 are maximum electric field and magnetic field component respectively, of electromagnetic waves.

15. Yes, electromagnetic waves carry energy and momentum.

Momentum, $p = \dfrac{U}{c}$ and energy density $= \dfrac{1}{2}\varepsilon_0 E^2$

16. Electromagnetic wave carries energy and momentum. Since, it has momentum due to this reason it exert pressure, called radiation pressure.

17. A wave of frequency 5×10^{19} Hz belongs to γ-rays of electromagnetic spectrum.

18. Gamma(γ) rays are used in food industry to will pathogenic microorganism. Its frequency ranges from 3×10^{18} Hz to 5×10^{22} Hz.

19. Speed remains same but wavelength changes.

20. Microwaves are generally used in RADAR system and aircraft due to the fact that, they have longer wavelengths and low frequencies, so they can be focused along a straight line without much deviation. Also, they do not bend around the corners of the abstacles

21. (i) Infrared rays (ii) Microwaves

22. In LASIK eye surgery, ultraviolet rays are used because of their short wavelength they can be focused into very narrow beam.

23. (a) Ultraviolet radiation
(b) Infrared radiation

24. Microwave oven heats up the food items containing water molecules most efficiently because the frequency of microwaves matches the resonant frequency of water molecules.

25. If an ideal capacitor is charged by DC battery, current flows momentarily till capacitor gets fully charged after that no current flow. However, when an AC source is connected, then conduction current, $I_c = \dfrac{dq}{dt}$ starts flowing in the connecting wire. As charging polarity of AC current changes, the capacitor is alternatively charged and discharged with time. This causes change in electric field between plates of the capacitor which causes electric flux to change and gives rise to displacement current in the region between plates of capacitor, as displacement current, $I_d = E_0 \dfrac{d\phi_E}{dt}$ and $I_d = I_c$ at all instants.

[redacted]

26. Let the alternating emf charging the plates of capacitor be $V = V_0 \sin \omega t$ ...(i)

Charge on the capacitor,

$$q = CV = CV_0 \sin \omega t \quad \text{[from Eq.(i)]}$$

and instantaneous current, $I = \frac{dq}{dt} = \frac{d}{dt}(CV_0 \sin \omega t)$

$$= \omega CV_0 \cos \omega t = I_0 \cos \omega t$$

where, $I_0 = \omega CV_0$

Displacement current, $I_d = \varepsilon_0 \frac{d\phi_E}{dt}$

$$\Rightarrow \varepsilon_0 A \frac{d(E)}{dt} = \varepsilon_0 A \frac{d}{dt}\left(\frac{q}{\varepsilon_0 A}\right) = \varepsilon_0 A \frac{d}{dt}\left(\frac{CV_0 \sin \omega t}{\varepsilon_0 A}\right)$$

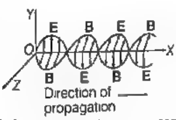
$$= \frac{d}{dt}(CV_0 \sin \omega t)$$

$$= \omega CV_0 \cos \omega t$$

$$= I_0 \cos \omega t$$

Thus, the displacement current inside the capacitor is the same as the current charging the capacitor.

27. (i) Given that velocity $\mathbf{v} = v\,\hat{\mathbf{i}}$, i.e. the wave is propagating along X-axis, so electric field $\mathbf{E}$ is along Y-axis and magnetic field $\mathbf{B}$ is along Z-axis. The propagation of electromagnetic wave is shown in the figure.



(ii) Speed of electromagnetic wave can be given as

$$c = \frac{E_0}{B_0} = \frac{E}{B}$$

where, E_0 and B_0 are peak values of E and B or instantaneous values of E and B.

28. Electric field of an electromagnetic wave is an oscillating field which causes force on the charged particle. This electric force averaged over an integral number of cycles is zero, because its direction changes with every half cycle. So, electric field is not responsible for radiation pressure.

29. Pressure $= \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$

Force is the rate of change of momentum,

i.e. $F = \frac{dp}{dt}$

Energy in time dt, $U = p \cdot c \Rightarrow p = \frac{U}{c}$

$$\therefore \text{Pressure} = \frac{1}{A} \cdot \frac{U}{c \cdot dt}$$

$$= \frac{I}{c} \quad \left[\because I = \text{intensity} = \frac{U}{A \cdot dt}\right]$$

30. Consider an electromagnetic waves with $\mathbf{E}$ be varying along Y-axis, $\mathbf{B}$ be along Z-axis and propagation of wave be along X-axis. Then, $\mathbf{E} \times \mathbf{B}$ will indicate the direction of propagation of energy flow in electromagnetic wave which will be along X-axis.

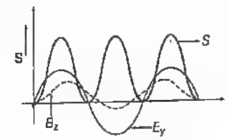
Let $\mathbf{E} = E_0 \sin(\omega t - kx)\hat{\mathbf{j}}$

$$\mathbf{B} = B_0 \sin(\omega t - kx)\,\hat{\mathbf{k}}$$

$$\mathbf{S} = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B}) = \frac{1}{\mu_0} E_0 B_0 \sin^2(\omega t - kx)[\hat{\mathbf{j}} \times \hat{\mathbf{k}}]$$

$$= \frac{E_0 B_0}{\mu_0} \sin^2(\omega t - kx)\hat{\mathbf{i}} \quad [\because \hat{\mathbf{j}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}]$$

The variation of $|\mathbf{S}|$ with time t will be as given in the figure below



31. (i) 10^{-12} m - 10^{-8} m = 0.01 Å-100 Å $\rightarrow$ X-ray.

It is used in crystallography

(ii) 10^{-3} m - 10^{-1} m = 0.1 cm-10 cm $\rightarrow$ Radio waves.

It is used in radio communication.

32. (a) Infrared waves have frequencies lower than those of visible light, vibrate not only the electrons, but also the entire atoms or molecules in the structure of the surface.

This vibration increases the internal energy and hence the temperature of the structure, which is why infrared waves are often called heat waves.

(b) Electromagnetic wave transports linear momentum as it travels through space. If an electromagnetic wave transfer a total energy U to a surface in time t, then total linear momentum delivered to the surface is given as

$$p = \frac{U}{c}$$

where, c is the speed of electromagnetic wave.

33. In the given figure, I_d is the displacement current in the region between two plates of parallel plate capacitor.

The magnetic field induction at a point in a region between two plates of capacitor at a perpendicular distance r from the axis of plates is given by

$$B = \frac{\mu_0 2I_d}{4\pi r} = \frac{\mu_0}{2\pi r} I_d$$

$$= \frac{\mu_0}{2\pi r} \times \varepsilon_0 \frac{d\phi_E}{dt} \quad \left[\because I_d = \frac{\varepsilon_0 d\phi_E}{dt}\right]$$

[redacted]

$$= \frac{\mu_0 \varepsilon_0}{2\pi r} \frac{d}{dt}(E\pi r^2) \qquad [\because \phi_E = E\pi r^2]$$

$$= \frac{\mu_0 \varepsilon_0}{2\pi r} \pi r^2 \frac{dE}{dt} = \frac{\mu_0 \varepsilon_0 r}{2} \cdot \frac{dE}{dt}$$

34. Yes, the ammeter will show the momentary deflection. This momentary deflection occurs due to the fact that the conducting current flows through connecting wires during the charging of capacitor. This leads to deposition of charge at two plates and hence, varying electric field of increasing nature is produced between the plates which in turn produces displacement current in space between two plates, which maintains the continuity with the conduction current.

$$I_c = I_d$$

i.e. Current inside the capacitor = Displacement current.

where, $I_d = \varepsilon_0 \frac{d\phi_E}{dt}$

35. Refer to text on page 329.

36. Radiant flux density,

$$S = \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B}) = c^2\varepsilon_0(\mathbf{E} \times \mathbf{B}) \quad \left[\because c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right]$$

Suppose electromagnetic waves are propagating along X-axis, the electric field vector of electromagnetic waves be along Y-axis and magnetic field vector be along Z-axis. Therefore,

$$\mathbf{E} = \mathbf{E}_0 \cos(kx - \omega t)$$

and $$\mathbf{B} = \mathbf{B}_0 \cos(kx - \omega t)$$

$$\therefore \frac{1}{\mu_0}(\mathbf{E} \times \mathbf{B}) = \frac{1}{\mu_0}(\mathbf{E}_0 \times \mathbf{B}_0)\cos^2(kx - \omega t)$$

$$S = c^2\varepsilon_0(E \times B) = c^2\varepsilon_0 |(E_0 \times B_0)| \cos^2(kx - \omega t)$$

Average value of the magnitude of radiant flux density over complete cycle,

$$S_{av} = c^2\varepsilon_0 |\mathbf{E}_0 \times \mathbf{B}_0| \frac{1}{T}\int_0^T \cos^2(kx - \omega t)\,dt$$

$$= c^2\varepsilon_0 E_0 B_0 \times \frac{1}{T} \times \frac{T}{2} \quad \left[\because \int_0^T \cos^2(kx - \omega t)\,dt = \frac{T}{2}\right]$$

$$= \frac{c^2}{2}\varepsilon_0 E_0\left(\frac{E_0}{c}\right) = \frac{c}{2}\varepsilon_0 E_0^2 \quad \left[\text{as, } c = \frac{E_0}{B_0}\right]$$

$$= \frac{c}{2} \times \frac{1}{c^2\mu_0} E_0^2 \quad \left[\because c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \text{ or } \varepsilon_0 = \frac{1}{c^2\mu_0}\right]$$

$$= \frac{E_0^2}{2\mu_0 c}$$

37. Refer to text on page 330.

38. (i) Refer to text on page 330.
(ii) Refer to text on pages 332 and 333.

39. Refer to text on pages 332 and 333.

40. (i) γ-rays. Its frequency range is from 3×10^{19} Hz to 3×10^{23} Hz.

(ii) The thin ozone layer on top of stratosphere absorbs most of the harmful ultraviolet rays coming from the sun towards the earth. They include UVA, UVB and UVC radiations which can destroy the life system on the earth. Hence, this layer is crucial for human survival.

(iii) Momentum transferred = Energy/Speed of light

$$= \frac{E}{c} = \frac{h\nu}{c} = 10^{-22}$$

Thus, the amount of the momentum transferred by the electromagnetic waves incident on the surface is very small.

41. (i) γ-rays. Its frequency range is from 3×10^{19} Hz to 3×10^{23} Hz.

(ii) Welders wear special glass goggles to protect the eyes from ultraviolet rays.

(iii) Infrared waves are produced by hot bodies, so they are called heat waves. They are used in physical therapy, weather forecasting, etc.

42. (i) Microwaves are suitable for RADAR systems that are used in aircraft navigation. These rays are produced by special vacuum tubes, namely klystrons, magnetrons and gunn diodes.

(ii) Infrared rays are used to treat muscular strain. These rays are produced by hot bodies and molecules.

(iii) X-rays are used as a diagnostic tool in medicine. These rays are produced when high energy electrons are stopped suddenly on a metal of high atomic number.

43. Refer to text on pages 332 and 333.

44. Refer to text on pages 332 and 333.

45. (i) Gamma rays has the highest frequency in the electromagnetic waves. These rays are of the nuclear origin and are produced in the disintegration of radioactive atomic nuclei and in the decay of certain subatomic particles. They are used in the treatment of cancer and tumours.

(ii) Ultraviolet rays lie near the high frequency end of visible part of EM spectrum. These rays are used to preserve food stuff. The harmful effect from exposure to ultraviolet (UV) radiation can be life threatening and include premature aging of the skin, suppression of the immune systems, damage to the eyes and skin cancer.

46. We know that, conduction current I_c is equal to the displacement current I_d.

i.e. $$I_c = I_d = 0.15 \text{ A} \quad \text{[given]}$$

It exists across the capacitor plates.

47. Refer to Example 2 on pages 329 and 330.

So, by applying a varying potential difference of 500V/s, we would produce a displacement current of desired value.

48. We know that, $I_d = \varepsilon_0 A \frac{dE}{dt}$

where, $\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$

[redacted]

Area, $A = 1\ cm^2 = 10^{-4}\ m^2$

$\because \quad \frac{dE}{dt} = 3 \times 10^6\ Vm^{-1}\ s^{-1}$

$\therefore \quad I_d = 8.85 \times 10^{-12} \times 10^{-4} \times 3 \times 10^6 = 2.7 \times 10^{-9}\ A$

49. Given, radius of plates, $r = 12\ cm = 12 \times 10^{-2}\ m$

Separation of two circular plates, $d = 5\ cm$

$$= 5 \times 10^{-2}\ m$$

Current, $I = 0.15\ A$

(i) Capacitance of parallel plate capacitor, $C = \frac{\varepsilon_0 A}{d}$

where, A is the area of plates.

$$\therefore \quad C = \frac{8.854 \times 10^{-12} \times 3.14\ (12 \times 10^{-2})^2}{5 \times 10^{-2}}$$

$$= 800.68 \times 10^{-14}\ F$$

$$= 8.01 \times 10^{-12} F = 8.01\ pF$$

Charge on the plates of the capacitor, $q = CV$

$$\Rightarrow \quad \frac{dq}{dt} = C \cdot \frac{dV}{dt}$$

$$\Rightarrow \quad I = C\ \frac{dV}{dt} \qquad \left[\because \frac{dq}{dt} = I\right]$$

$$\Rightarrow \quad \frac{dV}{dt} = \frac{I}{C} = \frac{0.15}{8.01 \times 10^{-12}}$$

$$= 18.7 \times 10^9\ V/s$$

Thus, the rate of change of potential is 18.7×10^9 V/s.

(ii) The displacement current is equal to the conduction current, i.e. $I_d = 0.15$ A.

(iii) Yes, Kirchhoff's first rule is valid because we take the current to be the sum of conduction current and the displacement current.

50. Given, radius of plates, $R = 6\ cm = 6 \times 10^{-2}\ m$

Capacitance of capacitor,

$$C = 100\ pF = 100 \times 10^{-12}\ F = 10^{-10}\ F$$

Voltage of capacitor, $V = 230$ V

Frequency of capacitor, $\omega = 300$ rad/s

(i) The rms value of current, $I_{rms} = \frac{V_{rms}}{X_C}$

$$\therefore \quad X_C = \frac{1}{\omega C} = \frac{1}{300 \times 10^{-10}} = \frac{10^{10}}{300}\ \Omega$$

$$\therefore \quad I_{rms} = \frac{230 \times 300}{10^{10}} = 3 \times 23 \times 1000 \times 10^{-10}$$

$$= 69 \times 10^{-7}$$

$$= 6.9 \times 10^{-6} A$$

$$= 6.9\ \mu A$$

(ii) Yes, the conduction current is equal to displacement current.

(iii) Given, the distance of point from the axis between the plates, $\quad r = 3\ cm = 3 \times 10^{-2}\ m$

Radius of plates, $R = 6\ cm = 6 \times 10^{-2}\ m$

The magnetic field at a point between the plates,

$$B = \frac{\mu_0}{2\pi R^2} \cdot r \cdot I_d$$

$$\Rightarrow \quad B = \frac{\mu_0 r}{2\pi R^2} I \qquad [\because I_d = I]$$

If $I = I_0$ is maximum value of current, then

$$I = \sqrt{2} I_{rms}$$

$$\therefore \quad B = \frac{\mu_0 r}{2\pi R^2} \sqrt{2} I_{rms}$$

$$= \frac{4\pi \times 10^{-7} \times 0.03 \times \sqrt{2} \times 6.9 \times 10^{-6}}{2\pi \times 0.06 \times 0.06}$$

$$= 1.63 \times 10^{-11}\ T$$

51. E and B vectors must be in x and y-directions.

As we know, $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{30 \times 10^6} = 10$ m

52. For 7.5 MHz band,

Wavelength, $\lambda_1 = \frac{c}{\nu} = \frac{3 \times 10^8}{7.5 \times 10^6} = 40$ m

For 12 MHz band,

Wavelength, $\lambda_2 = \frac{c}{\nu} = \frac{3 \times 10^8}{12 \times 10^6} = 25$ m

So, wavelength range is from 25 m to 40 m.

53. Magnetic field, $B = B_0 \sin \omega t$

Given equation,

$$B = 12 \times 10^{-8} \sin\ (1.20 \times 10^7 z - 3.60 \times 10^{15} t) T$$

On comparing this equation with standard equation, we get

$$B_0 = 12 \times 10^{-8}\ T$$

The average intensity of the beam,

$$I_{av} = \frac{B_0^2}{2\mu_0} \cdot c$$

$$= \frac{1}{2} \times \frac{(12 \times 10^{-8})^2 \times 3 \times 10^8}{4\pi \times 10^{-7}}$$

$$\Rightarrow \quad I_{av} = 1.71\ W/m^2$$

54. (i) $\therefore$ Intensity, $I = \frac{\text{Power of visible light}}{\text{Area}}$

$$= \frac{100 \times (5/100)}{4\pi (1)^2} = 0.4\ W/m^2$$

(ii) $I = \frac{100 \times \left(\frac{5}{100}\right)}{4\pi (10)^2} = 4 \times 10^{-3}\ W/m^2$

55. Given, amplitude of the magnetic field part of harmonic electromagnetic wave,

$$B_0 = 510\ nT = 510 \times 10^{-9}\ T$$

Speed of light in vacuum, $c = \frac{E_0}{B_0}$

[redacted]

where, E_0 is the amplitude of electric field part of the wave.

$$\Rightarrow \quad 3\times 10^8 = \frac{E_0}{510\times 10^{-9}}$$

$$\Rightarrow \quad E_0 = 153\text{ N/C}$$

Thus, the amplitude of the electric field part of wave is 153 N/C.

56. Given, frequency of oscillation, $\nu = 2\times 10^{10}$ Hz

Speed of wave, $c = 3\times 10^8$ m/s

Electric field amplitude, $E_0 = 48$ V/m

(i) Wavelength of wave,

$$\lambda = \frac{c}{\nu} = \frac{3\times 10^8}{2\times 10^{10}} = 1.5\times 10^{-2}\text{ m}$$

(ii) The amplitude of the oscillating magnetic field

$$B_0 = \frac{E_0}{c} = \frac{48}{3\times 10^8} = 1.6\times 10^{-7}\text{ T}$$

(iii) The average energy density of electric field,

$$u_e = \frac{1}{4}\varepsilon_0 E_0^2 \quad \text{...(i)}$$

As, $E_0 = cB_0$

Putting the value of E_0 in Eq. (i), we get

$$u_e = \frac{1}{4}\varepsilon_0 \cdot c^2 B_0^2 \quad \text{...(ii)}$$

Speed of electromagnetic waves, $c = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$ (1)

Putting the value of c in Eq. (ii), we get

$$u_e = \frac{1}{4}\varepsilon_0 B_0^2 \cdot \frac{1}{\mu_0\varepsilon_0} = \frac{1}{4}\cdot\frac{B_0^2}{\mu_0} = u_b$$

(u_b is average energy density of magnetic field)

Thus, the average energy density of the E field equals the average energy density of B field

57. Given, amplitude of an electromagnetic wave, $E_0 = 120$ N/C

Frequency of wave, $\nu = 50\text{ MHz} = 50\times 10^6$ Hz

(i) Speed of light in vacuum,

$$c = \frac{E_0}{B_0}$$

$$B_0 = \frac{E_0}{c} = \frac{120}{3\times 10^8} = 40\times 10^{-8}$$

$$= 400\times 10^{-9}\text{ T} = 400\text{ nT}$$

Angular frequency of electromagnetic wave,

$$\omega = 2\pi\nu = 2\times 3.14\times 50\times 10^6$$

$$\omega = 3.14\times 10^8\text{ rad/s}$$

Wave number of electromagnetic wave,

$$k = \frac{\omega}{c} = \frac{3.14\times 10^8}{3\times 10^8} = 1.05\text{ rad/m}$$

Wavelength of electromagnetic wave,

$$\lambda = \frac{c}{\nu} = \frac{3\times 10^8}{50\times 10^6} = 6.00\text{ m}$$

(ii) Expression of electric field, $E = E_0 \sin(kx - \omega t)$

$$E = 120\sin(1.05x - 3.14\times 10^8 t)$$

Expression of magnetic field B,

$$B = B_0\sin(kx - \omega t)$$

$$B = 4\times 10^{-7}\sin(1.05x - 3.14\times 10^8 t)$$

58. (i) The given equation signifies that the electromagnetic wave is moving along Y-axis and also in negative direction, so it moves in $-\hat{\mathbf{j}}$-direction.

(ii) The electric part of electromagnetic wave in vacuum,

$$E = [3.1\cos\{1.8y + (5.4\times 10^6 t)\}]\,\hat{\mathbf{i}}$$

Comparing with standard equation,

$E = E_0\cos(ky + \omega t)$, we get

Angular frequency, $\omega = 5.4\times 10^6$ rad/s

Wave number, $k = 1.8$ rad/m

The amplitude of the electric field part of the wave,

$$E_0 = 3.1\text{ N/C}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{1.8} = 3.491\text{ m}$$

$$\Rightarrow \quad \lambda = 3.5\text{ m}$$

(iii) Angular frequency, $\omega = 2\pi\nu$

$$\nu = \frac{\omega}{2\pi} = \frac{5.4\times 10^6\times 7}{2\times 22} = 0.86\times 10^6\text{ Hz}$$

(iv) As, $c = \frac{E_0}{B_0}$

Amplitude of magnetic field,

$$B_0 = \frac{E_0}{c} = \frac{3.1}{3\times 10^8}$$

$$= 1.03\times 10^{-8}\text{ T}$$

(v) Expression for the magnetic field part of wave,

$$B = B_0\cos(ky + \omega t)\hat{k}$$

$$B = 1.03\times 10^{-8}\cos(1.8y + 5.4\times 10^6 t)\hat{k}$$

59. Given, $\lambda_m T = 0.29$ cm-K

$$\Rightarrow \quad \lambda_m = \frac{0.29}{T}\text{ cm}$$

Let us take, $\lambda_m = 10^{-6}\text{ m} = 10^{-4}$ cm

Required absolute temperature,

$$T = \frac{0.29}{10^{-4}} = 2900\text{ K}$$

Let us take, $\lambda_m = 5\times 10^{-5}$ cm for visible region.

Required absolute temperature,

$$T = \frac{0.29}{5\times 10^{-5}} = 5800\text{ K} \approx 6000\text{ K}$$

Hence, we can find the temperature for other parts of the electromagnetic spectrum in the same way. So, these numbers tell us about the temperature ranges for which atomic vibrations can produce these parts of electromagnetic waves

60. (i) This wavelength (21 cm) corresponds to the radio waves.

(ii) This frequency (1057 MHz) also corresponds to the radio waves (short wavelength).

(iii) As, $T = 2.7$ K

Using the formula, $\lambda_m T = b = 0.29$ cm-K

$$\lambda_m = \frac{0.29}{2.7}\text{ cm} = 0.11\text{ cm}$$

This wavelength corresponds to the microwaves region of the electromagnetic spectrum

(iv) This wavelength lies in the visible region of the electromagnetic spectrum

(v) Energy, $E = 14.4$ keV $= 14.4 \times 10^3 \times 1.6 \times 10^{-19}$ J

Frequency of wave,

$$\nu = \frac{E}{h} = \frac{14.4 \times 1.6 \times 10^{-16}}{6.6 \times 10^{-34}}$$

$$= 3.5 \times 10^{18}\text{ Hz}$$

This frequency lies in the X-ray region of the electromagnetic spectrum.

61. Given, energy of photon, $E = h\nu$

For γ-rays

Frequency of γ-rays, $\nu = 3 \times 10^{20}$ Hz

Energy of γ-rays, $E = h\nu = 6.6 \times 10^{-34} \times 3 \times 10^{20}$

$$= 19.8 \times 10^{-14}\text{ J}$$

$$\Rightarrow \quad E = \frac{19.8 \times 10^{-14}}{1.6 \times 10^{-19}} = 1.24 \times 10^{6}\text{ eV}$$

The source of γ-rays is nuclear origin.

For X-rays

Frequency of X-rays, $\nu = 3 \times 10^{18}$ Hz

Energy of X-rays, $E = h\nu = 6.6 \times 10^{-34} \times 3 \times 10^{18}$

$$= 19.8 \times 10^{-16}\text{ J}$$

$$\Rightarrow \quad E = \frac{19.8 \times 10^{-16}}{1.6 \times 10^{-19}}$$

$$= 1.24 \times 10^{4}\text{ eV}$$

The retardation of high energy electron produces X-rays.

Similarly, we can find for ultraviolet rays, visible rays, infrared rays, microwaves and radio waves.

They originate by oscillating current.

Types of radiation	Photon energy
γ-rays	1.24×10^{6} eV
X-rays	1.24×10^{4} eV
Ultraviolet rays	4.12 eV
Visible rays	2.475 eV
Infrared waves	4.125×10^{-2} eV
Microwaves	4.125×10^{-5} eV
Radio waves	1.24×10^{-6} eV

SUMMARY

- **Displacement Current** When a capacitor is charged, then electric field is produced due to flow of current inside it. This current is called displacement current. It is given by $I_d = \frac{\varepsilon_0 \, d\phi_E}{dt}$
- **Basic Idea of Displacement Current** Ampere's circuital law for conduction current during a charging of capacitor was found to be inconsistent. Therefore, Maxwell modified Ampere's circuital by introducing displacement current.
- **Maxwell's Equations** Maxwell's equations are the basic laws of electricity and magnetism.
- **Gauss's Law in Electrostatics** The total electric flux through any closed surface is always equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed by the surface, i.e. $\oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$.
- **Gauss's Law in Magnetostatics** The net magnetic flux through any closed surface is always zero.
- **Faraday's Law of EMI** The induced emf produced in a circuit is numerically equal to the rate of change of magnetic flux through it.

  i.e. $\oint \mathbf{E} \cdot d\mathbf{l} = \frac{-d\phi_B}{dt}$
- **Ampere's-Maxwell Circuital Law** The line integral of the magnetic field along a closed path is equal to μ_0 times the total current threading the surface bounded by that closed path, i.e.

  $$\oint \mathbf{B} \cdot d\mathbf{l} = \mu(I_c + I_d)$$
- **Electromagnetic Waves** These waves produced due to change in electric field and magnetic field sinusoidally and propagates through space such that the two fields are perpendicular to each other and also to the direction of wave propagation.
- **Source of EM Waves** Accelerating charges produces EM waves.
- **Transverse Nature of EM Waves** Since, the electric and magnetic fields in an electromagnetic wave are perpendicular to each other and also to the direction of wave propagation. Hence, electromagnetic waves are transverse in nature.
- **Electromagnetic Spectrum** The orderly arrangement of EM wave in increasing or decreasing order of wavelength or frequency is called electromagnetic spectrum. The range varies from 10^{-12} m to 10^4 m, i.e. from γ-rays to radiowaves.

[redacted]

CHAPTER PRACTICE (UNSOLVED)

OBJECTIVE Type Questions

1. Out of the following options which one can be used to produce a propagating electromagnetic wave?
   (a) A stationary charge
   (b) A charge less particle
   (c) An accelerating charge
   (d) A charge moving at constant velocity

2. If E and B represent electric and magnetic field vectors of the electromagnetic wave, the direction of propagation of electromagnetic wave is along **NCERT Exemplar**
   (a) E (b) B
   (c) B × E (d) E × B

3. The range of wavelength of the visible light is
   (a) 10 Å to 100 Å (b) 4000 Å to 8000 Å
   (c) 8000 Å to 10000 Å (d) 10000 Å to 15000 Å

4. If ε_0 and μ_0 are the electric permittivity and magnetic permeability of free space and ε and μ are the corresponding quantities in the medium, the index of refraction of the medium in terms of above parameter is
   (a) $\dfrac{\varepsilon\mu}{\varepsilon_0\mu_0}$ (b) $\left(\dfrac{\varepsilon\mu}{\varepsilon_0\mu_0}\right)^{1/2}$
   (c) $\left(\dfrac{\varepsilon_0\mu_0}{\varepsilon\mu}\right)$ (d) $\left(\dfrac{\varepsilon_0\mu_0}{\varepsilon\mu}\right)^{1/2}$

5. The ratio of contributions made by the electric field and magnetic field components to the intensity of an EM wave is **NCERT Exemplar**
   (a) $c:1$ (b) $c^2:1$
   (c) $1:1$ (d) $\sqrt{c}:1$

6. Frequency of wave is 6×10^{10} Hz. The wave is
   (a) radiowave (b) microwave
   (c) X-ray (d) None of these

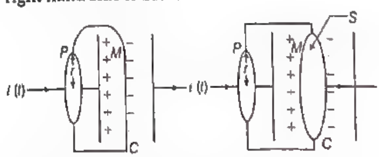
7. In the following waves, which is not electromagnetic wave?
   (a) α-rays (b) γ-rays
   (c) Infrared rays (d) X-rays

8. The largest wavelength of electromagnetic wave is
   (a) X-rays (b) radio waves
   (c) ultraviolet rays (d) infrared rays

Directions (Q. Nos. 9-12) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

9. **Assertion** While applying Ampere's circuital law to given surfaces with same perimeter, the left hand side $\oint B\cdot dl=\mu_0 i(t)$ has not changed but the right hand side is zero.



Reason No current passes through the surface.

10. **Assertion** We needed to do was to set up an AC circuit in which the current oscillate at the frequency of visible light *i.e.*, yellow.
    Reason The above experiment demonstrates electromagnetic wave.

[redacted]

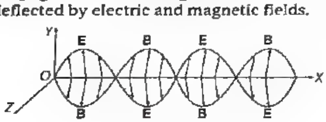
11. **Assertion** An oscillating charge produces an electric field in space, which produces an oscillating magnetic field, which in turn, is a source of electric field and so on.
Reason The oscillating electric and magnetic field thus regenerate each other, so to speak, as the wave propagates through the space.

12. **Assertion** When the sun shines on our hand, we feel the energy being absorbed from the electromagnetic waves (our hands get warm).
Reason Electromagnetic waves also transfer momentum to our hand but because c is very large, the amount of momentum transferred is extremely small and we do not feel the pressure.

Directions (Q.No. 13) *This question is case study based question. Attempt any 4 sub-parts from this question. Each question carries 1 mark.*

13. **EM Wave**

Electromagnetic waves are transverse in nature. *i.e.*, electric and magnetic fields are perpendicular to each other and to the direction of wave propagation. Electromagnetic waves are not deflected by electric and magnetic fields.



Direction of propagation ⟶

The magnetic field in a plane electromagnetic wave is given by
$B_y = 2\times10^{-7}\sin(0.5\times10^3x + 1.5\times10^{11}t)$T.

(i) What is the angular frequency of wave?
(a) 0.5×10^3 rad s^{-1}
(b) 1.5×10^{11} rad s^{-1}
(c) 3×10^8 rad s^{-1}
(d) 2×10^{-7} rad s^{-1}

(ii) What is the wavelength of the wave?
(a) 12.6 cm (b) 1.26 cm
(c) 1.26 m (d) 6.12 m

(iii) What is the frequency of the wave?
(a) 2.39 GHz (b) 23.9 MHz
(c) 23.9 GHz (d) 20.3 MHz

(iv) The maximum value of electric field is
(a) 6×10^2 Vm^{-1}
(b) 6×10^3 Vm^{-1}
(c) 6×10^1 Vm^{-1}
(d) 6 Vm^{-1}

(v) Write an expression for the electric field?
(a) $E_y = 60\sin(0.5\times10^3x + 1.5\times10^{11}t)$ Vm^{-1}
(b) $E_x = 60\sin(0.5\times10^3x + 1.5\times10^{11}t)$ Vm^{-1}
(c) $E_z = 60\sin(0.5\times10^3x + 1.5\times10^{11}t)$ Vm^{-1}
(d) $E_y = 60\cos(0.5\times10^3x + 1.5\times10^{11}t)$ Vm^{-1}

VERY SHORT ANSWER Type Questions

14. Depict the fields diagram of an electromagnetic wave propagating along positive X-axis with its electric field along Y-axis.

15. A capacitor is attached to a variable frequency of an AC source. What will happen to the displacement current with the increase in frequency?

16. To which part of electromagnetic spectrum do the waves emitted by radioactive nuclei belong? What is its frequency range?

17. Which part of the electromagnetic spectrum is used in radar? Give its frequency range.

Or How are electromagnetic waves produced by accelerating charges?

18. Write two uses of infrared rays.

19. Give the ratio of velocity of the two light waves of wavelengths 4000Å and 8000Å travelling in vacuum.

20. Mention one use of part of electromagnetic spectrum to which a wavelength of 21 cm (emitted by hydrogen in interstellar space) belongs.

21. What is common between difference types of electromagnetic radiations?

22. A laser beam has an intensity of 4×10^{14} W/m^2. What will the amplitude of electric field in the beam?

SHORT ANSWER Type Questions

23. When a plane electromagnetic wave travels in vacuum along y-direction. Write the
(i) ratio of the magnitudes and
(ii) the direction of its electric and magnetic field vectors.

24. How are electromagnetic waves produced by oscillating charges? What is the source of the energy associated with the EM waves?

[redacted]

25. Gamma rays and radiowaves travel with the same velocity in free space. Distinguish between them in terms of their origin and the main application.

26. A radio can tune into any station from 5.5 MHz to 16 MHz band. What is the corresponding wavelength band?

27. Green light of mercury has a wavelength 5×10^{-5} cm.
   (i) What is the frequency in MHz and period in second in vacuum?
   (ii) What is the wavelength in glass, if refractive index of glass is 1.5?

28. Name the electromagnetic radiation to which waves of wavelength in the range of 10^{-2}m belong. Give one use of this part of electromagnetic spectrum.

29. Find the wavelength of electromagnetic wave of frequency 5×10^{10}Hz in free space. Give its two applications.

LONG ANSWER Type I Questions

30. (a) Identify the part of the electromagnetic spectrum used in (i) radar and (ii) eye surgery. Write their frequency range.
   (b) Prove that the average energy density of the oscillating electric field is equal to that of the oscillating magnetic field.

31. (a) We feel the warmth of the sunlight but not the pressure on our hands. Explain.
   (b) Which out of wavelength, frequency and speed of an electro-magnetic wave does not change on passing from one medium to another?
   (c) A thin ozone layer in the upper atmosphere is crucial for human survival on earth, why?

32. (a) How are electromagnetic waves produced? Depict an electromagnetic wave propagation in z-direction with its magnetic field B oscillating along x-direction.
   (b) Write two characteristics of electromagnetic waves.

33. Name the constituent radiation of electromagnetic spectrum which
   (i) is used in satellite communication.
   (ii) is used for studying crystal structure.
   (iii) is emitted during decay of radioactive nuclei.
   Write two more uses of each.

34. How are X-rays different from γ-rays? Give a detailed description.

ANSWERS

1. (c) 2. (d) 3. (b) 4. (b) 5. (c)
6. (b) 7. (a) 8. (b)

9. (a) On applying Ampere's circuital law to such surfaces with the same perimeter, we find that the left hand side of equation $\oint B.dl = \mu_0 i(t)$ has not changed but the right hand side is zero and not $\mu_0 i$. Since, no current passes through the surface.

10. (c) We needed to set up an AC circuit in which the current oscillate at the frequency of visible light, *i.e.*, yellow. The frequency of yellow light is about 6×10^{14} Hz, while the frequency that we get even with modern electronic circuits is hardly about 10^{11} Hz. This is why the experimental demonstration of electromagnetic wave had to come in the low frequency region (the radio wave region), as in the Hertz's experiment (1887)

11. (a) An oscillating charge produces an electric field in space, which produces an oscillating magnetic field, which in turn, is a source of electric field.

12. (a) When the sun shines on your hand, you feel the energy being absorbed from the electromagnetic waves (your hands get warm). Electromagnetic waves also transfer momentum to your hand but because c is very large, the amount of momentum transferred is extremely small and you do not feel the pressure.

13. (i) (b) Comparing with given equation, we get

$$\omega = 1.5\times10^{11} \text{ rad s}^{-1}$$

(ii) (b) Comparing the given equation with magnetic field in a plane

i.e., $$B_y = B_0 \sin\left[2\pi\left(\frac{x}{\lambda}+\frac{t}{T}\right)\right]$$

we get $$\lambda = \frac{2\pi}{0.5\times10^3} \text{ m} = 1.26 \text{ cm}$$

(iii) (c) As we know, frequency *i.e.*, $\nu = \frac{1}{\text{Time taken}}$

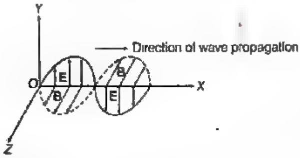
$$\nu = \frac{1}{T} = \frac{\omega}{2\pi} = (1.5\times10^{11}) / 2\pi = 23.9 \text{ GHz}$$

(iv) (c) According to Maxwell equation, electric field *i.e.*, $E_0 = B_0 c = 2\times10^{-7}\times3\times10^8 \text{ ms}^{-1} = 6\times10^1 \text{ Vm}^{-1}$

(v) (c) The electric field component is perpendicular to the direction of propagation and the direction of magnetic field. Therefore, the electric field component along the Z-axis is obtained as $E_z = 60\sin(0.5\times10^3 x + 1.5\times10^{11} t) \text{ Vm}^{-1}$

[redacted]

14. An electromagnetic wave propagating along positive X-axis with its electric field Y-axis, will have its magnetic field along Z-axis, as shown below



15. When frequency of AC source increases, then displacement current also increases.
16. Gamma rays, its frequency range is 3×10^{18} Hz to 5×10^{22} Hz.
17. Microwaves are used in radar systems of aircraft navigation. Its frequency range is 1 GHz to 300 GHz.

Or

Refer to text on page 330.
(Source of Electromagnetic Waves)
18. Two uses of infrared rays are
    (i) in weather forecasting.
    (ii) in production of dehydrated fruits.
19. As, light waves travel in free space or vacuum with the speed of light ($3 \times 10^8\ ms^{-1}$), irrespective of their wavelengths. So, the ratio of velocity of given light waves is 1 : 1.
20. Given, wavelength, $\lambda = 21\ cm = 0.21\ m$

The range of the wavelength of microwaves is approximately 30 cm to 1 mm. Thus, the given wavelength emitted by hydrogen interstellar space belongs to microwaves These are used in RADAR systems for aircraft navigation.
21. Speed, in vacuum all types of electromagnetic wave travels with same speed, i.e. 3×10^8 m/s.
22. We know that, $E_0 = \sqrt{\frac{2I}{\varepsilon_0 c}}$

$$= \sqrt{\frac{2 \times 4 \times 10^{14}}{8.85 \times 10^{-12} \times 3 \times 10^8}}$$

$$= 5.489 \times 10^8\ N/C$$
23. Refer to text on pages 330 and 331.
24. Refer to text on page 330.
(Source of Electromagnetic Waves)
25. Refer to text on pages 332 and 333.
26. Here, $\nu_1 = 5.5\ MHz = 5.5 \times 10^6\ Hz$

$$\nu_2 = 16\ MHz = 16 \times 10^6\ Hz$$

$$\therefore \quad \lambda_1 = \frac{c}{\nu_1} = \frac{3 \times 10^8}{5.5 \times 10^6}$$

$$= 0.545 \times 10^2\ m = 54.5\ m$$

$$\Rightarrow \quad \lambda_2 = \frac{c}{\nu_2} = \frac{3 \times 10^8}{16 \times 10^6} = 0.1875 \times 10^2$$

$$= 18.75\ cm$$

Hence, corresponding wavelengths of above frequencies are 54.5 m and 18.75 m.
27. (i) Frequency of the wave, $\nu = \frac{c}{\lambda} = \frac{3 \times 10^8\ m/s}{5 \times 10^{-5}\ cm}$

$$= \frac{3 \times 10^8\ m/s}{5 \times 10^{-7}\ m/s} = 6 \times 10^{14}\ Hz$$

Time period, $T = \frac{1}{\nu} = \frac{1}{6 \times 10^{14}\ Hz} = 0.16 \times 10^{-14}\ s$

(ii) Refractive index, $\mu = \frac{c}{v}$

$$\Rightarrow \quad v = \frac{c}{\mu} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8\ m/s$$

Also, $\nu = \frac{v}{\lambda}$

$$\rightarrow \quad \lambda = \frac{v}{\nu} = \frac{2 \times 10^8}{6 \times 10^{14}} = 0.33 \times 10^{-6}\ m$$

$$= 33 \times 10^{-3}\ m$$
28. Refer to text on page 332.
29. Refer to text on pages 332 and 333.
30. (a) (i) Microwave - 1 GHz to 300 GHz.
(ii) Ultraviolet (by LASIK eye surgery) -10^{14} Hz to 10^{16} Hz.
(b) Refer to text on page 331 [Important Characteristics of Electromagnetic Waves (vii)]
31. Refer to text on pages 332 and 333.
32. Refer to text on page 331.
33. Refer to text on pages 332 and 333.
34. Refer to text on page 333.

[redacted]

09

Light is a non-mechanical form of energy (i.e. it requires no medium for propagation), due to which we have sensation of vision. Light always travels in a straight line and its speed is very high. In vacuum, light travels with a speed of 3×10^8 m/s.

RAY OPTICS AND OPTICAL INSTRUMENTS

|TOPIC 1|
Ray Optics

A light wave can be considered to travel from one point to another, along a straight line joining them. The path is called a ray of light and a bundle of such rays constitutes a beam of light. The branch of study of light is called Optics.

Broadly optics is divided into three groups

(i) Geometrical optics (Ray optics)

(ii) Wave optics

(iii) Quantum optics

CHAPTER CHECKLIST

- Ray Optics
- Refraction
- Refraction at Spherical Surfaces and Lenses
- Prism and Optical Instruments

GEOMETRICAL OPTICS (RAY OPTICS)

In this, light is considered as a ray which travels in straight line. Geometrical optics states that for each and every object, there is an image. It works on following assumptions:

(i) Rectilinear propagation of light, i.e. light ray travels in straight line.

(ii) Laws of reflection.

(iii) Laws of refraction.

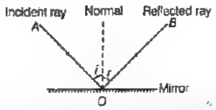
(iv) Physical independence of light rays, i.e. two light rays are totally independent of each other.

[redacted]

Reflection of Light

Reflection is the phenomenon of change in the path of light without any change in the medium.

The returning back of light in the same medium from which it has come after striking a surface is called reflection of light.



The incident ray reflected ray and the normal to the reflecting surface lie in the same plane

Laws of Reflection

The laws of reflection are as given below

(i) The incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence, all lie in the same plane.

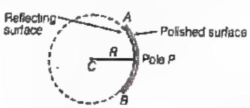
(ii) The angle of reflection (r) is equal to the angle of incidence (i), i.e. $i = r$. For normal incidence, $\angle i = 0°$, therefore $\angle r = 0°$. Hence, a ray of light falling normally on a mirror, retraces its path on reflection.

Spherical Mirrors

Spherical mirror is a mirror whose reflecting surface is a part of a hollow sphere. Spherical mirrors are of two types

Concave Spherical Mirror

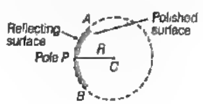
A spherical mirror whose reflecting surface is towards the centre of the sphere is called concave spherical mirror.



Concave mirror

Convex Spherical Mirror

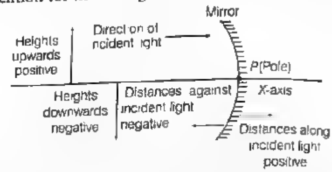
A spherical mirror whose reflecting surface is away from the centre of the sphere is called convex spherical mirror.



Convex mirror

Sign Convention

To derive the relevant formulae for reflection by spherical mirrors and refraction by spherical lenses (which we will study later in this chapter), we must first adopt a sign convention for measuring distances.



The cartesian sign convention

According to the cartesian sign convention,

(i) All the distances are measured from pole (P) of the mirror or the optical centre(O) of the lens.

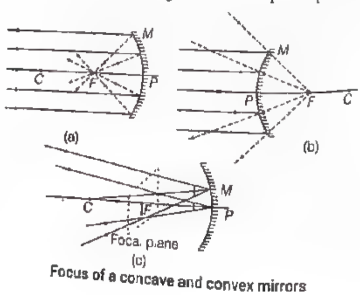
(ii) The principal axis of the mirror or lens is taken as X-axis and the pole or optical centre as origin.

(iii) Distances measured in the direction of the incident light are taken as positive and opposite to the direction of incident light as negative.

(iv) The heights measured upwards with respect to X-axis and normal to the principal axis of the mirror or lens are taken as positive and the heights measured downwards are taken as negative.

Focal Length of Spherical Mirrors

When a parallel beam of light is incident on a concave or convex mirror, the reflected rays converge or appear to diverge from a point F on principal axis called principal focus of the mirror. We assume that the rays are paraxial, i.e. they are incident at points close to the pole P of the mirror and make small angles with the principal axis.



Focus of a concave and convex mirrors

[redacted]

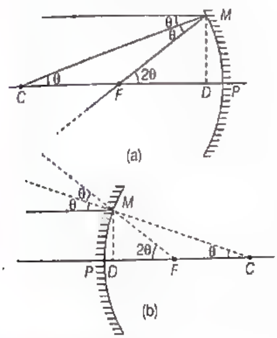
If the paraxial beam of light were incident, making some angle with principal axis, the reflected rays would converge or appear to diverge from a point in a plane through F normal to the principal axis. This is called the focal plane of the mirror.

The relation between Focal Length (f) and radius of Curvature (R)

$$\boxed{f = \frac{R}{2}}$$

Proof

Consider a ray parallel to the principal axis striking the mirror at point M. Then CM will be perpendicular to the mirror at point M. Let θ be the angle of incidence and MD be perpendicular to the principal axis.



Geometry of reflection of an incident ray on
(a) concave spherical mirror and
(b) convex spherical mirror

Then, $\angle MCP = \theta$

and $\angle MFP = 2\theta$

Now, $\tan\theta = \dfrac{MD}{CD}$

and $\tan 2\theta = \dfrac{MD}{FD}$ ...(i)

For small θ (condition true for paraxial rays),
$\tan\theta \approx \theta$ and $\tan 2\theta \approx 2\theta$

Therefore, from Eq.(i), we get

$$\frac{MD}{FD} = 2\frac{MD}{CD} \quad \text{or} \quad FD = \frac{CD}{2} \quad \text{...(ii)}$$

Again, for small θ, we can observe that the point D is very close to the point P. Therefore, $FD = f$ and $CD = R$.
From Eq.(ii), we have

$$\boxed{f = \frac{R}{2}}$$

Real Image If rays emanating from a point actually converge at another point after reflection/refraction, that point is called the real image of the first point.

Virtual Image If rays emanating from a point do not actually meet but appear to diverge from another point, that point is called the virtual image of the first point.

Mirror Formula

Mirror formula (or equation) is a relation between focal length of the mirror and distances of object and image from the mirror.

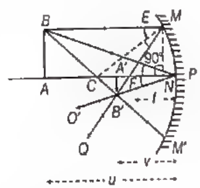
In principle, we can take any two rays originating from a point on an object, trace their paths, find their point of intersection and thus, obtain the image of the point due to reflection at a spherical mirror. However in practice, it is convenient to choose any two of the following rays.

(i) The ray from the point, which is parallel to the principal axis after reflection goes through the focus of the mirror.

(ii) The ray passing through the centre of curvature of a concave mirror or appearing to pass through it for a convex mirror simply retraces the path.

(iii) The ray passing through (or directed towards) the focus of the concave mirror or appearing to pass through (or directed towards) the focus of a convex mirror after reflection is parallel to the principal axis.

(iv) The ray incident at any angle at the pole is reflected following the laws of reflection.



In the above figure, the ray diagram is considering three rays for image formation by a concave mirror. In the figure, triangles $A'B'F$ and NEF are similar.

Then, $\dfrac{A'B'}{NE} = \dfrac{A'F}{NF}$

As, the aperture of the concave mirror is small and the points N and P lie very close to each other, then

$NF \approx PF$

and $NE = AB$

$\Rightarrow \dfrac{A'B'}{AB} = \dfrac{A'F}{PF}$

[redacted]

Since, all the distances are measured from the pole of the concave mirror, we have

$$A'F = PA' - PF$$

$$\therefore \quad \frac{A'B'}{AB} = \frac{PA' - PF}{PF} \quad \ldots\text{(i)}$$

Also, triangles ABP and $A'B'P$ are similar, then

$$\frac{A'B'}{AB} = \frac{PA'}{PA} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{PA' - PF}{PF} = \frac{PA'}{PA} \quad \ldots\text{(iii)}$$

Applying new Cartesian sign convention, we have

$$PA = -u$$

[$\because$ distance of object is measured against incident ray]

$$PA' = -v$$

[$\because$ distance of image is measured against incident ray]

$$PF = -f$$

[$\because$ focal length of concave mirror is measured against incident ray]

Substituting these values in Eq. (iii), we have

$$\frac{-v-(-f)}{-f} = \frac{-v}{-u} \Rightarrow \frac{v-f}{f} = \frac{v}{u} \Rightarrow \frac{v}{f} - 1 = \frac{v}{u}$$

Dividing both sides by v, we get

$$\therefore \quad \boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}$$

The above relation is called **mirror formula.**

Relation between u, v and R

$\because$ Focal length of the mirror, $f = \frac{R}{2}$

$$\therefore \quad \frac{1}{u} + \frac{1}{v} = \frac{1}{R/2} \Rightarrow \boxed{\frac{1}{u} + \frac{1}{v} = \frac{2}{R}}$$

Important Points

(i) An object is always placed in front of a spherical mirror, so the distance of the object (u) is always negative.

(ii) In a spherical mirror, real image is formed in front of the mirror. Therefore, its distance (v) is taken as negative. However, virtual image is formed at the back of the mirror. So, its distance (v) is taken as positive, as per the new Cartesian sign convention.

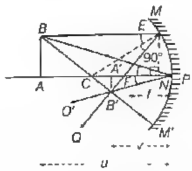
(iii) Similarly, focal length of a concave mirror is taken as negative, while that of convex mirror is taken as positive.

LINEAR MAGNIFICATION

The ratio of the height of the image (h') formed by a spherical mirror to the height of the object (h) is called the linear magnification produced by the spherical mirror.

It is denoted by m, i.e.

$$m = \frac{h'}{h}$$



In the above figure, triangles ABP and $A'B'P$ are similar.

$$\frac{A'B'}{AB} = \frac{PA'}{PA}$$

Applying new Cartesian sign conventions, we have

$A'B' = -h'$ [$\because$ height of image measured downward]

$AB = +h$ [$\because$ height of object measured upward]

$PA = -u$ [$\because$ object distance measured against incident ray]

$PA' = -v$ [$\because$ image distance measured against incident ray]

The above equation becomes

$$\frac{-h'}{h} = \frac{-v}{-u}$$

or

$$\frac{h'}{h} = \frac{-v}{u} \quad \ldots\text{(iv)}$$

$$\therefore \quad \boxed{\text{Linear magnification, } m = \frac{h'}{h} = -\frac{v}{u}}$$

The expression for magnification is same for both the concave and convex mirrors.

(i) When $m > 1$, image formed is enlarged.

(ii) When $m < 1$, image formed is diminished.

(iii) When m is positive, image must be erect, i.e. virtual.

(iv) When m is negative, image must be inverted, i.e. real.

(v) In case of concave mirror, m can be either positive or negative but in case of convex mirror, m is positive only.

[redacted]

EXAMPLE |1| A candle flame is held 3 cm away from a concave mirror of radius of curvature 24 cm. Where is the image formed? What is the nature of the image?

Sol. Given, object distance, $u = -3$ cm

Radius of curvature, $R = -24$ cm

$$\therefore \quad f = \frac{R}{2} = \frac{-24}{2} = -12 \text{ cm}$$

According to mirror formula,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{(-12)} - \frac{1}{-3}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{-12} + \frac{1}{3} = \frac{-1+4}{12}$$

$$\Rightarrow \quad v = 4 \text{ cm}$$

$$\because \quad \text{Magnification, } m = -\frac{v}{u} = \frac{-4}{-3} = +1.33$$

i.e. The image formed is virtual, erect and magnified.

EXAMPLE |2| An object is placed in front of a convex mirror of focal length 30 cm. If the image is a quarter of the size of the object, find the position of the image.

Sol. Given, focal length, $f = +30$ cm

Magnification, $m = \frac{1}{4}, v = ?$

From mirror's formula,

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \qquad \left[\because m = -\frac{v}{u} \Rightarrow u = -\frac{v}{m}\right]$$

$$\Rightarrow \quad \frac{1}{f} = -\frac{m}{v} + \frac{1}{v}$$

$$\Rightarrow \quad m = \frac{f-v}{f} \Rightarrow \frac{1}{4} = \frac{30-v}{30}$$

$$\Rightarrow \quad 30 = 120 - 4v$$

$$\Rightarrow \quad v = \frac{90}{4} = +22.5 \text{ cm}$$

As, v is positive, therefore a virtual and erect image will be formed on other side of the object.

Uses of Spherical Mirrors

(i) Convex mirror is used as a reflector in street lamps to diverge the light over a large area.

(ii) Convex mirror is used as rear view mirror (or driver's mirror) in vehicles, because it has a wider field of view.

(iii) Concave mirror is used as a reflector in search light, head lights of vehicles, etc.

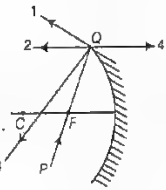
(iv) Concave mirror is also used as face looking mirror because it forms erect and magnified image.

(v) Spherical mirrors are also used as trick mirrors.

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. The direction of ray of light incident on a concave mirror is shown by PQ while directions in which the ray would travel after reflection is shown by four rays marked 1, 2, 3 and 4 (figure). Which of the four rays correctly shows the direction of reflected ray? **NCERT Exemplar**



(a) 1 (b) 2 (c) 3 (d) 4

2. In reflection over a spherical mirror, ray parallel to principal axis, after reflection from mirror pass through

(a) focus (b) centre of curvature
(c) pole of mirror (d) any point

3. A ray passing through or directed towards centre of curvature of a spherical mirror is reflected such that it trace back of its path, because

(a) it does not follow law of reflection
(b) angle of incidence is 0°
(c) centre of curvature is midway between object and pole
(d) distance of centre of curvature from focus is equal to its distance from pole

4. If lower half of a concave mirror is blackened, then

(a) image distance increases
(b) image distance decreases
(c) image intensity increases
(d) image intensity decreases

5. An object 2 cm high is placed at a distance of 16 cm from a concave mirror, which produces a real image 3 cm high. What is the focal length of the mirror?

(a) – 9.6 cm (b) – 3.6 cm
(c) – 6.3 cm (d) – 8.3 cm

[redacted]

VERY SHORT ANSWER Type Questions

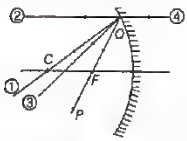
6. A mirror is turned through 15°. With what angle will the reflected ray turn?

7. A thick plane mirror forms a number of images of a point source of light. Which image is the brightest?

8. A boy is running towards a plane mirror with a speed of 2 m/s. With what speed the image of the boy approach him? **Foreign 2011**

9. How can the real image of an object be obtained with a convex mirror? **Delhi 2011**

10. The direction of ray of light incident on a concave mirror is shown by PQ while directions in which the ray would travel after reflection is shown by four rays marked as 1, 2, 3 and 4 in the figure? Which of the four rays correctly shows the direction of reflected rays?



11. Give the effect on image, if lower half of the concave mirror is blackened.

SHORT ANSWER Type Questions

12. A concave mirror of small aperture forms a sharper image. Why?

13. Choose the statement as wrong or right and justify.

(i) Linear magnification of a spherical mirror is given by $\frac{v}{u}$.

(ii) Focal length of plane mirror is zero.

(iii) $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ can be applied to all types of mirror.

14. Use the mirror equation to show that an object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$. **All India 2015**

15. You read a newspaper because of the light that it reflects. Then, why do not you see even a faint image of yourself in the newspaper?

16. "Mirrors used in search lights are parabolic but not concave spherical". Explain, why?

17. A short object of length L is placed along the principal axis of a concave mirror away from focus. The object distance is u. If the mirror has a focal length f, what will be the length of the image? You may take $L << |v - f|$.

NCERT Exemplar

> **Hints:** The length of image is the separation between the images formed by mirror of the extremities of object.

LONG ANSWER Type I Questions

18. (i) A mobile phone lies along the principal axis of a concave mirror. Show with the help of a suitable diagram the formation of its image. Explain, why magnification is not uniform?

(ii) Suppose the lower half of the concave mirror's reflecting surface is covered with an opaque material. What effect this will have on the image of the object? Explain. **Delhi 2014**

19. An object AB is kept in front of a concave mirror as shown in the figure.

(i) Complete the ray diagram showing the image formation of the object.

(ii) How will the position and intensity of the image be affected, if the lower half of the mirror's reflecting surface is painted black? **All India 2012**

20. An infinitely long rod lies along the axis of concave mirror of focal length f. The near end of the rod is at a distance $x > f$ from the mirror f. Then, what will be the length of the image of the rod?

21. Show that spherical mirror formula is applicable to a plane mirror.

22. Use the mirror equation to show that

(i) an object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$.

[redacted]

(ii) a convex mirror always produces a virtual image independent of the location of the object.

(iii) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image. **All India 2011**

23. (a) Draw a ray diagram to show image formation when the concave mirror produces a real, inverted and magnified image of the object.

(b) Obtain the mirror formula and write the expression for the linear magnification. **CBSE 2018**

LONG ANSWER Type II Questions

24. State and derive mirror formula for a concave mirror. State the sign convention used.

25. Use the mirror equation to deduce that,

(i) an object placed between f and $2f$ of a concave mirror produces a real image beyond $2f$.

(ii) a convex mirror always produces a virtual image independent of the location of the object.

(iii) the virtual image produced by a convex mirror is always diminished in size and is located between the focus and the pole.

(iv) an object placed between the pole and focus of a concave mirror produces a virtual and enlarged image. **NCERT**

NUMERICAL PROBLEMS

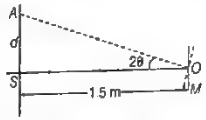
26. A square wire of side 3 cm is placed 25 cm away from a concave mirror of focal length 10 cm. What is the area enclosed by the image of the wire? Given, the centre of the wire is on the axis of the mirror, with its two sides normal to the axis.

27. An erect image 3 times the size of the object is obtained with a concave mirror of radius of curvature 36 cm. What is the position of the object?

28. A 12 m tall tree is to be photographed with a pin hole camera. It is situated 15 m away from the pin hole. How far should the screen be placed from the pin hole to obtain a 12 cm tall image of the tree?

29. Light of wavelength 5000 Å falls on a plane reflecting surface. What are the wavelength and frequency of reflected light? For what angle of incidence is the reflected ray normal to the incident ray?

30. Light incident normally on a plane mirror attached to a galvanometer coil retraces backwards as shown in the figure. A current in the coil produces a deflection of 3.5° of the mirror. What is the displacement of the reflected spot of light on a screen placed 1.5 m away? **NCERT**



31. Suppose while sitting in a parked car, you notice a Jogger approaching towards you in the rear view mirror having $R = 2$ m. If the Jogger is running at a speed of 5 m/s, how fast is the image of Jogger moving, when the Jogger is

(i) 39 m

(ii) 29 m

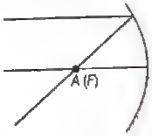
(iii) 19 m

(iv) 9 m away ?

HINTS AND SOLUTIONS

1. (b) The PQ ray of light passes through focus F and incident on the concave mirror, after reflection, should become parallel to the principal axis and shown by ray-2 in the figure.

2. (a) Parallel beam passes through focus after reflection. This can be shown in the figure given below.



3. (b) As we know, angle $i = 0°$ and angle $r = 0°$, when light ray is passes through centre of curvature of a spherical mirror is reflected such that it trace back its path.

4. (d) If lower half of a concave mirror is blackened, then image will be now only half of the object, but taking the laws of reflection to be true for all points of the remaining part of the mirror, the image will be that of the whole object. However, as the area of the reflecting surface has been reduced, the intensity of the image will be low i.e., half.

[redacted]

5. (a) Here, $h_1 = 2$ cm, $u = -16$ cm $h_2 = -3$ cm

(since image is real and inverted)

$$\therefore \quad m = \frac{-h_2}{h_1} = \frac{v}{u}$$

$$\Rightarrow \quad v = \frac{-h_2}{h_1}u = \frac{3}{2}\times(-16) = -24\text{ cm}$$

Now, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = -\frac{1}{24} - \frac{1}{16}$

$$\Rightarrow \frac{-2-3}{48} = \frac{-5}{48} \Rightarrow f = \frac{-48}{5} = -9.6\text{ cm}$$

6. The reflected ray turns twice the angle through which mirror is turned, i.e. 30°.

7. A thick plane mirror consist of two surfaces (top and bottom), where the reflection takes place. The images are formed after reflection from both the surfaces, except for the first image. The second image is the brightest of all as minimum absorption takes place and bounces of the silvery layers which makes the bottom surface.

8. The image of the object in a plane mirror is as far behind the mirror as the object is in front of it. Therefore, the image of the boy comes near the mirror through the distance equal to that moved by the boy towards the plane mirror. Hence, the image of the boy will approach him with double his speed, i.e. with 4 m/s.

9. A convex mirror produces a real image of a virtual object. Therefore, if a beam of light from a virtual object converges to a point behind the convex mirror, then its real image will be formed in front of the mirror.

10. The incident ray PQ passes through the focus, so the reflected ray is parallel to the principal axis. So, the answer is ray 2.

11. If the lower half of the concave mirror is blackened, then there is no change in the position of image, only intensity will get reduced.

12. The rays of light travelling parallel to the principal axis after reflection from a concave mirror meet at a single point only, if the beam of light is narrow or if the mirror is of small aperture. In case, a wide beam of light falls on a concave mirror of large aperture, the rays after reflection from the mirror do not come to focus at a single point. Therefore, it follows that, if the aperture of the concave mirror is small, the image formed will be sharper.

13. (i) Wrong, linear magnification of spherical mirror is $-\frac{v}{u}$ (using sign conventions).

(ii) Wrong, as the plane mirror can be considered to be the limit of either a concave or convex spherical curved mirror as the radius, therefore the focal length of plane mirror becomes infinite.

(iii) Right, but for plane mirror using this formula, focal length becomes infinite.

14. According to the mirror equation, we have

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

where, u = distance of the object from the mirror,

v = distance of the image from the mirror

and f = focal length of the mirror.

From the mirror equation, we have

$$v = \frac{uf}{u-f} \quad \text{...(i)}$$

Applying new cartesian sign convention, we get

$f = -ve$ and $u = -ve$

Given, $f < u < 2f$

$\Rightarrow \quad v = \quad ve$ [from Eq. (i)]

Magnification is given by $m = \left(\frac{-v}{-u}\right) = -ve$

Hence, the image formed is real

From the mirror formula, when $u = -2f$,

$$\Rightarrow \quad \frac{1}{-2f} + \frac{1}{v} = \frac{1}{-f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{2f} - \frac{1}{f} = -\frac{1}{2f}$$

When the object is at f, then image is formed at infinity. This shows that when $f < u < 2f$, then $\infty > v > 2f$.

15. We can an image, if it is caused by regular reflection. In the case of newspaper, the inhomogeneities of the surface cause diffuse reflection. So, the incident parallel beam is scattered in all directions, hence no image is seen.

16. A search light produces an intense parallel beam of light. This require a reflector of large aperture. When a source is placed at the focus of large concave mirror only the paraxial rays are reflected as parallel beam but when a source is placed at the focus of parabolic mirror All the rays are reflected as an intense parallel beam.

17. Since, the object distance is u. Let us consider the two ends of the object be at distance $u_1 = u - L/2$ and $u_2 = u + L/2$ respectively, so that $|u_1 - u_2| = L$. Let the image of the two ends be formed at v_1 and v_2 respectively, so that the image length would be

$$L' = |v_1 - v_2| \quad \text{...(i)}$$

Applying mirror formula, we have

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad \text{or } v = \frac{fu}{u-f}$$

On solving, the positions of two images are given by

$$v_1 = \frac{f(u - L/2)}{u - f - L/2},\ v_2 = \frac{f(u + L/2)}{u - f + L/2}$$

For length, substituting these values in Eq. (i), we get

$$L' = |v_1 - v_2| = \frac{f^2 L}{(u-f)^2 - L^2/4}$$

[redacted]

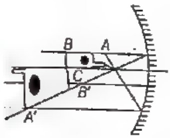
Since, the object is short and kept away from focus, we have

$$L^2/4 << (u-f)^2$$

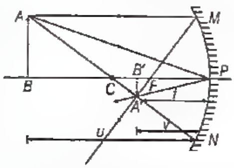
Hence, finally, $L' = \frac{f^2}{(u-f)^2} L$

This is the required expression of length of an image.

18. (i) The ray diagram for the formation of the image of the mobile phone is shown below. The image of the part which is on the plane perpendicular to principal axis will be on the same plane. It will be of the same size, i.e. $B'C = BC$

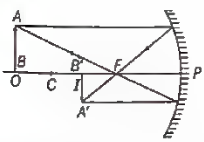


(ii) We may think that the image will now show only half of the object, but considering the laws of reflection to be true for all points of the remaining part of the mirror, the image will be that of the whole object.



However, as the area of the reflecting surface has been reduced, the intensity of the image will be low, i.e. half.

19. (i) The ray diagram showing the image formation of the object.



(ii) The position of image is unaffected but the intensity of image is reduced.

20. Using mirror formula, $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

Here, $u = -u, v = ?, f = -f$

$\therefore \quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

or $\quad \frac{1}{v} = \frac{1}{-f} + \frac{1}{u}$

$\Rightarrow \quad v = \frac{-fu}{-f+u}$

$\therefore$ Length of the image $= \frac{-fu}{-f+u} - f$

$$= \frac{-fu + f^2 - fu}{-f+u} = \frac{f^2}{u-f}$$

21. The spherical mirror formula is given by

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad \text{...(i)}$$

For a plane mirror, $R = \infty$

$\therefore \quad f = \frac{R}{2} = \infty$

From Eq. (i), we get

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{\infty} = 0 \Rightarrow \frac{1}{v} = \frac{-1}{u} \Rightarrow v = -u$$

As, u is negative, v becomes positive.

Thus, image is formed behind the mirror at the same distance as the object is in front of it. This happens in a plane mirror and is the desired result. Also, note that magnification, $m = -\frac{v}{u}$ is 1.

22. (i) Refer to Q. 14 on page 354.

(ii) For convex mirror, $f > 0$

Also, $\quad u < 0$

But from mirror equation,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{v} + \frac{1}{|-u|} \quad \text{[taking } u \text{ with sign]}$$

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

If f and u to be positive, then $\frac{1}{v} > 0 \Rightarrow v > 0$

Hence, virtual image is formed.

(iii) For concave mirror,

$$f < 0, u < 0, \quad f| > |u| > 0$$

But from mirror equation,

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{-1}{|f|} = \frac{1}{v} - \frac{1}{|u|}$$

$$\frac{1}{v} = \frac{1}{|u|} - \frac{1}{|f|}$$

$\because \quad |v| < |f| \Rightarrow \frac{1}{|u|} > \frac{1}{|f|}$

$\Rightarrow \quad \frac{1}{v} > 0 \Rightarrow v > 0$

Image is formed on RHS of mirror, i.e. virtual image.

Also, $\quad \frac{1}{f} = \frac{1}{|v|} - \frac{1}{|u|}$

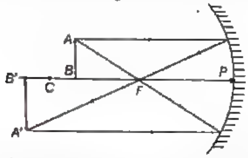
For concave mirror, f is negative.

$\Rightarrow \quad \frac{1}{|v|} < \frac{1}{|u|} \Rightarrow \frac{v|}{|u|} > 1 \Rightarrow m > 1$

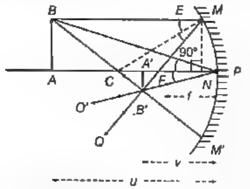
Enlarged virtual image formed on the other side of mirror.

[redacted]

23. (a) Concave mirror form real, inverted and magnified image of an object when it is placed between C and F. The ray diagram is given as



(b) In the given figure, the ray diagram considering three rays for image formation by a concave mirror.



In the figure, triangles $A'B'F$ and NEF are similar.

Then, $$\frac{A'B'}{NE}=\frac{A'F}{NF}$$

As the aperture of the concave mirror is small, the points N and P lie very close to each other.

$$NF = PF \text{ and } NE = AB$$

$$\frac{A'B'}{AB}=\frac{A'F}{PF}$$

Since, all the distances are measured from the pole of the concave mirror, we have

$$A'F = PA' - PF$$

$$\therefore \quad \frac{A'B'}{AB}=\frac{PA'-PF}{PF} \quad \text{...(i)}$$

Also, triangles ABP and $A'B'P$ are similar, then

$$\frac{A'B'}{AB}=\frac{PA'}{PA} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{PA'-PF}{PF}=\frac{PA'}{PA} \quad \text{...(iii)}$$

Applying new Cartesian sign convention, we have

$PA = -u$ ($\because$ distance of object is measured against incident ray)

$PA' = -v$ ($\because$ distance of image is measured against incident ray)

$PF = -f$ ($\because$ focal length of concave mirror is measured against incident ray)

Substituting these values in Eq. (iii), we get

$$\frac{-v-(-f)}{-f}=\frac{-v}{-u}$$

$$\Rightarrow \quad \frac{v-f}{f}=\frac{v}{u} \Rightarrow \frac{v}{f}-1=\frac{v}{u}$$

Dividing both sides by v, we get

$$\therefore \quad \boxed{\frac{1}{f}=\frac{1}{u}+\frac{1}{v}}$$

The above relation is called **mirror formula.**

Linear magnification

The ratio of the height of the image (h') formed by a spherical mirror to the height of the object (h) is called the linear magnification produced by the spherical mirror.

It is denoted by m.

$$m=\frac{h'}{h}$$

24. Refer to the text on pages 351 and 352.

25. For parts (i), (ii) and (iv), refer to Q. 22 on page 354.

(iii) For convex mirror, $f>0, u<0$

From mirror formula, $\frac{1}{v}=\frac{1}{f}-\frac{1}{u}$

So, $\frac{1}{v}>\frac{1}{f}$ or $v<f$

Also, $\frac{1}{v}>\frac{-1}{u}$ or $\frac{-v}{u}<1$,

i.e. $m<1$

Thus, image is always located between pole and focus of the mirror and is always diminished in size.

26. Here, $u=-25$ cm, $f=-10$ cm

Using mirror formula, $\frac{1}{f}=\frac{1}{u}+\frac{1}{v}$

$$-\frac{1}{10}=-\frac{1}{25}+\frac{1}{v}$$

$$\Rightarrow \quad v=\frac{-50}{3}$$

Now, magnification, $m=\frac{-v}{u}=-\left[\frac{\left(-\frac{50}{3}\right)}{(-25)}\right]=-\frac{2}{3}$

Length and breadth both change in the same proportion.

Area of the object, $A_o=3\times 3=9\text{ cm}^2$

$$\therefore \quad \frac{A_i}{A_o}=\left(-\frac{2}{3}\right)^2$$

$$\Rightarrow \quad A_i=\left(\frac{4}{9}\right)\times 9=4\text{ cm}^2$$

[redacted]

27. Given, magnification, $m = +3$, $R = -36$ cm

Object distance, $u = ?$

Let $u = -x$

$$m = \frac{h_2}{h_1} = \frac{+v}{-u} = 3$$

$$\Rightarrow \quad v = -3u \Rightarrow v = 3x$$

Applying mirror formula, we have

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{2}{R} \quad \Rightarrow \quad \frac{1}{-x} + \frac{1}{3x} = \frac{2}{-36}$$

$$\Rightarrow \quad \frac{-3+1}{3x} = \frac{-1}{18} \Rightarrow 3x = 36$$

$$\Rightarrow \quad x = 12 \text{ cm or } u = -12 \text{ cm}$$

28. Given, $h_1 = 12$ m, $u = -15$ m, $v = ?$,

$h_2 = 12$ cm $= 0.12$ m (symbols have their usual meanings)

As, $\frac{h_2}{h_1} = -\frac{v}{u}$

$$\Rightarrow \quad v = -\frac{h_2}{h_1} \times u$$

$$= -\frac{0.12}{12} \times -15 = 0.15 \text{ m} = 15 \text{ cm}$$

Thus, the screen should be placed 15 cm from the pin hole to obtain a 12 cm tall image of the tree.

29. Given, $\lambda = 5000$ Å $= 5 \times 10^{-7}$ m

Frequency of incident light,

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{5 \times 10^{-7}} \quad [\because c = 3 \times 10^8 \text{ m/s}]$$

$$= 6 \times 10^{14} \text{ Hz}$$

On reflection, there is no change in wavelength or frequency. Therefore, $\lambda' = \lambda = 5000$ Å

or $\nu' = \nu = 6 \times 10^{14}$ Hz

For reflected ray to be normal to incident ray,

$$i + r = 90^\circ \quad \Rightarrow \quad i + i = 90^\circ$$

$$\Rightarrow \quad i = \frac{90^\circ}{2} = 45^\circ$$

30. Given, deflection of the mirror, $\theta = 3.5^\circ$

Distance between screen and mirror, $x = 1.5$ m

As we know that when mirror turns by angle θ, the reflected light may turn by 2θ.

$$2\theta = 2 \times 3.5^\circ = 7^\circ = \frac{7\pi}{180} \text{ radians}$$

Again, in ΔAOS, $\tan 2\theta = \frac{AS}{SM}$

$$\tan\left(\frac{7\pi}{180}\right) = \frac{AS}{1.5} = \frac{d}{1.5}$$

or $d = 1.5 \tan\left(\frac{7\pi}{180}\right)$

For small angle, $\tan \frac{7\pi}{180} \approx \frac{7\pi}{180}$

$$d = 1.5 \times \frac{7\pi}{180} = 0.18 \text{ m}$$

31. Here, $R = 2$ m, $f = \frac{R}{2} = \frac{2}{2} = 1$ m

Using mirror formula, we have

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{u-f}{fu}$$

$$\Rightarrow \quad v = \frac{fu}{u-f} \quad \text{...(i)}$$

When Jogger is 39 m away, then $u = -39$ m

Using Eq. (i), we get

$$v = \frac{fu}{u-f} = \frac{1\,(-39)}{-39-1}$$

or $v = \frac{39}{40}$ m

As the Jogger is running at a constant speed of 5 m/s, after 1 s, the position of the image (v) for

$$u = -39 + 5$$

$$\Rightarrow \quad u = -34 \text{ m}$$

Again, using Eq. (i), we get

$$\Rightarrow \quad v = \frac{1(-34)}{-34-1}$$

$$\Rightarrow \quad v = \frac{34}{35} \text{ m}$$

Difference in apparent position of Jogger in 1s

$$= \frac{39}{40} - \frac{34}{35}$$

$$= \frac{1365 - 1360}{1400} = \frac{1}{280} \text{ m}$$

Average speed of Jogger's image $= \frac{1}{280}$ m/s

Similarly, for $u = -29$ m, -19 m and -9 m, average speed of Jogger image is $\frac{1}{150}$ m/s, $\frac{1}{60}$ m/s, $\frac{1}{10}$ m/s, respectively.

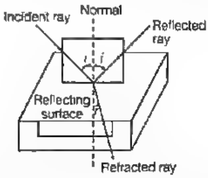
The speed increases as the Jogger approaches the car. This can be experienced by the person in the car.

[redacted]

|TOPIC 2|

Refraction

Reflection involves change in path of light without any change in the medium, whereas refraction involves change in the path of light due to change in the medium.



When a beam of light encounters another transparent medium, a part of light gets reflected back into the first medium while the rest enters the other. The direction of propagation of an obliquely incident ray of light, that enters the other medium, changes at the interface of two media. This phenomenon is called refraction of light.

Laws of Refraction

(i) The incident ray, the refracted ray and the normal to the refracting surface (or interface) at the point of incidence, all lie in the same plane.

(ii) The ratio of the sine angle of incidence to the sine angle of refraction is constant for the two given media. This constant is denoted by ${}^a\mu_b$ and is called the relative refractive index of medium b with respect to medium a.

$$\therefore \qquad \boxed{\frac{\sin i}{\sin r} = {}^a\mu_b}$$

This law is also called Snell's law of refraction.

Refractive Index

The refractive index or index of refraction μ of a material is the ratio of the speed of light (c) in vacuum to the speed of light in the medium (v).

Mathematically, refractive index is given by the relation

$$\mu = \frac{\text{Speed of light in the vacuum}}{\text{Speed of light in the material}} = \frac{c}{v}$$

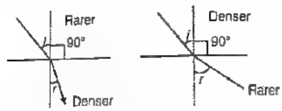
It is also referred as absolute refractive index of the substance.

Refractive Index of Some Substance Media

Substance medium	Refractive Index
Ethyl alcohol	1.362
Water, H_2O	1.333
Air	1.000293
Oxygen O_2	1.000271

Relative refractive index is a measure of how much light bends, when it travels from one medium to another medium.

If light travels from optical rarer medium to optical denser medium, then it bends towards the normal, i.e. $i > r$. On the other hand, if light travels from optical denser medium to optical rarer medium, then light bends away from the normal, i.e. $i < r$.



The medium in which the speed of light is higher with respect to other medium, is said to be optically denser medium. Optical density is the ratio of the speed of light in two media.

Optical density should not be confused with mass density, which is mass per unit volume. It is possible that, mass density of an optically denser medium may be less than that of an optically rarer medium. e.g. Turpentine and water. Mass density of turpentine is less than that of water but its optical density is higher.

Principle of Reversibility of Light

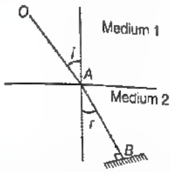
When a light ray, after suffering any number of reflections and refractions, has its final path reversed, it travels back along its entire initial path. This is called principle of reversibility of light. In the figure, OA is an incident ray in medium 1 and AB is the refracted ray in medium 2. By Snell's law, the refractive index of medium 2 relative to medium 1 is given by

$${}^1\mu_2 = \frac{\sin i}{\sin r} \qquad \text{...(i)}$$

where, i and r are the angles of incidence and refraction, respectively.

Suppose the ray AB is reflected back by a plane mirror. Now, BA is the incident ray and AO is the refracted ray.

[redacted]



Correspondingly, r is angle of incidence and i is angle of refraction. Again, by Snell's law, the refractive index of medium 1 relative to medium 2 is given by

$$^2\mu_1 = \frac{\sin r}{\sin i} \qquad \text{...(ii)}$$

Multiplying Eqs. (i) and (ii), we get

$$^1\mu_2 \times {}^2\mu_1 = \frac{\sin i}{\sin r} \times \frac{\sin r}{\sin i} = 1$$

or

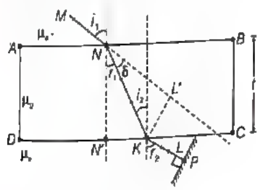
$$\boxed{^1\mu_2 = \frac{1}{^2\mu_1}}$$

Thus, the refractive index of medium 2 relative to medium 1 is equal to the reciprocal of the refractive index of medium 1 relative to medium 2.

Refraction of Light Through a Rectangular Glass Slab

Let $ABCD$ be a rectangular glass slab. A ray of light is incident along MN on the face AB of the rectangular slab at $\angle i_1$. It is refracted along NK with $\angle r_1$.

The refracted ray NK falls on face CD with $\angle i_2$ and emerges out along KL with $\angle r_2$.



Applying Snell's law at N,

$$\mu_a \times \sin i_1 = \mu_g \times \sin r_1$$

or

$$\frac{\sin i_1}{\sin r_1} = \frac{\mu_g}{\mu_a} = {}^a\mu_g \qquad \text{....(i)}$$

Again, applying Snell's law at K,

$$\mu_g \times \sin i_2 = \mu_a \times \sin r_2$$

$\Rightarrow$

$$\frac{\mu_a}{\mu_g} = \frac{\sin i_2}{\sin r_2} = {}^g\mu_a \qquad \text{.... (ii)}$$

According to the principle of reversibility of light, when final path of a light ray after suffering a number of reflections and refractions is reversed, then the ray retraces its entire path.

Now, imagine a plane mirror P held normal to KL so that on reflection from mirror, path KL is reversed. The ray would retrace its entire path. For the reversed ray, the application of Snell's law at K gives

$$\mu_a \times \sin r_2 = \mu_g \times \sin i_2$$

$$\frac{\mu_g}{\mu_a} = \frac{\sin r_2}{\sin i_2} = {}^a\mu_g \qquad \text{...(iii)}$$

Multiplying Eqs. (ii) and (iii), we get

$$\frac{\sin i_2}{\sin r_2} \times \frac{\sin r_2}{\sin i_2} = {}^g\mu_a \times {}^a\mu_g$$

$$1 = {}^g\mu_a \times {}^a\mu_g \;,\; {}^a\mu_g = \frac{1}{^g\mu_a}$$

From Eqs. (i) and (iii), we get

$$\frac{\sin i_1}{\sin r_1} = \frac{\sin r_2}{\sin i_2} \qquad \text{...(iv)}$$

As, $\quad i_2 = r_1 \quad$ [alternate angles]

$\therefore \quad \sin i_2 = \sin r_1$

From Eq. (iv), we get

$$\sin r_2 = \sin i_1 \text{ or } r_2 = i_1$$

Hence, the emergent ray KL is parallel to the incident ray MN as shown in the figure. We observe that the incident ray MN is displaced laterally, on suffering two refractions through a glass slab.

Expression for Lateral Displacement

Now, from K, draw $KL' \perp MN$ produced.

$\therefore$ Lateral displacement of the ray on passing through the parallel slab $= KL'$.

Let $\angle KNL' = \delta =$ deviation on first refraction.

In $\Delta NKL'$, $\quad \sin\delta = \dfrac{KL'}{NK}$

$$\therefore \quad KL' = NK \sin\delta \qquad \text{...(v)}$$

In $\Delta NN'K$, $\quad \cos r_1 = \dfrac{NN'}{NK}$

$$\therefore \quad NK = \frac{NN'}{\cos r_1} = \frac{t}{\cos r_1}$$

where, $\quad t = NN' =$ thickness of glass slab.

From Eq. (v), we get, $\quad KL' = \dfrac{t}{\cos r_1}\sin\delta$

or

$$\boxed{KL' = \frac{t\sin(i_1 - r_1)}{\cos r_1}} \qquad \text{...(vi)}$$

[redacted]

This is the required expression for lateral displacement (or shift), which is obviously proportional to thickness (t) of glass slab. Further, lateral displacement (or shift) will increase with increasing angle of incidence (i_1).

EXAMPLE |1| A ray of light is incident at an angle of 60° on one face of a rectangular glass slab of thickness 0.1 m and refractive index 1.5. Calculate the lateral shift produced.

Sol. Given, angle of incidence, $i_1 = 60°$

Thickness of glass slab, $t = 0.1$ m

Refractive index, $\mu = 1.5$

Since, $\dfrac{\sin i_1}{\sin r_1} = \mu$

$$\therefore \quad \sin r_1 = \frac{\sin i_1}{\mu} = \frac{\sin 60°}{1.5} = 0.5773 \qquad \left[\because \sin 60° = \frac{\sqrt{3}}{2} \text{ and } \sqrt{3} = 1.732\right]$$

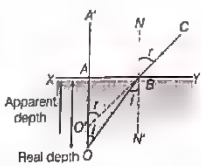
$$r_1 = \sin^{-1}(0.5773) = 35.3°$$

$$\therefore \text{ Lateral shift} = \frac{t \sin(i_1 - r_1)}{\cos r_1} = \frac{0.1 \sin(60° - 35.3°)}{\cos 35.3°} = \frac{0.1 \sin 24.7°}{\cos 35.3°}$$

$$= \frac{0.1 \times 0.418}{0.816} = 0.0513 \text{ m}$$

Apparent Depth and Normal Shift

The depth of an object immersed in water appears to be lesser than its actual depth. Let O be a point object at an actual depth OA below the free surface of water XY.



A ray of light incident normally on XY, along OA passes straight along OAA'. Another ray of light from O incident at $\angle i$ on surface XY along OB deviates away from normal. It is refracted at $\angle r$ along BC. On producing backwards BC meets OA at O'. Therefore, O' is virtual image of O.

Apparent depth = AO'

Real depth = OA

Clearly, $AO' < OA$

Now, $\angle BOA = \angle OBN' = i$ [alternate angles]

$\angle AO'B = \angle CBN = r$ [corresponding angles]

In ΔOAB, $\sin i = \dfrac{AB}{OB}$

In $\Delta O'AB$, $\sin r = \dfrac{AB}{O'B}$

As, light ray is travelling from denser medium to rarer medium,

$$\therefore \quad {}^a\mu_w = \frac{\sin r}{\sin i}$$

$$\text{or} \quad {}^a\mu_w = \frac{AB}{O'B} \times \frac{OB}{AB} = \frac{OB}{O'B}$$

B is close to A (as angles are very small). So, $OA \approx OB$ and $O'A \approx O'B$

$$\therefore \quad \boxed{{}^a\mu_w = \frac{OA}{O'A} = \frac{\text{Real depth}}{\text{Apparent depth}}}$$

If x is the real depth of water surface and ${}^a\mu_w$ is the refractive index of water with respect to air, then the normal shift (d) in position of point object is given by

d = Real depth − Apparent depth

$$\therefore \quad d = x - \frac{x}{{}^a\mu_w} \qquad \left[\because \text{apparent depth} = \frac{\text{real depth}}{{}^a\mu_w} = \frac{x}{{}^a\mu_w}\right]$$

$$\text{or} \quad \boxed{d = x\left(1 - \frac{1}{{}^a\mu_w}\right)}$$

EXAMPLE |2| Velocity of light in glass is 2×10^8 m/s and that in air is 3×10^8 m/s. By how much would an ink dot appear to be raised, when covered by a glass plate 6 cm thick?

Sol. Given, velocity of light in glass, $v = 2 \times 10^8$ m/s

Velocity of light in air, $c = 3 \times 10^8$ m/s

$\therefore$ Refractive index of glass with respect to air,

$${}^a\mu_g = \frac{c}{v} = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$$

$\therefore$ Normal shift in the position of ink dot,

$$d = t\left(1 - \frac{1}{{}^a\mu_g}\right) \qquad [\because t = 6 \text{ cm}]$$

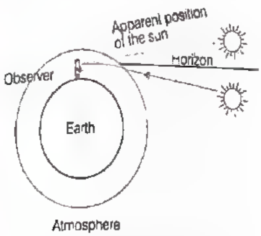
$$= 6\left(1 - \frac{1}{1.5}\right) = \frac{6 \times 0.5}{1.5} = 2 \text{ cm}$$

Effect of Atmospheric Refraction at Sunrise and Sunset

The density of atmosphere around the earth is not uniform throughout due to which, it has layers of different densities. The refraction of light due to variation in optical density of atmospheric layers is called atmospheric refraction.

[redacted]

Due to refraction of sunlight from atmosphere, the sun is visible a little before the actual sunrise and a little after the actual sunset.

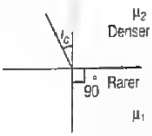


Advance sunrise and delayed sunset due to atmospheric refraction

The refractive index of air with respect to vacuum is 1.00029. Due to this, the apparent shift in the direction of the sun is about half a degree and the corresponding time difference between actual sunset and apparent sunset is about 2 min. The apparent flattening (oval shape) of the sun at sunset and sunrise is also due to atmospheric refraction.

Critical Angle

Critical angle for a pair of given media in contact can be defined as, "the angle of incidence in denser medium for which angle of refraction in rarer medium is 90°". The value of critical angle depends on the nature of two media in contact.



From Snell's law, $\mu_2 \times \sin i_c = \mu_1 \times \sin 90°$

$\therefore \quad \dfrac{\mu_1}{\mu_2} = \dfrac{\sin i_c}{\sin 90°} \Rightarrow \dfrac{\mu_1}{\mu_2} = \sin i_c \qquad [\because \sin 90° = 1]$

or $\quad \dfrac{\mu_2}{\mu_1} = \dfrac{1}{\sin i_c} \Rightarrow {}^1\mu_2 = \dfrac{1}{\sin i_c}$

Critical Angle of Some Transparent Media

Substance medium	Refractive Index	Critical angle
Water	1.33	48.75°
Crown glass	1.52	41.14°
Dense flint glass	1.62	37.31°
Diamond	2.42	24.41°

EXAMPLE |3| If a ray of light travelling in air is incident on a glass surface with an angle of incidence 40°, it deviates through 15°, determine the critical angle for a glass-air interface.

Sol. Given, angle of incidence, $i = 40°$

Angle of deviation, $\delta = 15°$

Since, ray deviates towards the normal, therefore

$r = i - \delta = 40° - 15° = 25°$

As we know that,

$$\mu = \frac{\sin i}{\sin r} = \frac{1}{\sin i_c}$$

$$\Rightarrow \sin i_c = \frac{\sin r}{\sin i} = \frac{\sin 25°}{\sin 40°} = \frac{0.4226}{0.6428}$$

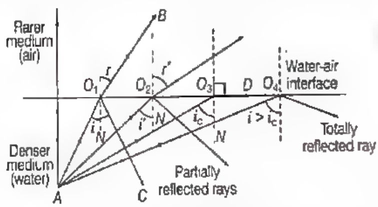
$$\Rightarrow \sin i_c = 0.6574$$

$$\therefore \quad i_c = \sin^{-1}(0.6574) = 41.1°$$

$$\Rightarrow \quad i_c = 41.1°$$

TOTAL INTERNAL REFLECTION (TIR)

When a ray of light travelling from denser medium to rarer medium, is incident at the interface of two media at an angle greater than the critical angle for the two media, the ray is totally reflected back to denser medium, this phenomena is called **Total Internal Reflection (TIR).**



Refraction and internal reflection of rays from a point A in the denser medium (water) incident at different angles at the interface with a rarer medium (air)

Necessary conditions for total internal reflection to take place are as follows

(i) The ray incident on the interface of two media should travel in the denser medium.

(ii) The angle of incidence should be greater than critical angle for the two media.

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. Which of the following quantity remains unchanged after refraction?

(a) Speed of light (b) Intensity of light

(c) Wavelength of light (d) Frequency of light

[redacted]

2. A ray of light strikes an air-glass interface at an angle of incidence ($i = 60°$) and gets refracted at an angle of refraction r. On increasing the angle of incidence ($i > 60°$), the angle of refraction r
(a) decreases (b) remains same
(c) is equal to $60°$ (d) increases

3. A ray of light strikes a transparent rectangular slab of refractive index $\sqrt{2}$ at an angle of incidence of 45°. The angle between the reflected and refracted ray is
(a) 75° (b) 90° (c) 105° (d) 120°

4. Speed of light in air is 3.0×10^8 m/s. Speed of light in the glass of refractive index 1.5 will be
(a) 1.5×10^8 m/s (b) 2.0×10^8 m/s
(c) 1.8×10^8 m/s (d) 2.5×10^8 m/s

5. The refractive indices of water and glass with respect to air are 4/3 and 5/3, respectively. The refractive index of glass with respect to water will be
(a) 1/3 (b) 4/3 (c) 5/4 (d) 20/9

6. If the value of critical angle is 30° for total internal reflection from any medium to vacuum, then speed of light in that medium
(a) 3×10^8 m/s (b) 1.5×10^8 m/s
(c) 6×10^8 m/s (d) 4.5×10^8 m/s

7. If in denser medium, incidence angle is equal to critical angle, then refraction angle will be
(a) 0° (b) 45°
(c) 90° (d) 180°

8. The ratio $\dfrac{\text{real depth}}{\text{apparent depth}}$ is equal to
(a) refractive index of denser medium with respect to air
(b) refractive index of denser medium with respect to rare medium
(c) refractive index of rare medium with respect to air
(d) refractive index of rare medium with respect to denser medium

9. The phenomena involved in the reflection of radiowaves by ionosphere is similar to **NCERT Exemplar**
(a) reflection of light by a plane mirror
(b) total internal reflection of light in air during a mirage
(c) dispersion of light by water molecules during the formation of a rainbow
(d) scattering of light by the particles of air

VERY SHORT ANSWER Type Questions

10. When monochromatic light travels from one medium to another, its wavelength changes, but frequency remains same. Explain. **Delhi 2011**

11. For the same value of angle of incidence, the angles of refraction in three media *A*, *B* and *C* are 15°, 25° and 35°, respectively. In which medium would the velocity of light be minimum? **All India 2012**

12. A ray of light strikes on air-glass interface at an angle of incidence ($< i = 60°$) and gets refracted at an angle of refraction ($< r$). What will happen to the angle of refraction on increasing the angle of incidence?

13. Why does a crack in a glass window pane appear silvery?

14. The refractive index of diamond is much higher than that of glass. How does a diamond cutter make use of this fact? **All India 2011**

15. Why prisms are used in many optical instruments?

16. Which of the two main parts of an optical fibre has a higher value of refractive index?

SHORT ANSWER Type Questions

17. When monochromatic light travels from a rarer to a denser medium, explain the following, giving reasons.
(i) Is the frequency of reflected and refracted light same as the frequency of incident light?
(ii) Does the decrease in speed imply a reduction in the energy carried by light wave? **Delhi 2013**

18. Mention any two situations in which Snell's law of refraction fails.

19. A ray of light is incident at a glass-water interface at an angle of i, it emerges finally parallel to the surface water, then what will be the value of μ_g?

20. Why does the sun rising in the sky appear oval in shape?

21. Choose the statement as wrong or right and justify.
(i) Snell's law is verified for all types of surface.

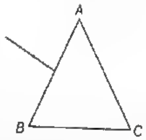
[redacted]

(ii) Total internal reflection only takes place, when light travels from rarer to denser medium.

12. (i) Write the necessary conditions for the phenomenon of total internal reflection to occur.

(ii) Write the relation between refractive index and critical angle for a given pair of optical media. **Delhi 2013**

23. The figure shows a ray of light falling normally on the face AB of an equilateral glass prism having refractive index 3/2, placed in water of refractive index 4/3. Will this ray suffer total internal reflection on striking the face AC? Justify your answer. **CBSE 2018**

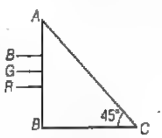


LONG ANSWER Type I Questions

24. Define the following with required formula.
(i) Apparent depth
(ii) Lateral displacement (or shift)
(iii) Critical angle

25. A beaker contains water upto height h_1 and kerosene of height h_2 above water surface, so that the total height of (water + kerosene) is $h_1 + h_2$. Refractive index of water is μ_1 and that of kerosene is μ_2. What will be the apparent shift in position of the bottom of the beaker as viewed from above?

26. Three light rays, red (R), green (G) and blue (B) are incident on a right angled prism ABC at face AB. The refractive indices of the material of the prism for red, green and blue wavelengths are 1.39, 1.44 and 1.47, respectively. Out of the three, which colour of ray will emerge out of face AC? Justify your answer. Trace the path of these rays after passing through face AB.



27. Show that for a material with refractive index $\mu \geq \sqrt{2}$, light incident at any angle shall be guided along a length perpendicular to the incident face. **NCERT Exemplar**

28. Three immiscible liquids of densities $d_1 > d_2 > d_3$ and refractive indices $\mu_1 > \mu_2 > \mu_3$ are put in a beaker. The height of each liquid column is $\frac{h}{3}$. A dot is made at a bottom of the beaker. For near normal vision, find the apparent depth of the dot. **NCERT Exemplar**

> Hints: The image formed by first medium acts as an object for second medium.

LONG ANSWER Type II Question

29. Explain the phenomenon of total internal reflection. Describe how TIR takes place in optical fibre. State any two uses of it.

NUMERICAL PROBLEMS

30. What is the ratio of the velocities of two light waves travelling in vacuum and having wavelengths 4000 Å and 8000 Å?

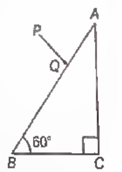
31. What is the critical angle for a material of refractive index $\sqrt{2}$?

32. Determine the lateral displacement of the ray of light passing through a 15 cm thick glass slab with opposite sides parallel, if the angle of incidence of the ray is 60°. Given, $n = 1.5$.

33. A ray of light is incident at an angle of 45° on one face of a rectangular glass slab of thickness 10 cm and refractive index 1.5. Calculate the lateral shift produced.

34. What is the apparent position of an object below a rectangular block of glass 6 cm thick, if a layer of water 4 cm thick is on the top of the glass? Given, $n_{ga} = 1.5$ and $n_{wa} = 1.33$.

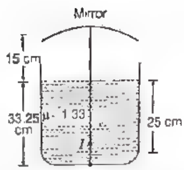
35. A ray PQ incident normally on the refracting face BA is refracted in the prism BAC made of material of refractive index 1.5. Complete the path of ray through the prism. From which face will the ray emerge? Justify your answer. **All India 2016**



36. A tank is filled with water to a height of 12.5 cm. The apparent depth of a needle lying at the bottom of the tank is measured by a microscope to be 9.4 cm. What is the refractive index of water? If water is replaced by a liquid of refractive index 1.63 upto the same height, by what distance would the microscope have to be moved to focus on the needle again? **NCERT**

[redacted]

37. A container is filled with water ($\mu = 1.33$) upto a height of 33.25 cm. A concave mirror is placed 15 cm above the water level and the image of an object placed at the bottom is formed 25 cm below the water level. What will be the focal length?]



HINTS AND SOLUTIONS

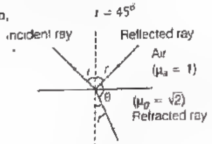
1. (d) Refraction does not change the frequency of light.

2. (d) From Snell's law of refraction,

$$^{a}\mu_g = \frac{\sin i}{\sin r} = \text{constant} \quad ...(i)$$

Since, angle of incidence increase, the angle of refraction has to increase. So, that the ratio $\left(\frac{\sin i}{\sin r}\right)$ is a constant according to Eq. (i).

3. (c) Given, $i = 45^\circ$



From Snell's law, $\frac{\sin i}{\sin r'} = \frac{\mu_g}{\mu_a}$

$$\Rightarrow \quad \frac{\sin 45^\circ}{\sin r'} = \frac{\sqrt{2}}{1}, \ \sin r' = \frac{1}{2} \Rightarrow r' = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$$

From diagram, $r + \theta + r' = 180^\circ$

$$i + \theta + 30^\circ = 180^\circ \qquad (\because i = r)$$

$$45 + \theta + 30^\circ = 180^\circ$$

$$\theta = 180^\circ - 75^\circ = 105^\circ$$

Hence, the angle between reflected and refracted ray is 105°.

4. (b) Refractive index of glass

$$= \frac{\text{Speed of light in air } (3\times10^8)}{\text{Speed of light in glass } (x)}$$

$$\Rightarrow \quad x = \frac{3\times10^8}{1.5} = 2\times10^8 \text{ m/s}$$

5. (c) Given, $_an_w = \frac{4}{3}, \ _an_g = \frac{5}{3}$

$$\therefore \quad _an_w \times {_wn_g} = {_an_g}$$

$$_wn_g = \frac{_an_g}{_an_w} = \frac{5/3}{4/3} = \frac{5}{3}\times\frac{3}{4} = \frac{5}{4}$$

6. (b) From Snell's law,

$$\sin C = {_1n_2} = \frac{v_1}{v_2}$$

where, C = critical angle = 30°

v_1 and v_2 are speed of light in medium and vacuum, respectively

We know that, $v_2 = 3\times10^8$ m/s

$$\therefore \quad \sin 30^\circ = \frac{v_1}{3\times10^8}$$

$$\Rightarrow v_1 = 3\times10^8 \times \frac{1}{2} \Rightarrow v_1 = 1.5\times10^8 \text{ m/s}$$

7. (c) If incidence angle, i = critical angle C, then refraction angle, $r = 90^\circ$

8. (b) As we know, refractive index of denser medium w.r.t. rare medium $= \frac{\text{Real depth}}{\text{Apparent depth}}$

9. (b) The phenomenon involved in the reflection of radiowaves by ionosphere is similar to total internal reflection of light in air during a mirage i.e., angle of incidence is greater than critical angle

10. Because refractive index for a given pair of media depends on the ratio of wavelengths and velocity of light in two media but not on frequency. So, frequency remains constant during refraction of light

11. From Snell's law, $\mu = \frac{\sin i}{\sin r} = \frac{c}{v}$

$\Rightarrow v \propto \sin r$, for given value of i

Smaller the angle of refraction, smaller the velocity of light in medium

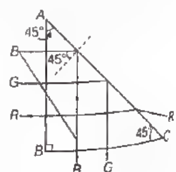
Velocity of light is minimum in medium A as the angle of refraction is minimum, i.e. 15°

12. From Snell's law, $\mu = \frac{\sin i}{\sin r}$ = constant

Since, angle of incidence increases, the angle of refraction has to increase, so that the ratio remains constant

13. Whenever rays of light travels through glass, they strike the glass-air interface at an angle greater than critical angle of glass. They are totally reflected, hence crack appears silvery

14. The refractive index of diamond is much higher than that of glass. Due to high refractive index, the critical angle for diamond air interface is low. The diamond is cut suitably, so that the light entering the diamond from any face suffers multiple total internal reflections at the various surfaces. This gives sparkling effect to the diamonds.



[redacted]

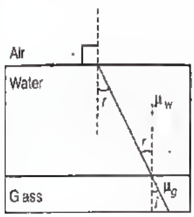
15. Since, prisms can bend the light rays by 90° and 180° by total internal reflection, so they are used in many optical instruments.

16. There are two main parts of the optical fibre
(i) Core
(ii) Cladding
The refractive index of core is greater than that of cladding such that TIR can occurs

17. (i) The frequency of reflected and refracted light remains same as that of incident light because frequency only depends on the source of light.
(ii) Since, the frequency remains same, hence there is no reduction in energy.

18. Snell's law of refraction fails in two situations
(i) When TIR (total internal reflection) takes place at angle greater than the critical angle.
(ii) When light is incident normally on a surface, as $i = 0$, $r = 0$.

19. For glass-water interface, applying Snell's law,



$$\frac{\sin i}{\sin r} = \frac{\mu_w}{\mu_g} \Rightarrow \mu_g = \left(\frac{\mu_w \sin r}{\sin i}\right) \quad \text{...(i)}$$

For water-air interface,

$$\frac{\sin r}{\sin 90^\circ} = \frac{1}{\mu_w} \Rightarrow \sin r = \frac{1}{\mu_w} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\mu_g = \frac{\mu_w \times \frac{1}{\mu_w}}{\sin i} \Rightarrow \mu_g = \frac{1}{\sin i}$$

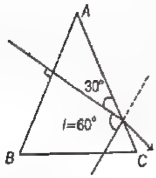
20. It is due to the refraction of sunlight as it travels through the earth's atmosphere. Refraction of light by these layers can make the sun appear flattened or distorted. Objects closer to the horizon are raised upwards most and the lower limb of the sun is raised more than the top making it appear oval.

21. (i) Refer to text on pages 360 and 361.
(ii) Refer to text on page 363.

22. (i) Refer to text on page 363.
(ii) Refer to text on page 363.

23. Given, refractive index of water, $\mu_w = 4/3$

Refractive index of glass prism, $\mu_g = \frac{3}{2}$



For total internal reflection occurrence the incident angle must be greater than critical angle.

∴ Let us calculate critical angle C.

As we know that, $\sin C = \frac{1}{\mu}$

where, $\mu = \frac{\text{refractive index of glass } ({}_a\mu_g)}{\text{refractive index of water } ({}_a\mu_w)}$

$$\therefore \quad \sin C = \frac{1}{\left(\frac{{}_a\mu_g}{{}_a\mu_w}\right)} = \frac{1}{\left(\frac{3/2}{4/3}\right)} = \frac{1}{9/8}$$

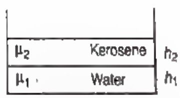
or $\sin C = \frac{8}{9} = 0.88 \Rightarrow C = 61.6^\circ$

[As $\sin 60^\circ = \sqrt{3}/2 = 0.86$]

As the critical angle, i.e, 61.6° is greater than the angle of incidence, i.e. 60°, hence TIR will not occurs.

24. (i) Refer to text on page 362.
(ii) Refer to text on page 361.
(iii) Refer to text on page 363.

25. ∴ Apparent depth, $d = d_1 + d_2 = \left(1 - \frac{1}{\mu_1}\right)h_1 + \left(1 - \frac{1}{\mu_2}\right)h_2$

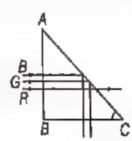


26. By geometry, angle of incidence (i) at face AC for all three rays is 45°. Light suffers total internal reflection for which this angle of incidence is greater than critical angle.

$$i > i_c \Rightarrow \sin i > \sin i_c \text{ or } \sin 45^\circ > \sin i_c$$

$$\Rightarrow \frac{1}{\sin 45^\circ} < \frac{1}{\sin i_c} \Rightarrow \sqrt{2} < \mu$$

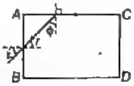
Total internal reflection takes place on AC for rays with $\mu > \sqrt{2} = 1.414$, i.e. green and blue colour suffer total internal reflection, whereas red undergoes refraction.



[redacted]

27. Any ray entering at an angle i shall be guided along AC, if the ray makes an angle ϕ with the face AC greater than the critical angle as per the principle of total internal reflection, $\phi + r = 90^\circ$, therefore $\sin\phi = \cos r$.

$$\Rightarrow \quad \sin\phi \geq \frac{1}{\mu} \quad \Rightarrow \quad \cos r \geq \frac{1}{\mu}$$



$$\text{or } 1-\cos^2 r \leq 1-\frac{1}{\mu^2} \Rightarrow \sin^2 r \leq 1-\frac{1}{\mu^2} \quad \left[\because 1-\cos^2 r = \sin^2 r\right]$$

Since, $\sin i = \mu \sin r$

$$\frac{1}{\mu^2}\sin^2 i \leq 1-\frac{1}{\mu^2}$$

or $$\sin^2 i \leq \mu^2 - 1$$

When $i = \frac{\pi}{2}$, then we have smallest angle ϕ.

If the angle ϕ is greater than the critical angle, then all other angles of incidence shall be more than the critical angle.

Thus, $\quad 1 \leq \mu^2 - 1$ or $\mu^2 \geq 2$

$\Rightarrow \quad \mu \geq \sqrt{2}$

This is the required result.

28. Let the apparent depth be O_1 for the object seen from m_2, then $O_1 = \frac{\mu_2}{\mu_1}\cdot\frac{h}{3}$.

Since, apparent depth = real depth/refractive index (μ).

Since, the image formed by medium 1 acts as an object for medium 2.If seen from μ_3, the apparent depth is O_2.

Similarly, the image formed by medium 2 acts as an object for medium 3.

$$O_2 = \frac{\mu_3}{\mu_2}\left(\frac{h}{3} + O_1\right)$$

$$= \frac{\mu_3}{\mu_2}\left(\frac{h}{3} + \frac{\mu_2 h}{\mu_1 3}\right) - \frac{h}{3}\left(\frac{\mu_3}{\mu_2} + \frac{\mu_3}{\mu_1}\right)$$

As, seen from outside, the apparent height is

$$O_3 = \frac{1}{\mu_3}\left(\frac{h}{3} + O_2\right) = \frac{1}{\mu_3}\left[\frac{h}{3} + \frac{h}{3}\left(\frac{\mu_3}{\mu_2} + \frac{\mu_3}{\mu_1}\right)\right]$$

$$= \frac{h}{3}\left(\frac{1}{\mu_1} + \frac{1}{\mu_2} + \frac{1}{\mu_3}\right)$$

This is the required expression of apparent depth.

29. Refer to text on pages 363.

30. Since, light travels in vacuum with a constant velocity, i.e. 3×10^8 m/s, hence ratio of velocities of all wavelengths remains same.

31. We know that, $\mu = \frac{1}{\sin C}$

$$\Rightarrow \quad \sin C = \frac{1}{\mu} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad C = 45^\circ$$

32. Using lateral shift, $d = \frac{t\sin(i_1 - r_1)}{\cos r_1}$

Refer to Example 1 on page 362.

33. Given, $i_1 = 45^\circ$, $t = 10$ cm $= 0.1$ m, $\mu = 1.5$

Lateral shift = ?

By Snell's law, $\mu = \frac{\sin i_1}{\sin r_1} \Rightarrow \sin r_1 = \frac{\sin i_1}{\mu} = \frac{\sin 45^\circ}{1.5}$

$$\Rightarrow \quad \sin r_1 = \frac{0.707}{1.5} \quad \left[\because \sin 45^\circ = 1/\sqrt{2},\ \sqrt{2} = 1.414\right]$$

$$\Rightarrow \quad \sin r_1 = 0.4713$$

$$\Rightarrow \quad r_1 = \sin^{-1}(0.4713) \Rightarrow r_1 = 28.12^\circ$$

$$\text{Lateral shift} = \frac{t\sin(i_1 - r_1)}{\cos r_1} = \frac{0.1\sin(45^\circ - 28.12^\circ)}{\cos 28.12^\circ}$$

$$= \frac{0.1\sin 16.88^\circ}{\cos 28.12^\circ} = \frac{0.1\times0.2904}{0.8819} = 0.033\text{ m}$$

34. Here, $\quad \mu = \frac{\text{real depth / thickness of object}}{\text{apparent depth}}$

Now, due to refraction at two different boundaries, the apparent depth of object is

$$\text{apparent depth} = \frac{\text{thickness of glass}}{\mu_{glass}} + \frac{\text{thickness of water}}{\mu_{water}}$$

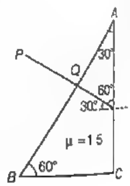
$$= \frac{6}{1.5} + \frac{4}{1.3} = 3 + 4 = 7\text{ cm}$$

35. Given, refractive index of the material of the prism, $\mu = 1.5$

$\therefore$ Critical angle for the material,

$$\sin C = \frac{1}{\mu} = \frac{1}{1.5} = 2/3$$

$$\Rightarrow C = \sin^{-1}\left(\frac{2}{3}\right) = 42^\circ.$$



From the ray diagram, it is clear that angle of incidence $i = 30^\circ < C$.

Therefore, the ray incident at the face AC will not suffer total internal reflection and merges out through this face.

36. *Case I* When tank is filled with the water.

Given, the apparent depth = 9.4 cm

Height of water, t = 12.5 cm

So, real depth = 12.5 cm

Refractive index of water,

$$\mu_w = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{12.5}{9.4} = 1.33$$

Case II When tank is filled with the liquid.

Refractive index of liquid, $\mu_l = 1.63$

[redacted]

Again, $\mu_l = \frac{\text{Real depth}}{\text{Apparent depth}}$

$\Rightarrow \quad 1.63 = \frac{12.5}{\text{Apparent depth}}$

Apparent depth $= \frac{12.5}{1.63} = 7.67$ cm

$\therefore$ The microscope is shifted by $9.4 - 7.67 = 1.73$ cm.

37. Distance of object from mirror

$$= 15 + \frac{33.25}{4} \times 3 = 39.93 \text{ cm}$$

Distance of image from the mirror

$$= 15 + \frac{25}{4} \times 3$$
$$= 33.75 \text{ cm}$$

Using mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

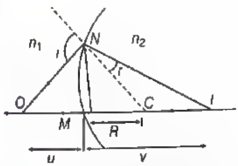
$\Rightarrow \quad \frac{1}{-33.75} - \frac{1}{39.93} = \frac{1}{f}$

$\therefore \quad f = -18.3$ cm

|TOPIC 3|
Refraction at Spherical Surfaces and Lenses

REFRACTION AT A SPHERICAL SURFACE

A refracting surface which forms a part of a sphere of transparent refracting material is called a spherical refracting surface.



Refraction at a spherical surface

In the figure, the geometry of formation of image I of an object O and the principal axis of a spherical surface with centre of curvature C and radius of curvature R.

Assumptions

(i) The aperture of the surface is small as compared to other distances involved.

(ii) NM will be taken to be nearly equal to the length of the perpendicular from the point N on the principal axis.

$$\tan NOM = \frac{MN}{OM}, \quad \tan NCM = \frac{MN}{MC},$$

$$\tan NIM = \frac{MN}{MI}$$

For small angles, $\tan\theta \approx \sin\theta \approx \theta$

So, $\quad \angle NOM = \frac{MN}{OM}$

$$\angle NCM = \frac{MN}{MC}$$

$$\angle NIM = \frac{MN}{MI}$$

For ΔNOC, i is the exterior angle.

$$\therefore \quad i = \angle NOM + \angle NCM = \frac{MN}{OM} + \frac{MN}{MC} \quad \text{...(i)}$$

For ΔNIC, $\angle NCM$ is the exterior angle.

$\therefore \quad \angle NCM = r + \angle NIM$

or $\quad r = \angle NCM - \angle NIM$

i.e. $\quad r = \frac{MN}{MC} - \frac{MN}{MI} \quad \text{...(ii)}$

By Snell's law, $n_1 \sin i = n_2 \sin r$

For small angles, $\quad n_1 i = n_2 r$

Substituting the values of i and r from Eqs. (i) and (ii), we get

$$n_1\left(\frac{MN}{OM} + \frac{MN}{MC}\right) = n_2\left(\frac{MN}{MC} - \frac{MN}{MI}\right)$$

or $\quad \frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC} \quad \text{...(iii)}$

Applying new Cartesian sign conventions,

$$OM = -u, \quad MI = +v$$
$$MC = +R$$

Substituting these values in Eq. (iii), we get

[redacted]

$$\boxed{\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}}$$

This equation holds for any curved spherical surface.

EXAMPLE |1| Light from a point source in air falls on a spherical glass surface ($n = 1.5$ and radius of curvature $= 20$ cm). The distance of the light source from the glass surface is 100 cm. At what position the image is formed?

Sol. Given, object distance, $u = -100$ cm,

$R = +20$ cm, $n_1 = 1$, $n_2 = 1.5$, image distance, $v = ?$

We know that, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

$$\Rightarrow \quad \frac{1.5}{v} + \frac{1}{100} = \frac{1.5 - 1}{20}$$

$$\Rightarrow \quad \frac{1.5}{v} = \frac{0.5}{20} - \frac{1}{100}$$

$$= \frac{2.5 - 1}{100} = \frac{1.5}{100}$$

$$v = +100 \text{cm}$$

Thus, the image is formed at a distance of 100 cm from the glass surface in the direction of incident light.

Cartesian Sign Convention for Spherical Surfaces

(i) The principal axis of the spherical surface is taken as X-axis and the optical centre as origin. Here, the principal axis is the diameter extended.

(ii) The direction of the incident light is taken as the positive direction of X-axis and opposite to it is taken as negative.

(iii) The upward direction is taken as positive and the downward direction as negative.

LENS

Lens is a transparent medium bounded by two surfaces of which one or both surfaces are spherical.

Lenses are of two types

(i) Convex or converging lens

(ii) Concave or diverging lens

Convex or Converging Lens

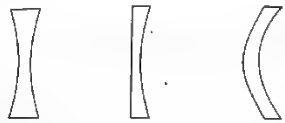
A lens which is thicker at the centre and thinner at its ends is called convex lens. Convex lenses are of three types as shown below.

(a) Double convex lens (b) Plano-convex lens (c) Concavo-convex lens

Note A convex lens is also known as converging lens because it converges a parallel beam of light rays passing through it. A double convex lens is simply called convex lens.

Concave or Diverging Lens

A lens which is thinner at the centre and thicker at its ends is called a concave lens. Concave lenses are of three types as shown below.

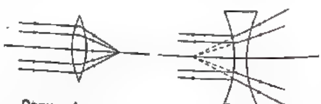


(a) Double concave lens (b) Plano-concave lens (c) Convexo-concave lens

Note A concave lens is also known as diverging lens because it diverges a parallel beam of light rays passing through it. A double concave lens is simply called concave lens.

Converging and Diverging Action of Lenses

As convex lens converges all the light rays, coming parallel to its principal axis at a point, it is also called converging lens. Concave lens diverges all the light rays coming parallel to its principal axis. So, it is also called diverging lens.



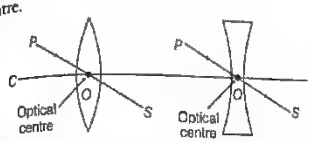
Converging lens Diverging lens

The converging and diverging action of lens can be explained by considering a lens made up of large number of different small angle prisms. In a convex lens, the base of prism is towards principal axis and in concave lens, base of prism is away from the principal axis.

[redacted]

Some Definitions Related to Lenses

(i) **Optical centre** The optical centre is a point lying on the principal axis of the lens, directed to which incident rays pass without any deviation in the path, i.e. the centre point of a lens is known as its optical centre.



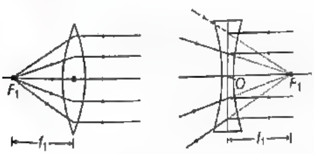
(ii) **Centre of curvature** The centres of the two imaginary spheres of which the lens is a part, are called centres of curvature of the lens. A lens has two centres of curvature with respect to its two curved surfaces.

(iii) **Radii of curvature** The radii of the two imaginary spheres of which the lens is a part are called radii of curvature of the lens. A lens has two radii of curvature. These may or may not be equal.

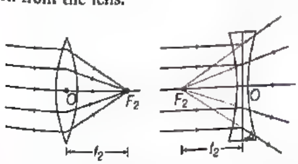
(iv) **Principal axis** The imaginary line joining the two centres of curvature is called principal axis of lens. Principal axis also passes through the optical centre.

(v) **Principal focus** Lens has two principal foci.

(a) **First principal focus** It is a point on the principal axis of lens, the rays starting from this point in convex lens or rays directed to this point in concave lens become parallel to principal axis after refraction.



(b) **Second principal focus** It is a point on the principal axis at which the rays coming parallel to the principal axis converge (convex lens) or passing through it appear to diverge (concave lens) at this point after refraction from the lens.



Both the foci of convex lens are real, while that of concave lens are virtual.

(vi) **Aperture** The effective diameter of the circular outline of a spherical lens is called its aperture.

(vii) **Refractive axis** It is an imaginary axis at the optical centre perpendicular to the principal axis which represents the lens.

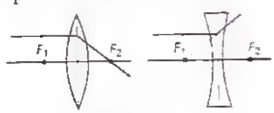
(a) Real path of ray (b) Path of ray as shown with reference to refractive axis

Note When the object is at infinity, the distance of image from the lens will be equal to the focal length of the lens.

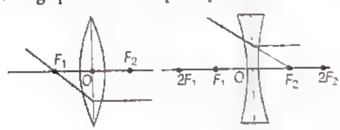
Image Formation in Lenses Using Ray Diagrams

We can represent image formation in lenses using ray diagrams. For drawing ray diagrams in lenses like spherical mirrors, we consider any two of the following rays.

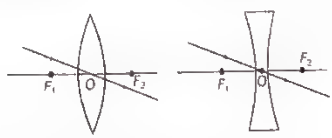
(i) Rays which are parallel to the principal axis after refraction, will pass through principal focus in case of convex lens and will appear to be coming from principal focus in case of concave lens.



(ii) Rays passing through or directed to the focus will emerge parallel to the principal axis.



(iii) Rays directed to optical centre will emerge out undeviated.



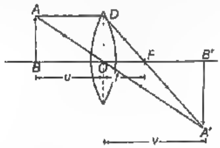
THIN LENS FORMULA

It is a relation between focal length of a lens and distances of object and image from optical centre of the lens.

Let O be the optical centre and f be the principal focus of a convex lens of focal length $OF = f$. AB is an object held

[redacted]

perpendicular to the principal axis of the lens at a distance beyond focal length of the lens. A real, inverted and magnified image $A'B'$ is formed as shown in the figure. As, $\Delta A'B'O$ and ΔABO are similar.



$$\therefore \quad \frac{A'B'}{AB} = \frac{OB'}{OB} \qquad ...(i)$$

Again, $\Delta A'B'F$ and ΔDOF are similar.

$$\therefore \quad \frac{A'B'}{OD} = \frac{FB'}{OF}$$

But $\quad OD = AB$

$$\therefore \quad \frac{A'B'}{AB} = \frac{FB'}{OF} \qquad ...(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{OB'}{OB} = \frac{FB'}{OF} = \frac{OB' - OF}{OF}$$

Using new cartesian sign conventions,

Let $\quad OB = -u, \quad OB' = +v,$

$$OF = +f$$

$$\therefore \quad \frac{v}{-u} = \frac{v-f}{f}$$

$$\Rightarrow \quad vf = -uv + uf$$

or $\quad uv = uf - vf$

Dividing both sides by uvf, we get

$$\frac{uv}{uvf} = \frac{uf}{uvf} - \frac{vf}{uvf} \Rightarrow \boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$$

This is the thin lens formula.

This formula can also be proved for concave lens and for virtual images in the same way.

EXAMPLE |2| A convergent beam of light passes through the diverging lens of focal length 0.2 m and comes to focus 0.3 m behind the lens. Find the position of the point at which the beam would converge in the absence of the lens.

Sol. Given, focal length, $f = 0.2$ m

Image distance, $v = -0.3$ m

Object distance, $u = ?$

From thin lens formula,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

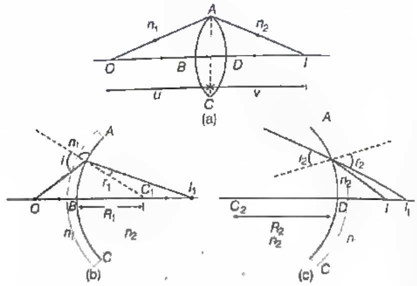
$$\Rightarrow \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f}$$

$$= \frac{1}{-0.3} - \frac{1}{0.2} = \frac{-0.5}{0.06}$$

$$\Rightarrow \quad u = \frac{-0.06}{0.5}$$

$\therefore$ Object distance, $u = -0.12$ m

Refraction by a Lens : Lens Maker's Formula



The above figures show the image formation by a convex lens.

Assumptions Made in the Derivation

Some assumptions made from the derivation are as

(i) The lens is thin, so that distances measured from the poles of its surfaces can be taken as equal to the distance from the optical centre of the lens.

(ii) The aperture of the lens is small.

(iii) The object considered as a point lying on the principal axis of the lens.

(iv) The incident ray and refracted ray make small angles with the principal axis of the lens.

(v) A convex lens is made up of two convex spherical refracting surfaces.

(vi) The first refracting surface forms image I_1 of the object O [Fig. (b)].

(vii) Image I_1 acts as virtual object for the second surface that forms the image at I [Fig. (c)].

Applying the equation for spherical refracting surface to the first interface ABC, we get

[redacted]

$$\frac{n_1}{OB}+\frac{n_2}{BI_1}=\frac{n_2-n_1}{BC_1} \quad \ldots(i)$$

A similar procedure applied to the second interface ADC, we get

$$-\frac{n_2}{DI_1}+\frac{n_1}{DI}=\frac{n_2-n_1}{DC_2} \quad \ldots(ii)$$

For a thin lens, $BI_1 = DI_1$

Adding Eqs. (i) and (ii), we get

$$\frac{n_1}{OB}+\frac{n_1}{DI}=(n_2-n_1)\left(\frac{1}{BC_1}+\frac{1}{DC_2}\right) \quad \ldots(iii)$$

Suppose the object is at infinity, i.e.

$$OB \to \infty \quad \text{and} \quad DI \to f$$

So, Eq. (iii) can be written as,

$$\frac{n_1}{f}=(n_2-n_1)\left(\frac{1}{BC_1}+\frac{1}{DC_2}\right) \quad \ldots(iv)$$

The point where image of an object placed at infinity is formed is called the focus (f) of the lens and the distance f gives its focal length. A lens has two foci, F and F' on either side of it by sign convention.

$$BC_1 = R_1$$

or

$$DC_2 = -R_2$$

Therefore, Eq. (iv) can be written as

$$\boxed{\frac{1}{f}=({}_1n_2-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)} \quad \left[\because {}_1n_2=\frac{n_2}{n_1}\right] \ldots(v)$$

Eq. (v) is known as the lens Maker's formula.

Putting $\frac{n_2}{n}=n$, refractive index of material of lens w.r.t. its surroundings, we get

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

From Eqs. (iii) and (iv), we get

$$\frac{n_1}{OB}+\frac{n_1}{DI}=\frac{n_1}{f} \quad \ldots(vi)$$

As, B and D both are close to the optical centre of the lens,

$OB=-u$, $DI=+v$, we get

$$\boxed{\frac{1}{v}-\frac{1}{u}=\frac{1}{f}} \quad \ldots(vii)$$

Eq. (vii) is the thin lens formula.

From lens maker's formula, it is clear that focal length of lens depends upon radii of curvature of lens and refractive index of material of lens w.r.t. its surroundings.

EXAMPLE |3| **The radii of curvature of the surfaces of a double convex lens are 20 cm and 40 cm, respectively and its focal length is 20 cm. What is refractive index of the material of the lens?**

Sol. Given, $R_1 = 20\text{ cm}$, $R_2=-40\text{ cm}$, $f = 20\text{cm}$, $n=?$

We know that, $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\Rightarrow \quad \frac{1}{20}=(n-1)\left(\frac{1}{20}+\frac{1}{40}\right)$$

$$\Rightarrow \quad \frac{1}{n-1}=20\left(\frac{2+1}{40}\right)=\frac{3}{2}$$

$$\Rightarrow \quad n-1=\frac{2}{3} \Rightarrow 3n=5 \Rightarrow n=\frac{5}{3}$$

Hence, the refractive index of the material of the lens is 5/3.

Dependence of Focal Length on Refractive Index

Refractive index of material of lens depends upon the medium in which it is kept. Generally, the lens is placed in air, so in the above formula, n is the refractive index of material of lens with respect to air. If lens is placed in a medium other than air, then due to change in refractive index (n), focal length of the lens changes. If lens is immersed in a liquid whose refractive index with respect to air is less than the refractive index of material of lens with respect to air, then focal length of the lens increases.

If lens is immersed in a liquid whose refractive index with respect to air is more than the refractive index of material of the lens with respect to air, then focal length will become negative. That means, the nature of lens will change in such a medium, convex lens will behave like concave lens and concave lens will behave like convex lens.

If lens is immersed in a liquid whose refractive index with respect to air is equal to the refractive index of material of lens with respect to air, then focal length of the lens will become infinite and it will behave like plane glass sheet. Also, in such medium, lens will become invisible.

Dependence of Focal Length on the Radii of Curvature

From lens maker's formula, it is clear that the focal length of a lens of large radii of curvature is large and that of a lens of small radii of curvature is small. In simple words, the focal length of thin lens is large and that of thick lens is small. For plano-convex or plano-concave lens, $R_1 = R$ and $R_2=\infty$ (for plane surface).

[redacted]

Linear Magnification Produced by a Lens (m)

Linear magnification of a lens is defined as, the ratio of the height of the image formed by the lens to height of the object.

$$\text{Linear magnification } (m) = \frac{\text{Height of image} (I)}{\text{Height of object } (O)}$$

For Convex Lens

When image is real, $m = \frac{-I}{O} = \frac{v}{-u}$

When image is real, it is inverted and forms on the other side of object.

When image is virtual, $m = \frac{I}{O} = \frac{v}{u}$

When image is virtual, it is erect and forms on the same side of object. Thus, it can be said that convex lens gives positive linear magnification for virtual image and negative linear magnification for real image.

For Concave Lens

Concave lens always forms virtual image, so linear magnification of concave lens, $m = \frac{I}{O} = \frac{v}{u}$.

Concave lens always gives positive linear magnification. Other formulae for linear magnification are

$$m = \frac{v}{u} = \frac{f - v}{f} = \frac{f}{f + u}$$

EXAMPLE |4| **The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. Find the ratio of $\frac{m_{25}}{m_{50}}$.**

Sol. Since, magnification, $m = \frac{f}{f + u}$

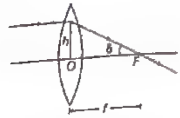
$$\Rightarrow \quad m_{25} = \frac{20}{20 - 25} = -4$$

Similarly, $$m_{50} = \frac{20}{20 - 50} = \frac{-2}{3}$$

Therefore, $$\frac{m_{25}}{m_{50}} = (-4)\left(\frac{-3}{2}\right) = 6$$

Power of a Lens

The ability of a lens to converge or diverge the rays of light incident on it is called the power of the lens.



Power of a lens is defined as the tangent of the angle by which it converges or diverges a beam of light falling at unit distance from the optical centre.

According to the figure.

$$\tan\delta = \frac{h}{f}, \quad \text{if } h = 1, \text{ then}$$

$$\tan\delta = \frac{1}{f}$$

For small values of δ, $\tan\delta \approx \delta$

$$\therefore \quad \delta = \frac{1}{f}$$

Thus, power of a lens, $P = \frac{1}{f}$.

The SI unit of power of lens is dioptre (D). The power of a lens is measured as the reciprocal of its focal length (in metre).

$$P = \frac{1}{f \text{ (in m)}}$$

If $f = 1\,\text{m}$, then $P = 1\,\text{m}^{-1} = 1\,\text{dioptre (D)}$

According to the lens Maker's formula for a lens,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \left[\because P = \frac{1}{f}\right]$$

We have, $$\boxed{P = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$$

Here, R_1 and R_2 are to be measured in metre.

For converging (convex) lens, power is positive and for diverging (concave) lens, power is negative.

EXAMPLE |5| **If the radii of curvature of the faces of a double convex lens are 9 cm and 15 cm, respectively and the refractive index of glass is 1.5, then determine the focal length and the power of the lens.**

Sol. Given, radii of curvature, $R_1 = 9\,\text{cm}$, $R_2 = -15\,\text{cm}$, refractive index, $\mu = 1.5$, $f = ?$, $P = ?$

According to lens Maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f} = (1.5 - 1)\left[\frac{1}{9} - \left(\frac{-1}{15}\right)\right]$$

[redacted]

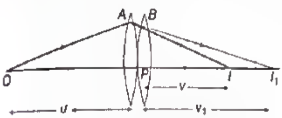
$$\Rightarrow \quad \frac{1}{f} = (0.5)\left(\frac{1}{9} + \frac{1}{15}\right) = (0.5)\left(\frac{5+3}{45}\right)$$

$$\Rightarrow \quad \frac{1}{f} = 0.5 \times \frac{8}{45} \Rightarrow f = \frac{45}{4} = 11.25\text{ cm}$$

$$\therefore \text{ Power,} \quad P = \frac{1}{f} = \frac{1}{11.25 \times 10^{-2}} = \frac{10000}{1125} = 8.88\text{ D}$$

Combination of Thin Lenses in Contact

Consider two lenses A and B of focal lengths f_1 and f_2 placed in contact with each other. An object is placed at a point O beyond the focus of the first lens A. The first lens produces an image at I_1 (virtual image), which serves as a virtual object for the second lens B, producing the final image at I.



Since, the lenses are thin, we assume the optical centres (P) of the lenses to be co-incident.

For the image formed by the first lens A, we obtain

$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1} \quad \ldots\text{(i)}$$

For the image formed by the second lens B, we get

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2} \quad \ldots\text{(ii)}$$

Adding Eqs. (i) and (ii), we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} \quad \ldots\text{(iii)}$$

If the two lens system is regarded as equivalent to a single lens of focal length f. We have,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \ldots\text{(iv)}$$

From Eqs. (iii) and (iv), we get

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}} \quad \ldots\text{(v)}$$

For several thin lenses of focal lengths $f_1, f_2, f_3, \ldots$, the effective focal length is

$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots} \quad \ldots\text{(vi)}$$

In terms of power, Eq. (vi) can be written as

$$\boxed{P = P_1 + P_2 + P_3 + \ldots}$$

EXAMPLE |6| Two thin lenses are in contact and the focal length of the combination is 80 cm. If the focal length of one lens is 20 cm, then what would be the power of the another lens?

Sol. Given, combined focal length, $F = 80$ cm,

$f_1 = 20$ cm, $P_2 = ?$

$$\therefore \quad P = \frac{100}{F\text{ (cm)}} = \frac{100}{80} = 1.25\text{ D}$$

$$\Rightarrow \quad P_1 = \frac{100}{f_1} = \frac{100}{20} = 5\text{ D}$$

We know that, $P_1 + P_2 = P$

$$\therefore \quad P_2 = P - P_1 = 1.25 - 5 = -3.75\text{ D}$$

Magnification by Combination of Lenses

Suitable combination of lenses helps to obtain diverging or converging lens of desired magnification. It also enhances sharpness of the image. Since, the image formed by the first lens becomes the object for second lens and so on. So, the magnification of combination (m) is the product of magnification (m_1, m_2, m_3) of individual lenses.

Magnification of combination of lenses,

$$m = m_1 \times m_2 \times m_3 \times \ldots$$

(i) If combination of lenses consists of one convex lens (f_1) and one concave lens ($-f_2$), then for combination of lenses,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{-f_2} = \frac{1}{f_1} - \frac{1}{f_2}$$

$$\Rightarrow \quad \boxed{f = \frac{f_1 f_2}{f_2 - f_1}}$$

(ii) If $f_1 > f_2$, then f is negative, i.e. combination will behave like concave lens, when focal length of convex lens is larger. If $f_1 < f_2$, then f is positive, i.e. combination will behave like convex lens, when focal length of convex lens is smaller. If $f_1 = f_2$, then f is infinite, i.e. combination will behave like plane glass sheet.

If the lenses are placed d distance apart, then

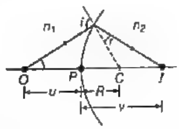
$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}}$$

[redacted]

TOPIC PRACTICE 3

OBJECTIVE Type Questions

1. For the refraction shown below the correct relation is,



(a) $\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$ (b) $\frac{n_1}{v}-\frac{n_2}{u}=\frac{n_2-n_1}{R}$

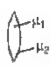
(c) $\frac{n_1}{v}-\frac{n_2}{u}=\frac{n_1-n_2}{R}$ (d) $\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_1-n_2}{R}$

2. Light from a point source in air falls on a spherical glass surface ($n = 1.5$ and radius of curvature $= 20$ cm). The distance of the light source from the glass surface is 100 cm. Image distance from the glass surface is
(a) 20 cm (b) 50 cm
(c) 100 cm (d) 75 cm

3. First and second focal lengths of spherical surface of n refractive index are f_1 and f_2 respectively. The relation between them, is
(a) $f_2 = f_1$ (b) $f_2 = -f_1$ (c) $f_2 = nf_1$ (d) $f_2 = -nf_1$

4. A magician during a show makes a glass lens with $n = 1.47$ disappear in a trough of liquid. Refractive index of the liquid is
(a) 1.47 (b) 1.33 (c) $\frac{4}{3}$ (d) $\frac{12}{5}$

5. Which of the following is true for rays coming from infinity?



(a) Two images are formed
(b) Continuous image is formed between focal points of upper and lower lens
(c) One image is formed
(d) None of the above

6. Two thin lenses are in contact and that combination has 15 cm focal length. If one lens has focal length 30 cm, then what is the second lens focal length?
(a) 15 cm (b) 25 cm
(c) 20 cm (d) 30 cm

7. The radius of curvature of the curved surface of a plano-convex lens is 20 cm. If the refractive index of the material of the lens be 1.5, it will **NCERT Exemplar**
(a) act as a convex lens only for the objects that lie on its curved side
(b) act as a concave lens for the objects that lie on its curved side
(c) act as a convex lens irrespective of the side on which the object lies
(d) act as a concave lens irrespective of side on which the object lies

8. Two lenses are in contact having focal length 25 cm and − 40 cm. Find power of this combination.
(a) − 6.67 D (b) − 2.5 D (c) + 1.5 D (d) + 4 D

9. Two lenses are in contact having powers of 5 D and − 3 D. The focal length of this combination will be
(a) 50 cm (b) 75 cm (c) 25 cm (d) + 20 cm

VERY SHORT ANSWER Type Questions

10. A beam of light is converging towards a certain point. A parallel sided glass plate is introduced in the path of the converging beam. How will the point of convergence be shifted?

11. What type of lens is an air bubble inside water?

12. A concave lens of refractive index 1.5 is immersed in a medium of refractive index 1.65. What is the nature of the lens? **All India 2015**

13. A biconvex lens made of a transparent material of refractive index 1.25 is immersed in water of refractive index 1.33. Will the lens behave as a converging or a diverging lens? Give reason. **All India 2014**

14. Under what condition, does a biconvex lens of glass having a certain refractive index act as a plane glass sheet when immersed in a liquid? **Delhi 2012**

15. A glass lens is immersed in water. How is power of the lens affected?

SHORT ANSWER Type Questions

16. The lens shown in the given figure is made of two different materials. A point object is placed on the principal axis of this lens. How many images will be obtained?

[redacted]

17. Show analytically from the lens equation that when the object is at the principal focus, the image is formed at infinity.

18. A student measures the focal length of a convex lens by putting an object pin at a distance u from the lens and measuring the distance v of the image pin. What will be the graph drawn between u and v?

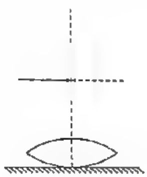
19. A magician during a show makes a glass lens $n = 1.47$ disappear in a trough of liquid. What is the refractive index of the liquid? Could the liquid be water?

LONG ANSWER Type I Questions

20. An equiconvex lens of focal length f is cut into two equal halves in thickness. What is the focal length of each half?

21. Define power of a lens. Write its units. Deduce the relation $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$ for two thin lenses kept in contact coaxially. **Foreign 2012**

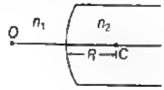
22. A symmetric biconvex lens of radius of curvature R and made of glass of refractive index 1.5, is placed on a layer of liquid placed on the top of a plane mirror as shown in the figure. An optical needle with its tip on the principal axis of the lens is moved along the axis until its real, inverted image coincides with the needle itself. The distance of the needle from the lens is measured to be x. On removing the liquid layer and repeating the experiment, the distance is found to be y. Obtain the expression for the refractive index of the liquid in terms of x and y. **CBSE 2018**



23. The objective of an astronomical telescope has a diameter of 150 mm and a focal length of 4 m. The eyepiece has a focal length of 25 mm. Calculate the magnifying and resolving power of telescope ($\lambda = 6000$ Å for yellow colour). **Delhi 2011**

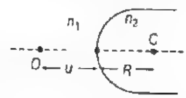
LONG ANSWER Type II Questions

24. Figure shows a convex spherical surface with centre of curvature C, separating the two media of refractive indices n_1 and n_2. Draw a ray diagram showing the formation of the image of a point object O lying on the principal axis. Derive the relationship between the object and image distance in terms of refractive indices of the media and the radius of curvature R on the surface.



All India 2014

25. (i) A point object O is kept in a medium of refractive index n_1 in front of a convex spherical surface of radius of curvature R which separates the second medium of refractive index n_2 from the first one, as shown in the figure.
Draw the ray diagram showing the image formation and deduce the relationship between the object distance and the image distance in terms of n_1, n_2 and R.



(ii) When the image formed above acts as a virtual object for a concave spherical surface separating the medium n_2 from $n_1 (n_2 > n_1)$, draw this ray diagram and write the similar [similar to (i)] relation. Hence obtain the expression for the lens Maker's formula. **All India 2015**

NUMERICAL PROBLEMS

26. A converging lens of refractive index 1.5 is kept in a liquid medium having the same refractive index. What would be the focal length of lens in the medium?

27. The radii of curvature of the faces of a double convex lens are 10 cm and 15 cm. If focal length of the lens is 12 cm, find the refractive index of the material of the lens. **Delhi 2010**

28. Find the radius of curvature of the convex surface of a plano-convex lens, whose focal length is 0.3 m and the refractive index of the material of the lens is 1.5. **Delhi 2010**

29. A biconvex lens has a focal length 2/3 times the radius of curvature of either surface. Calculate the refractive index of lens material. **Delhi 2010**

30. What is the focal length of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20 cm? Is the system a

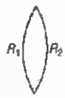
[redacted]

converging or a diverging lens? Ignore thickness of the lenses. **NCERT**

31. (i) Monochromatic light of wavelength 589 nm is incident from air on a water surface. If μ for water is 1.33, find the wavelength, frequency and speed of the refracted light.

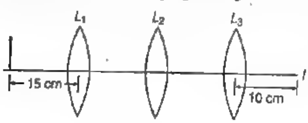
(ii) A double convex lens is made of a glass of refractive index 1.55 with both faces of the same radius of curvature. Find the radius of curvature required, if the focal length is 20 cm. **All India 2017**

32. Double convex lenses are to be manufactured from a glass of refractive index 1.55, with both faces of the same radius of curvature. What is the radius of curvature required, if the focal length is to be 20 cm? **NCERT**



33. The image obtained with a convex lens is erect and its length is four times the length of the object. If the focal length of the lens is 20 cm, calculate the object and image distances. **All India 2010**

34. You are given three lenses L_1, L_2 and L_3 each of focal length 10 cm. An object is kept at 15 cm in front of L_1, as shown in figure. The final real image is formed at the focus of L_3. Find the separation between L_1, L_2 and L_3. **All India 2012**



HINTS AND SOLUTIONS

1. (a) As refraction formula for curved surface is

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

2. (c) Here, $u = -100$ cm, $v = ?$, $R = +20$ cm, $n_1 = 1$ and $n_2 = 1.5$

As, refraction formula for curved surface, we have

$$\frac{1.5}{v} + \frac{1}{100} = \frac{0.5}{20} \Rightarrow v = +100 \text{ cm}$$

The image is formed at a distance of 100 cm from the glass surface, in the direction of incident light.

3. (b) When medium is equal on both sides of lens, then the numerical value of both focal length is equal, hence $f_2 = -f_1$.

4. (a) The refractive index of the liquid must be equal to 1.47 in order to make the lens disappear. This means $n_1 = n_2$. This give $1/f = 0$ or $f \to \infty$.

5. (a) Since, lens is made of two layers of different refractive indices, for a given wavelength of light it will have two different focal lengths or will have two images at two different points as $\frac{1}{f} \propto (\mu - 1)$ (from Lens maker's formula).

6. (d) Given, $F = 15$ cm, $f_1 = 30$ cm

We know that, $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$

$$\Rightarrow \frac{1}{15} = \frac{1}{30} + \frac{1}{f_2}$$

$$\frac{1}{f_2} = \frac{1}{15} - \frac{1}{30} = \frac{2-1}{30}$$

$$\Rightarrow f_2 = 30 \text{ cm}$$

7. (c) Here, $R = 20$ cm, $\mu = 1.5$, on substituting the values in $f = \frac{R}{\mu - 1} = \frac{20}{1.5 - 1} = 40$ cm of converging nature as $f > 0$.

Therefore, lens act as a convex lens irrespective of the side on which the object lies.

8. (c) Given, $f_1 = 25$ cm, $f_2 = -40$ cm

$$\therefore \quad P_1 = \frac{100}{f_1} = \frac{100}{25} = +4 \text{ D}$$

and $$P_2 = \frac{100}{f_2} = \frac{100}{-40} = -2.5 \text{ D}$$

$$\therefore \quad P = P_1 + P_2 = 4 + (-2.5) \text{ D} = +1.5 \text{ D}$$

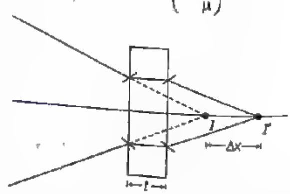
9. (a) Given, $P_1 = 5$ D, $P_2 = -3$ D

$$\therefore \quad P = P_1 + P_2 = 5 + (-3) = 2 \text{ D}$$

$$\therefore \quad P = \frac{1}{f} \Rightarrow 2 = \frac{1}{f}$$

$$\Rightarrow \quad f = \frac{1}{2} \text{ m} = \frac{100}{2} \text{ cm} = 50 \text{ cm}$$

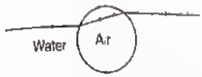
10. Here, shift is given as $\Delta x = \left(1 - \frac{1}{\mu}\right)t$



which takes place in the direction of ray.

[redacted]

11. It is clearly visible that air bubble acts as a diverging lens (concave lens) in water.



12. A concave lens behaves as a diverging lens, when it is placed in a medium of refractive index less than the refractive index of the material of the lens and behaves as a converging lens, when it is placed in a medium of refractive index greater than the refractive index of the material of the lens

In the given case, concave lens is immersed in a medium having refractive index greater than the refractive index of the material of the lens ($1.65 > 1.5$). Therefore, it will behave as a converging lens.

13. When a lens is placed in a liquid, where refractive index is more than that of the material of lens, then the nature of the lens changes. So, when a biconvex lens of refractive index 1.25 is immersed in water (refractive index 1.33), i.e. in the liquid of higher refractive index, its nature will change. So, biconvex lens will act as converging or diverging lens.

14. When refractive index of lens is equal to the refractive index of liquid, it will behave like plane glass sheet.

15. We know that, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

For glass $n_2 = 1.5$, for air, $n_1 = 1$, for water $n = 1.33$

$$\therefore \quad \frac{1}{f} = \left(\frac{1.5}{1.33} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

So, focal length becomes 4 times, hence power becomes $\frac{1}{4}$th of the initial value.

16. Since, refractive index of each material is different, so the lens will have two different focal lengths, one for each material. Hence, two images will formed.

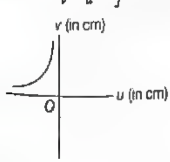
17. Given, $u = -f$

$\therefore$ Lens equation is, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\Rightarrow \quad \frac{1}{v} + \frac{1}{f} = \frac{1}{f} \Rightarrow \frac{1}{v} = 0$$

$$\Rightarrow \quad v = \frac{1}{0} = \text{infinity}$$

18. As we know that, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$



$\therefore$ Correct answer is 1.

19. If $\mu_1 = \mu_2$, then $f = \infty$

Hence, the lens in the liquid acts like a plane sheet, when refractive index of the lens and the surrounding medium is the same. Therefore, $\mu_1 = \mu_2 = 1.47$

Hence, the liquid medium is not water, refractive index for water $= 1.33$.

20. Focal length can be given as

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where, μ is the refractive index of the lens medium. R_1 and R_2 are radii of curvature.

Equiconvex lens have the same radius of curvature,

i.e. $\quad R_1 = -R_2$

$$\therefore \quad \frac{1}{f'} = (\mu - 1)\left[\frac{1}{R} - \left(-\frac{1}{R}\right)\right] \Rightarrow \frac{1}{f'} = \frac{2(\mu - 1)}{R}$$

$$\therefore \quad f' = 2f$$

Hence, focal length of each half becomes twice of the original value.

21. Refer to text on pages 374 and 375.

22. First measurement gives the focal length ($f_{eq} = x$) combination of the convex lens and the plano-convex liquid lens. Second measurement gives the focal length ($f_1 = y$) of the convex lens.

Focal length (f_2) of plano-convex lens is given by

$$\frac{1}{f_2} = \frac{1}{f_{eq}} - \frac{1}{f_1} = \frac{1}{x} - \frac{1}{y}$$

$$\Rightarrow \quad f_2 = \frac{xy}{y - x} \quad \text{...(i)}$$

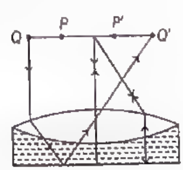
For equiconvex glass lens using Lens Maker's formula, we get

$$\frac{1}{f_1} = (n_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{y} = (1.5 - 1)\left(\frac{2}{R}\right)$$

(As $R_1 = R$ and $R_2 = -R$)

$$\Rightarrow \quad \frac{1}{y} = \frac{1}{2} \times \frac{2}{R} \Rightarrow R = y$$



Now, we apply Lens Maker's formula for plano-convex lens.

Here $R_1 = R$ and $R_2 = \infty$ and let n_l = refractive index of liquid

[redacted]

$$\frac{1}{f_2} = (n_l - 1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$$\Rightarrow \quad \frac{1}{f_2} = (n_l - 1)\left(\frac{1}{R}\right)$$

$$\Rightarrow \quad n_l = 1 + \frac{R}{f_2} = 1 + \frac{y}{\left(\frac{xy}{y-x}\right)}$$

$$= 1 + \frac{y-x}{x} = \frac{y}{x}$$

23. The diameter of objective of the telescope

$$= 150 \times 10^{-3} \text{ m}, f_o = 4 \text{ m}$$

$$f_e = 25 \times 10^{-3} \text{ m and } D = 0.25 \text{ m}$$

Magnifying power, $m = -\frac{f_o}{f_e}\left(1 + \frac{D}{f_e}\right)$

$$= -\frac{4}{25 \times 10^{-3}}\left(1 + \frac{0.25}{25 \times 10^{-3}}\right) = -1760$$

Now, $\quad d\theta = \frac{1.22\lambda}{D} = \frac{1.22 \times 6 \times 10^{-7}}{0.25}$

$$= 2.9 \times 10^{-6} \text{ rad}$$

$\therefore$ Resolving power $= \frac{1}{d\theta} = \frac{1}{2.9 \times 10^{-6}}$

$$= 0.34 \times 10^{6}$$

24. Refer to text on pages 369 and 370.

25. Let a spherical surface separate a rarer medium of refractive index n_1 from the second medium of refractive index n_2. Let C be the centre of curvature and $R = MC$ be the radius of the surface.

Consider a point object O lying on the principal axis of the surface. Let a ray starting from O incident normally on the surface along OM and pass straight. Let another ray of light incident on NM along ON and refract along NI. From M, draw MN perpendicular to OI.

The above figure shows the geometry of the formation of image I of an object O and the principal axis of a spherical surface with centre of curvature C and radius of curvature R.

Here, we have to make following assumptions,

(i) the aperture of the surface is small as compared to the other distance involved

(ii) NM will be taken as nearly equal to the length of the perpendicular from the point N on the principal axis.

$$\tan \angle NOM = \frac{MN}{OM}, \quad \tan \angle NCM = \frac{MN}{MC}$$

$$\tan \angle NIM = \frac{MN}{MI}$$

For ΔNOC, is the exterior angle.

$$\therefore \quad \angle i = \angle NOM + \angle NCM$$

For small angles, $\quad i = \frac{MN}{OM} + \frac{MN}{NC} \quad$ ...(i)

Similarly, $\quad r = \angle NCM - \angle NIM$

$$\Rightarrow \quad r = \frac{MN}{NC} - \frac{MN}{NI} \quad \text{...(ii)}$$

By Snell's law, we get

$$n_1 \sin i = n_2 \sin r$$

For small angles, $n_1 i = n_2 r$

Put the values of i and r from Eqs. (i) and (ii), we get

$$n_1\left(\frac{MN}{OM} + \frac{MN}{MC}\right) = n_2\left(\frac{MN}{MC} - \frac{MN}{MI}\right)$$

$$\Rightarrow \quad \frac{n_1}{OM} + \frac{n_2}{MI} = \frac{n_2 - n_1}{MC} \quad \text{...(iii)}$$

Applying new cartesian sign conventions, we get

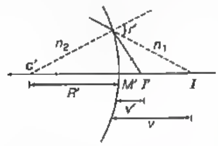
$$OM = -u, \; MI = +v$$

and $\quad MC = +R$

Substituting this in Eq. (iii), we get

$$\frac{n_2}{v} - \frac{n_2}{u} = \frac{n_2 - n_1}{R} \quad \text{...(iv)}$$

Now, the image I' acts as a virtual object for the second surface that will form a real at I. As, refraction takes place from denser to rarer medium,



$$\therefore \quad \frac{-n_2}{v} + \frac{n_1}{v'} = \frac{n_2 - n_1}{-R'} \quad \text{...(v)}$$

On adding Eqs. (iv) and (v), we get

$$\frac{1}{f} = (n_{21} - 1)\left(\frac{1}{R} - \frac{1}{R'}\right) \quad \left[\because n_{21} = \frac{n_2}{n_1}, \frac{1}{f} = \frac{1}{v} - \frac{1}{u}\right]$$

26. When lens is immersed in a liquid, then

$$\frac{1}{f_L} = ({}^L\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

where, ${}^L\mu_g$ = refractive index of lens material (glass) w.r.t. liquid.

$$\therefore \quad \frac{\mu_g}{\mu_L} = \frac{1.5}{1.5} = 1$$

Hence, $\frac{1}{f_L} = (1-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0 \Rightarrow f_L = \infty$

27. Given, $R_1 = +10$ cm, $R_2 = -15$ cm, $f = +12$ cm, $\mu = ?$

Applying lens Maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{12} = (\mu - 1)\left(\frac{1}{10} + \frac{1}{15}\right) = (\mu - 1)\frac{5}{30}$$

$$\Rightarrow \quad (\mu - 1) = \frac{1}{2} \Rightarrow \mu = \frac{3}{2}$$

[redacted]

28. For a plano-convex lens, $R_1 = \infty$

$R_2 = -R$, $f = 0.3\ \text{m} = 30\ \text{cm}$

$\mu = 1.5$

Radius of curvature of plano-convex lens, $R = ?$

Applying lens Maker's formula, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$$\Rightarrow \quad \frac{1}{30} = (\mu - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$$

$$= \frac{(1.5 - 1)}{R} \Rightarrow R = 15\ \text{cm}$$

29. Given, $f = \frac{2}{3}R$, $R_1 = +R$, $R_2 = -R$

$\therefore$ Using lens Maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{3}{2R} = (\mu - 1)\left(\frac{2}{R}\right)$$

$$\Rightarrow \quad \mu - 1 = \frac{3}{4}$$

$$\Rightarrow \quad \mu = 1 + \frac{3}{4} = \frac{7}{4}$$

30. Given, focal length of convex lens, $f_1 = 30$ cm

Focal length of concave lens, $f_2 = -20$ cm

Using the formula of combination of lenses,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{30} - \frac{1}{20} = \frac{2-3}{60} = -\frac{1}{60}$$

$$\Rightarrow \quad f = -60\ \text{cm}$$

Since, the focal length of combination is negative in nature. So, the combination behaves like a diverging lens, i.e. as a concave lens.

31. (i) In refraction, frequency remains same, so

$f_{\text{refracted beam}} = f_{\text{incident beam}}$

Also, $\mu_{21} = \frac{v_1}{v_2} = \frac{f\lambda_1}{f\lambda_2} = \frac{\lambda_1}{\lambda_2}$ $[\because v = f\lambda]$

$$\Rightarrow v_2 = \frac{v_1}{\mu_{21}} = \frac{3 \times 10^8}{1.33} = 2.25 \times 10^8\ \text{ms}^{-1}$$

$$\therefore \quad \lambda_2 = \frac{\lambda_1}{\mu_{21}} = \frac{589}{1.33} = 442.85 \approx 443\ \text{nm}$$

So, wavelength of reflected beam = 443 nm and its speed = $2.25 \times 10^8\ \text{ms}^{-1}$

(ii) For a biconvex lens, using lens Maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $f = 20$ cm, $\mu = 1.55$

$\Rightarrow$ $R_1 = +R$ and $R_2 = -R$

We have, $\frac{1}{f} = (\mu - 1)\frac{2}{R}$

$$\Rightarrow \quad R = 2(\mu - 1)f = 2 \times (1.55 - 1) \times 20 = 22\ \text{cm}$$

$\therefore$ Radius of 22 cm is required.

32. Given, the refractive index of glass with respect to air,

$${}^a\mu_g = 1.55$$

For double convex lenses, $R_1 = R$, $R_2 = -R$

[$\because$ both faces have same radius of curvature]

[for double convex lens, one radius is taken as positive and other negative]

Focal length of lens, $f = +20$ cm

Using the lens Maker's formula,

$$\frac{1}{f} = ({}^a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{20} = (1.55 - 1)\left(\frac{1}{R} + \frac{1}{R}\right)$$

$$\Rightarrow \quad \frac{1}{20} = 0.55 \times \frac{2}{R} \Rightarrow R = 0.55 \times 2 \times 20 = 22\ \text{cm}$$

Thus, the required radius of curvature is 22 cm.

33. As magnification, $m = \frac{I}{O} = \frac{v}{u} \Rightarrow I = 4 \times$ length of object

$$\Rightarrow \quad \frac{I}{O} = 4 \Rightarrow \frac{v}{u} = 4 \Rightarrow v = 4u$$

Using lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{(-4u)} - \frac{1}{(-u)}$

$$\Rightarrow \quad \frac{1}{f} = -\frac{1}{4u} + \frac{1}{u} \Rightarrow \frac{1}{20} = \frac{4-1}{4u} = \frac{3}{4u}$$

$$\Rightarrow \quad u = \frac{20 \times 3}{4} = 15\ \text{cm}$$

$$\Rightarrow \quad v = 4u = 15 \times 4 = 60\ \text{cm}$$

Distance of the object, $u = 15$ cm

Distance of the image, $v = 60$ cm

The image is on the same side of the object.

34. For lens L_1, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

Given, $u = -15$ cm, $f = +10$ cm, $v = ?$

$$\therefore \quad \frac{1}{10} = \frac{1}{v} + \frac{1}{15} \Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{15} \Rightarrow \frac{1}{v} = \frac{1}{30}$$

Distance of image from lens L_1, $v = 30$ cm

For lens L_3, $\frac{1}{f''} = \frac{1}{v''} - \frac{1}{u''}$

Distance of image from lens L_3, $v'' = 10$ cm

$$\therefore \quad \frac{1}{10} = \frac{1}{10} - \frac{1}{u''} \Rightarrow \frac{1}{u''} = 0 \Rightarrow u'' = \infty$$

The refracted rays from lens L_2 becomes parallel to principal axis. It is possible only when image formed by L_1 lies at first focus of L_2, i.e. at a distance of 10 cm from L_2.

$\therefore$ Separation between L_1 and $L_2 = 30 + 10 = 40$ cm

The distance between L_2 and L_3 may take any value.

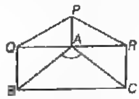
[redacted]

|TOPIC 4|

Prism and Optical Instruments

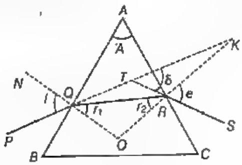
A prism is a portion of a transparent medium bounded by two plane faces inclined to each other at a suitable angle.

In the given figure, *ABQP* and *ACRP* are the two refracting faces and $\angle A$ is called angle of prism.



REFRACTION OF LIGHT THROUGH A PRISM

The figure below shows the passage of light through a triangular prism *ABC*.



The angles of incidence and refraction at first face *AB* are i and r_1.

The angle of incidence at the second face *AC* is r_2 and the angle of emergence is e.

The angle between the emergent ray *RS* and incident ray *PQ* is called angle of deviation (δ).

Here, $\angle PQN = i, \angle SRK = e$

$\angle RQO = r_1, \angle QRO = r_2$

$\angle KTS = \delta, \ \angle TQO = i$

and $\angle TQR = i - r_1$

or $\angle TRQ = e - r_2$

In ΔTQR, the side *QT* has been produced outwards. Therefore, the exterior angle δ should be equal to the sum of the interior opposite angles.

i.e.
$$\delta = \angle TQR + \angle TRQ$$
$$= (i - r_1) + (e - r_2)$$
$$\Rightarrow \quad \delta = (i + e) - (r_1 + r_2) \quad \text{...(i)}$$

In ΔQRO,
$$r_1 + r_2 + \angle ROQ = 180° \quad \text{...(ii)}$$

From quadrilateral *AROQ*, we have the sum of angles
$$\angle AQO + \angle ARO = 180°.$$

This means that the sum of the remaining two angles should be 180°.

i.e.
$$\angle A + \angle ROQ = 180° \quad \text{...(iii)}$$
[$\angle A$ is called the angle of prism]

From Eqs. (ii) and (iii), we get
$$r_1 + r_2 = A \quad \text{...(iv)}$$

Substituting the value from Eq. (iv) in Eq. (i), we obtain
$$\delta = (i + e) - A$$

If μ is the refractive index of material of the prism, then according to Snell's law,
$$\mu = \frac{\sin i_1}{\sin r_1}$$

When angles are small, $\sin i_1 \approx i_1$ and $\sin r_1 \approx r_1$

$$\therefore \quad \mu = \frac{i_1}{r_1} \Rightarrow i = \mu r_1 \quad [\text{here, } i_1 = i]$$

Similarly,
$$\mu = \frac{i_2}{r_2} \text{ or } \mu = \frac{e}{r_2} \quad [\because i_2 = e]$$
$$\Rightarrow \quad e = \mu r_2$$
$$\therefore \quad \delta = i + e - A$$
$$= \mu r_1 + \mu r_2 - A = \mu(r_1 + r_2) - A$$

But $r_1 + r_2 = A$
$$\therefore \quad \delta = \mu A - A$$
or
$$\boxed{\delta = (\mu - 1) A}$$

This is the angle through which a ray deviates on passing through a thin prism of small refracting angle *A*.

EXAMPLE |1| A thin prism of 5° angle gives a deviation of 3.2°. What is the value of refractive index of the material of the prism?

Sol. Given, $A = 5°, \delta = 3.2°, \mu = ?$

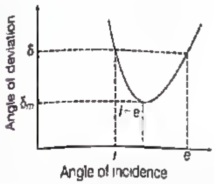
We know that, $\delta = A(\mu - 1)$

$$\therefore \quad \mu = 1 + \frac{\delta}{A} = 1 + \frac{3.2°}{5°} = 1 + 0.64 = 1.64$$

Prism Formula

If the angle of incidence is increased gradually, then the angle of deviation first decreases, attains a minimum value (δ_m) and then again starts increasing.

[redacted]



When angle of deviation is minimum, the prism is said to be placed in the minimum deviation position. There is only one angle of incidence for which the angle of deviation is minimum.

When $\delta = \delta_m$ [prism in minimum deviation position]

$$e = i \quad \text{and} \quad r_2 = r_1 \qquad \text{...(i)}$$

$\because \quad r_1 + r_2 = A$

$\Rightarrow \quad r + r = A$ or $r = \dfrac{A}{2}$

Also, we have

$$A + \delta = i + e \qquad \text{...(ii)}$$

Putting $\delta = \delta_m$ and $e = i$ in Eq. (ii), we get

$$A + \delta_m = i + i$$

$$\Rightarrow \quad i = \left(\frac{A + \delta_m}{2}\right)$$

From Snell's law, $\mu = \dfrac{\sin i}{\sin r}$

$$\therefore \quad \boxed{\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}}}$$

This relation is called a prism formula.

For thin prisms (i.e. A is very small), the value of δ_m is also very small.

So,
$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}} \approx \frac{\frac{A+\delta_m}{2}}{A/2}$$

$$\Rightarrow \quad \delta_m = (\mu - 1)A$$

EXAMPLE |2| A ray of light suffers minimum deviation, while passing through a prism of refractive index 1.5 and refracting angle 60°. Calculate the angle of deviation and angle of incidence.
(Given, $\sin^{-1}(0.75) = 48.6°$)

Sol. Given, refractive index, $\mu = 1.5$

Angle of prism, $A = 60°$, angle of deviation, $\delta_m = ?$

Angle of incidence, $i = ?$

We know that, $\mu = \dfrac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$$\Rightarrow \quad 1.5 = \frac{\sin\left(\frac{60° + \delta_m}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$$

$$\Rightarrow \quad 1.5 \sin 30° = \sin\left(\frac{60° + \delta_m}{2}\right)$$

$$\Rightarrow \quad 1.5 \times 0.5 = \sin\left(\frac{60° + \delta_m}{2}\right)$$

$$\Rightarrow \quad \frac{60° + \delta_m}{2} = \sin^{-1}(0.75)$$

$$\Rightarrow \quad \frac{60° + \delta_m}{2} = 48.6°$$

$$\delta_m = 48.6° \times 2 - 60° = 37.2°$$

Also, the angle of incidence,

$$i = \frac{(A + \delta_m)}{2}$$

$$= \frac{60° + 37.2°}{2} = 48.6°$$

OPTICAL INSTRUMENTS

Using the reflecting and refracting properties of mirrors, lenses and prisms, many optical instruments have been designed like microscopes and telescopes. Our eye is a natural optical device.

The Eye

The structure and working of eye were already learnt in your younger classes. The eye lens is a convex lens whose focal length can be modified by the ciliary muscles. This property of eye is called **accommodation**. The image is formed on a film of nerve fibres called **retina**.

The closest distance for which the lens can form image is called the near point and its value is 25 cm for a normal eye. The far point of a normal eye is infinity. It is the farthest point upto which the eye can see clearly.

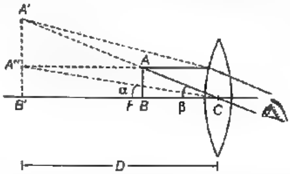
Simple Microscope

Microscope is an optical instrument which forms large image of close and minute objects. A simple microscope is a converging lens of small focal length. When an object is at a distance less than the focal length of the lens, the image obtained is virtual, erect and magnified.

When the object is at a distance equal to the focal length of the lens, the image is formed at infinity.

[redacted]

Case I When the image is formed at the near point



The angular magnification or magnifying power of a simple microscope is defined as the ratio of the angle β subtended at the eye by image at the near point and the angle α subtended at the unaided eye by the object at the near point.

∴ Magnifying power, $m = \frac{\beta}{\alpha}$ ...(i)

In $\Delta A'B'C$, $\tan\beta = \frac{A'B'}{D}$

In $\Delta A''B''C$, $\tan\alpha = \frac{A''B'}{D} = \frac{AB}{D}$

Since, the angles are small, then

$\tan\alpha = \alpha$ and $\tan\beta = \beta$

∴ $\beta = \frac{A'B'}{D}$ and $\alpha = \frac{AB}{D}$

From Eq. (i), we have

$$m = \frac{A'B'}{D} \times \frac{D}{AB} = \frac{A'B'}{AB}$$

This gives the linear magnification produced by the lens.

It can be proved that, $\frac{A'B'}{AB} = \frac{v}{u}$

We know that, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Multiplying both sides by v, we have $\frac{v}{v} - \frac{v}{u} = \frac{v}{f}$

$\Rightarrow 1 - m = \frac{v}{f} \Rightarrow m = 1 - \frac{v}{f}$

∴ $\boxed{m = 1 + \frac{D}{f}}$

[∵ $v = -D$ because image is formed at near point]

In this case, the eye is placed behind the lens at a distance a, then

$$\boxed{m = 1 + \frac{D-a}{f}}$$

Case II When the image is formed at infinity

i.e. $v = \infty$

In this case, $\beta = \frac{AB}{f}$ and $\alpha = \frac{AB}{D}$

∴ $m = \frac{AB}{f} \times \frac{D}{AB} = \frac{D}{f}$

$$\boxed{m = \frac{D}{f}}$$

EXAMPLE |3| A convex lens of focal length 5 cm is used as a simple microscope. What will be the magnifying power when the image is formed at the least distance of distinct vision?

Sol. Given, focal length, $f = 5$ cm

Least distance of distinct vision, $D = 25$ cm

∴ Magnification, $m = \left(1 + \frac{D}{f}\right) = \left(1 + \frac{25}{5}\right) = 6$

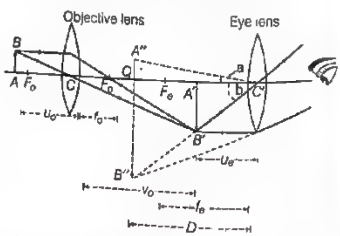
Compound Microscope

A compound microscope consists of two convex lenses coaxially separated by some distance. The lens nearer to the object is called the objective. The lens through which the final image is viewed is called the eyepiece.

Working

The objective of compound microscope forms the real, inverted and magnified image of the object. This image serves as the object for the second lens, i.e. eyepiece which produces the final image, which is enlarged and virtual.

The first inverted image is thus near the focal plane of the eyepiece, at a distance appropriate for final image formation at infinity or a little closer for image formation at the near point. The final image is inverted with respect to the original object.



Angular magnification or magnifying power of a compound microscope is defined as the ratio of the angle β subtended by the final image at the eye to the

[redacted]

angle α subtended by the object seen directly, when both are placed at least distance of distinct vision.

$\therefore$ Angular magnification, $m = \dfrac{\beta}{\alpha}$

Since, the angles are small, then

$$\alpha \approx \tan\alpha \text{ or } \beta \approx \tan\beta$$

$$\therefore \quad m = \frac{\tan\beta}{\tan\alpha} \quad \text{...(i)}$$

From right angled $\Delta C'QB''$, we have

$$\tan\beta = \frac{B''Q}{C'Q} = \frac{B''Q}{D} = \frac{A''B''}{D}$$

Also, from right angled $\Delta C'A''Q$, we have

$$\tan\alpha = \frac{A''Q}{C'Q} = \frac{AB}{D} \quad [\because A''Q = AB]$$

Substituting the values of $\tan\alpha$ and $\tan\beta$ in Eq. (i), we have

$$m = \frac{B''Q}{D} \times \frac{D}{AB} = \frac{B''Q}{AB}$$

$$\Rightarrow \quad m = \frac{B''Q}{A'B'} \times \frac{A'B'}{AB}$$

Thus, the magnification produced by the compound microscope is the product of the magnification produced by the eyepiece and objective.

$$\therefore \quad m = m_e \times m_o \quad \text{...(ii)}$$

where, m_e and m_o are the magnifying powers of the eyepiece and objective, respectively.

The linear magnification of the real inverted image produced by the eyepiece is $\dfrac{A''B''}{A'B'}$.

Case I **When the final image is formed at near point**

Linear magnification is given by

$$m_e = 1 + \frac{D}{f_e} \quad \text{...(iii)}$$

where, f_e is focal length of the eyepiece

$\dfrac{A'B'}{AB}$ is the linear magnification of the object produced by the objective.

$$m_o = \frac{v_o}{u_o} \quad \text{...(iv)}$$

From Eqs. (ii), (iii) and (iv), we have

$$m = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right) \quad \text{...(v)}$$

We know that, $\dfrac{1}{v_o} - \dfrac{1}{u_o} = \dfrac{1}{f_o}$

Multiplying both sides by v_o, we have

$$\frac{v_o}{v_o} - \frac{v_o}{u_o} = \frac{v_o}{f_o}$$

$$\Rightarrow \quad -\frac{v_o}{u_o} = -1 + \frac{v_o}{f_o} \Rightarrow \frac{v_o}{u_o} = 1 - \frac{v_o}{f_o}$$

Substituting the value of $\dfrac{v_o}{u_o}$ in Eq. (v), we have

$$\boxed{m = \left(1 - \frac{v_o}{f_o}\right)\left(1 + \frac{D}{f_e}\right)}$$

Case II **When the final image is at infinity**

If u_o is the distance of the object from the objective and v_o is the distance of the image from the objective, then the magnifying power of the objective is $m_o = \dfrac{v_o}{u_o}$

When the final image is at infinity, then angular magnification is given by

$$\boxed{m_e = \frac{D}{f_e}}$$

The total magnification when image is at infinity is given by

$$m = m_o \times m_e = \left(\frac{v_o}{u_o} \times \frac{D}{f_e}\right)$$

If the object is very close to the principal focus of the objective and the image formed by the objective is very close to the eyepiece, then

$$m = \frac{-L}{f_o} \cdot \frac{D}{f_e}$$

where, L = length of the tube of microscope

In this case, the microscope is said to be in normal adjustment.

EXAMPLE |4| **A compound microscope has an objective of focal length 1 cm and an eyepiece of focal length 2.5cm. An object has to be placed at a distance of 1.2 cm away from the objective for the normal adjustment. Determine the angular magnification and length of microscope tube.**

Sol. Given, focal length of objective, $f_o = 1$ cm

Focal length of eyepiece, $f_e = 2.5$ cm

Object distance, $u_o = -1.2$ cm

$$\because \quad \frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{f_o}$$

[redacted]

$$\Rightarrow \quad \frac{1}{v_o} = \frac{1}{u_o} + \frac{1}{f_o}$$

$$\Rightarrow \quad \frac{1}{v_o} = 1 - \frac{1}{1.2} = \frac{0.2}{1.2}$$

$$\Rightarrow \quad v_o = \frac{1.2}{0.2} \Rightarrow v_o = 6 \text{ cm}$$

$\because$ Angular magnification, $m = \frac{v_o}{|u_o|}\left(1 + \frac{D}{f_e}\right)$

$$\Rightarrow \quad m = \frac{6}{|-1.2|}\left(1 + \frac{25}{2.5}\right) = 55$$

$\therefore$ Length of microscope tube,

$$L = v_o + f_e = (6 + 2.5) = 8.5 \text{ cm}$$

Astronomical (Refracting) Telescope

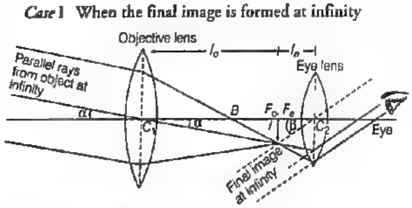
An astronomical telescope is an optical instrument which is used for observing distinct images of heavenly bodies like stars, planets, etc., when the final image is formed at infinity.

Astronomical telescope has two convex lenses coaxially separated by some distance. The lens towards the object is called objective and has much larger aperture than the eyepiece of the lens towards the eye.

Working

Light from the distant object enters the objective and real image is formed at second focal point of objective. The eyepiece magnifies this image producing a final inverted image.

Case I When the final image is formed at infinity



Angular magnification is given by

$$m = \frac{\beta}{\alpha}$$

Since, β and α are very small.

$\therefore \quad \beta \approx \tan\beta$

or $\quad \alpha \approx \tan\alpha$

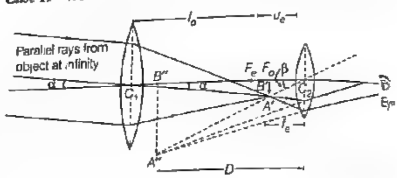
$$\Rightarrow \quad m = \frac{\tan\beta}{\tan\alpha} \qquad \text{...(i)}$$

Now, $\tan\alpha = \frac{I}{f_o}$ and $\tan\beta = \frac{I}{-f_e}$

where, I is the image formed by the objective, f_o and f_e are the focal lengths of objective and eyepiece, respectively. Substituting the values of $\tan\alpha$ and $\tan\beta$ in Eq. (i), we get

$$m = \frac{-\frac{I}{f_e}}{\frac{I}{f_o}} \quad \text{or} \quad \boxed{m = -\frac{f_o}{f_e}}$$

Case II When final image is formed at near point



Angular magnification, $m = \frac{\beta}{\alpha}$

$$\Rightarrow \quad m = \frac{\tan\beta}{\tan\alpha} \quad [\because \beta \text{ and } \alpha \text{ are small}]$$

$$\Rightarrow \quad m = \frac{\frac{A'B'}{C_2B'}}{\frac{A'B'}{C_1B'}} = \frac{C_1B'}{C_2B'}$$

$$\Rightarrow \quad m = \frac{f_o}{-u_e} \qquad \text{...(i)}$$

Using lens formula $\left(\frac{1}{v} - \frac{1}{u} = \frac{1}{f}\right)$ for the eyepiece, we have

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e}$$

$$\Rightarrow \quad \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e}\left(1 + \frac{f_e}{D}\right)$$

Putting the value of $\frac{1}{u_e}$ in Eq. (i), we have

$$\boxed{m = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)}$$

EXAMPLE |5| A telescope consists of two lenses of focal lengths 20 cm and 5 cm. Obtain its magnifying power when the final image is (i) at infinity (ii) at 25 cm from the eye.

Sol. (i) When the final image is at infinity,

$$m = -\frac{f_o}{f_e} = \frac{-20}{5}$$

[redacted]

$\Rightarrow \quad m = -4$

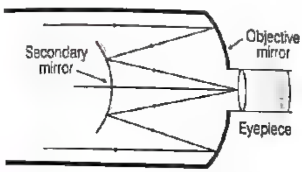
(ii) When the final image is at 25 cm from the eye, i.e. $D = 25$ cm,

$$m = \frac{-f_o}{f_e}\left(1 + \frac{f_e}{D}\right) = \frac{-20}{5}\left(1 + \frac{5}{25}\right)$$

$\Rightarrow \quad m = -4.8$

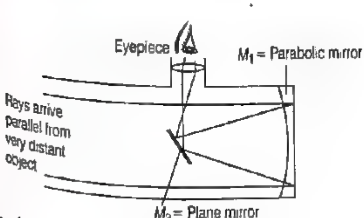
Reflecting Telescope

Reflecting telescope is also known as **Cassegrain Telescope,** which was designed by Guillaume Cassegrain, shown in figure below. Reflecting telescope is an improvement over refracting or astronomical telescope. To obtain a bright image of a distant star by refracting telescope, it is essential to have an objective of large aperture, so that it may collect more light coming from the object. But to deal with such a big lens is problem in terms of using and making and it is too costly. The same bright image of a distant object can be obtained by using a concave mirror of large aperture in place of objective.



Reflecting telescope consists of concave mirror of large aperture and large focal length (objective). A convex mirror is placed between the concave mirror and its focus. A small convex lens works as eyepiece.In the reflecting telescope, parallel rays from a distant object are intercepted and focused by a reflecting concave mirror rather than a refracting lens. One popular configuration of mirror and eyepiece is called the Newtonian reflecting type telescope, named after its designer Newton.

The parallel beam of light coming from the distant object (star) is reflected by concave parabolic mirror M_1, on the plane mirror M_2. The plane mirror M_2 is inclined at an angle of 45° to axis of the mirror M_1.



The plane mirror reflects the beam and a real image is formed in front of eyepiece. The eyepiece acts as a magnifier and the final magnified image of the distant object can be observed by the eye.

Advantages of Reflecting Telescope over Refracting Telescope

For astronomical telescope, the mirror affords several advantages over the objective lens. A mirror is easier to produce with a larger diameter, so that it can intercept rays crossing a larger area and direct them to the eyepiece.

The mirror can be made parabolic to reduce spherical aberration. Aberration is further reduced because passage through one layer of glass (the objective lens) is eliminated.

| TOPIC PRACTICE 4 |

OBJECTIVE Type Questions

1. A prism has refractive angle 60°. When a light ray is incident on 50°, then minimum deviation is obtained. What is the value of minimum deviation?
   (a) 40° (b) 45°
   (c) 50° (d) 60°

2. A ray of light passes through an equilateral prism such that, the angle of incidence is equal to the angle of emergence and the latter is equal to 3/4 the angle of prism. The angle of deviation is
   (a) 25° (b) 30°
   (c) 45° (d) 35°

3. A ray of light incident at an angle θ on a refracting face of a prism emerges from the other face normally. If the angle of the prism is 5° and the prism is made of a material of refractive index 1.5, the angle of incidence is **NCERT Exemplar**
   (a) 7.5° (b) 5° (c) 15° (d) 2.5°

4. The image formed by an objective of a compound microscope is
   (a) virtual and diminished
   (b) real and diminished
   (c) real and enlarged
   (d) virtual and enlarged

5. In order to increase the angular magnification of a simple microscope, one should increase
   (a) the object size
   (b) the aperture of the lens
   (c) the focal length of the lens
   (d) the power of the lens

[redacted]

6. F_1 and F_2 are focal lengths of objective and eyepiece respectively, of the telescope. The angular magnification of the given telescope is equal to

(a) $\frac{F_1}{F_2}$ (b) $\frac{F_2}{F_1}$

(c) $\frac{F_1F_2}{F_1+F_2}$ (d) $\frac{F_1+F_2}{F_1F_2}$

7. An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eyepiece is 36 cm and the final image is formed at infinity. The focal length f_o of the objective and the focal length f_e of the eyepiece are

(a) $f_o = 45$ cm and $f_e = -9$ cm
(b) $f_o = -7.2$ cm and $f_e = 5$ cm
(c) $f_o = 50$ cm and $f_e = 10$ cm
(d) $f_o = 30$ cm and $f_e = 6$ cm

8. Limitation of reflecting telescope is

(a) objective mirror focusses light inside the telescope tube
(b) objective mirror focusses light outside the telescope tube
(c) objective mirror has large focal length
(d) tube length is large

VERY SHORT ANSWER Type Questions

9. How does the angle of minimum deviation of a glass prism vary, if the incident violet light is replaced by red light? Give reason. **All India 2017**

10. Write the relationship between angle of incidence i, angle of prism A and angle of minimum deviation δ_m for a triangular prism. **Delhi 2013**

11. Why should the objective lens of a compound microscope have a small focal length?

12. How will you distinguish between a compound microscope and a telescope simply by seeing it?

SHORT ANSWER Type Questions

13. What should be the position of the object relative to the biconvex lens, so that this lens behaves like a magnifying glass?

14. How does the magnification of a magnifying glass differ from its magnifying power?

15. Is it possible to increase the range of a telescope by increasing the diameter of the objective lens?

16. Draw a schematic arrangement of a reflecting telescope (Cassegrain) showing how rays coming from a distant object are received at the eyepiece. Write its two important advantages over a refracting telescope. **Delhi 2013**

17. Explain two advantages of a reflecting telescope over a refracting telescope. **CBSE 2018**

LONG ANSWER Type I Questions

18. Choose the statement as wrong or right and justify.

(i) The intensity of scattered light varies inversely as square of wavelength.
(ii) Magnification of simple microscope when final image is at infinity is given by $m = 1 - \frac{d}{f}$.
(iii) In reflecting type telescope, objective lens is replaced by convex parabolic mirror.

19. (i) Draw a neat labelled ray diagram of a compound microscope. Explain briefly its working.

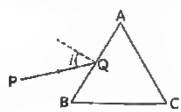
(ii) Why must both the objective and the eyepiece of a compound microscope have short focal lengths? **All India 2010**

20. Draw a ray diagram showing the image formation by a compound microscope. Hence, obtain the expression for total magnification, when the image is formed at infinity. **Delhi 2010**

21. Draw a labelled ray diagram on a refracting telescope. Define its magnifying power and write the expression for it. Write two important limitations of a refracting telescope over a reflecting type telescope. **All India 2013**

LONG ANSWER Type II Questions

22. (i) A ray PQ of light is incident on the face AB of a glass prism ABC (as shown in the figure) and emerges out of the face AC. Trace the path of the ray. Show that $\angle i + \angle e = \angle A + \angle \delta$



[redacted]

where, δ and e denote the angle of deviation and angle of emergence, respectively.

Plot a graph showing the variation of the angle of deviation as a function of angle of incidence. State the condition under which $\angle\delta$ is minimum.

(ii) Find out the relation between the refractive index (μ) of the glass prism and $\angle A$ for the case, when the angle of prism (A) is equal to the angle of minimum deviation (δ_m). Hence, obtain the value of the refractive index for angle of prism $A = 60°$. **Delhi 2015**

23. Draw a ray diagram to show refraction of a ray of monochromatic light passing through a glass prism. Deduce the expression for the refractive index of glass in terms of angle of prism and angle of minimum deviation. **Delhi 2012**

24. Define magnifying power of a telescope. Write its expression.
A small telescope has an objective lens of focal length 150 cm and an eyepiece of focal length 5 cm. If this telescope is used to view a 100 m high tower 3 km away, find the height of the final image, when it is formed 25 cm away from the eyepiece. **Delhi 2012**

NUMERICAL PROBLEMS

25. White light is incident on one of the refracting surface of a prism of angle 5°. If the refractive indices for red and blue colours are 1.641 and 1.659 respectively, then what will be the angular separation between these two colours when they emerge out?

26. Consider a telescope whose objective lens has a focal length of 100 cm and the eyepiece has focal length 1 cm. What will be the magnification of the given telescope?

27. Two lenses of focal lengths 6 cm and 50 cm are to be used for making a telescope. Which will you see for the objective?

28. A ray of light, incident on an equilateral glass prism ($\mu_g = \sqrt{3}$) moves parallel to the base line of the prism inside it. Find the angle of incidence for this ray. **Delhi 2012**

29. A ray PQ incident on the refracting face BA is refracted in the prism BAC and emerges from the other refracting face AC as RS, such that $AQ = AR$. If the angle of prism $\angle A = 160°$ and refractive index of the material of prism is $\sqrt{3}$, then what will be the angle of deviation of the ray?

30. The following table gives the values of the angle of deviation, for different values of the angle of incidence, for a triangular prism.

Angle of incidence	33°	38°	42°	52°	60°	71°
Angle of deviation	60°	50°	46°	40°	43°	50°

(i) For what value of the angle of incidence, is the angle of emergence likely to be equal to the angle of incidence itself?

(ii) Draw a ray diagram, showing the passage of a ray of light through this prism, when the angle of incidence has the above value.

31. The near vision of an average person is 25 cm. To view an object with an angular magnification of 10, what should be the power of the microscope? **NCERT Exemplar**

32. You are given two converging lenses of focal length 1.25 cm and 5 cm to design a compound microscope. If it is desired to have a magnification of 30, then find out the separation between the objective and eyepiece. **Delhi 2015**

33. A small telescope has an objective lens of focal length 144 cm and an eyepiece of focal length 6 cm. What is the magnifying power of the telescope? What is the separation between the objective and the eyepiece? **NCERT**

34. A telescope consists of two thin lenses of focal lengths 0.3 m and 3 cm, respectively. It is focused on moon which subtends an angle of 0.5° at the objective. Then, what will be the angle subtended at the eye by the final image?

35. A small telescope has an objective lens of focal length 150 cm and eyepiece of focal length 5 cm. What is the magnifying power of the telescope for viewing distant objects in normal adjustments?
If this telescope is used to view a 100 m tall tower 3 km away, then what is the height of the tower formed by the objective lens? **Delhi 2015**

[redacted]

36. (i) A ray of light incident of face AB of an equilateral glass prism, shows minimum deviation of 30°. Calculate the speed of light through the prism.

(ii) Find the angle of incidence at face AB, so that the emergent ray grazes along the face AC. **Delhi 2017**

37. For a glass prism($\mu = \sqrt{3}$), the angle of minimum deviation is equal to the angle of the prism. Find the angle of the prism. **NCERT Exemplar**

38. (i) Draw a labelled ray diagram showing the formation of a final image by a compound microscope at least distance of distinct vision.

(ii) The total magnification produced by a compound microscope is 20. The magnification produced by the eyepiece is 5. The microscope is focused on a certain object. The distance between the object and eyepiece is observed to be 14 cm. If least distance of distinct vision is 20 cm, calculate the focal length of the object and the eyepiece. **Delhi 2014**

39. A compound microscope uses an objective lens of focal length 4 cm and eyepiece lens of focal length 10 cm. An object is placed at 6 cm from the objective lens. Calculate the magnifying power of the compound microscope. Also, calculate the length of the microscope. **All India 2011**

40. (i) A giant reflecting telescope at an observatory has an objective lens of focal length 15 m. If an eyepiece lens of focal length 1.0 cm is used, find the angular magnification of the telescope.

(ii) If this telescope is used to view the moon, what is the diameter of the image of the moon formed by the objective lens? The diameter of the moon is 3.48×10^6 m and the radius of the lunar orbit is 3.8×10^8 m. **All India 2015, All India 2011; NCERT**

HINTS AND SOLUTIONS

1. (a) Given, incidence angle, $i = 50°$

Refraction angle, $A = 60°$

Minimum deviation, $\delta = 2i - A = 50° \times 2 - 60° = 40°$

2. (b) Given, equilateral prism i.e., $A = 60°$

$$i = e = \frac{3}{4}A = \frac{3}{4} \times 60° = 45°$$

From relation, $A + D = i + e$

We have, $60° + D = 2 \times 45°$

$\Rightarrow \quad D = 90° - 60° = 30°$

3. (a) Since, deviation $\delta = (\mu - 1)A = (1.5 - 1) \times 5° = 2.5°$

By geometry, angle of refraction by first surface is 5°.

But $\delta = \theta - r$, so, we have, $2.5° = \theta - 5°$ on solving $\theta = 7.5°$.

4. (c) Objective of a compound microscope is a convex lens. Convex lens forms real and enlarged image when an object is placed between focus and radius of curvature.

5. (d) For least distance of distinct vision, the angular magnification of simple microscope is

$$M = 1 + \frac{D}{f} \Rightarrow M = 1 + DP \quad \left(\because \text{Power}(P) = \frac{1}{f}\right)$$

and for normal adjustment $M = \dfrac{D}{f} \Rightarrow M = DP \Rightarrow M \propto P$.

6. (a) Given, $f_o = F_1, f_e = F_2$

We know, angular magnification for telescope

$$|M| = \left|\frac{f_o}{f_e}\right| = \left|\frac{F_1}{F_2}\right| \Rightarrow \frac{F_1}{F_2}$$

7. (d) For telescope $|m| = \dfrac{|f_o|}{|f_e|} = 5$ ...(i)

and length of the telescope

$L = |f_o| + |f_e| = 36$ ...(ii)

From Eqs. (i) and (ii),

$\Rightarrow f_e = 6\,\text{cm}$ and $f_o = 30\,\text{cm}$

8. (a) The main limitation of reflecting telescope is that the objective mirror focusses light inside the telescope tube.

9. Wavelength of violet light is smaller than that of red light. Also, angle of minimum deviation,

$$\delta_m = (\mu - 1)A$$

$\Rightarrow \quad \delta_m \propto \mu$

As, $\quad \mu_R < \mu_V$

$\Rightarrow \quad (\delta_m)_R < (\delta_m)_V$

As deviation is less for red light, hence angle of deviation decreases.

10. The relation between the angle of incidence i, angle of prism A and the angle of minimum deviation δ_m, for a triangular prism is given as $i = \dfrac{A + \delta_m}{2}$.

11. The angular magnification of eyepiece is $\left(1 + \dfrac{d}{f_e}\right)$.

Hence, as f_e decreases angular magnification increases.

Also, the magnification of the object lens is $\dfrac{v}{u}$.

[redacted]

The object lies close the focus of the objective lens $u = f_o$. Therefore, to increase the magnification f_o should be small.

12. In compound microscope objective lens has smaller aperture and smaller focal length than the eyepiece, while in telescope, the objective has a larger aperture and larger focal length than the eyepiece.

13. Whenever object is placed within the focus of the biconvex lens, we will obtain enlarged image, hence the biconvex lens behaves like a magnifying lens.

14. The **magnification** of a magnifying glass depends upon, where it is placed between the user's eye and the object being viewed and the total distance between them, while the **magnifying power** is equivalent to angular magnification.

15. By increasing the diameter of the objective lens, we can increase the range of the telescope because as the diameter of lens increases, the area covered by the lens also increases, i.e. lens is able to focus on a large area there by helping us to view the object better.

16. Refer to text on page 387.

17. Advantages of reflecting telescope over refracting telescope
   (i) In reflecting telescope, image formed is free from chromatic aberration defect. So, it is sharper than image formed by a retracting type telescope.
   (ii) A mirror is easier to produce with a large diameter, so that it can intercept rays crossing a large area and direct them to the eye-piece.

18. (i) Wrong; as it varies $I \propto \frac{1}{\lambda^4}$.
   (ii) Refer to text on pages 383 and 384.
   (iii) Refer to text on pages 387.

19. (i) Refer to text on pages 384 and 385.
   (ii) f_o and f_e of compound microscope must be small, so as to have large magnifying power as

$$m = \frac{-1}{f_o}\left(1 + \frac{D}{f_e}\right)$$

20. Refer to text on pages 384 and 385.

21. Refer to text on page 386 and 387.
   Magnifying power (in normal adjustment) of a reflecting telescope is the ratio of the focal length of concave reflector and the focal length of eyepiece.

$$m = \frac{f}{f_e} = \frac{R/2}{f_e}$$

Limitations of refracting telescope over a reflecting type telescope

   (i) Refracting telescope suffers from chromatic aberration as it uses large sized lenses.
   (ii) It is also difficult and expensive to make such large sized lenses.

22. (i) Refer to text on pages 382 and 383.
   (ii) Refer to text on pages 382 and 383.

Since, $\angle A = 60^\circ$ [given]

$$\therefore \quad \mu = \frac{\sin\left(\frac{60^\circ + 60^\circ}{2}\right)}{\sin(60^\circ/2)}$$

$$= \frac{\sin 60^\circ}{\sin 30^\circ} = \frac{\sqrt{3}}{2} \times \frac{2}{1}$$

$$= \sqrt{3} = 1.732$$

23. Refer to text on pages 382 and 383.

24. The magnifying power of a telescope is equal to the ratio of the visual angle subtended at the eye by final image formed at least distance of distinct vision to the visual angle subtended at naked eye by the object at infinity.

Magnification, $m = \frac{I}{O} = \frac{v_0}{u_0} = \frac{f_0}{u_0}$

$$\Rightarrow \quad \frac{I}{100} = \frac{150 \times 10^{-2}}{3 \times 10^3}$$

$$\Rightarrow \quad I = 5 \times 10^{-2}\ \text{m} = 5\ \text{cm}$$

25. Angle of prism, $A = 5^\circ, \mu_R = 1.641, \mu_B = 1.659$

As we know that, $\delta = (\mu - 1)A$

So, $\delta_B = (\mu_B - 1)A$ and $\delta_R = (\mu_R - 1)A$

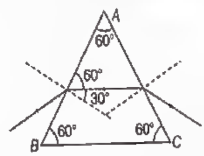
$\therefore \quad \delta_B - \delta_R = (\mu_B - \mu_R)A = (1.659 - 1.641) \times 5$
$= 0.09^\circ$

26. Magnification of telescope is given by

$$m = \frac{f_o}{f_e} = \frac{100}{1} = 100$$

27. Yes, these lenses can be used for making a telescope. Since, the objective lens has large aperture and focal length, hence the lens having focal length will be used as objective lens.

28. To draw the ray diagram for the refraction from the prism. Following things should be kept in mind.
   (i) Draw normal to the point of incidence.
   (ii) Consider each boundary of the prism as separate interface and draw the ray diagram for the refraction taking place.
   The reflection of light through prism is shown below.



By geometry, angle of refraction, $r = 30^\circ$

Refractive index, $\mu = \sqrt{3}$ $\left[\text{from Snell's law, } \mu = \frac{\sin i}{\sin r}\right]$

$$\Rightarrow \quad \sin i = \mu \sin r = (\sqrt{3})\sin(30^\circ) = \frac{\sqrt{3}}{2}$$

Angle of incidence, $i = 60^\circ = \frac{\pi}{3} \Rightarrow i = \frac{\pi}{3}$

[redacted]

29. Refer to the Example 2 on pages 383.

30. (i) $i = 52°$, when prism is adjusted at an angle of minimum deviation, then angle of incidence is equal to the angle of emergence.

Hence, $r = 0$

This ray pass unrefracted at AC interface and reaches AB interface. Here, we can see angle of incidence becomes 30°.

Thus, applying Snell's law, $\dfrac{\sin 30°}{\sin e} = \dfrac{\mu_a}{\mu_g} = \dfrac{1}{\sqrt{3}}$

$$\sin e = \sqrt{3} \times \sin 30° = \frac{\sqrt{3}}{2}$$

Thus, $e = 60°$

31. The least distance of distinct vision of an average person, (i.e. D) is 25cm, in order to view an object with magnification of 10.

Here, $v = D = 25$ cm and $u = f$

But the magnification, $m = v/u = D/f$

$\therefore \quad m = \dfrac{D}{f}$

$\Rightarrow \quad f = \dfrac{D}{m} = \dfrac{25}{10} = 2.5 = 0.025$ m

and $\quad P = \dfrac{1}{0.025} = 40$ D $\quad \left[\because P = \dfrac{1}{f}\right]$

This is the required power of lens .

32. Given, $f_o = 1.25$ cm, $f_e = -5$ cm

Magnification, $m = 30$, $D = 25$ cm

If the object is very close to the principal focus of the objective and the image formed by the objective is very close to eyepiece, then magnifying power of a microscope is given by

$$m = -\frac{L}{f_o} \cdot \frac{D}{f_e}$$

$\Rightarrow \quad 30 = \dfrac{L}{1.25} \cdot \dfrac{25}{5}$

$\Rightarrow \quad L = \dfrac{125 \times 30 \times 5}{25 \times 100}$

$\Rightarrow \quad L = \dfrac{25 \times 30}{100} \Rightarrow L = \dfrac{30}{4}$

$\Rightarrow \quad L = 7.5$ cm

This is a required separation between the objective and the eyepiece.

33. Given, focal length of objective lens, $f_o = 144$ cm

Focal length of eyepiece, $f_e = 6$ cm

Magnifying power of the telescope in normal adjustment (i.e. when the final image is formed at ∞),

$$m = -\frac{f_o}{f_e} = -\frac{144}{6} = -24$$

$\therefore$ Separation between lenses,

$$L = f_o + f_e = 144 + 6 = 150 \text{ cm}$$

34. Since, $\quad m = \dfrac{\tan\beta}{\tan\alpha} \approx \dfrac{\beta}{\alpha} = \dfrac{f_o}{f_e}$

$\therefore \quad \dfrac{\beta}{0.5°} = \dfrac{0.3}{0.03} = 5°$

35. When final image is at D, then

magnifying power, $m = \dfrac{-f_o}{f_e}\left(1 + \dfrac{f_e}{D}\right)$

In normal adjustment, $m = \dfrac{-f_o}{f_e}$

For telescope,

focal length of objective lens, $f_o = 150$ cm

Focal length of eye lens, $f_e = 5$ cm

When final image forms at D, i.e. 25 cm, then

magnification, $m = \dfrac{-f_o}{f_e}\left(1 + \dfrac{f_e}{D}\right)$

$$= \frac{-150}{5}\left(1 + \frac{5}{25}\right) = \frac{-150}{5} \times \frac{6}{5}$$

$\Rightarrow \quad m = -36$

Let height of final image be h cm.

$\Rightarrow \quad \tan\beta = \dfrac{h}{25}$ and $\tan\alpha = \dfrac{100\text{m}}{3000\text{m}} = \dfrac{1}{30}$

where, β = visual angle formed by final image at eye and α = visual angle subtended by object at objective.

But $\quad m = \dfrac{\tan\beta}{\tan\alpha} \Rightarrow 36 = \dfrac{\left(\dfrac{h}{25}\right)}{\left(\dfrac{1}{30}\right)}$

$\Rightarrow \quad -36 = \dfrac{h}{25} \times 30 \Rightarrow -36 = \dfrac{6h}{5}$

$\Rightarrow \quad h = \dfrac{-36 \times 5}{6}$

$h = -30$ cm

Negative sign indicates inverted image.

36. (i) Given, angle of minimum deviation, $\delta_m = 30°$

$\therefore$ Angle of prism, $A = 60°$

By prism formula, reflected index,

$$\mu = \frac{\sin\dfrac{\delta_m + A}{2}}{\sin A/2} = \frac{\sin\dfrac{30° + 60°}{2}}{\sin 30°} = \frac{\sin 45°}{\sin 30°}$$

$$= \frac{1}{\sqrt{2}} \times 2 = \sqrt{2}$$

Also, $\mu = \dfrac{\text{speed of light in vacuum } (c)}{\text{speed of light in prism } (v)}$

$\Rightarrow \quad v = c/\mu = (3 \times 10^8/\sqrt{2})$ m/s

Hence, speed of light through prism is $(3 \times 10^8/\sqrt{2})$ m/s

(ii) Critical angle i_c is given as

$$\sin i_c = \frac{1}{\sqrt{2}} \quad \left[\because \sin i_c = \frac{1}{\mu}\right]$$

[redacted]

$\Rightarrow$ $i_c = 45°$

$A = r + i_c = 60°$

$\Rightarrow$ $r = 60° - 45° = 15°$

Using Snell's law, $\frac{\sin i}{\sin r} = \sqrt{2}$

$\Rightarrow$ $\sin i = \sqrt{2} \sin r = \sqrt{2} \times \sin 15°$

$\therefore$ $i = \sin^{-1}(\sqrt{2}\sin 15°)$

37. The relationship between refractive index, prism angle A and angle of minimum deviation δ_m is given by

$$\mu = \frac{\sin[(A+\delta_m)/2]}{\sin(A/2)}$$

Given, $\delta_m = A$

Substituting the value of δ_m, we have

$$\therefore \quad \mu = \frac{\sin A}{\sin(A/2)}$$

On solving, we have, $\mu = \dfrac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}} = 2\cos\frac{A}{2}$

For the given value of refractive index, $\mu = \sqrt{3}$, we have

$$\cos\frac{A}{2} = \frac{\sqrt{3}}{2} \text{ or } \frac{A}{2} = 30° \text{ or } A = 60°$$

This is the required value of prism angle.

38. (i) Refer to text on page 384

(ii) Given, magnification, $m = 20$

Magnification of eyepiece, $m_e = 5$

Least distance vision, $D = 20$ cm

Distance between the object and eyepiece,

$L = 14$ cm

We know that, magnification, $m = m_e \times m_o$

$\Rightarrow$ $m_o = \frac{m}{m_e} = \frac{20}{5} = 4$

As, $m_e = 1 + \frac{D}{f_e}$

where, f_e is focal length of eyepiece.

$\Rightarrow$ $5 = 1 + \frac{20}{f_e} \Rightarrow f_e = 5$ cm

Using lens formula for eyepiece,

$$\frac{1}{u_e} = \frac{-1}{20} - \frac{1}{5} = \frac{-5}{20} = \frac{-1}{4}$$

$\Rightarrow$ $u_e = -4$ cm (object distance for eyepiece)

$\Rightarrow$ $L = v_o + |u_e|$

$\Rightarrow$ $v_o = L - |u_e|$

$= 14 - 4 = 10$ cm

Magnification produced by object, $m_o = -\frac{v_o}{u_o}$

Object distance for object,

$$u_o = \frac{-v_o}{m_o} = \frac{-10}{4} = -2.5 \text{ cm}$$

Using lens formula for object,

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o} = \frac{1}{10} - \frac{1}{-2.5} = \frac{1}{10} + \frac{1}{2.5}$$

$f_o = 2$ cm

39. For compound microscope, $f_o = 4$ cm, $f_e = 10$ cm,

$u_o = -6$ cm, $v_e = -D = -25$ cm

For objective lens, $\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o} \Rightarrow \frac{1}{4} = \frac{1}{v_o} + \frac{1}{6}$

$\Rightarrow$ $\frac{1}{v_o} = \frac{1}{4} - \frac{1}{6} = \frac{1}{12} \Rightarrow v_o = 12$ cm

$\therefore$ Magnifying power, $m = -\left(\frac{v_o}{u_o}\right)\left(1 + \frac{D}{f_e}\right)$

$$= -\left(\frac{12}{6}\right)\left(1 + \frac{25}{10}\right) = -2\left(\frac{7}{2}\right) = -7$$

Length of microscope $= |v_o| + |u_e|$

where, $v_o = 12$ cm

For eye lens, $v_e = -25$ cm, $f_e = 10$ cm, $u_e = ?$

$\therefore$ $\frac{1}{f_e} = \frac{1}{v_e} - \frac{1}{u_e}$

$\Rightarrow$ $\frac{1}{u_e} = \frac{1}{v_e} - \frac{1}{f_e} = \frac{1}{-25} - \frac{1}{10}$

$\Rightarrow$ $\frac{1}{u_e} = \frac{-2-5}{50} = -\frac{7}{50}$

$\Rightarrow$ $u_e = -7.14$ cm

$\therefore$ Length of microscope $= |v_o| + |u_e|$

$= 12 + 7.14 = 19.14$ cm

40. (i) For astronomical telescope,

$f_o = 15$ m $= 1500$ cm, $f_e = 1$ cm

Angular magnification, $m = -\frac{f_o}{f_e} = -\frac{15 \times 100 \text{ cm}}{1 \text{ cm}}$

$= -1500$

(ii) Given, diameter, $D = 3.48 \times 10^6$ m,

$f_o = 3.8 \times 10^8$ m

Let α be the angle subtended by the moon at objective.

$$\therefore \quad \alpha = \frac{D}{\text{Radius of lunar orbit}}$$

$$\alpha = \frac{3.48 \times 10^6 \text{ m}}{3.8 \times 10^8 \text{ m}} \quad \text{...(i)}$$

Also, then angle subtended by image formed by objective on itself,

$$\alpha = \frac{d}{f_o} \quad \text{...(ii)}$$

where, d = diameter of image.

From Eqs. (i) and (ii), we get

$$\frac{3.48 \times 10^6}{3.8 \times 10^8} = \frac{d}{1500} \quad \text{[given]}$$

$$\therefore \quad d = \frac{1500 \times 3.48 \times 10^6}{3.8 \times 10^8} = 13.7 \text{ cm}$$

[redacted]

SUMMARY

- **Reflection of Light** It is the phenomenon of change in the path of light without any change in medium.
- **Laws of Reflection** Incident ray, reflected ray and the normal to the reflecting surface at the point of incidence, all lies in the same plane. Angle of incidence is always equal to the angle of reflection.
- **Mirror formula** is given by, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$.
- **Linear Magnification** The ratio of size of the image formed by the spherical mirror to the size of the object is called linear magnification.

  i.e $\quad m=\frac{I}{O}=\frac{-v}{u}$
- **Refraction** It is the phenomenon of change in the path of light as it goes from one medium to another medium.
- **Laws of Refraction** Incident ray, refracted ray and the normal to the refracting surface at the point of incidence, all lies in the same plane.

  According to second law, $(\sin i / \sin r) = {}_1\mu_2$. This is called Snell's law of refraction.
- **Refractive Index** It is equal to the ratio of speed of light in vacuum to the speed of light in the material.
- **Principle of Reversibility of Light** When a light rays, after suffering any number of reflections and refractions, its final path is reversed and it travels back along its entire initial path.
- **Expression for Lateral Displacement** $D=\frac{t\sin(i_1-r_1)}{\cos r_1}$
- **Apparent Depth and Real Depth** ${}^a\mu_w=\frac{\text{Real depth}}{\text{Apparent depth}}$
- **Critical Angle** The angle of incidence in denser medium corresponding to which angle of refraction in rarer medium is 90°.
- **Total Internal Reflection** (TIR) The ray on the interface of two media should travel in the denser medium. The angle of incidence should be greater than the critical angle for the two media.
- **Refraction at Spherical Surfaces** The equation which holds the good for any curved spherical surface is given by $\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$
- **Lens** It is a transparent medium bounded by two surfaces of which one or both surfaces are spherical. It is of two types.

  Convex lens is thicker at the centre and thinner at its end.
  Concave lens is thinner at the centre and thicker at its end.
- **Lens formula** $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$
- **Lens Maker's formula** $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$
- **Power of a Lens** It is the ability to converge or diverge the rays of incident light.

  $$P=\frac{1}{f(\text{in m})}$$

  Also, $\quad P=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$
- **Prism** A prism is a portion of transparent medium bounded by two plane faces inclined to each other at a suitable angle.
- **Refraction of Light through a Prism** The relation between angle of deviation and angle of prism is $\delta=(\mu-1)A$.
- **Prism Formula** It is given by, $\mu=\frac{\sin\left[\frac{A+\delta_m}{2}\right]}{\sin\left[\frac{A}{2}\right]}$
- **Angle of minimum deviation** $\delta_m=(\mu-1)A$
- **Simple Microscope** It forms the large image of close and minute objects. It is a converging lens of small focal length.
- **Compound Microscope** It consists of two convex lenses coaxially separated by some distance. One is objective and another is eyepiece.
- **Astronomical Telescope** It has two convex lenses coaxially separated by some distance, which is used for observing distinct images of heavenly by bodies like stars planet, etc.
- **Refracting and Reflecting Telescope** Refracting telescope is used for observing the distinct images of heavenly bodies like stars, planets, etc.

  Reflecting telescope is an improvement over refracting telescope.

[redacted]

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. Relation between focal length (f) and radius of curvature (R) of a spherical mirror is
(a) $R = f/2$ (b) $f = 3R$
(c) $f = R/2$ (d) $f = R/4$

2. A convex mirror has focal length 20 cm. If an object is placed 20 cm away from the pole of mirror, then what is the distance between image formed and pole?
(a) 40 cm (b) 10 cm
(c) 20 cm (d) At infinity

3. In total internal reflection,
(a) light ray travelling through a denser medium is completely reflected back to denser medium
(b) light ray travelling through a denser medium is completely refracted to rare medium
(c) light ray is partially reflected back to denser medium and partially refracted to rare medium
(d) light ray is absorbed completely by denser medium

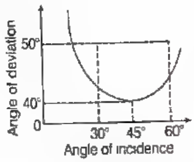
4. Ray of light transmitted from glass ($n = 3/2$) to water ($n = 4/3$). What is the value of critical angle?
(a) $\sin^{-1}\left(\frac{1}{2}\right)$ (b) $\sin^{-1}\sqrt{\frac{8}{9}}$
(c) $\sin^{-1}\left(\frac{8}{9}\right)$ (d) $\sin^{-1}\left(\frac{5}{7}\right)$

5. Two convex and concave lens are in contact and having focal length 12 cm and 18 cm, respectively. Focal length of joint lens will be
(a) 50 cm (b) 45 cm (c) 36 cm (d) 18 cm

6. Two lenses are kept in contact with powers +2 D and – 4 D. The focal length of this combination will be
(a) + 50 cm (b) – 50 cm
(c) – 25 cm (d) + 25 cm

7. A thin lens of glass ($\mu = 1.5$) of focal length $\pm$ 10 cm is immersed in water ($\mu = 1.33$). The new focal length is
(a) 20 cm (b) 40 cm (c) 48 cm (d) 12 cm

8. A plot of angle of deviation D *versus* angle of incidence for a triangular prism is shown below. The angle of incidence for which the light ray travels parallel to the base is



(a) 30° (b) 60°
(c) 45° (d) Data insufficient

9. An equilateral prism is in condition of minimum deviation. If incidence angle is 4/5 times of prism angle, then minimum deviation angle is
(a) 72° (b) 60° (c) 48° (d) 36°

10. Advantage of reflecting telescopes are
(a) no chromatic aberration
(b) parabolic reflecting surfaces are used
(c) weighs of mirror are much less than a lens of equivalent optical quality
(d) All of the above

11. When a wave undergoes reflection at an interface from rarer to denser medium, then change in its phase is **CBSE 2020**
(a) $\frac{\pi}{2}$ (b) zero (c) π (d) $\frac{\pi}{4}$

12. A bi-convex lens of focal length f is cut into two identical plano-convex lenses. The focal length of each part will be **CBSE 2020**
(a) f (b) $\frac{f}{2}$ (c) $2f$ (d) $4f$

13. biconcave lens of power P vertically splits into two identical plano-concave parts. The power of each part will be **CBSE 2020**
(a) $2P$ (b) $\frac{P}{2}$ (c) P (d) $\frac{P}{\sqrt{2}}$

[redacted]

ASSERTION AND REASON

Directions (Q. Nos. 14-19) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

14. **Assertion** A ray of light incident along the normal to the plane mirror retraces its path after reflection from the mirror.

Reason A ray of light along the normal has angle of incidence as $\pi/2$ and hence, it retraces its own path after reflection from mirror.

15. **Assertion** Refractive index of glass with respect to air is different for red light and violet light.

Reason Refractive index of a pair of media depends on the wavelength of light used.

16. **Assertion** Propagation of light through an optical fibre is due to total internal reflection taking place at the core-clade interface.

Reason Refractive index of the material of the core of the optical fibre is greater than that of air.

17. **Assertion** The refractive index of diamond is $\sqrt{6}$ and that of liquid is $\sqrt{3}$. If the light travels from diamond to the liquid, it will initially reflected when the angle of incidence is 45°.

Reason $\mu = \frac{1}{\sin C}$, where μ is the refractive index of diamond with respect to liquid.

18. **Assertion** Convergent lens property of converging remains same in mediums.

Reason Property of lens whether the ray is diverging or converging depends on the surrounding medium.

19. **Assertion** By roughening the surface of a glass at sheet its transparency can be reduced.

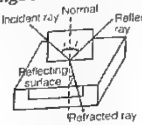
Reason Glass sheet with rough surface absorbs more light.

CASE BASED QUESTIONS

Directions (Q.No. 20) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

20. **Refraction of Light**

Refraction involves change in the path of light due to change in the medium.



When a beam of light encounters another transparent medium, a part of light gets reflected back into the first medium while the rest enters the other. The direction of propagation of an obliquely incident ray of light, that enters the other medium, changes at the interface of two media. This phenomenon is called refraction of light.

(i) Which of the following quantity remains unchanged after refraction?
(a) Speed of light
(b) Intensity of light
(c) Wavelength of light
(d) Frequency of light

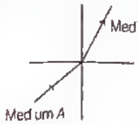
(ii) A ray of light strikes an air-glass interface at an angle of incidence ($i - 60°$) and gets refracted at an angle of refraction r. On increasing the angle of incidence ($i > 60°$), the angle of refraction r
(a) decreases (b) remains same
(c) is equal to 60° (d) increases

(iii) When an object lying in a denser medium is observed from rarer medium, then real depth of object is
(a) more than that observed
(b) less than that observed
(c) equals to observed depth
(d) depends on angle of vision

(iv) For the same angle of incidence, the angles of refraction in media P, Q and R are 35°, 25° and 15°, respectively. Which of the following relation hold true for the velocity of light in medium P, Q and R?
(a) $v_P < v_Q < v_R$ (b) $v_P < v_R < v_Q$
(c) $v_P > v_Q > v_R$ (d) $v_P > v_R > v_Q$

[redacted]

(v) A light ray enters from medium A to medium B as shwon in figure. The refractive index of medium B relative to A will be



(a) greater than unity (b) less than unity
(c) equal to unity (d) zero

VERY SHORT ANSWER Type Questions

21. At what angle, is a ray of light falling normally on a mirror reflected?

22. Does size of mirror affect the nature of the image?

23. Why are danger signals red in colour?

24. Why does a convex lens of glass $\mu = 1.5$ behave as a diverging lens when immersed in carbon disulphide of $\mu = 1.65$?

25. Why does rising sun appear oval shaped?

SHORT ANSWER Type Questions

26. Where should an object be placed from a convex lens to form an image of the same size? Can it happen in case of a concave lens?

27. Derive the expression for the effective focal length of two thin lenses in contact.

28. Discuss refraction of monochromatic light through a prism and derive its relation.

29. A 4 cm tall light bulb is placed at a distance of 8.30 cm from a double convex lens having a focal length of 15.2 cm. Calculate the position and size of the image of the bulb.

LONG ANSWER Type I Questions

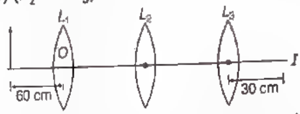
30. Minimum deviation suffered by violet, yellow and red beams passing through an equilateral transparent prism are 39.2°, 38.7° and 38.4°, respectively. Calculate the dispersive power in the medium.

31. A beam of light strikes a glass sphere of diameter 15 cm converging towards a point 30 cm behind the pole of the spherical surface. Find the position of the image, if μ of glass is 1.5.

LONG ANSWER Type II Questions

32. (i) Explain with reason, how the power of a diverging lens changes when (a) it is kept in a medium of refractive index greater than that of the lens. (b) incident red light is replaced by violet light.

(ii) Three lenses L_1, L_2 and L_3 each of focal length 30 cm are placed coaxially as shown in the figure. An object is held at 60 cm from the optic centre of lens L_1. The final real image is formed at the focus of L_3. Calculate the separation between (a) (L_1 and L_2) and (b) (L_2 and L_3). **All India 2017 C**



33. (i) Deduce the expression by drawing a suitable ray diagram for the refractive index of a triangular glass prism in terms of the angle of minimum deviation (D) and the angle of prism (A).
Draw a plot showing the variation of the angle of deviation with the angle of incidence.

(ii) Calculate the value of the angle of incidence when a ray of light incident on one face of an equilateral glass prism produces the emergent ray, which just grazes along the adjacent face. Refractive index of the prism is $\sqrt{2}$. **All India 2017C**

ANSWERS

1. (c)	2. (b)	3. (c)	4. (c)	5. (c)
6. (b)	7. (b)	8. (c)	9. (d)	10. (d)
11. (c)	12. (c)	13. (b)		

14. (c) Angle of incidence = Angle between incident ray and normal to the mirror = 0°

$\Rightarrow$ Angle of reflection = 0° (from laws of reflection)
Hence, the reflected ray retraces its path along the normal at an angle 0° with normal.

[redacted]

15. (*a*) Refractive index of any pair of media is inversely proportional to wavelength of light.

Hence, $\lambda_v < \lambda_r$

$\Rightarrow \mu_v > \mu_r$

where, λ_v and λ_r are the wavelengths of violet and red light and μ_r and μ_v are refractive index of violet and red light.

16. (*b*) Optical fibre communication is based on the phenomenon of total internal reflection at core-clad interface.

The refractive index of the material of the cladding, hence, light striking at core-cladding interface gets totally internally reflected. The light undergoes and reaches the other end of the fibre.

17. (*a*) Refractive index of diamond w.r.t. liquid

$${}^l\mu_d = \frac{1}{\sin C} = \frac{\mu_d}{\mu_l}$$

$$\Rightarrow \frac{\sqrt{6}}{\sqrt{3}} = \frac{1}{\sin C}$$

$$\Rightarrow \sin C = \frac{1}{\sqrt{2}} = \sin 45^\circ$$

$$\therefore C = 45^\circ$$

18. (*d*) In air or water a convex lens made of glass behaves as a convergent lens but when it is placed in carbon disulfide, it behaves as a divergent lens. Therefore, when a convergent lens is placed inside a transparent medium having refractive index greater than that of material of lens, it behaves as a divergent lens.

19. (*c*) When glass surface is made rough, then light incident on it is scattered in different directions. Due to which its transparency decreases. There is no effect of roughness on absorption of light

20. (i) (*d*) Refraction does not change the frequency of light.

(ii) (*d*) From Snell's law of refraction,

$${}^a\mu_g = \frac{\sin i}{\sin r} = \text{constant} \quad \text{...(i)}$$

Since, angle of incidence increase, the angle of refraction has to increase. So, that the ratio $\left(\frac{\sin i}{\sin r}\right)$ is a constant according to Eq. (i).

(iii) (*a*) When an object lying in a denser medium is observed from rarer medium, then real depth of object is more than that observed depth.

(iv) (*c*) As, $\mu = \frac{\sin i}{\sin r}$ ...(i)

or $\mu \propto \frac{1}{\sin r}$

$\Rightarrow \mu$ is maximum for R, since r is minimum and hence, $\sin r$ is minimum.

Also, $\mu = \frac{c}{v}$ ...(ii)

Therefore, if μ is maximum, v is minimum, i.e. velocity of light is minimum in medium R and order of velocity will be $v_P > v_Q > v_R$

(v) (*a*) We see that, ray of light bends towards the normal as we go from medium A to medium B. And we know that, when ray goes from rarer to denser medium, it bends towards normal.

So, that means refractive index of B is greater than A. Thus, refractive index of B relative than A = Refractive index of B/Refractive index of A. Since, Refractive index of B > Refractive of A. Therefore, refractive index of B relative than $A > 1$.

21. A ray of light which is incident normally on a mirror, is reflected along the same path, i.e. the angle of incidence as well as the angle of reflection is zero.

22. Nature of the image is independent of the size of mirror. Image can be real or virtual depending upon the position of object.

23. The colour red is used for danger signals because red light is scattered the least by air molecules. The effect of scattering is inversely, related to the fourth power of the wavelength, i.e. $I \propto \frac{1}{\lambda^4}$ of colour, so red light is able to travel the longest distance through fog, rain and the alike.

24. Here, $\mu = \frac{\mu_g}{\mu_c} = \frac{1.5}{1.65} < 1$

$\therefore \frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ Hence, it becomes negative.

So, it behaves like a diverging lens.

25. Sun appears oval shaped due to atmospheric refraction.

26. Refer to the text on page 374.

27. Refer to the text on page 375.

28. Refer to the text on pages 382 and 383.

29. $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \Rightarrow \frac{1}{15.2} = \frac{1}{8.30} + \frac{1}{d_i}$

$$0.0658 = 0.120 + \frac{1}{d_i}$$

$$\Rightarrow -0.0547 = \frac{1}{d_i}$$

$$-18.3 \text{ cm} = d_i$$

Also here, $d_i = -18.3$ cm

$$\therefore \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

$$\Rightarrow \frac{h_i}{(4.00)} = -\frac{(-18.3)}{8.30}$$

$$h_i = \frac{-(4.00)\cdot(-18.3)}{8.30}$$

$$h_i = 8.81 \text{ cm}$$

30. $\therefore$ Dispersive power $= \dfrac{\mu_v - \mu_r}{\mu_y - 1} = \dfrac{\left(\dfrac{\delta_v}{A}\right) - \left(\dfrac{\delta_r}{A}\right)}{\left(\dfrac{\delta_y}{A}\right)}$

$[\because \delta = (\mu - 1)A]$

$$= \frac{39.2 - 38.4}{38.7} = 0.0204$$

31. Here, $\mu_1 = 1, \mu_2 = 1.5, u = -\infty, R = \dfrac{15}{2} = 7.5\,\text{cm}$

$\therefore$ Using $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

32. (i) (a) Refer to text on page 373.

(b) Power of a lens increases if red light is replaced by violet light because

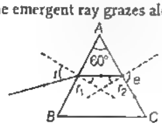
$$P = \frac{1}{f} = ({}_a\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

As refractive index is maximum for violet light in visible region of spectrum

(ii) Refer to Q. 34 on page 378.

33. (i) Refer to text on pages 382 and 383.

(ii) Given, the emergent ray grazes along the face AC.



$$e = 90^\circ$$

$$\mu = \sqrt{2}$$

$$\frac{\sin i}{\sin r_1} = \mu = \frac{\sin e}{\sin r_2}$$

$\Rightarrow \quad \dfrac{\sin 90^\circ}{\sin r_2} = \sqrt{2}$

i.e. $\quad \sin r_2 = \dfrac{1}{\sqrt{2}}$ or $r_2 = 45^\circ$

$\Rightarrow \quad r_1 + r_2 = \angle A = 60^\circ$

$$r_1 = 60 - r_2 = 15^\circ$$

$\Rightarrow \quad \dfrac{\sin i}{\sin 15^\circ} = \sqrt{2}$

$\Rightarrow \quad i = 21.47^\circ$

[redacted]

10

The connection between waves and rays of light is described by wave optics. The wave theory of light was put forward by Huygens' in 1678 and later on modified by Fresnel. According to this theory, light is a form of energy which travels in the form of transverse wave. The speed of light in a medium depends upon the nature of medium. In this chapter, we will study about the various phenomena (i.e. interference of light, diffraction of light) related to the wave nature of light.

WAVE OPTICS

|TOPIC 1|

Huygens' Principle

As discussed above, the speed of light in a medium depends upon the nature of the medium. Huygens supposed the existence of a hypothetical medium called "luminiferous ether" which filled the entire space.This medium was supposed to be massless with extremely high elasticity and very low density.

CHAPTER CHECKLIST

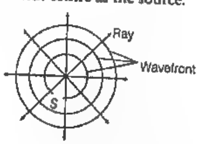
- Huygens' Principle
- Interference of Light
- Diffraction of Light

WAVEFRONT

It is the locus of points (wavelets) having the same phase (a surface of constant phase) of oscillations. A wavelet is the point of disturbance due to propagation of light. A line perpendicular to a wavefront is called a ray.

Depending on the shape of source of light, wavefronts can be of three types, which are given below

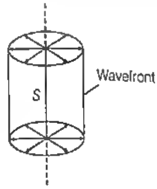
(i) **Spherical wavefront** When the source of light is a point source, the wavefront is a sphere with centre as the source.



Spherical wavefront

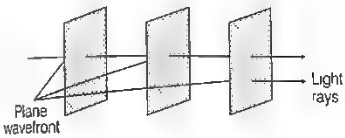
[redacted]

(ii) **Cylindrical wavefront** When the source of light is linear, e.g. a straight line source, slit etc. as shown in the figure. All the points equidistant from the source lie on a cylinder. Therefore, the wavefront is cylindrical in shape.



Cylindrical wavefront

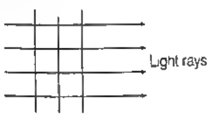
(iii) **Plane wavefront** When the point source or linear source of light is at very large distance, a small portion of spherical or cylindrical wavefront appears to be plane. Such a wavefront is called a plane wavefront.



Plane wavefront

Hence, the wavefront is a surface of constant phase.

The speed with which the wavefront moves outwards from the source is called the speed of the wave. The energy of the wave travels in a direction perpendicular to the wavefront.



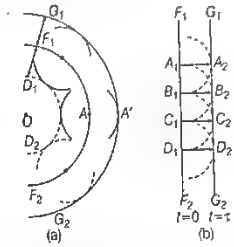
Plane wavefront

HUYGENS' PRINCIPLE

It is essentially a geometrical construction, which gives the shape of the wavefront at any time and allows us to determine the shape of the wavefront at a later time. According to Huygens' principle,

(i) Each point on the given wavefront (called primary wavefront) is the source of a secondary disturbance (called secondary wavelets) and the wavelets emanating from these points spread out in all directions with the speed of the wave.

(ii) A surface touching these secondary wavelets, tangentially in the forward direction at any instant gives the new wavefront at that instant. This is called secondary wavefront.



In Fig.(a), F_1F_2 is the section of the given spherical wavefront and G_1G_2 is the new wavefront in the forward direction. In Fig.(b), F_1F_2 is the section of the given plane wavefront and G_1G_2 is the new wavefront in the forward direction.

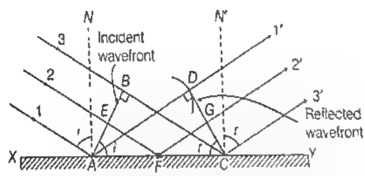
Note Huygens argued that the amplitude of the secondary wavelets is maximum in the forward direction and zero in the backward direction. Hence the backward secondary wavefront is absent.

Refraction and Reflection of Plane Waves Using Huygens' Principle

Huygens' principle can be used to explain the phenomena of reflection and refraction of light on the basis of wave theory of light.

Laws of Reflection at a Plane Surface

Let 1, 2, 3 be the incident rays and $1', 2', 3'$ be the corresponding reflected rays.



Laws of reflection by Huygens' principle

If c is the speed of the light, t is the time taken by light to go from B to C or A to D or E to G through F, then

$$t = \frac{EF}{c} + \frac{FG}{c} \qquad \text{...(i)}$$

In ΔAEF, $\sin i = \dfrac{EF}{AF}$

In ΔFGC, $\sin r = \dfrac{FG}{FC}$

[redacted]

or $t = \dfrac{AF \sin i}{c} + \dfrac{FC \sin r}{c}$

$\Rightarrow$ $t = \dfrac{AC \sin r + AF (\sin i - \sin r)}{c}$ $\quad [\because FC = AC - AF]$

For rays of light from different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the reflected wavefront.

So, t should not depend upon AF. This is possible only, if

$$\sin i - \sin r = 0$$

i.e. $\quad \sin i = \sin r$

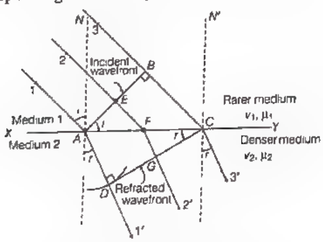
or $\quad \angle i = \angle r \quad$ ...(ii)

which is the first law of reflection.

Further, the incident wavefront AB, the reflecting surface XY and the reflected wavefront CD are all perpendicular to the plane of the paper. Therefore, incident ray, normal to the plane of the paper. Therefore, incident ray, normal to the mirror XY and reflected ray all lie in the plane of the paper. This proves the second law of reflection.

Laws of Refraction (Snell's Law) at a Plane Surface

Let 1, 2, 3 be the incident rays and 1′, 2′, 3′ be the corresponding refracted rays.



Laws of refraction by Huygens' principle

If v_1, v_2 are the speeds of light in the two media and t is the time taken by light to go from B to C or A to D or E to G through F, then $\quad t = \dfrac{EF}{v_1} + \dfrac{FG}{v_2}$

In ΔAFE, $\quad \sin i = \dfrac{EF}{AF}$

In ΔFGC, $\quad \sin r = \dfrac{FG}{FC}$

$\Rightarrow$ $\quad t = \dfrac{AF \sin i}{v_1} + \dfrac{FC \sin r}{v_2} \quad$ ...(iii)

$\Rightarrow$ $\quad t = \dfrac{AC \sin r}{v_2} + AF\left(\dfrac{\sin i}{v_1} - \dfrac{\sin r}{v_2}\right)$

For rays of light from different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the refracted wavefront. So, t should not depend upon AF. This is possible only, if

$$\frac{\sin i}{v_1} - \frac{\sin r}{v_2} = 0$$

$\Rightarrow$ $\quad \dfrac{\sin i}{\sin r} = \dfrac{v_1}{v_2} \quad$ ...(iv)

Now, if c represents the speed of light in vacuum, then $\mu_1 = \dfrac{c}{v_1}$ and $\mu_2 = \dfrac{c}{v_2}$ are known as the **refractive indices** of medium 1 and medium 2, respectively.

In terms of refractive indices, Eq. (iv) can be written as

$$\mu_1 \sin i = \mu_2 \sin r$$

$\Rightarrow$ $\quad \boxed{\mu = \dfrac{\sin i}{\sin r}}$

This is known as **Snell's law of refraction.**

Further, if λ_1 and λ_2 denote the wavelengths of light in medium 1 and medium 2, respectively and if the distance BC is equal to λ_1, then the distance AD will be equal to λ_2, thus

$$\frac{\lambda_1}{\lambda_2} = \frac{BC}{AD} = \frac{v_1}{v_2}$$

or $\quad \dfrac{v_1}{\lambda_1} = \dfrac{v_2}{\lambda_2}$

$\Rightarrow$ $\quad \nu_1 = \nu_2 \quad \left[\because \dfrac{v}{\lambda} = \nu\right]$

Hence, the frequency does not change on refraction.

Thus, frequency ν being a characteristic of the source, remains the same as light travels from one medium to another.

Also, wavelength is directly proportional to the (phase) speed and inversely proportional to refractive index.

$\therefore \quad \lambda' = \dfrac{\lambda}{\mu}, \; \mu = \dfrac{\lambda}{\lambda'} = \dfrac{c\lambda}{v\lambda} = \dfrac{c}{v}$

Behaviour of Prism, Lens and Spherical Mirror Towards Plane Wavefront

(i) **Behaviour of a prism** Since, the speed of light waves are less in glass, so the lower portion of the incoming wavefront (which travels through the greatest thickness

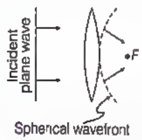
[redacted]

of glass prism) will get delayed resulting in a tilt in the emerging wavefront.

Reflection of plane wave by a thin prism

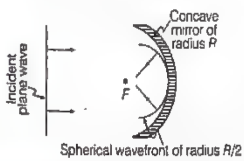
(ii) **Behaviour of a lens** The central part of the incident plane wave traverses the thickest portion of the lens and is delayed the most.

Due to this, the emerging wavefront has a depression at the centre. Therefore, the wavefront becomes spherical and converges to the point F which is known as the focus.



Reflection of plane wave by convex lens

(iii) **Behaviour of a spherical mirror** The central part of the incident wavefront travels the largest distance before reflection from the concave mirror. Hence, gets delayed, as a result of which the reflected wavefront is spherical which converges at the focal point F.



Reflection of plane wave by concave mirror

EXAMPLE |1|

(i) When monochromatic light is incident on a surface separating two media, the reflected and refracted lights both have the same frequency as the incident frequency. Explain, why?

(ii) When light travels from a rarer to a denser medium, the speed decreases. Does the reduction in speed imply a reduction in the energy carried by the light wave?

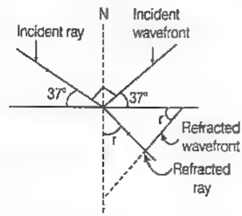
Sol. (i) Reflection and refraction arises through interaction of incident light with the atomic constituents of matter. Atoms may be viewed as oscillators, which take up the frequency of the external agency (light) causing forced oscillations.

The frequency of light emitted by a charged oscillator equals its frequency of oscillation. Thus, the frequency of scattered light equals the frequency of incident light.

(ii) No, the energy carried by a wave depends on the amplitude of the wave, not on the speed of wave propagation.

EXAMPLE |2| A plane wavefront is incident from air ($\mu = 1$) at an angle of 37° with a horizontal boundary of a refractive medium from air of refractive index $\mu = \frac{3}{2}$. Find the angle of refracted wavefront with the horizontal boundary.

Sol. It has been given that incident wavefront makes 37° with horizontal. Hence, incident ray makes 37° with normal as the ray is perpendicular to the wavefront.



Now, by Snell's law, $\frac{\sin 53^\circ}{\sin r} = \frac{3}{2}$

$$\Rightarrow \quad \sin r = \frac{2}{3} \times \sin 53^\circ$$

$$\therefore \quad r = \sin^{-1}(0.66 \times 0.79)$$

$$= 31.33^\circ$$

which is same as angle of refractive wavefront with horizontal.

DOPPLER'S EFFECT IN LIGHT

According to this effect, whenever there is a relative motion between a source of light and observer, the apparent frequency of light received by observer is different from the true frequency of light emitted actually from the source of light. Astronomers call the increase in wavelength due to Doppler effect as red shift, since a wavelength in the middle of the visible region of spectrum moves towards the red end of the spectrum.

When waves are received from a source moving towards the observer, there is an apparent decrease in wavelength, this is referred to as blue shift.

The fractional change in frequency is given by

$$\frac{\Delta \nu}{\nu} = -\frac{v_{\text{radial}}}{c}$$

[redacted]

where, v_{radial} is the component of the source velocity along the line joining the observer to the source relative to the observer. v_{radial} is considered positive, when the source moves away from the observer. The above formula is valid only when the speed of the source is small compared to that of light.

EXAMPLE |3| What speed should a galaxy move with respect to us so that the sodium light at 589 nm is observed at 589.6 nm?

Sol. Since, $v\lambda = c$,

$$\Rightarrow \quad \frac{\Delta v}{v} = -\frac{\Delta\lambda}{\lambda} \quad \text{[for small changes in } v \text{ and } \lambda]$$

Here, $\Delta\lambda = (589.6 - 589)\text{ nm} = 0.6\text{ nm}$

We know that, $\dfrac{\Delta v}{v} = -\dfrac{\Delta\lambda}{\lambda} = -\dfrac{v_{radial}}{c}$

$$\because \quad v_{radial} = +c\left(\frac{0.6}{589}\right)$$

$$= 3.06 \times 10^5 \text{ m/s}$$

$$= 306 \text{ km/s}$$

Therefore, the galaxy is moving 306 km/s away from us.

Applications of Doppler's Effect in Light

Some important applications of Doppler's effect are as given below

(i) Measuring the speed of stars and galaxies.

(ii) Measuring speed of rotation of the sun.

(iii) Estimation of velocity of aeroplanes, rockets and submarines, etc.

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. Huygens' principle of secondary wavelets may be used to
(a) find the velocity of light in vacuum
(b) explain the particle's behaviour of light
(c) find the new position of a wavefront
(d) explain photoelectric effect

2. Which one of the following phenomena is not explained by Huygens' construction of wavefront?
(a) Refraction (b) Reflection
(c) Diffraction (d) Origin of spectra

3. The direction of wavefront of a wave with the wave motion is
(a) parallel (b) perpendicular
(c) opposite (d) at an angle of θ

4. Ray diverging from a point source on a wavefront are
(a) cylindrical (b) spherical
(c) plane (d) cubical

5. According to Huygens' principle, each point of the wavefront is the source of
(a) secondary disturbance
(b) primary disturbance
(c) third disturbance
(d) fourth disturbance

6. When light is refracted into a denser medium
(a) its wavelength and frequency both increases
(b) its wavelength increases but frequency remains unchanged
(c) its wavelength decreases but frequency remains the same
(d) its wavelength and frequency both decreases

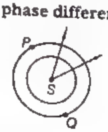
7. The Doppler effect is produced if
(a) the source is in motion
(b) the detector is in motion
(c) Both (a) and (b)
(d) None of the above

8. In the context of Doppler effect in light, the term red shift signifies
(a) decrease in frequency
(b) increase in frequency
(c) decrease in intensity
(d) increase in intensity

VERY SHORT ANSWER Type Questions

9. Define a wavefront. Foreign 2009

10. In the given figure, there are two points P and Q, what is the phase difference between them?



11. State Huygens' principles of secondary wavelets.

12. If a plane wavefront is incident on a prism, then draw the refracted wavefront.

[redacted]

13. Draw the wavefront coming out from a convex lens when a point source of light is placed at its focus. Foreign 2009

SHORT ANSWER Type Questions

14. (i) Differentiate between a ray and a wavefront.
(ii) What is the phase difference between any two points on a wavefront?

15. What is the shape of the wavefront on earth for sunlight? NCERT Exemplar

16. Construct a diagram to show the wave characteristics of light.

17. Light of wavelength 5000 Å propagating in air gets partly reflected from the surface of water. How will the wavelengths and frequencies of the reflected and refracted light be affected? All India 2015

18. Consider a point at the focal point of convergent lens. Another convergent lens of short focal length is placed on the other side. What is the nature of the wavefronts emerging from the final image? NCERT Exemplar

19. Discuss Doppler's effect in the electromagnetic waves.

20. Define a wavefront. Using Huygens' principle, verify the laws of reflection at a plane surface. CBSE 2018

21. Is Huygens' principle valid for longitudinal sound waves? NCERT Exemplar

LONG ANSWER Type I Questions

22. Define the following terms and give its source of origin.
(i) Spherical wavefront
(ii) Plane wavefront
(iii) Cylindrical wavefront

23. Choose the statement as right or wrong and justify.
(i) Light is longitudinal wave, which gives the sensation of vision.
(ii) A wavefront is a continuous locus of all points in which all particles vibrate in different phase.
(iii) Rays of light are always normal to its wavefront.

24. Using Huygens' geometrical construction of wavefronts, show how a plane wave gets reflected from a surface. Hence, verify laws of reflection. [CBSE 2019 Delhi]

25. Use Huygens' principle to show how a plane wavefront propagates from a denser to rarer medium. Hence, verify Snell's law of refraction. Delhi, 2015

26. Define a wavefront. Use Huygens' geometrical construction to show the propagation of plane wavefront from a rarer medium
(i) to a denser medium.
(ii) undergoing refraction, hence derive Snell's law of refraction. Foreign 2012

27. Use Huygens' principle to verify the laws of refraction. Delhi 2011

28. What is the shape of the wavefront in each of the following cases?
(i) Light diverging from point source.
(ii) Light emerging out of a convex lens when a point source is placed at its focus.
(iii) The portion of the wavefront of light from a distant star intercepted by the earth.
NCERT

29. Define the term wavefront. State Huygen's principle. Consider a plane wavefront incident on a thin convex lens. Draw a proper diagram to show how the incident wavefront traverses through the lens and after refraction focusses on the focal point of the lens, giving the shape of the emergent wavefront. All India 2016

30. (i) Use Huygens' geometrical construction to show the behaviour of a plane wavefront,
(a) passing through a biconvex lens
(b) reflected by a concave mirror.
(ii) When monochromatic light is incident on a surface separating two media, why does the refracted light have the same frequency as that of the incident light?

31. You have learnt in the text how Huygens' principle leads to the laws of reflection and refraction.Use the Huygens' principle to deduce directly that a point object placed in front of a plane mirror produces a virtual image whose distance from the mirror is equal to the distance of the object from the mirror. NCERT

[redacted]

32. Give three applications of Doppler's effect in light.

LONG ANSWER Type II Questions

33. (i) State Huygens' principle. Using this principle, draw a diagram to show how a plane wavefront incident at the interference of the two media gets refracted when it propagates from a rarer to a denser medium.
Hence, verify Snell's law of refraction.
(ii) Is the frequency of reflected and refracted light same as the frequency of incident light? **Delhi 2013**

34. (i) Use Huygens' geometrical construction to show how a plane wavefront at $t = 0$ propagates and produces a wavefront at a later time.
(ii) Verify, using Huygens' principle, Snell's law of refraction of a plane wave propagating from a denser to a rarer medium.
(iii) When monochromatic light is incident on a surface separating two media, the reflected and refracted light both have the same frequency. Explain, why ? **Delhi 2013C**

35. (i) A plane wavefront approaches a plane surface separating two media. If medium 1 is optically denser and medium 2 is optically rarer, using Huygens' principle, explain and show how a refracted wavefront is constructed?
(ii) Verify Snell's law.
(iii) When a light wave travels from a rarer to a denser medium, the speed decreases. Does it imply reduction in its energy? Explain.

NUMERICAL PROBLEMS

36. Light of wavelength 5000 Å falls on a plane reflecting surface. What are the wavelength and frequency of the reflected light? For what angle of incidence is the reflected ray normal to the incident ray? **NCERT**

37. (i) The refractive index of glass is 1.5. What is the speed of light in glass? (Speed of light in vacuum is 3×10^8 ms^{-1})
(ii) Is the speed of light in glass independent of the colour of light? If not, which of the two colours red and violet travels slower in a glass prism? **NCERT**

38. The 6563 Å H_α-line emitted by hydrogen in a star is found to be red shifted by 15 Å. Estimate the speed with which the star is receding from the earth. **NCERT**

39. The spectral line for a given element in light received from a distant star is shifted towards the longer wavelength by 0.032%. Deduce the velocity of star in the line of sight.

40. Monochromatic light of wavelength 589 nm is incident from air on a water surface. What are the wavelength, frequency and speed of
(i) reflected and
(ii) refracted light? (μ of water is 1.33) **NCERT**

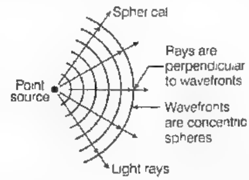
HINTS AND SOLUTIONS

1. (c) Every point on a given wavefront act as a secondary source of light and emits secondary wavelets which travels in all directions with the speed of light in the medium. A surface touching all these secondary wavelets tangentially in the forward direction, gives new wavefront at that instant of time.

2. (d) Huygens' construction does not explains quantisation of energy and it is not able to explain origin of spectrum.

3. (b) Wavefront is a surface perpendicular to a ray but a wavefront moves in the direction of the light.

4. (b) Wavefronts emitting from a point source are spherical wavefronts.



5. (a) According to Huygens' principle, each point of the wavefront is the source of a secondary disturbance and the wavelength emanating from these points spread out in all directions with the speed of the wave.

6. (c) Wavelength is dependent on refractive index medium by

$$\frac{\lambda_1}{\lambda_2} = \frac{\mu_2}{\mu_1}$$

So, in denser medium, $\mu_2 > \mu_1$, so $\lambda_1 > \lambda_2$ (i.e. wavelength decreases as the light travels from rarer to denser medium)

$\because$ $$c = v\lambda$$

7. (c) In the case of Doppler's effect, there is a relative motion between source and detector.

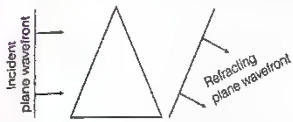
8. (a) When source moves away from the observer, frequency observed is smaller than that emitted from the source and (as if light emitted is yellow but it will be observed as red) this shift is called red shift.

[redacted]

9. Wavefront is the locus of points (wavelets) having the same phase of oscillations.

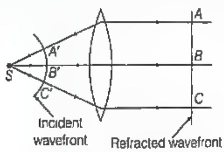
10. Since, the two points P and Q are at the same locus of the source S, so the phase difference between them equals to zero.

11. According to Huygens' principle,
   (i) Each point on the given wavefront is the source of secondary disturbance and the wavelet emanating from these points spread out in all directions with the speed of wave.
   (ii) A surface touching these secondary wavelets, tangentially in the forward direction at any instant gives the new wavefront at that instant.

12. 

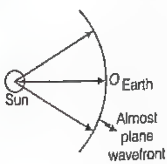
Reflection of plane wave by a thin prism

13. The wavefront in the given condition is shown below in figure.



14. Refer to the text on pages 400 and 401.

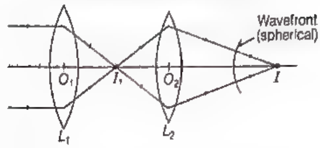
15. We know that, the sun is at very large distance from the earth. Assuming sun as spherical, it can be considered as point source situated at infinity.
   Due to the large distance the radius of wavefront can be considered as large (infinity) and hence, wavefront is almost plane.



16. Refer to the text on pages 400 and 401.

17. The frequency and wavelength of reflected wave will not change. The refracted wave will have same frequency. The velocity of light in water is given by $v = f\lambda$
   where, v = velocity of light
   f = frequency of light
   λ = wavelength of light
   If velocity will decrease, then wavelength (λ) will also decrease.

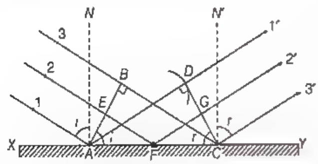
18. Consider the ray diagram shown below



The point image I_1 due to L_1 is at the focal point. Now, due to the converging lens L_2, let the final image formed be I which is a point image, hence the wavefront for this image will be of spherical symmetry.

19. Refer to the text on pages 403 and 404.

20. Wavefront It is the locus of points (wavelets) having the same phase (a surface of constant phase) of oscillations.
   Laws of reflection at a plane surface (On Huygens' principle)
   Let 1, 2, 3 be the incident rays and 1′, 2′, 3′ be the corresponding reflected rays.



Laws of reflection by Huygens' principle

If c is the speed of the light, t is the time taken by light to go from B to C or A to D or E to G through F, then

$$t = \frac{EF}{c} + \frac{FG}{c} \qquad \text{...(i)}$$

In ΔAEF, $\sin i = \dfrac{EF}{AF}$

In ΔFGC, $\sin r = \dfrac{FG}{FC}$

or $$t = \frac{AF \sin i}{c} + \frac{FC \sin r}{c}$$

or $$t = \frac{AC \sin r + AF(\sin i - \sin r)}{c}$$

$$(\because FC = AC - AF)$$

For rays of light from different parts on the incident wavefront, the values of AF are different. But light from different points of the incident wavefront should take the same time to reach the corresponding points on the reflected wavefront.

So, t should not depend upon AF. This is possible only if

$$\sin i - \sin r = 0$$

i.e. $$\sin i = \sin r$$

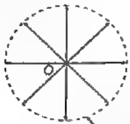
or $$\angle i = \angle r \qquad \text{...(ii)}$$

which is the **first law of reflection.**

[redacted]

Further, the incident wavefront AB, the reflecting surface XY and the reflected wavefront CD are all perpendicular to the plane of the paper. Therefore, incident ray, normal to the mirror XY and reflected ray all lie in the plane of the paper. This is second law of reflection.

21. When we are considering a point source of sound wave. The disturbance due to the source propagates in spherical symmetry, i.e. in all directions.



The formation of wavefront is in accordance with Huygens' principle. So, Huygens' principle is valid for longitudinal sound waves also.

22. Refer to text on pages 400 and 401.

23. Refer to text on pages 400 and 401.
(i) wrong (ii) wrong
(iii) Right

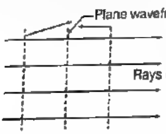
24. Refer to the text on pages 401 and 402.

25. Refer to text on page 402.

26. Refer to text on pages 400 and 402.

27. Refer to text on page 402.

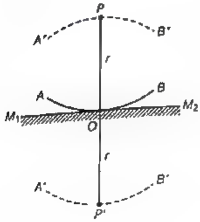
28. (i) Refer to text on page 402.
(ii) Refer to Q. 13 on page 405.
(iii) As, the star (i.e. source of light) is very far off, i.e. at infinity, the wavefront intercepted by the earth must be a plane wavefront.



29. Refer to the text on pages 400, 401 and 403.

30. (i) Refer to the text on pages 402 and 403.
(ii) Refer to the Example 1(i) on page 403.

31. In the figure, P is a point object placed at a distance r from a plane mirror M_1M_2. With P as centre and $PO = r$ as radius, draw a spherical arc AB. This is the spherical wavefront from the object, incident on M_1M_2.



32. Refer to text on page 404.

33. (i) Refer to text on pages 401 and 402.
(ii) The frequency of reflected and refracted light remains same as the frequency of incident light because frequency only depends on the source of light.

34. (i) Refer to text on page 401
(ii) Refer to text on page 402
(iii) Refer to Example 1(i) on page 403.

35. (i) and (ii) refer to text on page 402.
(iii) Refer to Example 1(ii) on page 403.

36. On the reflection, there is no change in wavelength and frequency. So, wavelength of reflected light will be 5000Å. Frequency of the reflected light,

$$\nu = \frac{c}{\lambda} = \frac{3\times10^8}{5\times10^{-7}} = 6\times10^{14}\,\text{Hz}$$

For $i = 45^\circ$, reflected ray becomes normal to the incident ray.

37. (i) Here, refractive index, $\mu = 1.5$

$c = 3\times 10^8\,\text{m s}^{-1}, v = ?$

As, $\mu = \dfrac{c}{v}$

$\Rightarrow \quad v = \dfrac{c}{\mu} = \dfrac{3\times 10^8}{1.5} = 2\times 10^8\,\text{ms}^{-1}$

(ii) No, the refractive index and the speed of light in a medium depend on wavelength, i.e. colour of light. We know that, $\mu_v > \mu_r$. Therefore, $v_{\text{violet}} < v_{\text{red}}$. Hence, violet component of white light travels slower than the red component.

38. Here, $\lambda = 6563\,\text{Å}, \Delta\lambda = +15\,\text{Å}$ and $c = 3\times10^8\,\text{ms}^{-1}$

Since, the star is receding away, hence its velocity v is negative.

$\therefore \quad \Delta\lambda = -\dfrac{v\lambda}{c}$ or $v = -\dfrac{c\Delta\lambda}{\lambda}$

$$= -\frac{3\times10^8\times15}{6563} = -6.86\times10^5\,\text{m s}^{-1}$$

Negative sign shows that star is receding away from the earth.

[redacted]

39. Here, $\frac{\Delta\lambda}{\lambda} = \frac{0.032}{100}, v = ?$

Since, the wavelength of light from a star is shifting towards longer wavelength side, then $\Delta\lambda$ is positive. Hence, star is moving away from the earth, i.e. v is negative.

$$\therefore \quad v = \frac{-\Delta\lambda}{\lambda}c = \frac{-0.032}{100} \times 3 \times 10^8$$

$$= -9.6 \times 10^4 \text{ m/s}$$

40. Here, wavelength, $\lambda = 589$ nm,

$c = 3 \times 10^8$ m/s, $\mu = 1.33$

(i) For reflected light,

Wavelength, $\lambda = 589 \text{ nm} = 589 \times 10^{-9}$ m

$$\Rightarrow \quad \nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{589 \times 10^{-9}}$$

$$= 5.09 \times 10^{14} \text{ Hz}$$

Speed, $v = c = 3 \times 10^8$ m/s

(ii) For refracted light,

$$\lambda' = \frac{\lambda}{\mu} = \frac{589 \times 10^{-9}}{1.33}$$

$$= 4.42 \times 10^{-7} \text{ m}$$

As, frequency remains unaffected on entering another medium, therefore

$$\nu' = \nu = 5.09 \times 10^{14} \text{ Hz}$$

$$\therefore \text{Speed, } v' = \frac{c}{\mu} = \frac{3 \times 10^8}{1.33} = 2.25 \times 10^8 \text{ m/s}$$

|TOPIC 2|

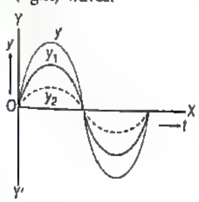
Interference of Light

SUPERPOSITION PRINCIPLE

According to this principle, at a particular point in the medium, the resultant displacement (y) produced by a number of waves is the vector sum of the displacements produced by each of the waves ($y_1, y_2, \ldots$).

i.e. $\quad y = y_1 + y_2 + y_3 + y_4 + \ldots$

Clearly, each wave contributes as if the other wave is not present. The superposition principle which was stated first for mechanical waves is equally applicable to the electromagnetic (light) waves.



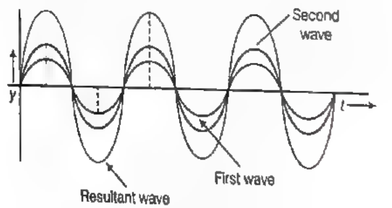
Superposition of waves

INTERFERENCE OF LIGHT WAVES

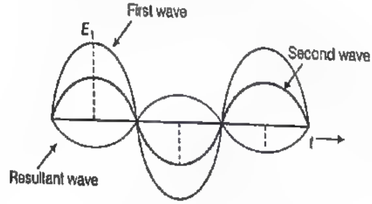
When two light waves of exactly equal frequency having constant phase difference w.r.t. time travelled on same direction and superimpose (overlap) with each other, then intensity of resultant wave does not remain uniform in space.

This phenomenon of formation of maximum intensity at some points and minimum intensity at some other points by two identical light waves travelling in same direction is called the **interference of light.**

At the points, where the resultant intensity of light is maximum, interference is said to be **constructive**. At the points, where the resultant intensity of light is minimum, the interference is said to be **destructive**.



(a) Constructive interference



(b) Destructive interference

[redacted]

Theory of Interference of Waves

Let the waves from two sources of light be represented as

$$y_1 = a \sin \omega t \quad \text{and} \quad y_2 = b \sin (\omega t + \phi)$$

where, a and b are the respective amplitudes of the two waves and ϕ is the constant phase angle by which second wave leads the first wave. Applying superposition principle, the magnitude of the resultant displacement of the waves is

$$y = y_1 + y_2$$

$$\Rightarrow \quad y = a \sin \omega t + b \sin (\omega t + \phi)$$

$$\Rightarrow \quad y = a \sin \omega t + b \sin \omega t \cdot \cos\phi + b \cos \omega t \cdot \sin \phi$$

$$[\because \sin (A + B) = \sin A \cos B + \cos A \sin B]$$

$$\Rightarrow \quad y = (a + b \cos \phi) \sin \omega t + b \sin \phi \cos \omega t$$

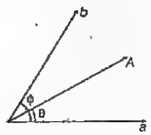
Putting $a + b \cos \phi = A \cos \theta$ and $b \sin \phi = A \sin \theta$

we get, $\quad y = A \cos \theta \cdot \sin \omega t + A \sin \theta \cdot \cos \omega t$

or $\quad y = A \sin (\omega t + \theta)$

where, A is the resultant amplitude and θ is the resultant phase difference.

$$\therefore \quad A = \sqrt{a^2 + b^2 + 2ab \cos \phi} \text{ and } \tan \theta = \frac{b \sin \phi}{a + b \cos \phi}$$



Resultant of amplitudes a and b

As, intensity is directly proportional to the square of the amplitude of the wave, i.e. $I \propto a^2$

So, for two different cases,

$$I_1 = ka^2, I_2 = kb^2$$

$$\therefore \quad I_R = kA^2 = k(a^2 + b^2 + 2ab \cos \phi)$$

$$\therefore \quad \boxed{I_R = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi}$$

For constructive interference

I should be maximum, for which

$$\cos \phi = \text{maximum} = +1$$

$\therefore$ Phase difference, $\phi = 0, 2\pi, 4\pi, \ldots$

i.e. $\quad \phi = 2n\pi$, where $n = 1, 2, \ldots$

If Δx be the path difference between the interfering waves, then

$$\Delta x = \frac{\lambda}{2\pi} \phi = \left(\frac{\lambda}{2\pi}\right)(2n\pi)$$

$$\Rightarrow \quad \Delta x = n\lambda$$

and $\quad \boxed{I_{max} \propto (a + b)^2}$

Hence, condition for constructive interference at a point is that, the phase difference between the two waves reaching the point should be zero or an even integral multiple of 2π. Equivalent path difference between the two waves reaching the point should be zero or an integral multiple of full wavelength.

For destructive interference

I should be minimum, for which

$$\cos \phi = \text{minimum} = -1$$

$\therefore$ Phase difference, $\phi = \pi, 3\pi, 5\pi, \ldots$

i.e. $\quad \phi = (2n - 1)\pi$

where, $\quad n = 1, 2, \ldots$

The corresponding path difference between the two waves is

$$\Delta x = \left(\frac{\lambda}{2\pi}\right)\phi = \left(\frac{\lambda}{2\pi}\right)(2n - 1)\pi$$

$$\Rightarrow \quad \Delta x = (2n - 1)\frac{\lambda}{2}$$

and $\quad \boxed{I_{min} \propto (a - b)^2}$

Hence, condition for destructive interference at a point is that, the phase difference between the two waves reaching the point should be an odd integral multiple of π or path difference between the two waves reaching the point should be an odd integral multiple of half wavelength.

Comparison of Intensities of Maxima and Minima

As, $\quad I_{max} \propto (a + b)^2$ and $I_{min} \propto (a - b)^2$

$$\therefore \quad \frac{I_{max}}{I_{min}} = \frac{(a + b)^2}{(a - b)^2} = \frac{\left(\frac{a}{b} + 1\right)^2}{\left(\frac{a}{b} - 1\right)^2}$$

$$\Rightarrow \quad \boxed{\frac{I_{max}}{I_{min}} = \frac{(r + 1)^2}{(r - 1)^2}}$$

where, $r = \frac{a}{b}$ (ratio of amplitudes)

Interference and Energy Conservation

In the interference pattern,

$$I_{max} = k(a + b)^2, \; I_{min} = k(a - b)^2$$

Average intensity of light in the interference pattern,

$$I_{av} = \frac{I_{max} + I_{min}}{2} = \frac{k(a + b)^2 + k(a - b)^2}{2}$$

[redacted]

$$= \frac{2k(a^2+b^2)}{2} = k(a^2+b^2)$$

Intensity of light is simply being redistributed, i.e. energy is being transferred from regions of destructive interference to the regions of constructive interference.

Thus, the principle of energy conservation is being obeyed in the process of interference of light.

EXAMPLE |1| Light waves from two coherent sources having intensities I and $2I$ cross each other at a point with a phase difference of 60°. What is the resultant intensity at that point?

Sol. Here, $I_1 = I$ and $I_2 = 2I$, $\phi = 60°$

$\therefore$ Resultant intensity,

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$
$$= I + 2I + 2\sqrt{I \times 2I}\cos 60°$$
$$= 3I + I\sqrt{2} = I(3+\sqrt{2}) = 4.414\, I$$

EXAMPLE |2| Light wave from two coherent sources of intensities in ratio 64:1 produces interference. Calculate the ratio of maxima and minima of the interference pattern.

Sol. Ratio of intensities of coherent sources,

i.e. $\frac{I_1}{I_2} = \frac{64}{1} = \frac{8^2}{1^2}; \frac{I_{max}}{I_{min}} = ?$

We have, $I_1 \propto a^2$, $I_{max} \propto (a+b)^2$

and $I_2 \propto b^2$, $I_{min} \propto (a-b)^2$

$\Rightarrow \frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{64}{1}$

$\Rightarrow \frac{a}{b} = \frac{8}{1}; \; a = 8b$

So, $\frac{I_{max}}{I_{min}} = \frac{(a+b)^2}{(a-b)^2} = \frac{(8b+b)^2}{(8b-b)^2} = \frac{81b^2}{49b^2}$

$= 81 : 49$

COHERENT AND INCOHERENT SOURCES

Light sources are of two types, i.e. coherent and non-coherent light sources. The sources of light which emit light waves of same wavelength, same frequency and are in same phase or having constant phase difference are known as coherent sources.

Two such sources of light, which do not emit light waves with constant phase difference are called incoherent sources.

Need of Coherent Sources for the Production of Interference Pattern

As discussed earlier, when two monochromatic waves of intensity I_1, I_2 and phase difference ϕ meet at a point, then the resultant intensity is given by

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

Here, the term $2\sqrt{I_1 I_2}\cos\phi$ is called interference term.

There are two possibilities.

(i) If $\cos\phi$ remains constant with time, then the total intensity at any point will be constant. The intensity will be maximum $(\sqrt{I_1}+\sqrt{I_2})^2$ at points, where $\cos\phi$ is 1 and minimum $(\sqrt{I_1}-\sqrt{I_2})^2$ at points, where $\cos\phi$ is -1. The sources in this case are coherent.

(ii) If $\cos\phi$ varies continuously with time assuming both positive and negative value, then the average value of $\cos\phi$ will be zero over time interval of measurement. The interference term averages to zero. There will be same intensity $I = I_1 + I_2$ at every point. The two sources in this case are incoherent.

Note In practice, coherent sources are produced either by dividing the wavefront or by dividing the amplitude of the incoming waves.

Requirements for Obtaining Two Coherent Sources of Light

Following are the requirements (conditions) for obtaining two coherent sources of light

(i) Coherent sources of light should be obtained from a single source by some device.

Two coherent sources can be obtained either by
- (a) the source and its virtual image (Lloyd's mirror).
- (b) the two virtual images of the same source (Fresnel's biprism)
- (c) two real images of the same source (Young's double slit).

(ii) The two sources should give monochromatic light.

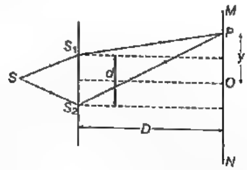
(iii) The path difference between light waves from two sources should be small.

(iv) Coherent sources can be produced by two methods:
- (a) By division of wavefront (Young's double slit experiment).
- (b) By division of amplitude (partial reflection or refraction in thin films).

[redacted]

YOUNG'S DOUBLE SLIT EXPERIMENT

Suppose S_1 and S_2 are two fine slits, a small distance d apart. They are illuminated by a strong source S of monochromatic light of wavelength λ. MN is a screen at a distance D from the slits.



Young's double slit arrangement to produce interference pattern

Consider a point P at a distance y from O, the centre of screen.

The path difference between two waves arriving at point P is equal to $S_2P - S_1P$.

Now,

$$(S_2P)^2 - (S_1P)^2 = \left[D^2 + \left(y + \frac{d}{2}\right)^2\right] - \left[D^2 + \left(y - \frac{d}{2}\right)^2\right]$$

$$= 2yd$$

Thus, $S_2P - S_1P = \dfrac{2yd}{S_2P + S_1P}$

But $\quad S_2P + S_1P \approx 2D$

$\therefore \quad S_2P - S_1P \approx \dfrac{dy}{D}$

For constructive interference (Bright fringes)

Path difference $= \dfrac{dy}{D} = n\lambda$, where, $n = 0, 1, 2, 3, \ldots$

$\therefore \quad y = \dfrac{nD\lambda}{d} \quad [\because n = 0, 1, 2, 3, \ldots]$

Hence, for $n = 0$, $y_0 = 0$ at O central bright fringe

for $n = 1$, $\quad y_1 = \dfrac{D\lambda}{d}$ for 1st bright fringe

for $n = 2$, $\quad y_2 = \dfrac{2D\lambda}{d}$ for 2nd bright fringe

for $n = n$, $\quad y_n = \dfrac{nD\lambda}{d}$ for nth bright fringe

The separation between two consecutive bright fringes is $\quad \beta = \dfrac{nD\lambda}{d} - \dfrac{(n-1)D\lambda}{d} = \dfrac{D\lambda}{d}$

For destructive interference (Dark fringes)

Path difference $= \dfrac{dy}{D} = (2n-1)\dfrac{\lambda}{2}$

or $\quad y = (2n-1)\dfrac{D\lambda}{2d}$, where, $n = 1, 2, 3, \ldots$

Hence, for $n = 1$, $\quad y_1' = \dfrac{D\lambda}{2d}$ for 1st dark fringe

for $n = 2$, $\quad y_2' = \dfrac{3D\lambda}{2d}$ for 2nd dark fringe

for $n = n$, $\quad y_n' = (2n-1)\dfrac{D\lambda}{2d}$ for nth dark fringe

The separation between two consecutive dark fringes is

$$\beta' = (2n-1)\frac{D\lambda}{2d} - \{2(n-1)-1\}\frac{D\lambda}{2d} = \frac{D\lambda}{d}$$

Fringe Width

The distance between two consecutive bright or dark fringes is called fringe width W.

$\therefore$ Fringe width, $\boxed{W = \dfrac{D\lambda}{d}}$

The above formula is free from n that means the width of all fringes is same.

Fringe width is directly proportional to λ. Hence, the fringes of red light (longer wavelength) are broader than the fringes of blue light (shorter wavelength).

Intensity of the Fringes

For a bright fringe, $\phi = 2n\pi$

and $\quad \cos\phi = \cos 2n\pi = 1$

So, $\quad I_R = I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2} = 4I$

[as, $I_1 = I_2 = I$ in YDSE]

$\therefore$ Intensity of a bright fringe $= 4I$

For a dark fringe, $\phi = (2n-1)\pi$

$\Rightarrow \quad \cos\phi = -1$

So, $\quad I_R = I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2} = 0$

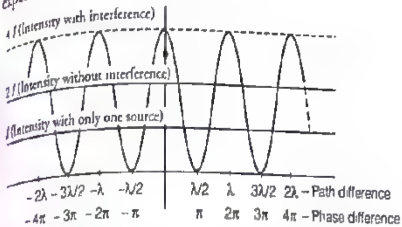
$\therefore$ Intensity of a dark fringe $= 0$

Note If YDSE apparatus is immersed in a liquid of refractive index μ, then wavelength of light and hence fringe width decreases μ times.

[redacted]

Distribution of Intensity

The distribution of intensity in Young's double slit experiment is shown below



Relation among Intensity, Amplitude of the Wave and Width of the Slit

If W_1 and W_2 are widths of two slits from which intensities of light I_1 and I_2 emanate, then

$$\boxed{\frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{W_1}{W_2}}$$

where, a and b are the respective amplitudes of two waves.

EXAMPLE |3| Two slits are made one millimetre apart and the screen is placed one metre away. What is the fringe separation when blue-green light of wavelength 500 nm is used? **NCERT**

Sol. Here, $d = 1\text{mm} = 1\times10^{-3}\text{m}$, $D = 1\text{m}$,

$$\lambda = 500\text{nm} = 500\times10^{-9}\text{m} = 5\times10^{-7}\text{m}$$

As, fringe width, $\beta = \dfrac{D\lambda}{d}$

$$\therefore \quad \beta = \frac{1\times5\times10^{-7}}{1\times10^{-3}} = 5\times10^{-4}\text{m}$$
$$= 0.5\text{ mm}$$

EXAMPLE |4| In Young's experiment, the width of the fringes obtained with light of wavelength 6000 Å is 2 mm. What will be the fringe width, if the entire apparatus is immersed in a liquid of refractive index 1.33?

Sol. As, $\beta = \dfrac{D\lambda}{d}$

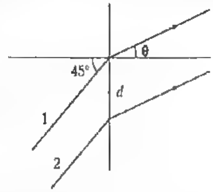
and $\beta_1 = \dfrac{D\lambda_1}{d}$

$$\therefore \quad \frac{\beta_1}{\beta} = \frac{\frac{D\lambda_1}{d}}{\frac{D\lambda}{d}} = \frac{\lambda_1}{\lambda} = \frac{1}{\mu}$$

or $\beta_1 = \dfrac{\beta}{\mu} = \dfrac{2}{1.33}$

$$= 1.5\text{ mm}$$

EXAMPLE |5| Distance between the slits, in YDSE, shown in figure is $d = 20\,\lambda$, where λ is the wavelength of light used.



Find the angle θ, where

(i) central maxima (where path difference is zero) is obtained.

(ii) third order maxima is obtained.

Sol. Ray 1 has a longer path than that of ray 2 by a distance $d\sin45°$, before reaching the slits. Afterwards ray 2 has a path longer than ray 1 by a distance $d\sin\theta$. The net path difference is therefore, $d\sin\theta - d\sin 45°$.

(i) Central maximum is obtained, where net path difference is zero

$$d\sin\theta - d\sin45° = 0 \Rightarrow \theta = 45°$$

(ii) Third order maxima is obtained, where net path difference is 3λ, i.e.

$$d\sin\theta - d\sin45° = 3\lambda$$

$$\Rightarrow \quad \sin\theta = \sin45° + \frac{3\lambda}{d}$$

Putting $d = 20\,\lambda$, we have

$$\sin\theta = \sin45° + \frac{3\lambda}{20\lambda}$$

$$\therefore \quad \theta = 59°$$

Conditions for Sustained Interference

In order to obtain a well-defined observable interference pattern, the intensity at points of constructive and destructive interference must be maintained maximum and almost zero, respectively.

For this, following conditions must be satisfied

(i) The two sources producing interference must be coherent.

(ii) The two interfering waves must have the same plane of polarisation.

(iii) The two sources must be very close to each other and the pattern must be observed at a larger distance to have sufficient width of the fringe $\left(\dfrac{D\lambda}{d}\right)$.

(iv) The sources must be monochromatic, otherwise the fringes of different colours will overlap.

(v) The two waves must be having same amplitude for better contrast between bright and dark fringes.

[redacted]

Fringe Shift

If refracting slab of thickness t is placed in front of one of the two slits of Young's double slit experiment, then fringe pattern gets shifted by n fringes and is given by

$$(\mu - 1)\,t = n\lambda$$

If both slits are covered by refracting surfaces of thicknesses t_1 and t_2 and refractive indices (μ_1, μ_2), then fringe pattern gets shifted by n fringe and is given by

$$(\mu_2 - \mu_1)\,t = n\lambda$$

Note The topic Young's double slit experiment has been asked frequently in the previous years 2015, 2014, 2012, 2011.

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. A thin film of oil is spread on the surface of water. The beautiful colours exhibited in the light of sun is due to
(a) dispersion of light (b) polarisation of light
(c) interference of light (d) diffraction of light

2. The phase difference between the two light waves reaching at a point P is 100π. Their path difference is equal to
(a) 10λ (b) 25λ (c) 50λ (d) 100λ

3. In the phenomenon of interference, energy is
(a) destroyed at destructive interference
(b) created at constructive interference
(c) conserved but it is redistributed
(d) same at all points

4. Two light waves superimposing at the mid-point of the screen are coming from coherent sources of light with phase difference π rad. Their amplitudes are 2 cm each. The resultant amplitude at the given point will be
(a) 8 cm (b) 2 cm (c) 4 cm (d) zero

5. The ratio of maximum and minimum intensities of two sources is 4 : 1. The ratio of their amplitudes is
(a) 1 : 81 (b) 3 : 1 (c) 1 : 9 (d) 1 : 16

6. The interference is produced by two waves of intensity ratio 16 : 9. The ratio of maximum and minimum intensities in interference pattern is
(a) 4 : 3 (b) 49 : 1
(c) 25 : 7 (d) 256 : 81

7. Two identical and independent sodium lamps act as
(a) coherent sources (b) incoherent sources
(c) Either (a) and (b) (d) None of these

8. In Young's double slit experiment, distance between slits in kept 1 mm and a screen is kept 1 m apart from slits. If wavelength of light used is 500 nm, then fringe spacing is
(a) 0.5 mm (b) 0.5 cm (c) 0.25 mm (d) 0.25 cm

9. In a Young's double-slit experiment, the source is white light. One of the holes is covered by a red filter and another by a blue filter. In this case, NCERT Exemplar
(a) there shall be alternate interference patterns of red and blue
(b) there shall be an interference pattern for red distinct from that for blue
(c) there shall be no interference fringes
(d) there shall be an interference pattern for red mixing with one for blue

VERY SHORT ANSWER Type Questions

10. Define the term "coherent sources" which are required to produce interference pattern in Young's double slit experiment.

11. Why we cannot obtain interference using two independent sources of light?

12. How will the fringe pattern change, if the screen is moved away from the slits?

13. How does the fringe width in Young's double slit experiment change when the distance of separation between the slits and screen is doubled? All India 2012

14. If the source of light used in Young's double slit experiment is changed from red to violet, what will happen to the fringe?

15. How does the angular separation of interference fringes change in Young's experiment, if the distance between the slits is increased? Delhi 2008

16. In Young's double slit experiment, the intensity of central maxima is I. What will be the intensity at the same place, if one slit is closed?

17. In a moving car, radio signals are interrupted sometimes. Why?

[redacted]

SHORT ANSWER Type Questions

18. Two light waves of amplitudes a and b interfere with each other. Calculate the ratio of intensities of a maxima to that of a minima.

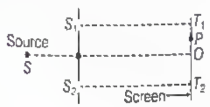
19. What are the conditions for obtaining two coherent sources of light?

20. In Young's double slit experiment, the two slits are illuminated by two different lamps having same wavelength of light. Explain with reason, whether interference pattern will be observed on the screen or not. **All India 2017 C**

21. How will the interference pattern in Young's double slit experiment get affected, when
 (i) distance between the slits S_1 and S_2 reduced.
 (ii) the entire set up is immersed in water?
 Justify your answer in each case. **Delhi 2011**

22. In Young's double slit experiment, two coherent sources S_1 and S_2 are placed at a distance D from screen, such that a bright fringe is obtained at a distance x from the centre line of screen. Give the value of x for which there is a bright fringe.

23. Consider a two slit interference arrangement (shown in figure) such that the distance of the screen from the slits is half the distance between the slits. Obtain the value of D in terms of λ such that the first minima on the screen falls at a distance D from the centre O.



NCERT Exemplar

24. Give four conditions for obtaining sustained interference.

25. If two waves of equal intensities $I_1 = I_2 = I_0$, meet at two locations P and Q with path differences Δ_1 and Δ_2, respectively. What will be the ratio of resultant intensity at points P and Q?

26. Answer the following questions:
 (i) When a low flying aircraft passes overhead, we sometimes notice a slight shaking of the picture on our TV screen. Suggest a possible explanation.
 (ii) As, you have learnt in the text, the principle of linear superposition of wave displacement is basic to understanding intensity distributions in diffraction and interference patterns.
 What is the justification of this principle? **NCERT**

LONG ANSWER Type I Questions

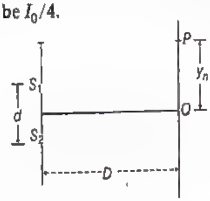
27. (i) Why are coherent sources necessary to produce a sustained interference pattern?
 (ii) In Young's double slit experiment using monochromatic light of wavelength λ, the intensity of light at a point on the screen, where path difference is λ, is K unit. Find out the intensity of light at a point, where path difference is $\lambda/3$. **Delhi 2012, NCERT**

28. Choose the statement as right or wrong and justify.
 (i) For a coherent source, initial phase difference between two sources must have same finite value.
 (ii) Resultant intensity of interference is given by $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$.
 (iii) Two independent light bulbs fail to produce interference.

29. In Young's double slit experiment, derive the condition for
 (i) constructive interference and
 (ii) destructive interference at a point on the screen.

30. In Young's double slit experiment, describe briefly how bright and dark fringes are obtained on the screen kept in front of a double slit. Hence, obtain the expression for the fringe width.

31. The intensity at the central maxima (O) in a Young's double slit experiment is I_0. If the distance OP equals one-third of fringe width of the pattern, show that the intensity at point P would be $I_0/4$.



Foreign 2011

[redacted]

32. (a) If one of two identical slits producing interference in Young's experiment is covered with glass, so that the light intensity passing through it is reduced to 50%, find the ratio of the maximum and minimum intensity of the fringe in the interference pattern.

(b) What kind of fringes do you except to observe, if white light is used instead of monochromatic light? **CBSE 2018**

LONG ANSWER Type II Questions

33. (i) What is the effect on the interference fringes to a Young's double slit experiment when

(a) the width of the source slit is increased?

(b) the monochromatic source is replaced by a source of white light? Justify your answer in each case.

(ii) The intensity at the central maxima in Young's double slit experiment set up is I_0. Show that the intensity at a point, where the path difference is $\lambda/3$ is $I_0/4$. **Foreign 2012**

34. (i) Consider two coherent sources S_1 and S_2 producing monochromatic waves to produce interference pattern.

Let the displacement of the wave produced by S_1 be given by $Y_1 = a\cos\omega t$ and the displacement by S_2 be $Y_2 = a\cos(\omega t + \phi)$.

Find out the expression for the amplitude of the resultant displacement at a point and show that the intensity at that point will be

$$I = 4a^2\cos^2\frac{\phi}{2}$$

Hence, establish the conditions for constructive and destructive interference.

(ii) What is the effect on the interference fringes in Young's double slit experiment when

(a) the width of the source slit is increased; (b) the monochromatic source is replaced by a source of white light? **Delhi 2015**

35. (i) (a) Two independent monochromatic sources of light cannot produce a sustained interference pattern. Give reason.

(b) Light waves each of amplitude a and frequency ω, emanating from two coherent light sources superpose at a point.

If the displacements due to these waves is given by $y_1 = a\cos\omega t$ and $y_2 = a\cos(\omega t + \phi)$, where, ϕ is the phase difference between the two, obtain the expression for the resultant intensity at the point.

(ii) In Young's double slit experiment, using monochromatic light of wavelength λ, the intensity of light at a point on the screen, where path difference is λ, is K units. Find out the intensity of light at a point where path difference is $\lambda/3$. **All India 2014**

36. State the importance of coherent sources in the phenomenon of interference. In Young's double slit experiment to produce interference pattern, obtain the conditions for constructive and destructive interference, hence deduce the expression for the fringe width. How does the fringe width get affected, if the entire experimental apparatus of Young's double slit experiment is immersed in water? **All India 2011**

37. (i) In a Young's double slit experiment, derive the conditions for constructive and destructive interference. Hence, write the expression for the distance between two consecutive bright or dark fringe.

(ii) What change in the interference pattern do you observe, if the two slits S_1 and S_2 are taken as point sources?

(iii) Plot a graph of intensity distribution *versus* path difference in this experiment.

NUMERICAL PROBLEMS

38. Light waves from two coherent sources having intensities I and $4I$ cross each other at a point with a phase difference of 90°. What is the resultant intensity at that point?

39. Light waves from coherent sources arrive at two points on a screen with path difference of 0 and $\lambda/2$. Find the ratio of intensities at the points. **All India 2017 C**

40. In a double slit experiment using light of wavelength 600 nm, the angular width of the fringe formed on a distant screen is 0.1°. Find the spacing between the two slits. **NCERT**

41. In Young's double slit experiment using light of wavelength 630 nm, angular width of a fringe formed on a distant screen is 0.2°. What is the spacing between the two slits?

42. In Young's double slit experiment, the slits are separated by 0.28 mm and the screen is placed 1.4 m away. The distance between the central bright fringe and the fourth bright fringe is measured to be 1.2 cm, determine the wavelength of light used in the experiment. **NCERT**

[redacted]

43. The amplitudes of light waves from two slits in Young's experiment are in ratio $\sqrt{2}:1$, what is the ratio of slit widths?

44. Widths of two slits in Young's experiment are in the ratio 4 : 1. What is the ratio of the amplitudes of light waves from them?

45. A beam of light, consisting of two wavelengths 560 nm and 420 nm, is used to obtain interference fringes in a Young's double slit experiment. Find the least distance from the central maxima, where the bright fringes due to both the wavelengths coincide. The distance between the two slits is 4 mm and the screen is at a distance of 1m from the slits. **Delhi 2010 C**

46. A beam of light consisting of two wavelengths 650 nm and 520 nm, is used to obtain interference fringes in a Young's double slit experiment.
   (i) Find the distance of the third bright fringe on the screen from the central maximum for wavelength 650 nm.
   (ii) What is the least distance from the central maximum, where the bright fringes due to both the wavelengths coincide? **NCERT**

47. A beam of light consisting of two wavelenghts, 800 nm and 600 nm is used to obtain the interference fringes in a Young's double slit experiment on a screen placed 1.4 m away. If the two slits are separated by 0.28 mm, calculate the least distance from the central bright maximum where the bright fringes of the two wavelengths coincide. **All India 2012**

48. Laser light of wavelength 630 nm incident on a pair of slits produces an interference pattern in which the bright fringes are separated by 7.2 mm. Calculate the wavelength of another source of laser light which produce interference fringes separated by 8.1 mm using same pair of slits. **All India 2011**

49. In a Young's double slit experiment, the angular width of the fringe is found to be 0.2° on a screen placed 1 m away. The wavelength of light used is 600 nm. What will be the angular width of the fringe, if the entire experimental apparatus is immersed in water? Take, refractive index of water to be 4/3. **NCERT**

50. The ratio of the intensities at minima to the maxima in the Young's double slit experiment is 9 : 25. Find the ratio of the widths of the two slits. **All India 2014**

51. In Young's double slit experiment, the two slits 0.15 mm apart are illuminated by monochromatic light of wavelength 450 nm. The screen is 1 m away from the slits.
Find the distance of the second
   (i) bright fringe.
   (ii) dark fringe from the central maxima.

HINTS AND SOLUTIONS

1. (c) The light reflected from the oil film produced two coherent waves and these waves are superposed (interference) and produce beautiful colours.

2. (c) Given, $\Delta\phi = 100\pi$

We know, change in phase difference,

i.e., $\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x$

where, Δx = path difference

$\Rightarrow \quad \Delta x = \Delta\phi \times \frac{\lambda}{2\pi} = 100\pi \times \frac{\lambda}{2\pi} = 50\lambda$

3. (c) In the phenomenon of interference, energy in conserved but it is redistributed.

4. (d) Resultant amplitude

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

Here $\quad A_1 = A_2 = 2\,\text{cm} \Rightarrow \phi = \pi\,\text{rad}$

$$A = \sqrt{(2)^2 + (2)^2 + 2\times2\times2\times\cos\pi}$$

$$A = \sqrt{4+4-8} \text{ or } A = 0$$

5. (b) Given, $\frac{I_{max}}{I_{min}} = \frac{4}{1}$

We know, $\frac{I_{max}}{I_{min}} = \left(\frac{r+1}{r-1}\right)^2 = \frac{4}{1}$

$\Rightarrow \quad \frac{r+1}{r-1} = \frac{2}{1}$

$\Rightarrow \quad r+1 = 2r-2$ or $r=3$

$\therefore$ The ratio of amplitudes $\frac{A_1}{A_2} = r = 3$

Hence, $\frac{r_1}{r_2} = 3$ or $3:1$

6. (b) Let the intensities of two waves is I_1 and I_2, respectively. Given, $I_1 : I_2 = 16 : 9$

$\therefore \quad \frac{I_1}{I_2} = \frac{a_1^2}{a_2^2} \Rightarrow \frac{16}{9} = \frac{a_1^2}{a_2^2}$

[redacted]

$$\frac{4}{3}=\frac{a_1}{a_2} \Rightarrow a_2=\frac{3}{4}\cdot a_1$$

$$\therefore \quad \frac{I_{max}}{I_{min}}=\frac{(a_1+a_2)^2}{(a_1-a_2)^2}=\frac{\left(a_1+\frac{3}{4}a_1\right)^2}{\left(a_1-\frac{3}{4}a_1\right)^2}=\frac{\left(\frac{7}{4}\right)^2}{\left(\frac{1}{4}\right)^2}=\frac{49}{1}$$

i.e., $I_{max}:I_{min}=49:1$

7. (b) Two identical and independent sodium lamps (i.e., two independent sources of light) can never be coherent. Hence, no coherence between the light emitted by different atoms

8. (a) Fringe spacing,

$$\beta=\frac{D\lambda}{d}=\frac{1\times5\times10^{-7}}{1\times10^{-3}}\text{ m}$$

$$(1\text{ nm}=10^{-9}\text{ m})$$

$$=5\times10^{-4}\text{ m}=0.5\text{ mm}$$

9. (c) For the interference pattern to be formed on the screen, the sources should be coherent and emits lights of same frequency and wavelength.

In a Young's double-slit experiment, when one of the holes is covered by a red filter and another by a blue filter. In this case due to filteration only red and blue lights are present. In YDSE monochromatic light is used for the formation of fringes on the screen. Hence, in this case there shall be no interference fringes.

10. Those sources of light which emit light waves of same wavelength, same frequency and are in same phase or having constant phase difference are called coherent sources.

11. This is because two independent sources of light cannot be coherent, as their relative phases are changing randomly.

12. With increase of D, fringe width also increases as,

$$\beta=\frac{D\lambda}{d}$$

or $\quad \beta\propto D$

13. As we know that, fringe width,

$$\beta=\frac{\lambda D}{d}$$

Here, $\quad D'=2D$

So, $\quad \beta'=\frac{2\lambda D}{d}$

$\Rightarrow \quad \beta'=2\beta$

14. If the source of light used in Young's double slit experiment is changed from red to violet, then their consecutive fringes will come closer.

15. Angular separation decreases with the increase of separation between the slits as, $\theta=\frac{\lambda}{d}$

where, d = separation between two slits.

16. When one slit is closed, amplitude becomes $\frac{1}{2}$ and hence intensity becomes $\frac{1}{4}$th and there will be no interference.

17. While moving in a car, there are periodic interruptions in a radio signal that we hear. It occurs on account of interfering of the radio waves, i.e. a common form of interference called multi path interference.

18. Refer to the text on page 410.

19. Refer to the text on page 411.

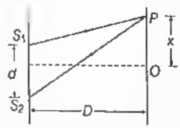
20. No, interference pattern will be observed on the screen. This is because, the source will serve as incoherent sources. For details refer to text on page 411.

21. (i) The fringe width of interference pattern increases with the decrease in separation between S_1 and S_2 as

$$\beta\propto\frac{1}{d}$$

(ii) The fringe width decreases as wavelength gets reduced, when interference set up is taken from air to water as, $\quad \beta\propto\lambda$

22.



For a bright fringe, path difference = $n\lambda$

and so $\quad n\lambda=\frac{x_n d}{D}$

$\Rightarrow \quad x_n=\frac{n\lambda D}{d}$, where $n=1,2,3,\ldots$

$\therefore \quad x_n=\frac{\lambda D}{d}$, when $n=1$

and $\quad x_n=\frac{2\lambda D}{d}$, when $n=2$

23. From the given figure, of two slit interference arrangement, we can write

$$T_2P=T_2O+OP=D+x$$

and $\quad T_1P=T_1O-OP=D-x$

$$S_1P=\sqrt{(S_1T_1)^2+(PT_1)^2}=\sqrt{D^2+(D-x)^2}$$

and $\quad S_2P=\sqrt{(S_2T_2)^2+(T_2P)^2}=\sqrt{D^2+(D+x)^2}$

The minima will occur when $S_2P-S_1P=(2n-1)\frac{\lambda}{2}$

i.e. $[D^2+(D+x)^2]^{1/2}-[D^2+(D-x)^2]^{1/2}=\frac{\lambda}{2}$

[for first minima, $n=1$]

If $x=D$, we can write, $[D^2+4D^2]^{1/2}-[D^2+0]^{1/2}=\frac{\lambda}{2}$

[redacted]

$$\Rightarrow \quad [5D^2]^{1/2} - [D^2]^{1/2} = \frac{\lambda}{2}$$

$$\Rightarrow \quad \sqrt{5}D - D = \frac{\lambda}{2}$$

$$\Rightarrow \quad D(\sqrt{5}-1) = \lambda/2 \text{ or } D = \frac{\lambda}{2(\sqrt{5}-1)}$$

Putting $\sqrt{5} = 2.236$

$$\Rightarrow \quad \sqrt{5} - 1 = 2.236 - 1 = 1.236$$

$$D = \frac{\lambda}{2(1.236)} = 0.404\,\lambda$$

24. Refer to the text on page 413.

25. By formula, $\frac{I_{max}}{I_{min}} = \frac{(r+1)^2}{(r-1)^2}$ [Ans. 1 : 1]

26. (i) We notice a slight shaking of the picture on our TV screen because a low flying aircraft reflects the TV signal and there may be an interference between the direct signal and the reflected signal which results in shaking.

(ii) The superposition principle follows the linear character of the differential equation governing wave motion. If y_1 and y_2 be the solutions of wave equation, so there can be any linear combination of y_1 and y_2. When the amplitudes are large and non-linear effects are important, then the situation is more complicated

27. (i) Refer to text on page 413.

(ii) Intensity of light at a point on the screen is given by

$$I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$$

For the path difference λ, phase difference is 2π.

As, sources are coherent and taken out of the same source in Young's double slit experiment,

$$I_1 = I_2 = I$$

$$\Rightarrow \quad I_R = 2I + 2I\cos 2\pi$$

$$\Rightarrow \quad I_R = 4I$$

$$\Rightarrow \quad 4I = K \text{ unit} \quad \text{...(i)}$$

For the path difference, $\frac{\lambda}{3}$ corresponding to phase difference of $\frac{2\pi}{3}$,

$$I_R = 2I + 2I\cos\frac{2\pi}{3} = 2I - I = I \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we conclude

$$I_R = \frac{K}{4} \text{ unit}$$

28. (i) Refer to text on page 411.

(ii) Refer to text on page 410

(iii) Two independent light bulbs are an example of incoherent sources of light and hence fail to produce interference. For details, refer to text on page 437.

29. Refer to text on page 410

30. Refer to text on page 412.

31. Given, $OP = y_n$

The distance OP equals one-third of fringe width of the pattern.

i.e. $y_n = \frac{\beta}{3} = \frac{1}{3}\left(\frac{D\lambda}{d}\right) = \frac{D\lambda}{3d}$

$$\Rightarrow \quad \frac{dy_n}{D} = \frac{\lambda}{3}$$

Path difference $= S_2P - S_1P = \frac{dy_n}{D} = \frac{\lambda}{3}$

$\therefore$ Phase difference, $\phi = \frac{2\pi}{\lambda} \times$ path difference

$$= \frac{2\pi}{\lambda} \times \frac{\lambda}{3} = \frac{2\pi}{3}$$

If intensity at central fringe is I_0, then intensity at a point P, where phase difference ϕ, is given by

$$I = I_0 \cos^2\phi$$

$$\Rightarrow \quad I = I_0\left(\cos\frac{2\pi}{3}\right)^2 = I_0\left(-\cos\frac{\pi}{3}\right)^2$$

$$= I_0\left(-\frac{1}{2}\right)^2 = \frac{I_0}{4}$$

Hence, the intensity at point P would be $\frac{I_0}{4}$.

32. (a) Given, $I_1 = I_0$

$I_2 = 50\%$ of I_1

i.e., $I_2 = \frac{I_0}{2}$

Now, ratio of maximum and minimum intensity is given as

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2$$

$$= \left(\frac{\sqrt{I_0}+\sqrt{\frac{I_0}{2}}}{\sqrt{I_0}-\sqrt{\frac{I_0}{2}}}\right)^2 = \frac{\left(\sqrt{I_0}+\sqrt{\frac{I_0}{2}}\right)^2}{\left(\sqrt{I_0}-\sqrt{\frac{I_0}{2}}\right)^2}$$

$$= \frac{\left(1+\frac{1}{\sqrt{2}}\right)^2}{\left(1-\frac{1}{\sqrt{2}}\right)^2} = \left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)^2 = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$= \left(\frac{3+2\sqrt{2}}{3-2\sqrt{2}}\right) \times \left(\frac{3+2\sqrt{2}}{3+2\sqrt{2}}\right) = \frac{(3+2\sqrt{2})^2}{(3)^2-(2\sqrt{2})^2}$$

$$= 17 + 12\sqrt{2}$$

(b) When a white light source is used, the interference patterns due to different component colours of white light overlap incoherently. The central bright fringe for different colours is at centre.

So, central bright fringe is white.

[redacted]

As $\lambda_{blue} < \lambda_{red}$, fringe closest on either side of central bright fringe is blue and the farthest is red.

After few fringes, no clear pattern of fringes will be visible.

33. (i) (a) For interference fringes to be seen, $\frac{s}{S} \le \frac{\lambda}{d}$ condition should be satisfied where, s = size of the source and d – distance of the source from the plane of two slits. As, the source slit width increases, fringe pattern gets less and less sharp. When the source slit is so wide, the above condition does not get satisfied and the interference pattern disappears.

(b) The interference pattern due to the different colour components of white light overlap. The central bright fringes for different colours are at the same position. Therefore, central fringes are white. And on the either side of the central fringe (i.e. central maxima), coloured bands will appear.The fringe closed on either side of central white fringe is red and the farthest will be blue. After a few fringes, would be clear fringe pattern is seen.

(ii) Refer to Q. 31 on page 415.

34. (i) Given, the displacements of two coherent sources $y_1 = a\cos\omega t$ and $y_2 = a\cos(\omega t + \phi)$

By principle of superposition,

$$y = y_1 + y_2 = a\cos\omega t + a\cos(\omega t + \phi)$$

$$y = a\cos\omega t + a\cos\omega t\cos\phi - a\sin\omega t\sin\phi$$

$$y = a(1+\cos\phi)\cos\omega t + (-a\sin\phi)\sin\omega t$$

Let $\quad a(1+\cos\phi) = A\cos\theta \quad$ ...(i)

and $\quad a\sin\phi = A\sin\theta \quad$ ...(ii)

$\therefore \quad y = A\cos\theta\cos\omega t - A\sin\theta\sin\omega t$

$\Rightarrow \quad y = A\cos(\omega t + \theta) \quad$ ...(iii)

Squaring and adding Eqs. (i) and (ii), we get

$$(A\cos\theta)^2 + (A\sin\theta)^2 = a^2(1+\cos\phi)^2 + (a\sin\phi)^2 \quad A^2(\cos^2\theta + \sin^2\theta)$$

$$= a^2(1+\cos^2\phi + 2\cos\phi) + a^2\sin^2\phi$$

$$\Rightarrow A^2 \times 1 = a^2 + a^2 + 2a^2\cos\phi = 2a^2(1+\cos\phi)$$

$$\Rightarrow \quad A^2 = 2a^2\left(2\cos^2\frac{\phi}{2}\right) = 4a^2\cos^2\left(\frac{\phi}{2}\right)$$

If I is the resultant intensity, then $I = 4a^2\cos^2\frac{\phi}{2}$

Refer to text on page 434 for conditions for constructive and destructive interference.

(ii) (a) Refer to Q. 21 (i) on page 415 (b) Q. 32 (b) on page 416

35. (i) (a) Two independent monochromatic sources of light cannot produce a sustained interference pattern because their relative phases are changing randomly. When d is negligibly small, fringe width β which is proportional to $1/d$ may become too large. Even a single fringe may occupy the screen. Hence, the pattern cannot be detected.

(b) Refer to Q. 34 (i) on page 416.

(ii) Refer to Q. 27 (ii) on page 415.

36. Refer to text on page 411.

Refer to text on pages 410 and 412.

Refer to Q. 21 (ii) on page 415.

37. Refer to text on pages 410, 412 and 413.

38. $5I$, refer to Example 1 on page 411.

39. As, $I = I_0\cos^2\phi$

$\because$ Phase difference, $\phi = \frac{2\pi x}{\lambda}$

So, $\quad I_1 = I_0\cos^2\left(\frac{2\pi\times 0}{\lambda}\right) = I_0$

and $\quad I_2 = I_0\cos^2\left(\frac{2\pi}{\lambda}\times\lambda/2\right) = I_0\cos^2(\pi) = I_0$

$\therefore \quad \frac{I_1}{I_2} = \frac{1}{1}$

40. Here, $\lambda = 600\text{ nm} = 600\times10^{-9}\text{ m} = 6\times10^{-7}\text{ m}$

$\Rightarrow 0.1° = \frac{0.1\pi}{180}$ rad

From angular width, $\theta = \frac{\lambda}{d}$

$$\Rightarrow \quad d = \frac{\lambda}{\theta} = \frac{6\times10^{-7}}{\frac{\pi}{180}\times0.1} = 3.44\times10^{-4}\text{ m}$$

41. Angular width, $\theta = \frac{\lambda}{d}$

Here, $\theta = 0.2°\ \lambda = 630\text{ nm} = 630\times10^{-9}\text{m}$

$$\Rightarrow \quad d = \frac{630\times10^{-9}\times180}{0.2\times\pi} = 1.80\times10^{-4}\text{m}$$

42. Here, slit width, $d = 0.28\text{ mm} = 0.28\times10^{-3}$ m

Distance between slit and screen, $D = 1.4$ m

$y = 1.2\text{ cm} = 1.2\times10^{-2}\text{ m}, n = 4, \lambda = ?$

For constructive interference, $y = n\lambda\frac{D}{d}$ or $\lambda = \frac{yd}{nD}$

$$= \frac{1.2\times10^{-2}\times0.28\times10^{-3}}{4\times1.4} = 6\times10^{-7}\text{ m}$$

43. We know that, $\frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{W_1}{W_2}$

$$\Rightarrow \quad \frac{W_1}{W_2} = \frac{(\sqrt{2})^2}{(1)^2} = \frac{2}{1}$$

44. We know that $\frac{W_1}{W_2} = \frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{4}{1} \Rightarrow \frac{a}{b} = 2:1$

45. Given, distance between the screen and slit, $D = 1$ m

Slit width, $\quad d = 4\times10^{-3}$ m

$\lambda_1 = 560\text{ nm}, \lambda_2 = 420\text{ nm}$

Let nth order bright fringe of λ_1 coincides with $(n+1)$th order bright fringe of λ_2.

[redacted]

$\therefore \quad \frac{Dn\lambda_1}{d} = \frac{D(n+1)\lambda_2}{d} \quad (\lambda_1 > \lambda_2)$

$\Rightarrow \quad n\lambda_1 = (n+1)\lambda_2$

$\Rightarrow \quad \frac{n+1}{n} = \frac{\lambda_1}{\lambda_2}$

$\therefore \quad 1 + \frac{1}{n} = \frac{560 \times 10^{-9}}{420 \times 10^{-9}} \Rightarrow 1 + \frac{1}{n} = \frac{4}{3}$

$\therefore \quad n = 3$

$\therefore$ Least distance from the central fringe where bright fringe of two wavelengths coincides

= distance of 3rd order bright fringe of λ_1

$\therefore \quad y_n = \frac{3D\lambda_1}{d} = \frac{3 \times 1 \times 560 \times 10^{-9}}{4 \times 10^{-3}}$

$= 0.42 \times 10^{-3}$ m

$\therefore \quad y_n = 0.42$ mm

3rd bright fringe of λ_1 and 4th bright fringe of λ_2 coincide at 0.42 mm from central fringe.

46. Here, $\lambda_1 = 650$ nm $= 650 \times 10^{-9}$ m,

$\lambda_2 = 520$ nm $= 520 \times 10^{-9}$ m

Suppose, d = distance between two slits

D = distance of screen from the slits

(i) For third bright fringe, $n = 3$

$\therefore \quad y = n\lambda_1 \frac{D}{d}$

$= 3 \times 650 \frac{D}{d}$ nm $= 1950 \frac{D}{d}$ nm

(ii) Refer to Q. 33 of topic practice 2. on page 416.

$$y = 2600 \frac{D}{d} \text{ nm}$$

47. 12 mm; refer to Q. 45 on page 417.

48. According to the question, $\frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2}$

[$\because$ D and d are same]

Here, $\beta_1 = 7.2 \times 10^{-3}$ m, $\beta_2 = 8.1 \times 10^{-3}$ m

and $\lambda_1 = 630 \times 10^{-9}$ m

Wavelength of another source of laser light,

$$\lambda_2 = \frac{\beta_2}{\beta_1} \times \lambda_1$$

$$= \frac{8.1 \times 10^{-3}}{7.2 \times 10^{-3}} \times 630 \times 10^{-9} \text{ m}$$

$\therefore \quad \lambda_2 = 708.75 \times 10^{-9}$ m

$= 708.75$ nm

49. Here, $\theta_1 = 0.2°$, $D = 1$ m, $\lambda_1 = 600$ nm, $\mu = 4/3$

Angular width of a fringe, $\theta = \frac{\lambda}{d}$

$$\frac{\theta_2}{\theta_1} = \frac{\lambda_2}{\lambda_1} = \frac{1}{\mu} = \frac{3}{4}$$

$\Rightarrow \quad \theta_2 = \frac{3}{4} \times \theta_1 = \frac{3}{4} \times 0.2° = 0.15°$

50. $\frac{I_{min}}{I_{max}} = \frac{9}{25} = \frac{(a-b)^2}{(a+b)^2}$

$\Rightarrow \quad \frac{a-b}{a+b} = \frac{3}{5}$

$\Rightarrow \quad a = 4b$

As, $\frac{W_1}{W_2} = \frac{a^2}{b^2} = \frac{(4b)^2}{b^2} = \frac{16}{1}$

51. (i) The distance of nth order bright fringe from central fringe is given by

$$y_n = \frac{nD\lambda}{d}$$

For second bright fringe,

$$y_2 = \frac{2D\lambda}{d} = \frac{2 \times 1 \times 4.5 \times 10^{-7}}{1.5 \times 10^{-4}}$$

$$y_2 = 6 \times 10^{-3} \text{ m}$$

The distance of the second bright fringe

$$y_2 = 6 \text{ mm}$$

(ii) The distance of nth order dark fringe from central fringe is given by

$$y_n' = (2n-1)\frac{D\lambda}{2d}$$

For second dark fringe, $n = 2$

$$y_n' = (2 \times 2 - 1)\frac{D\lambda}{2d} = \frac{3D\lambda}{2d}$$

$\Rightarrow \quad y_n' = \frac{3}{2} \times \frac{1 \times 4.5 \times 10^{-7}}{1.5 \times 10^{-4}} = 4.5 \times 10^{-3}$ m

$\therefore$ The distance of the second dark fringe

$$y_n' = 4.5 \text{ mm}$$

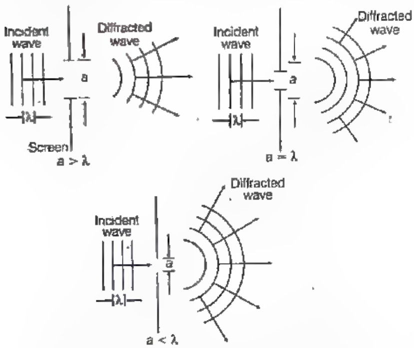
[redacted]

|TOPIC 3|

Diffraction of Light

DIFFRACTION OF LIGHT

The phenomenon of bending of light around the sharp corners and the spreading of light within the geometrical shadow of the opaque obstacles is called diffraction of light. The light thus deviates from its linear path. The deviation becomes much more pronounced, when the dimensions of the aperture or the obstacle are comparable to the wavelength of light.



Diffraction of waves for slits of different width

Note Diffraction is a general characteristic exhibited by all types of waves. For visible light, λ is very small ($\approx 10^{-6}$ m).
Therefore, diffraction of visible light is not so common as obstacles/apertures of this size are hardly available.

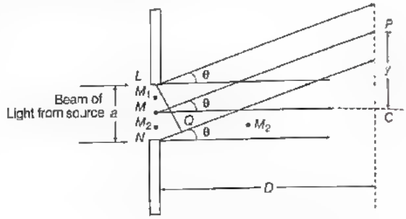
According to Fresnel, diffraction occurs on the account of mutual interference of secondary wavelets starting from portions of the wavefront which are not blocked by the obstacle or from portions of the wavefront which are allowed to pass through the aperture.

Diffraction of Light at a Single Slit

A parallel beam of light with a plane wavefront is made to fall on a single slit LN. As width of the slit $LN = a$ is of the order of wavelength of light, therefore diffraction occurs when beam of light passes through the slit.

The wavelets from the single wavefront reach the centre C on the screen in same phase. Hence, interfere constructively to give central maximum (bright fringe).

The diffraction pattern obtained on the screen consists of a central bright band, having alternate dark and weak bright bands of decreasing intensity on both sides.



Geometry of single slit diffraction

Consider a point P on the screen at which wavelets travelling in a direction, make an angle θ with MC. The wavelets from points L and N will have a path difference equal to NQ.

From the right angled ΔLNQ, we have

$$NQ = LN \sin\theta$$

or

$$NQ = a \sin\theta$$

To establish the condition for secondary minima, the slit is divided into 2, 4, 6,... equal parts such that corresponding wavelets from successive regions interfere with path difference of $\lambda/2$.

Or for nth secondary minima, the slit can be divided into $2n$ equal parts.

Hence, for nth secondary minima,

$$\text{Path difference} = \frac{a}{2}\sin\theta = \frac{\lambda}{2}$$

or

$$\boxed{\sin\theta_n = \frac{n\lambda}{a},\ (n = 1, 2, 3, \ldots)}$$

To establish the condition for secondary maxima, the slit is divided into 3, 5, 7, ... equal parts such that corresponding wavelets from alternate regions interfere with path difference of $\lambda/2$.

[redacted]

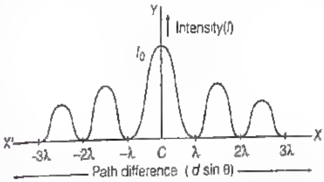
Or for nth secondary maxima, the slit can be divided into $(2n+1)$ equal parts.

Hence, for nth secondary maxima,

$$a \sin \theta_n = (2n+1)\frac{\lambda}{2} \qquad [n = 1, 2, 3, \ldots]$$

or
$$\boxed{\sin \theta_n = (2n+1)\frac{\lambda}{2a}}$$

Hence, the diffraction pattern can be graphically shown as below



The point C corresponds to the position of central maxima. And the position $-3\lambda, -2\lambda, -\lambda, \lambda, 2\lambda, 3\lambda\ldots$ are secondary minima. The above conditions for diffraction maxima and minima are exactly reverse of mathematical conditions for interference maxima and minima.

Width of Central Maximum

It is the distance between first secondary minimum on either side of the central bright fringe C.

For first secondary minimum,

$$a \sin \theta = \lambda \quad \text{or} \quad \sin \theta = \frac{\lambda}{a} \qquad \ldots\text{(i)}$$

If θ is small, $\sin \theta \approx \theta = \frac{y}{D}$ ...(ii)

From Eqs. (i) and (ii), we get

$$\frac{y}{D} = \frac{\lambda}{a} \quad \text{or} \quad y = \frac{D\lambda}{a}$$

$$\boxed{\text{Width of central maximum} = 2y = \frac{2D\lambda}{a}}$$

As, the slit width a increases, width of central maximum decreases.

$\therefore$ $$\boxed{\text{Angular width of central maxima, } 2\theta = \frac{2\lambda}{a}}$$

EXAMPLE |1| A beam of light whose wavelength is 4000 Å is diffracted by a single slit of width 0.2 mm. Give the angular position of the second minima.

Sol. Given, wavelength, $\lambda = 4000$ Å

Slit width, $a = 0.2$ mm $= 0.2 \times 10^{-3}$ m

Width of second minima = ?

For nth minima, $a \sin\theta = n\lambda$

$\therefore \quad 0.2 \times 10^{-3} \sin\theta = 2 \times 4000 \times 10^{-10}$

For θ to be very small; $\sin\theta \approx \theta$

$$\therefore \quad \theta = \frac{2 \times 4000 \times 10^{-10}}{0.2 \times 10^{-3}} = 4 \times 10^{-3} \text{ rad}$$

EXAMPLE |2| A slit 4 cm wide is irradiated with microwaves of wavelength 2 cm. Find the angular spread of central maximum, assuming incidence normal to the plane of the slit.

Sol. Here, $a = 4$ cm $= 4 \times 10^{-2}$ m,

$\lambda = 2$ cm $= 2 \times 10^{-2}$ m

$\therefore$ Angular spread of central maximum (2θ) is

$$2\theta = \frac{2\lambda}{a} = \frac{2 \times 2 \times 10^{-2}}{4 \times 10^{-2}} = 1 \text{ rad}$$

EXAMPLE |3| Angular width of central maximum in the Fraunhoffer diffraction pattern of a slit is measured. The slit is illuminated by light of wavelength 6000 Å. When the slit is illuminated by light of another wavelength, then the angular width decreases by 30%. Calculate the wavelength of this light. The same decrease in the angular width of central maximum is obtained when the original apparatus is immersed in a liquid. Find the refractive index of the liquid.

Sol. Angular width of central maximum $= \frac{2\lambda}{a}$

$$\lambda_1 = 600\,\text{nm}, \theta_1 = \frac{2\lambda_1}{a} \qquad \ldots\text{(i)}$$

$$\theta_2 = \theta_1 \times 0.7 = \frac{2\lambda_2}{a} \qquad \ldots\text{(ii)}$$

On dividing Eq. (i) by Eq. (ii), we get

$$\frac{1}{0.7} = \frac{\lambda_1}{\lambda_2} = \frac{600}{\lambda_2}$$

Wavelength, $\lambda_2 = 420$ nm

When immersed in liquid, $\lambda_2 = \lambda_1/\mu$

$$\theta_2 = \theta_1 \times 0.7 \qquad \ldots\text{(iii)}$$

$$\Rightarrow \quad \frac{2\lambda_1/\mu}{a} = \frac{2\lambda_1}{a} \times 0.7$$

$$\frac{1}{0.7} = \mu$$

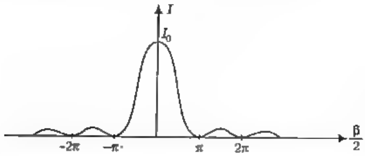
$\therefore$ Refractive index of the liquid,

$$\mu = \frac{10}{7} = 1.42$$

EXAMPLE |4| Intensity curve for a single slit diffraction pattern is shown below. Find the ratio of the intensities of the secondary maxima to the intensity of

[redacted]

the central maximum for the single-slit Fraunhofer diffraction pattern.



Sol. To a good approximation, the secondary maxima lie midway between the zero points. From figure, we see that this corresponds to $\beta/2$ values of $3\pi/2, 5\pi/2, 7\pi/2$,

$$\therefore \quad \frac{I_1}{I_0}=\left[\frac{\sin(3\pi/2)}{(3\pi/2)}\right]^2=\frac{1}{9\pi^2/4}=0.045$$

and
$$\frac{I_2}{I_0}=\left[\frac{\sin(5\pi/2)}{5\pi/2}\right]^2=\frac{1}{25\pi^2/4}=0.016$$

i.e. the first secondary maxima (the ones adjacent to the central maximum) have an intensity of 4.5% that of the central maximum and the next secondary maxima have an intensity of 1.6% that of the central maximum.

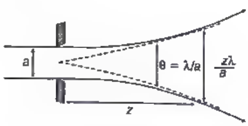
Difference in Diffraction Pattern at a Single Slit due to Monochromatic Light and White Light

For monochromatic light, the diffraction is of alternate bright and dark bands of unequal widths. The central bright fringe has maximum intensity and the intensity of successive secondary maxima decreases rapidly.

If the source is of white light, the diffraction pattern is coloured. The central maximum is white but other bands are coloured. As band width $\propto \lambda$, therefore red band width is wider than the violet band width.

Validity of Ray Optics/ Fresnel's Distance

When a slit or hole of size a is illuminated by a parallel beam, it is diffracted with an angle $\theta=\dfrac{\lambda}{a}$.



Diffraction of a parallel beam

In travelling a distance z, size of beam is $z\lambda/a$.

So, taking $\quad \dfrac{z\lambda}{a}\geq a$

or $\quad z\geq\dfrac{a^2}{\lambda}$

Now, distance z_F is called Fresnel's distance $[z_F=a^2/\lambda]$.

Spreading of light due to diffraction is comfortable upto distance $z_F/2$. For distance much greater than z_F, spreading due to diffraction is also prominant. So, the image formation can be explained by ray optics for distances less than z_F.

EXAMPLE |5| For what distance is ray optics a good approximation when the aperture is 3 mm wide and wavelength is 500 nm?

Sol. Here, $a=3\,\text{mm}=3\times10^{-3}\,\text{m}$

and $\lambda=500\,\text{nm}=5\times10^{-7}\,\text{m}$

According to Fresnel's distance,

$$z_F=\frac{a^2}{\lambda}=\frac{(3\times10^{-3})^2}{5\times10^{-7}}=18\,\text{m}$$

Difference between Interference and Diffraction

(i) The interference pattern has a number of equally spaced bright and dark bands. Where as the diffraction pattern has a central bright maximum, which is twice as wide as the other maxima. The intensity falls as we go to successive maxima away from the centre on either side.

(ii) We calculate the interference pattern by superposing two waves originating from the two narrow slits. The diffraction pattern is a superposition of a continuous family of waves originating from each point on a single slit.

(iii) For a single slit of width a, the first null of the interference pattern occurs at an angle of λ/a. At the same angle of λ/a, we get a maxima (not a null) for two narrow slits separated by a distance a.

One must understand that both distances between two slits in Young's double slit experiment, d and a width of each slit have to be quite small, to be able to observe good interference and diffraction patterns respectively. e.g. d must be of the order of a millimetre or so and must be even smaller of the order of 0.1 of 0.2 mm.

[redacted]

| TOPIC PRACTICE 3 |

OBJECTIVE Type Questions

1. Light seems to propagate in rectilinear path because
(a) its speed is very large
(b) its wavelength is very small
(c) reflected from the upper surface of atmosphere
(d) it is not absorbed by atmosphere

2. In diffraction from a single slit the angular width of the central maxima does not depends on
(a) λ of light used
(b) width of slit
(c) distance of slits from the screen D
(d) ratio of λ and slit width

3. What should be the slit width to obtain 10 maxima of the double slit pattern within the central maxima of the single slit pattern of slit width 0.4 mm?
(a) 0.4 mm (b) 0.2 mm
(c) 0.6 mm (d) 0.8 mm

4. In a single diffraction pattern observed on a screen placed at D m distance from the slit of width d m, the ratio of the width of the central maxima to the width of other secondary maxima is
(a) 2 : 1 (b) 1 : 2
(c) 1 : 1 (d) 3 : 1

VERY SHORT ANSWER Type Questions

5. What is the condition for first minima in case of diffraction due to a single slit?

6. How does the angular separation between fringes in single slit diffraction experiment change when the distance of separation between the slit and screen doubled?

7. Explain how the intensity of diffraction pattern changes as the order(n) of the diffraction band varies? **All India 2017**

8. If the wavelength of light decreases, then Fresnel's distance increases or decreases?

9. What is the basic difference between diffraction and interference of light?

SHORT ANSWER Type Questions

10. For a single slit of width a, the first minimum of the interference pattern of a monochromatic light of wavelength λ occurs at an angle of $\frac{\lambda}{a}$. At the same angle of $\frac{\lambda}{a}$, we get a maximum for two narrow slits separated by a distance a. Explain. **Delhi 2014**

11. Draw the intensity pattern for single slit diffraction and double slit interference. Hence, state two differences between interference and diffraction patterns. **All India 2017**

12. State briefly two features which can distinguish the characteristic features of an interference pattern from those observed in the diffraction pattern due to a single slit. **All India 2011**

13. Why is the diffraction of sound waves more evident in daily experience than that of light wave? **NCERT Exemplar**

14. In deriving the single slit diffraction pattern, it was stated that the intensity is zero at angles $\frac{n\lambda}{d}$. Justify this by suitably dividing the slit to bring out the cancellation. **NCERT**

LONG ANSWER Type I Question

15. (a) In a single slit diffraction experiment, the width of the slit is made double the original width. How does this affect the size and intensity of the central diffraction band? Explain.
(b) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the obstacle. Explain, why? **CBSE 2018**

LONG ANSWER Type II Questions

16. (i) Using Huygens' construction of secondary wavelets explain how a diffraction pattern is obtained on a screen due to a narrow slit on which a monochromatic beam of light is incident normally.
(ii) Show that the angular width of the first diffraction fringe is half that of the central fringe.
(iii) Explain why the maxima at $\theta = \left(n + \frac{1}{2}\right)\frac{\lambda}{a}$ become weaker and weaker with increasing n? **All India 2015**

[redacted]

17. (i) Obtain the conditions for the bright and dark fringes in diffraction pattern due to a single narrow slit illuminated by a monochromatic source. Explain clearly, why the secondary maxima go on becoming weaker with increasing of their order?
(ii) When the width of the slit is made double, how would this affect the size and intensity of the central diffraction band? Justify your answer. **Foreign 2012**

NUMERICAL PROBLEMS

18. What should be the width of each slit to obtain 10th maxima of the double slit pattern within the central maxima of single slit pattern?

19. A parallel beam of light of wavelength 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minimum is at a distance of 2.5 mm from the centre of the screen. Find the width of the slit. **All India 2013, NCERT**

20. Yellow light ($\lambda = 6000$ Å) illuminates a single slit of 1×10^{-4} m. Calculate the distance between two dark lines on either side to the central maximum, when the diffraction pattern is viewed on a screen kept 1.5 m away from the slit. **All India 2011**

21. A parallel beam of light of 500 nm falls on a narrow slit and the resulting diffraction pattern is observed on a screen 1 m away. It is observed that the first minima is at a distance of 2.5 mm from the centre of the screen. Calculate the width of the slit. **All India 2013**

22. Two wavelength of sodium light of 590 nm and 596 nm are used in turn to study the diffraction taking place at a single slit of aperture 2×10^{-6} m. The distance between the slit and the screen is 1.5 m.Calculate the separation between the position of first maxima of the diffraction pattern obtained in the two cases. **All India 2014**

23. Two wavelengths of sodium light 590nm and 596 nm are used in turn to study the diffraction at a single slit 4mm. The distance between the slit and the screen is 2m. Calculate the separation between the positions of the first maximum of diffraction pattern in two cases.

HINTS AND SOLUTIONS

1. (b) The wavelength of visible light is very small, so it seems to propagate in rectilinear path.

2. (c) Angular width of central maxima,
$$2\theta = 2\lambda / e.$$
Thus, θ does not depend on D i.e., distance between the slit and the screen.

3. (b) As, the path difference $a\theta$ is λ,
then $\theta = \dfrac{\lambda}{a}$
$\Rightarrow \quad \dfrac{10\lambda}{d} = \dfrac{2\lambda}{a}$
$\Rightarrow \quad a = \dfrac{d}{5} = \dfrac{10}{5} = 0.2$ mm
So, the width of each slit is 0.2 mm.

4. (a) Width of central maxima $= 2\lambda D / e$
Width of other secondary maxima $= \lambda D / e$
$\therefore$ Width of central maxima : Width of other secondary maxima = 2:1.

5. For first minima, $\sin\theta_1 = 1 \times \lambda / a = \lambda / a$

6. Angular width, $2\theta = \dfrac{2\lambda}{a}$
i.e. it is independent of the distance of separation between the slit and screen.

7. In diffraction pattern, intensity of alternate dark and weak bright bands decreases as compare to central bright band

8. As, Fresnel distance $\propto \dfrac{1}{\lambda}$, hence Fresnel's distance increases as λ decreases.

9. The interference pattern has a number of equally spaced bright and dark bands whereas the diffraction pattern has a central bright maximum, which is twice as wide as the other maxima.

10. In diffraction, angular position, $\theta = \dfrac{\Delta x}{a}$
For first minima, $\Delta x = \lambda$,
$\therefore \quad \theta = \dfrac{\lambda}{a}$
In interference, $d = a$ (given) and angular position,
$$\theta = \frac{\Delta x}{a}$$
$\therefore$ Angular position of first maxima ($\Delta x = \lambda$)

11. For intensity pattern curve Refer to text on pages 413 and 423.
For difference Refer to text on page 424.

12. Refer to text on page 424.

[redacted]

13. As, we know that the frequencies of sound waves lie between 20 Hz to 20 kHz, so their wavelength ranges between 15 m to 15 mm. The diffraction occurs, if the wavelength of waves is nearly equal to slit width.

As, the wavelength of light waves is 7000×10^{-10} m to 4000×10^{-10} m, the slit width is very near to the wavelength of sound waves as compared to light waves. Thus, the diffraction of sound waves is more evident in daily life than that of light waves.

14. Let the slit be divided into n smaller slits each of width,

$$d' = \frac{d}{n}.$$

The angle, $\theta = \frac{n\lambda}{d} = \frac{n\lambda}{d'n} = \frac{\lambda}{d'}$

Therefore, each of the smaller slit would send zero intensity in the direction of θ. Hence, for the entire single slit, intensity at angle $\frac{n\lambda}{d}$ would be zero.

15. (a) We know that, width of central maximum is given as

$$2y = \frac{2D\lambda}{a}$$

where, a = width of slit.

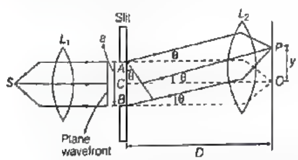
when $a = 2a$

$\therefore$ Width of central maximum $= \frac{2D\lambda}{2a} = \frac{\lambda D}{a}$

Thus, the width of central maximum became half. But in case of diffraction, intensity of central maxima does not changes with slit width. Thus, the intensity remains same in both cases.

(b) When a tiny circular obstacle is placed in the path of light from a distant source, a bright spot is seen at the centre of the obstacle because the waves diffracted from the edge of circular obstacle interfere constructively at the centre of the shadow resulting in the formation of a bright spot.

16. (i)



Consider a parallel beam of light from a lens falling on a slit AB. As, diffraction occurs, the pattern is focused on the screen with the help of lens L_2. We will obtain a diffraction pattern that is central maximum at the centre O, flanked by a number of dark and bright fringes called secondary maxima and minima.

Each point on the plane wavefront AB sends out the secondary wavelets in all directions. The waves from points equidistant from the centre C, lying on the upper and lower half, reach point O with zero path difference and hence, reinforce each other producing maximum intensity at O.

(ii) Let λ and a be the wavelength and slit width of diffracting system respectively. Let O be the position of central maximum.

Condition for the first minimum is given by

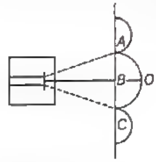
$$a \sin\theta = m\lambda \quad \text{... (i)}$$

Let θ be the angle of diffraction.

As, diffraction angle is small

$\therefore \quad \sin\theta \approx \theta$

For first diffraction minimum, $\theta = \theta_1$ (let)



For the first minimum, take $m = 1$

$$a\theta_1 = \lambda \Rightarrow \theta_1 = \frac{\lambda}{a}$$

Now, angular width, $AB = \theta_1$

Angular width, $BC = \theta_1$

Angular width, $AC = 2\theta_1$

(iii) On increasing the value of n, the part of slit contributing to the maximum decreases. Hence, the maximum becomes weaker.

17. (i) Refer to text on page 412

On increasing the order, the part of slit contributing to the maximum decreases. Hence, the secondary maxima becomes weaker.

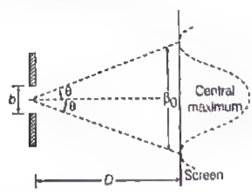
(ii) As, the number of point sources increases, their contribution towards intensity also increases. intensity varies as square of the slit width. Thus, when the width of the slit is made double the original width, intensity will get four times of its original value.

Width of central maximum is given by $\beta = \frac{2D\lambda}{b}$

where, D = distance between screen and slit,

λ = wavelength of the light

and b = size of slit.



So, with the increase in size of slit, the width of central maxima decreases. Hence, double the size of the slit would result as half the width of the central maxima.

[redacted]

18. As, the path difference is λ

So, $a\theta = \lambda$

$\Rightarrow \theta = \lambda / a$

$\Rightarrow \frac{10\lambda}{d} = \frac{2\lambda}{a}$

$\Rightarrow a = \frac{d}{5} = \frac{10}{5} = 2\text{m}$

19. Given, $D = 1$ m, $n = 1$, $y = 2.5$ mm

$= 2.5 \times 10^{-3}$ m

$\lambda = 500$ nm $= 5 \times 10^{-7}$ m, $d = ?$

The condition for minima is $d\frac{y}{D} = n\lambda$

$\Rightarrow d = \frac{n\lambda D}{y} = \frac{1 \times 5 \times 10^{-7} \times 1}{2.5 \times 10^{-3}}$

$= 2 \times 10^{-4}$ m $= 0.2$ mm

20. Given, $\lambda = 6000\ \text{Å} = 6 \times 10^{-7}$ m and $d = 1 \times 10^{-4}$ m

Separation between slit and screen, $D = 1.5$ m

$\therefore$ The separation between two dark lines on either side of the central maximum

= fringe width of central maximum

$= \frac{2D\lambda}{d} = \frac{2 \times 1.5 \times 6 \times 10^{-7}}{1 \times 10^{-4}}$

$= 18 \times 10^{-3}$ m $= 18$ mm

21. The distance of the nth minima from the centre of the screen is given by $x_n = \frac{nD\lambda}{a}$ ...(i)

where, D = distance of slit from screen = 1 m

λ = wavelength of the light = 500 nm

$= 500 \times 10^{-9}$ m,

$n = 1$,

$x_n = 2.5$ mm $= 2.5 \times 10^{-3}$ m,

and a = width of the slit for first minima = ?

Putting these values in Eq. (i), we get

$2.5 \times 10^{-3} = \frac{(500 \times 10^{-9})}{a}$

$\Rightarrow a = 2 \times 10^{-4}$ m $= 0.2$ m

22. For $\lambda_1 = 590$ nm

Location of 1st maxima, $y_1 = (2n+1)\frac{D\lambda_1}{2a}$

If $n = 1 \Rightarrow y_1 = \frac{3D\lambda_1}{2a}$

For $\lambda_2 = 596$ nm

Location of 2nd maxima, $y_2 = (2n+1)\frac{D\lambda_2}{2a}$

If $n = 1 \Rightarrow y_2 = \frac{3D\lambda_2}{2a}$

$\therefore$ Path difference $= y_2 - y_1 = \frac{3D}{2a}(\lambda_2 - \lambda_1)$

$= \frac{3 \times 1.5}{2 \times 2 \times 10^{-6}}(596 - 590) \times 10^{-9} = 6.75 \times 10^{-3}$ m

23. Given, $\lambda_1 = 590$ nm, $\lambda_2 = 596$ nm,

$D = 2$ m, $d = 4$ mm

Refer to Sol. 50 (page 459) location of 1st maxima,

$y_1 = (2n+1)\frac{D\lambda_1}{2a}$

if $1 = 1, \Rightarrow y_1 = \frac{3D\lambda_1}{2a}$

For $\lambda_2 = 596$ nm, location of 2nd maxima,

$y_2 = (2n+1)\frac{D\lambda_2}{2a}$

if $n = 1, \Rightarrow y_2 = \frac{3D\lambda_2}{2a}$

$y_2 - y_1 = \frac{3D}{2a}(\lambda_2 - \lambda_1)$

$= \frac{3 \times 2}{2 \times 4 \times 10^{-3}}(596 - 590) \times 10^{-9}$

$= 4.5 \times 10^{-6}$ m

[redacted]

SUMMARY

- **Wave optics** describes the connection between the waves and rays of light.
- **Wavefront** is the locus of points (wavelets) having the same phases of oscillations.
- Wavefronts can be of three types
  (i) Spherical wavefront
  (ii) Cylindrical wavefront
  (iii) Plane wavefront
- **Huygens' Principle** is essentially a geometrical construction, which gives the shape of a wavefront at any time allows us to determine the shape of the wavefront at a later time.
- **Doppler's Effect in Light** According to this effect, whenever there is a relative motion between a source of light and observer, then the apparent frequency of light emitted from the light source
- **Superposition Principle of Waves** states that, at a particular point in the medium, the resultant displacement produced by the number of the displacements produced by each of the waves
- **Interference of Light Waves** is the phenomenon of redistribution of light energy in a medium on the account of superposition of light waves from two coherent sources
- **Relation Between Intensity, Amplitude of the Wave and Width of Slit**
  It is given by, $\frac{I_1}{I_2} = \frac{a^2}{b^2} = \frac{W_1}{W_2}$
- **Conditions for Interference** The intensity at the points of constructive and destructive interference must be maintained maximum and zero, respectively
- **Diffraction of Light** Bending of light around the sharp corners an spreading of light with in the geometrical shadow of opaque obstacles is called diffraction of light
- In diffraction pattern, angular width of the central maxima is
  $$2\theta = \frac{2\lambda}{a}$$
- **Difference between Interference and Diffraction** The interference pattern has number of equally spaced dark and bright bands while the diffraction pattern has central bright maximum
- **Interference and Energy Conservation** Intensity of light is simply redistributed, i.e., energy is being transferred from the regions of destructive interference. So, the principle of energy conservation is obeyed in interference process
- **Coherent Sources** of light emit the light waves with constants phase difference
- **Incoherent Sources** of light emit the light do not waves with a constants phase difference
- **Young's Double Slit Experiment**
  (i) For constructive Interference (bright fringes),
  $$\text{Path difference} = \frac{dy}{D} = n\lambda$$
  $$\Rightarrow \quad y = \frac{nD\lambda}{d}$$
  (ii) For destructive interference (dark fringes),
  path difference
  $$\frac{dy}{D} = (2n-1)\frac{\lambda}{2}$$
  $$\Rightarrow \quad y = (2n-1)\frac{D\lambda}{2d}$$
- **Intensity of Fringes**
  (i) For bright fringe, $I_R = I_1 + I_2 + 2\sqrt{I_1 I_2}$
  (ii) For dark fringe, $I_R = I_1 - I_2 - 2\sqrt{I_1 I_2}$
- **Fringe Width** is given as, $W = \frac{D\lambda}{d}$
  (i) Resolving power of a microscope is
  $$RP = \frac{1}{\Delta d} = \frac{2\mu \sin\beta}{1.22\lambda}$$
  (ii) Resolving power of a telescope is
  $$RP = \frac{1}{d\theta} = \frac{D}{1.22\lambda}$$

[redacted]

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. If a source is at infinity, then wavefronts reaching to observer are

(a) cylindrical (b) spherical
(c) plane (d) conical

2. In Huygens' wave theory, the locus of all points in the same state of vibration is called

(a) a half period zone (b) oscillator
(c) a wavefronts (d) a ray

3. A monochromatic light refracts by the medium of refractive index 1.5 in vacuum. The wavelength of refracted wave will be

(a) equal
(b) increase
(c) decrease
(d) depend upon the intensity of refracted light

4. The source of light is moving towards observer with relative velocity of 3 kms^{-1}. The fractional change in frequency of light observed is

(a) 3×10^{-3} (b) 3×10^{-5}
(c) 10^{-5} (d) None of these

5. The reason of interference is

(a) phase difference
(b) change of amplitude
(c) change of velocity
(d) intensity

6. Two distinct light bulbs as sources

(a) can produce an interference pattern
(b) cannot produce a sustained interference pattern
(c) can produce an interference pattern, if they produce light of same frequency
(d) can produce an interference pattern only when the light produced by them is monochromatic in nature

7. From a single slit, the first diffraction minima is obtained at 30° for a light of 6500 Å wavelength. The width of the slit is

(a) 3250 Å (b) 1.3 μ
(c) 5.4×10^{-4} km (d) 1.2×10^{-2} cm

8. In Young's double slit experiment two disturbance arriving at a point P have phase difference of $\pi/2$. The intensity of this point expressed as a fraction of maximum intensity I_0 is

(a) $\frac{3}{2}I_0$ (b) $\frac{1}{2}I_0$ (c) $\frac{4}{3}I_0$ (d) $\frac{3}{4}I_0$

9. If the width of slit is decreased in a single slit diffraction, then the width of central maxima will

(a) increase
(b) decrease
(c) remain unchanged
(d) not depend on the width of slit.

ASSERTION AND REASON

Directions (Q. Nos. 10-16) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

10. **Assertion** In the field of geometrical optics, light can in assumed to approximately travel in straight line.

Reason The wavelength of visible light is very small in comparison to the dimensions of typical mirrors and lenses, then light can be assumed to approximately travel in straight line.

11. **Assertion** When monochromatic light is incident on a surface separating two media, the reflected and refracted light both have the same frequency as the incident frequency.

Reason Speed of light and wavelength of light both changes in refraction and hence, the ratio $v=c/\lambda$ is a constant.

[redacted]

12. **Assertion** The emergent plane wavefront is tilted on refraction of a plane wave by a thin prism.
Reason The speed of light waves is more in glass and the base of the prism is thicker than the top.

13. **Assertion** If we have a point source emitting waves uniformly in all directions, the locus of point which have the same amplitude and vibrate in the same phase are spheres.
Reason Each point of the wavefront is the source of a secondary disturbance and the wavelets emanating from these points spread out in all directions with the speed of the wave.

14. **Assertion** Increase in the wavelength of light due to Doppler's effect is red shift.
Reason When the wavelength increases, then wavelength in the middle of the visible region of the spectrum moves towards the red end to the spectrum.

15. **Assertion** No interference pattern is detected when two coherent sources are infinitely close to each other.
Reason The fringe width is inversely proportional to the distance between the two slits.

16. **Assertion** If the initial phase difference between the light waves emerging from the slits of Young's double slit experiment is π-radian, the central fringe will be dark.
Reason Phase difference is equal to $\frac{2\pi}{\lambda}$ times the path difference.

CASE BASED QUESTIONS

Directions (Q.No. 17) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

17. **The Wavefront**

In 1678, a Dutch scientist, Christian Huygens' propounded the wave theory of light. According to him, wave theory introduced the concepts of wavefront. Light travels in the form of waves.
A wavefront is the locus of points (wavelets) having the same phase (a surface of constant phase) of oscillations. A wavelet is the point of disturbance due to propagation of light. Wavefront may also be defined as the hypothetical surface on which the light waves are in the same phase.

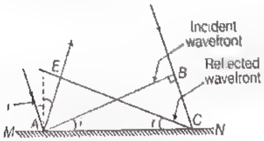
(i) Huygens' original theory of light assumed that, light propagates in the form of
(a) minute elastic particles
(b) transverse electromagnetic wave
(c) transverse mechanical wave
(d) longitudinal mechanical wave

(ii) A wave normal
(a) is parallel to a surface at the point of incidence of a wavefront
(b) is the line joining the source of light and an observer
(c) gives the direction of propagation of a wavefront at a given point
(d) is the envelope that is tangential to the secondary wavelets

(iii) Ray diverging from a point source form a wavefront that is
(a) cylindrical
(b) spherical
(c) plane
(d) cubical

(iv) According to Huygens' principle, a wavefront propagates through a medium by
(a) pushing medium particles
(b) propagating through medium with speed of light
(c) carrying particles of same phase along with it
(d) creating secondary wavelets which forms a new wavefront

(v) In case of reflection of a wavefront from a reflecting surface,



I. points A and E are in same phase.
II. points A and C are in same phase.
III. points A and B are in same phase.
IV. points C and E are in same phase.
Which of the following is correct?
(a) Both I and II
(b) Both II and III
(c) Both III and IV
(d) Both I and IV

[redacted]

VERY SHORT ANSWER Type Questions

18. State the conditions that must be satisfied for two light sources to be coherent.

19. In Young's double slit experiment, if the distance between the slits is halved, what changes in the fringe width will take place?

20. Suppose while performing double slit experiment, the space between the slits and the screen is filled with water. How does the interference pattern change?

21. 5000 Å monochromatic light passes through a slit having 0.05 mm width. How much does it spread?

SHORT ANSWER Type Questions

22. Derive fringe separation formula for YDSE.

23. Obtain the expression for the maximum slit width for diffraction.

LONG ANSWER Type II Questions

24. What do you understand by coherent sources? Obtain expression for fringe width of a bright fringe. Write expression for the angular width of fringe.

25. Describe Young's experiment for interference of light. Obtain the formula for fringe width. What is the shape of the fringes?

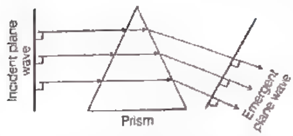
ANSWERS

1. (c)	2. (c)	3. (c)	4. (c)	5. (a)
6. (b)	7. (b)	8. (b)	9. (a)	

10. (a) The branch of optics in which one completely neglects the finiteness of the wavelength is called geometrical optics. The wavelength of light is very small as compared to the dimensions of objects (such as mirror, lenses etc.) and hence, it can be neglected and assumed to travel in a straight line.

11. (b) Reflection and refraction arise through interaction of incident light with constituents of matter. Atoms may be viewed as oscillators, which take up the frequency of the external agency (light) causing forced oscillations. The frequency of light emitted by a charged oscillator equals its frequency of oscillation. Thus, the frequency of scattered light equals the frequency of incident light.

12. (c) Since, the speed of light waves is less in glass, the lower portion of the incoming wavefront (which travels through the greatest thickness of glass) will get delayed resulting in a tilt in the emerging wavefront.



13. (a) According to Huygens' principle each point of the wavefront is the source of a secondary disturbance and the wavelets emanating from these point spread out in all directions with the space of wave.

These wavelets emanating from the wavefront are usually referred to as secondary wavelets and if we draw a common tangent to all these spheres, we obtain the new position of the wavefront at a later time.

14. (a) Increase in wavelength of light when the source move away from the observer due to Doppler's effect is called red shift. The visible regions shifts towards red end of electromagnetic spectrum and hence called red shift.

15. (a) As, we know, fringe width β i.e., $= \frac{\lambda D}{d}$

So, smaller the distance between the slits (d), then larger will be fringe width (β).

Hence, single fringe will cover whole screen and pattern will not be visible.

16. (b) Given, initial phase difference $= \phi_{S_1} - \phi_{S_2} = \pi$

At central maximum, $\Delta x = 0$ (path difference $= \Delta x$)

$\Rightarrow$ Total phase difference

$$= \phi_{S_1} - \phi_{S_2} + \frac{2\pi}{\lambda}\Delta x$$

At central maximum,

$$\Delta\phi = \pi + \frac{2\pi}{\lambda} \times \Delta x = \pi + 0$$

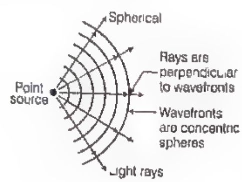
or $\Delta\phi = \pi =$ odd multiple of π.

Hence, at central maximum dark band is obtained.

17. (i) (d) Huygens' original wave theory of light assumes that light propagates in the form of longitudinal mechanical wave.

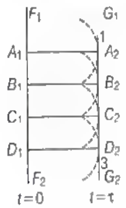
(ii) (c) A wave normal is a line perpendicular to the wavefront. It gives the direction of a moving wave.

(iii) (b) Wavefronts emanating from a point source are spherical wavefronts.

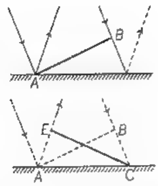


[redacted]

(iv) (d) According to Huygens' principle, each point of the wavefront is the source of a secondary disturbance and the wavelets emanating from these points spread out in all directions with the speed of the wave. These wavelets emanating from the wavefront ($F_1 F_2$) are usually referred to as secondary wavelets and if we draw a common tangent to all these spheres as shown below, we obtain the new position of the wavefront ($G_1 G_2$) at a later time (τ).



(v) (c) Figure shows AB as incident wavefront, so A and B are in same phase.



By the time B reaches C, secondary wavelet from A reaches E.

So, points C and E are same time intervals apart as they are in same phase.

18. Refer to text on page 411.

19. Fringe width, $W \propto \frac{1}{d}$, hence W is doubled.

20. The pattern will remain same, but slightly shifted.

21. Refer to Example 2 on page 423. [**Ans.** 0.2 rad]

22. Refer to text on page 412.

23. Refer to text on page 423.

24. Refer to text on page 411 and 412.

25. Refer to text on page 412 and 413.

[redacted]

11

The Maxwell's equations of electromagnetism and Hertz experiments on generation and detection of electromagnetic waves established the wave nature of light in 1887. But the discoveries of photoelectric effect by Hertz, Compton effect by Compton, established the particle nature of light. Hence, it was concluded that light has dual nature.

DUAL NATURE OF RADIATION AND MATTER

ELECTRON EMISSION

In metals, the electrons in the outer shells (valence electrons) are loosely bound to the atoms, hence they are free to move easily within the metal surface but cannot leave it. Such electrons are called **free electrons**.

These free electrons can be emitted from the metals, if they have sufficient energy to overcome the attractive pull of metal surface. The phenomenon of emission of electrons from the surface of a metal is called **electron emission**.

The minimum required energy for the electron emission from the metal surface can be supplied to the free electrons by anyone of the following physical processes

(i) **Thermionic Emission** Sufficient thermal energy can be imparted to the free electrons by suitable heating, so that they can come out of the metal. This process of emission of electrons is known as thermionic emission and the electrons so emitted are known as **thermions** or **thermal electrons** . The number of thermions emitted depends on the temperature of the metal surface.

(ii) **Field Emission or Cold Cathode Emission** It is the phenomenon of emission of electrons from the surface of a metal by applying a very strong electric field ($\sim 10^8$ Vm^{-1}) to a metal. One of the examples of cold emission is spark plug.

(iii) **Photoelectric Emission** It is the phenomenon of emission of electrons from the surface of metal when light radiations of suitable frequency fall on it. Here, the energy to the free electrons for their emission is being supplied by light photons. The emitted electrons are called **photoelectrons**. The number of photoelectrons emitted depends on the intensity of the incident light.

CHAPTER CHECKLIST
- Photoelectric Effect
- Matter Wave

[redacted]

(iv) **Secondary Emission** It is the phenomenon of emission of electrons from the surface of metal in large number when fast moving electrons (called primary electrons) or other particles strike the metal surface.

Work Function

A certain minimum amount of energy is required to be given to an electron to pull it out from the surface of the metal. This minimum energy required by an electron to escape from the metal surface is called the **work function** of the metal. It is generally denoted by ϕ_0 or W_0 and measured in eV (electron volt). It depends on the properties of the metal and the nature of its surface. It decreases with the increase on temperature.

One electron volt (1 eV) is the energy gained by an electron when it has been accelerated by a potential difference of one volt (1 V), so that $1 \text{ eV} = 1.602 \times 10^{-19}$ J.

The values of work function of some metals are given below

Work Function of Some Metals

Metal	Work function, ϕ_0 (eV)	Metal	Work function, ϕ_0 (eV)
Cs	2.14	Al	4.28
K	2.30	Hg	4.49
Na	2.75	Cu	4.65
Ca	3.20	Ag	4.70
Mo	4.17	Ni	5.15
Pb	4.25	Pt	5.65

From the above table, it can be concluded that, the work function of platinum is the highest ($\phi_0 = 5.65$ eV), while it is the lowest for caesium ($\phi_0 = 2.14$ eV).

|TOPIC 1|
Photoelectric Effect

As discussed earlier, the phenomenon of emission of electrons from the surface of metal, when radiations of suitable frequency fall on it, is called photoelectric effect. The emitted electrons are called photoelectrons and the current, so produced is called photoelectric current.

Alkali metals like lithium, sodium, etc. show photoelectric effect with visible light, whereas the metals like zinc, cadmium, etc. are sensitive only to ultraviolet light.

Note Non-metals also show photoelectric effect. Liquids and gases can also show this effect but to limited extent

Hertz's Observations

In 1887, Heinrich Hertz discovered the phenomenon of photoelectric emission while working with his electromagnetic wave experiment, by means of spark discharge. He observed that high voltage sparks across the detector loop were enhanced when the emitter plate was illuminated by ultraviolet light from an arc lamp. It was accounted as follows:

When suitable radiations fall on a metal surface, some electrons near the surface absorb enough energy from the incident radiations, to overcome the attraction of the positive ions in the material of the surface. This helps them to escape from the surface of the material to the surrounding space.

Hallwachs' and Lenard's Observations

During 1886-1902, Wilhelm Hallwachs and Philipp Lenard made a detailed study of photoelectric effect. Lenard observed that, if a potential difference is applied across the two metal plates enclosed in an evacuated tube, then there is no flow of current in the circuit. However, when one plate (called emitter plate) enclosed in the evacuated tube, kept at negative potential is exposed with ultraviolet radiations, current begins to flow in the circuit.

As soon as ultraviolet radiations falling on the emitter plate are stopped, the current flowing is also stopped. Thus, light falling on the surface of emitter causes current in the external circuit. From his observation, Hallwachs concluded that negatively charged particles were ejected out from the zinc plate under the action of ultraviolet radiations. After the discovery of electron in 1897, it became evident that the exposure of emitter plate with the incident light causes the electrons to emit also. Due to negative charge, the emitted electrons are pushed towards the collector plate by the applied electric field.

Hallwachs and Lenard also observed that when ultraviolet light fell on the emitter plate, no electrons were emitted, until the frequency of the incident light was smaller than a certain minimum value, called the threshold frequency. This minimum frequency depends on the nature of the material of the emitter plate.

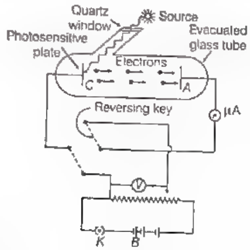
EXPERIMENTAL STUDY OF PHOTOELECTRIC EFFECT

The figure given below shows the experimental setup for the study of photoelectric effect.

The setup consists of an evacuated glass or quartz tube which encloses a photosensitive plate C (called **emitter**) and a metal plate A (called **collector**). A transparent quartz

[redacted]

window is sealed onto the glass tube which permits ultraviolet radiation to pass through it and irradiate the photosensitive plate C.



Experimental arrangement for the study of photoelectric effect

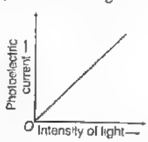
The electrons are emitted by the plate C and are collected by the plate A. When the collector plate A is positive with respect to the emitter plate C, then the electrons are attracted to it. Hence, photoelectric current is constituted. This emission of electrons causes flow of electric current in the circuit.

The potential difference between the emitter and collector plates is measured by a voltmeter (V), whereas the resulting photocurrent flowing in the circuit is measured by a microammeter (μA).

The experimental arrangement given above is used to study the variations of photocurrent with intensity of radiation, frequency of radiation and the potential difference between the plates A and C.

Effect of Intensity of Light on Photoelectric Current

For a fixed frequency of incident radiation and accelerating potential, the photoelectric current increases linearly with increase in intensity of incident light.

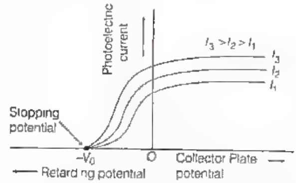


Variation of photoelectric current with intensity of light

As, the photoelectric current is directly proportional to the number of photoelectrons emitted per second. So, the number of photoelectrons emitted per second is directly proportional to the intensity of the incident radiation.

Effect of Potential on Photoelectric Current

For a fixed frequency and intensity of incident light, photoelectric current increases with increase in potential applied to the collector as shown in the graph.



Variation of photoelectric current versus potential for different intensities but constant frequency

From the above graph, we can observe that,

(i) After a certain value of accelerating potential, when all photo electrons reach the plate A, and the photocurrent ceases. On increasing the value of accelerating potential, this maximum value of photoelectric current is called Saturation current.

(ii) When the potential is decreased, the current decreases but does not become zero at zero potential. This shows that even in the absence of accelerating potential, few photoelectrons manages to reach the plate A on their own due to their kinetic energy.

(iii) For a particular frequency of incident radiation, when minimum negative potential V_0 is applied to the plate A w.r.t. C, photoelectric current becomes zero at a particular value of negative potential V_0 called **stopping potential or cut-off potential.**

In this condition, the stopping potential is sufficient to repel even the most energetic photoelectron with maximum kinetic energy K_{max}. Photoelectric current becomes zero whenever no electron even the fastest photoelectrons cannot reach the plate A. Hence, maximum kinetic energy is given as,

$$K_{max} = eV_0 = \frac{1}{2} m v_{max}^2$$

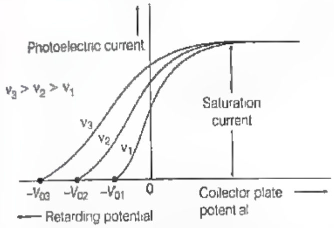
where, m is the mass of photoelectron and v_{max} is the maximum velocity of emitted photoelectron.

Note For the radiation of a given frequency and material of plate C, the value of stopping potential V_0 is independent of the intensity of the incident radiation. It means, the maximum kinetic energy of emitted photoelectron depends on the light source and the emitter plate material but is independent of intensity of incident radiation.

[redacted]

Effect of Frequency of Incident Radiation on Stopping Potential

If we take radiations of different frequencies but of same intensity. For each radiation, we study the variation of photoelectric current against the potential difference between the plates as shown in the graph below.



Variation of photoelectric current versus potential for different frequencies but constant intensity of incident radiation

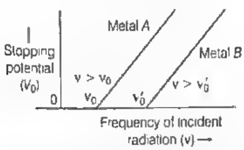
From the above graph, we observe that

(i) the value of stopping potential is different for radiation of different frequencies but the value of saturation current (for given intensity) remains constant.

(ii) the value of stopping potential is more negative for incident radiation of higher frequency. This means that the energy of the emitted electrons depends on the frequency of incident radiations. Greater the frequency of incident radiation, greater is the maximum kinetic energy of photoelectrons, consequently greater retarding potential or stopping potential is required to stop them completely.

(iii) the value of saturation current depends upon the intensity of incident radiation but is independent of the frequency of the incident radiation.

If we plot a graph between stopping potential and the frequency of the incident radiation for two different metals A and B, we get the graph as shown below.



Variation of stopping potential versus frequency of incident radiation

From the graph, are observe that

(i) the stopping potential V_0 varies linearly with the frequency of incident radiation for a given photosensitive material.

(ii) there exists a certain minimum cut-off frequency ν_0 for which the stopping potential is zero. This frequency is called threshold frequency.

Note The minimum frequency of light which can emit photoelectrons from a material is called **threshold frequency** or **cut-off frequency** of that material. It is a characteristic property of material.

For a frequency lower than cut-off frequency, no photoelectric emission is possible even if the intensity is large. If frequency of incident radiation is more than the threshold frequency, the photoelectric emission starts instantaneously without any apparent time lag ($\sim 10^{-9}$s or less) even when the incident radiation is very dim.

LAWS OF PHOTOELECTRIC EMISSION

The laws of photoelectric emission are as follows

(i) For a given material and a given frequency of incident radiation, the photoelectric current or number of photoelectrons ejected per second is directly proportional to the intensity of the incident light.

(ii) For a given material and frequency of incident radiation, saturation current is found to be proportional to the intensity of incident radiation, whereas the stopping potential is independent of its intensity.

(iii) For a given material, there exists a certain minimum frequency of the incident radiation below which no emission of photoelectrons takes place. This frequency is called threshold frequency.

Above the threshold frequency, the maximum kinetic energy of the emitted photoelectrons or equivalent stopping potential is independent of the intensity of the incident light but depends upon only the frequency (or wavelength) of the incident light.

(iv) The photoelectric emission is an instantaneous process. The time lag between the incidence of radiations and emission of photoelectrons is very small, less than even 10^{-9}s.

[redacted]

PHOTOELECTRIC EFFECT AND WAVE THEORY OF LIGHT

Huygens' wave theory of light could not explain the photoelectric emission due to the following main reasons:

(i) According to the wave nature of light, the free electrons at the surface of the metal absorb the radiant energy continuously.

The greater the intensity of radiation, the greater should be the energy absorbed by each electron. The maximum kinetic energy of the photoelectrons on the surface is then expected to increase with increase in intensity.

But according to experimental facts, the maximum kinetic energy of ejected photoelectrons is independent of intensity of incident radiation.

(ii) According to wave theory of light, no matter what the frequency of radiation is, a sufficiently intense beam of radiation should be able to impart enough energy to the electrons, so that they exceed the minimum energy needed to escape from metal surface.

A threshold frequency, therefore should not exist which contradicts the experimental fact that, no photoelectric emission takes place below that threshold frequency, no matter whatsoever may be its intensity.

(iii) According to the wave theory of light, the absorption of energy by electron takes place continuously over the entire wavefront of the radiation. Since, a large number of electrons absorb energy, the energy absorbed per electron per unit time turns out to be small.

Hence, it will take hours or more for a single electron to come out of the metal which contradicts the experimental fact that photoelectron emission is instantaneous.

EINSTEIN'S PHOTOELECTRIC EQUATION

Energy Quantum of Radiation

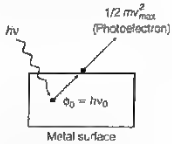
In 1905, Albert Einstein explained the various laws of photoelectric emission on the basis of Planck's quantum theory. According to that theory, the energy of an electromagnetic wave is not continuously distributed over the wavefront of waves. Instead of this, these waves travel in the form of discrete packets or bundels of energy called quanta of energy of radiation

Each quantum of energy radiate an energy, which is given by

$$E = h\nu$$

where, h is Planck's constant and ν is the frequency of light radiation.

When a quantum of light radiation of energy $h\nu$ falls on a metal surface, then this energy is absorbed by the electron and is used in following two ways



Emission of photoelectron by a metal surface when a quantum of light is absorbed by it

(i) A part of energy is used to overcome the surface barrier and come out of the metal surface. This part of energy is called work function. It is expressed as $\phi_0 = h\nu_0$.

(ii) The remaining part of the energy is used in giving a velocity v to the emitted photoelectron. This is equal to the maximum kinetic energy of the photoelectrons $\left(\frac{1}{2}mv_{max}^2\right)$, where m is the mass of the photoelectron.

According to the law of conservation of energy,

$$h\nu = \phi_0 + \frac{1}{2}mv_{max}^2 = h\nu_0 + \frac{1}{2}mv_{max}^2$$

$$\therefore \quad \frac{1}{2}mv_{max}^2 = K_{max} = h(\nu - \nu_0) = h\nu - \phi_0$$

$$\therefore \quad \boxed{K_{max} = h\nu - \phi_0}$$

This equation is called **Einstein's photoelectric equation.**

EXAMPLE |1| The electric field associated with a monochromatic beam of light becomes zero, with frequency 2.4×10^{15} times per second. Find the maximum kinetic energy of the photoelectrons when this light falls on a metal surface whose work function is 2.0 eV.

[redacted]

Sol. Given, $\phi_0 = 2.0$ eV, $h = 6.63 \times 10^{-34}$ J-s, $KE_{max} = ?$

In one complete vibration twice the electric field becomes zero, so the frequency of incident light is given by

$$\nu = \frac{1}{2} \times 2.4 \times 10^{15} = 1.2 \times 10^{15} \text{ Hz}$$

Hence, maximum kinetic energy,

$$KE_{max} = h\nu - \phi_0 = \frac{6.63 \times 10^{-34} \times 1.2 \times 10^{15}}{1.6 \times 10^{-19}} - 2 = 2.97 \text{ eV}$$

Relation between Stopping Potential (V_0) and Threshold Frequency (ν_0)

Maximum kinetic energy is given by

$$K_{max} = h(\nu - \nu_0)$$

Also, $$K_{max} = eV_0$$

$$\therefore \quad \boxed{eV_0 = h(\nu - \nu_0)} \quad \ldots(i)$$

If λ = wavelength of the incident radiation,

λ_0 = threshold wavelength of the metal surface and

c = velocity of light.

Then, $$\nu = \frac{c}{\lambda} \text{ and } \nu_0 = \frac{c}{\lambda_0}$$

Putting these values in Eq. (i), we get

$$eV_0 = h\left(\frac{c}{\lambda} - \frac{c}{\lambda_0}\right)$$

$$\therefore \quad \boxed{eV_0 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)} \quad \ldots(ii)$$

EXAMPLE |2| The work function of caesium is 2.14 eV. Calculate

(i) the threshold frequency for caesium and

(ii) the wavelength of the incident light, if the photocurrent is brought to zero by a stopping potential of 0.60 V. Given, $h = 6.63 \times 10^{-34}$ J-s.

Sol. Here, $V_0 = 0.60$ V, $\phi_0 = 2.14$ eV $= 2.14 \times 1.6 \times 10^{-19}$ J

(i) Threshold frequency, $\nu_0 = \frac{\phi_0}{h}$ $\quad [\because \phi_0 = h\nu_0]$

$$= \frac{2.14 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$= 5.16 \times 10^{14} \text{ Hz}$$

(ii) We have, $eV_0 = \frac{hc}{\lambda} - \phi_0 \Rightarrow \lambda = \frac{hc}{(eV_0 + \phi_0)}$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{(1.6 \times 10^{-19} \times 0.60 + 2.14 \times 1.6 \times 10^{-19})}$$

$$= 454 \times 10^{-9} \text{ m} = 454 \text{ nm}$$

Verification of Laws of Photoelectric Emission Based on Einstein's Photoelectric Equation

Einstein's photoelectric equations is

$$K_{max} = \frac{1}{2} mv_{max}^2 = h(\nu - \nu_0)$$

This equation successfully explains the laws of photoelectric emission. These are as follows

(i) If $\nu < \nu_0$, then $\frac{1}{2} mv_{max}^2$ is negative which is not possible therefore, for photoelectric emission to take place, $\nu > \nu_0$.

(ii) Since, one photon emits one electron, so the number of photoelectrons emitted per second is directly proportional to the intensity of incident light.

(iii) It is clear that $\frac{1}{2} mv_{max}^2 \propto \nu$, as h and ν_0 are constants. This shows that kinetic energy of the photoelectrons is directly proportional to the frequency of the incident light.

(iv) Photoelectric emission is due to elastic collisions between a photon and an electron. As such there cannot be any significant time lag between the incidence of photon and emission of photoelectron.

Graphs Related to Photoelectric Effect From Einstein Photoelectric Equation

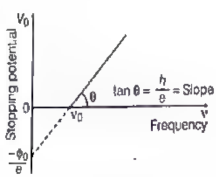
The important graphs related to photoelectric effect are as follows

(i) Frequency ν and stopping potential V_0 graph

We know that, $eV_0 = h\nu - \phi_0$

$$\Rightarrow \quad V_0 = \frac{h\nu}{e} - \frac{\phi_0}{e}$$

So, $$V_0 \propto \nu$$



Graph of V_0 versus ν

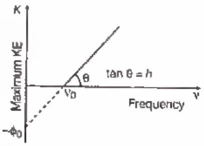
It could be seen that, V_0 *versus* ν curve is a straight line with slope = (h/e) and is independent of the nature of material.

[redacted]

(ii) **Frequency ν and maximum kinetic energy graph**

As, $K_{max} = h\nu - \phi_0$

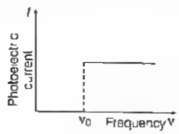
$\Rightarrow$ $K_{max} \propto \nu$



Graph of K_{max} *versus* ν

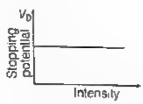
(iii) **Frequency ν and photoelectric current I graph**

The graph given below shows that, the photoelectric current I is independent of frequency of the incident light, till intensity remains constant.



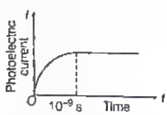
Graph of I *versus* ν

(iv) **Intensity and stopping potential V_0 graph**



Graph of V_0 *versus* intensity

(v) **Photoelectric current I and time lag t graph**



Graph of I *versus* t

PARTICLE NATURE OF LIGHT : THE PHOTON

Photoelectric effect thus gave evidence that light consists of packets of energy. These packets of energy were called **light quantum** that are associated with particles named as **photons**. So, photons confirm the particle nature of light.

Energy of a photon is given by

$$E = h\nu = \frac{hc}{\lambda}$$

where, h is the Planck's constant, ν is the frequency of radiation or photon, c is the speed of light and λ is the wavelength of photon.

The momentum of photon is given by

$$p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda} \text{ kg ms}^{-1}$$

EXAMPLE |3| The momentum of photon of electromagnetic radiation is 3.3×10^{-29} kg ms^{-1}. Find out the frequency and wavelength of the wave associated with it.

Sol. Given, $p = 3.3 \times 10^{-29}$ kg ms^{-1}

$h = 6.63 \times 10^{-34}$ J-s

$c = 3 \times 10^8$ m/s, $\nu = ?$

and $\lambda = ?$

Since, $E = h\nu = mc^2 = mc \times c = p \times c$

$$\therefore \quad \nu = \frac{pc}{h} = \frac{3.3 \times 10^{-29} \times 3 \times 10^8}{6.63 \times 10^{-34}}$$

$$= 1.5 \times 10^{13} \text{ Hz}$$

and $$\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{1.5 \times 10^{13}}$$

$$= 2 \times 10^{-5} \text{ m}$$

Characteristic Properties of Photons

Different characteristic properties of photons are given below

(i) In interaction of radiation with matter, radiation behaves as if it is made up of particles called photons.

(ii) A photon travels at a speed of light c in vacuum (i.e. 3×10^8 m/s).

(iii) It has zero rest mass, i.e. the photon cannot exist at rest. According to the theory of relativity, the mass m of a particle moving with velocity v, comparable with the velocity of light c is given by

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} \Rightarrow m_0 = m\sqrt{1 - \frac{v^2}{c^2}}$$

where, m_0 is the mass of the particle at rest.

As, a photon moves with the speed of light, i.e. $v = c$, hence $m_0 = 0$. So, rest mass of photon is zero.

[redacted]

(iv) The inertial mass of a photon is given by

$$m = \frac{E}{c^2} = \frac{h}{c\lambda} = \frac{h\nu}{c^2}$$

(v) Photons travel in a straight line.

(vi) Irrespective of the intensity of radiation, all the photons of a particular frequency ν or wavelength λ have the same energy $E\left(= h\nu = \frac{hc}{\lambda}\right)$ and momentum, $p\left(= \frac{h\nu}{c} = \frac{h}{\lambda}\right)$.

(vii) Energy of a photon depends upon frequency of the photon, so the energy of the photon does not change when photon travels from one medium to another.

(viii) Wavelength of the photon changes in different media, so velocity of a photon is different in different media.

(ix) Photons are not deflected by electric and magnetic fields. This shows that photons are electrically neutral.

(x) In a photon-particle collision (such as photoelectron collision), the energy and momentum are conserved. However, the number of photons may not be conserved in a collision.

(xi) Photons may show diffraction under given conditions.

EXAMPLE |4| Monochromatic light of wavelength 632.8 nm is produced by a helium-neon laser. The power emitted is 9.42 mW.

(i) Find the energy and momentum of each photon in the light beam.

(ii) How many photons per second, on the average, arrive at a target irradiated by this beam? (Assume the beam to have uniform cross-section, which is less than the target area.)

(iii) How fast does a hydrogen atom have to travel in order to have the same momentum as that of the photon?

NCERT

Sol. Given, wavelength of monochromatic light,

$$\lambda = 632.8 \text{ nm} = 632.8 \times 10^{-9} \text{ m}$$

$$\text{Power} = 9.42 \text{ mW} = 9.42 \times 10^{-3} \text{ W}$$

(i) Energy of each photon, $E = \frac{hc}{\lambda}$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{632.8 \times 10^{-9}}$$

$$= 3.14 \times 10^{-19} \text{ J}$$

We know that momentum of each photon, $p = \frac{h}{\lambda}$

$$p = \frac{6.63 \times 10^{-34}}{632.8 \times 10^{-9}} = 1.05 \times 10^{-27} \text{ kg-m/s}$$

(ii) Let n be the number of photons per second.

So, $n = \frac{\text{Power}}{\text{Energy of each photon}} = \frac{9.42 \times 10^{-3}}{3.14 \times 10^{-19}}$

$= 3 \times 10^{16}$ photons /s

(iii) Momentum, $p = mv$

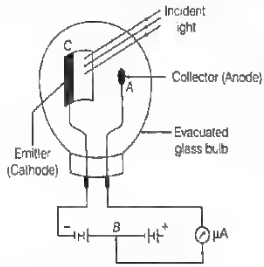
$\therefore$ Velocity of hydrogen atom, $v = \frac{p}{m} = \frac{1.05 \times 10^{-27}}{1.66 \times 10^{-27}}$

$= 0.63$ m / s

[$\because m = 1.66 \times 10^{-27}$ kg (mass of electron)]

PHOTOCELL

It is a device which converts light energy into electrical energy. It is also called an electric eye. As, the photoelectric current sets up in the photoelectric cell corresponding to incident light, it provides the information about the objects as has been seen by our eye in the presence of light.



Photocell

A photocell consists of a semi-cylindrical photosensitive metal plate C (emitter) and a wire loop A (collector) supported in an evacuated glass or quartz bulb. When light of suitable wavelength falls on the emitter C, photoelectrons are emitted.

Applications of Photocell

Some applications of photocell are given below

(i) Used in television camera for telecasting scenes and in photo telegraphy.

(ii) Reproduction of sound in cinema film.

(iii) Used in counting devices.

(iv) Used in burglar alarm and fire alarm.

(v) To measure the temperature of stars.

(vi) Used for the determination of Planck's constant.

[redacted]

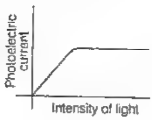
TOPIC PRACTICE 1

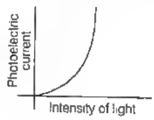
OBJECTIVE Type Questions

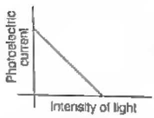
1. Lenard observed that no electrons are emitted when frequency of light is less than a certain minimum frequency. This minimum frequency depends on
   (a) potential difference of emitter and collector plates
   (b) distance between collector and the emitter plate
   (c) size (area) of the emitter plate
   (d) material of the emitter plate

2. The work function of a metal is hc/λ_0. If light of wavelength λ is incident on its surface, then the essential condition for the electron to come out from the metal surface is
   (a) $\lambda \geq \lambda_0$ (b) $\lambda \geq 2\lambda_0$
   (c) $\lambda \leq \lambda_0$ (d) $\lambda \leq \lambda_0/2$

3. Variation of photoelectric current with intensity of light is

(a)


(b)


(c)


(d)


4. A photon of energy 3.4 eV is incident on a metal surface whose work function is 2 eV. Maximum kinetic energy of the photoelectron emitted by the metal surface will be
   (a) 1.4eV (b) 1.7 eV
   (c) 5.4 eV (d) 6.8 eV

5. If photons of frequency ν are incident on the surfaces of metals A and B of threshold frequencies $\frac{\nu}{2}$ and $\frac{\nu}{3}$ respectively, the ratio of the maximum kinetic energy of electrons emitted from A to that from B is **CBSE All India 2020**
   (a) 2 : 3 (b) 3 : 4
   (c) 1 : 3 (d) $\sqrt{3} : \sqrt{2}$

6. Consider a beam of electrons (each electron with energy E_0) incident on a metal surface kept in an evacuated chamber. Then, **NCERT Exemplar**
   (a) no electrons will be emitted as only photons can emit electrons
   (b) electrons can be emitted but all with an energy, E_0
   (c) electrons can be emitted with any energy, with a maximum of $E_0 - \phi$ (ϕ is the work function)
   (d) electrons can be emitted with any energy, with a maximum of E_0

7. The formula for kinetic mass of a moving photon is
   (a) $h\nu/\lambda$ (b) $h\lambda/e$
   (c) $h\nu/e$ (d) $h/c\lambda$

   where, h is Planck constant and ν, λ, c are frequency, wavelength and speed of photon, respectively.

8. The wavelength of a photon needed to remove a proton from a nucleus which is bound to the nucleus with 1 MeV energy is nearly **NCERT Exemplar**
   (a) 1.2 nm
   (b) 1.2×10^{-3} nm
   (c) 1.2×10^{-6} nm
   (d) 1.2×10 nm

9. A photocell connected in an electrical circuit is placed at a distance d from a source of light. As a result, current I flows in the circuit. What will be the current in the circuit when the distance is reduced to $d/2$? **CBSE All India 2020**
   (a) I (b) $2I$
   (c) $4I$ (d) $I/2$

10. A photocell connected in an electrical circuit is placed at a distance d from a source of light. As a result, current I flows in the circuit. What will be the current in the circuit when the distance is reduced to $d/3$?
    (a) I (b) $6I$
    (c) $9I$ (d) $\frac{1}{3}I$

ASSERTION AND REASON

11. **Assertion** Cathode rays produce fluorescence in glass and colour of glow depends on nature of glass.

    Reason Cathode rays excite glass electrons and they on de-excitation emits radiation in visible region.

[redacted]

12. **Assertion** In photoelectric effect, cathode or emitter plate is usually coated with barium oxide, barium sulphide or strontium oxide.

Reason Coating prevents cathode from erosion.

13. **Assertion** According to wave theory of light, if intensity of incident radiation is increased, then energy of emitted photoelectrons increases.

Reason Energy of a wave is proportional to its intensity.

14. **Assertion** Photoelectric current depends on the intensity of incident light.

Reason Number of photoelectrons emitted per second is directly proportional to the intensity of incident radiation.

15. **Assertion** Photosensitivity of a metal is high if its work-function is small.

Reason Work-function $= h\nu_0$ where, ν_0 is the threshold frequency.

16. **Assertion** In photon-particle collision, the total energy and total momentum are conserved.

Reason The number of photons are conserved in a collision.

17. **Assertion** Photocell is also called electric eye.

Reason Photocell can see the things placed in front of it.

CASE STUDY BASED QUESTIONS

18. **Photoelectric Effect**

When a beam of 10.6 eV photons of intensity $2.0\ Wm^{-2}$ falls on a surface of platinum of surface area $1.0 \times 10^{-4}\ m^2$ and the work-function of the material is 5.6 eV. Given that, 0.53% of the incident photons eject photoelectrons.

(i) What is the energy of incident photon in joules?

(a) 10.6 J (b) 1.6×10^{-19} J
(c) 16.96×10^{-19} J (d) 2×10^{-21} J

(ii) Find the number of photons incident on given area.

(a) 1.18×10^{18} (b) 1.18×10^{14}
(c) 2×10^{16} (d) 2.3×10^{18}

(iii) Find number of photoelectrons emitted per second.

(a) 7×10^{11} (b) 6.25×10^{11}
(c) 9×10^{10} (d) 11×10^{11}

(iv) Find maximum energy of photoelectrons emitted.

(a) 5.0 eV (b) 6.0 eV
(c) 2.5 eV (d) 0 eV

(v) Find minimum energy of photoelectrons emitted.

(a) 6.0 eV (b) 5.0 eV
(c) 5.8 eV (d) 0 eV

VERY SHORT ANSWER Type Questions

19. Define the term intensity in photon picture of electromagnetic radiation. **CBSE 2019**

20. Do all the electrons that absorb a photon come out as photoelectrons? **NCERT Exemplar**

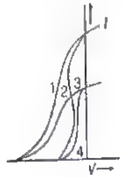
21. When radiations of frequency 10^{14} Hz is incident on certain surface, no photoemission takes place. What does this statement mean?

22. Two metals X and Y, when illuminated with appropriate radiation, emit photoelectrons. The work function of X is higher than of Y. Which metal will have higher value of threshold frequency?

23. Two metals A and B have work functions 2 eV and 4 eV respectively. Which metal has lower threshold wavelength for photoelectric effect?

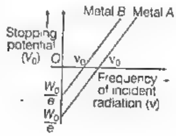
24. For a given photosensitive material and with a source of constant frequency of incident radiation, how does the photocurrent vary with the intensity of incident light? **All India 2011**

25. The given graph shows the variation of photoelectric current I versus applied voltage V for two different photosensitive materials and for two different intensities of the incident radiations. Identify the pairs of curves that correspond to different materials but same intensity of incident radiation. **Delhi 2013**



[redacted]

26. The graph shows the variation of stopping potential with frequency of incident radiation for two photosensitive metals A and B.



Which one of the two has higher value of work function? Justify your answer. **All India 2014**

27. Ultraviolet radiations of different frequencies v_1 and v_2 are incident on two photosensitive materials having work functions W_{01} and W_{02} ($W_{01} > W_{02}$) respectively. The kinetic energy of the emitted electrons is same in both the cases. Which one of the two radiations will be of higher frequency?

28. If the frequency of incident radiation is equal to the threshold frequency, what will be the value of stopping potential?

29. All the photoelectrons are not emitted with same energy. The energies of photoelectrons are distributed over a certain range. Why?

30. The photoelectric current at distances r_1 and r_2 of light source from photoelectric cell are I_1 and I_2, respectively. Find the value of $\frac{I_1}{I_2}$.

31. Draw graphs showing variation of photoelectric current with applied voltage for two incident radiations of equal frequency and different intensities. Mark the graph for the radiation of higher intensity. **CBSE 2018**

SHORT ANSWER Type Questions

32. There are materials which absorb photons of shorter wavelength and emit photons of longer wavelength. Can there be stable substances which absorb photons of larger wavelength and emit light of shorter wavelength. **NCERT Exemplar**

33. In the wave picture of light, intensity of light is determined by the square of the amplitude of the wave. What determines the intensity in the photon picture of light? **All India 2016**

34. Why does the existence of a cut-off frequency in the photoelectric effect favor a particle theory of light rather than a wave theory? Explain.

35. Two monochromatic beams A and B of equal intensity I, hit a screen. The number of photons hitting the screen by beam A is twice that by beam B. Then, what inference can you make about their frequencies? **NCERT Exemplar**

36. Two monochromatic radiations, blue and violet, of the same intensity are incident on a photosensitive surface and cause photoelectric emission. Would
    (i) the number of electrons emitted per second and
    (ii) the maximum kinetic energy of the electrons be equal in the two cases? Justify your answer. **Delhi 2010**

37. (i) In the explanation of photoelectric effect, we assume one photon of frequency v collides with an electron and transfers its energy. This leads to the equation for the maximum energy E_{max} of the emitted electron as, $E_{max} = hv - \phi_0$, where ϕ_0 is the work function of the metal. If an electron absorbs 2 photons (each of frequency v), what will be the maximum energy for the emitted electron?
    (ii) Why is this fact (two photon absorption) not taken into consideration in our discussion of the stopping potential? **NCERT Exemplar**

38. Draw a graph to show the variation of stopping potential with frequency of radiation incident on a metal plate. How can the value of Planck's constant be determined from this graph?

39. Consider figure for photoemission. How would you reconcile with momentum conservation? Note light (photons) have momentum in a different direction than the emitted electrons. **NCERT Exemplar**

40. If light of wavelength 412.5 nm is incident on each of the metals given below, which ones will show photoelectric emission and why? **CBSE 2018**

Metal	Work Function (eV)
Na	1.92
K	2.15
Ca	3.20
Mo	4.17

[redacted]

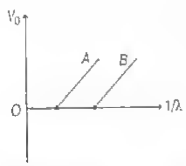
41. Why is wave theory of electromagnetic radiation not able to explain photoelectric effect? How does photon picture resolve this problem? **CBSE 2019**

LONG ANSWER Type I Questions

42. Light of same wavelength is incident on three photo-sensitive surfaces A, B and C. The following observations are recorded.
    (a) From surface A, photoelectrons are not emitted.
    (b) From surface B, photoelectrons are just emitted.
    (c) From surface C, photoelectrons with some kinetic energy are emitted.
    Compare the threshold frequencies of the three surfaces and justify your answer. **CBSE 2020**

43. (i) Describe briefly three experimentally observed features in the phenomenon of photoelectric effect.
    (ii) Discuss briefly how wave theory of light cannot explain these features. **Delhi 2015, 16**

44. Figure shows the stopping potential (V_0) for the photoelectron *versus* $\left(\frac{1}{\lambda}\right)$ graph, for two metals A and B, λ being the wavelength of incident light. **CBSE 2020**



    (a) How is the value of Planck's constant determined from the graph?
    (b) If the distance between the light source and the surface of metal A is increased, how will the stopping potential for the electrons emitted from it be effected? Justify your answer.

45. Predict and Explain
    Light of a particular wavelength does not eject electrons from the surface of a given metal.
    (i) Should the wavelength of the light be increased or decreased in order to make ejection of electrons possible?
    (ii) Choose the best explanation from among the following:
    (a) The energy of a photon is proportional to its frequency, i.e. inversely proportional to its wavelength. To increase the energy of the photons, so they can eject electrons, one must decrease their wavelength.
    (b) The photons have too little energy to eject electrons. To increase their energy, their wavelength should be increased.

46. Explain how does (i) photoelectric current and (ii) kinetic energy of the photoelectrons emitted in a photocell vary, if the frequency of incident radiation is doubled but keeping the intensity same? Show the graphical variation in the above two cases. **CBSE SQP Term-II**

47. Sketch the graphs showing variation of stopping potential with frequencies of incident radiations for two photosensitive materials A and B having threshold frequencies $\nu_A > \nu_B$.
    (i) In which case is the stopping potential more and why?
    (ii) Does the slope of the graph depend on the nature of the material used? Explain. **All India 2016**

48. Define the terms cut-off voltage and threshold frequency in relation to the phenomenon of photoelectric effect. Using Einstein's photoelectric equation, show how the cut-off voltage and threshold frequency for a given photosensitive material can be determined with the help of a suitable plot/graph. **All India 2012**

49. Define the term "cut-off frequency" in photoelectric emission. The threshold frequency of a metal is f. When the light of frequency $2f$ is incident on the metal plate, the maximum velocity of photo-electron is v_1. When the frequency of the incident radiation is increased to $5f$, the maximum velocity of photoelectrons is v_2. Find the ratio $v_1 : v_2$. **Foreign 2016**

50. Plot a graph showing the variation of stopping potential with frequency of incident radiation for two different photosensitive materials having work functions W_{01} and W_{02} ($W_{01} > W_{02}$). On what factors does the
    (i) slope and
    (ii) intercept of the lines depend?

[redacted]

51. (i) State two important features of Einstein's photoelectric equation.
(ii) Radiation of frequency 10^{15} Hz is incident on two photosensitive surfaces P and Q. There is no photoemission from surface P. Photoemission occurs from surface Q but photoelectrons have zero kinetic energy. Explain these observations and find the value of work function for surface Q. **Delhi 2017**

52. (i) Write the important properties of photons which are used to establish Einstein's photoelectric equation.
(ii) Use this equation to explain the concept of
(a) threshold frequency and
(b) stopping potential. **Delhi 2015**

53. Write Einstein's photoelectric equation and mention which important features in photoelectric effect can be explained with the help of this equation.The maximum kinetic energy of the photoelectrons gets doubled when the wavelength of light incident on the surface changes from λ_1 to λ_2. Derive the expressions for the threshold wavelength λ_0 and work function for the metal surface. **All India 2015**

54. In case of photoelectric effect experiment, explain the following facts, giving reasons.
(a) The wave theory of light could not explain the existence of the threshold frequency.
(b) The photoelectric current increases with increase of intensity of incident light. **CBSE 2020**

55. If the frequency of light incident on the cathode of a photo-cell is increased, how will the following be affected? Justify your answer.
(a) Energy of the photoelectrons.
(b) Photocurrent. **CBSE 2020**

56. State the main implications of observations obtained from various photoelectric experiments. Can these implications be explained by wave nature of light? Justify your answer. **CBSE SQP**

NUMERICAL PROBLEMS

57. The photoelectric cut-off voltage in a certain experiment is 1.5 V. What is the maximum kinetic energy of photoelectrons emitted? **NCERT**

58. The work function for a certain metal is 4.2 eV. Will this metal give photoelectric emission for incident radiation of wavelength 330 nm? **NCERT**

59. The maximum kinetic energy of photoelectrons emitted from a surface, when photons of energy 6 eV fall on it is 4 eV. What is the stopping potential (in volt) for the fastest photoelectrons.

60. In an experiment on photoelectric effect, the slope of the cut-off voltage *versus* frequency of incident light is found to be 4.12×10^{-15} V-s. Calculate the value of Planck's constant. **NCERT**

61. Find the
(i) maximum frequency and
(ii) minimum wavelength of X-rays produced by 30 kV electrons. **NCERT**

62. (i) An X-ray tube produces a continuous spectrum of radiation with its short wavelength of 0.45 Å. What is the maximum energy of a photon in the radiation?
(ii) From your answer to (i), guess what order of accelerating voltage (for electrons) is required in such a tube? **NCERT**

63. The threshold frequency for a certain metal is 3.3×10^{14} Hz. If light of frequency 8.2×10^{14} Hz is incident on the metal, predict the cut-off voltage for the photoelectric emission. **NCERT**

64. If radiation of wavelength 5000 Å is incident on a surface of work function 1.2 eV, find the value of stopping potential. Given, $h = 6.62 \times 10^{-34}$ J-s.

65. Light of frequency 7.21×10^{14} Hz is incident on a metal surface. Electrons with a maximum speed of 6×10^5 m/s are ejected from the surface. What is the threshold frequency for photoemission of electrons? **NCERT**

66. Consider a metal exposed to light of wavelength 600 nm. The maximum energy of the electron doubles when light of wavelength 400 nm is used. Find the work function in eV. **NCERT Exemplar**

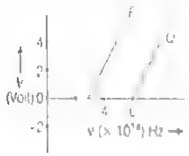
67. In an accelerator experiment on high energy collisions of electrons with positrons, a certain event is interpreted as annihilation of an electron-positron pair of total energy 10.2 BeV into two γ-rays of equal energy. What is the wavelength associated with each γ-ray? (1 BeV = 10^9 eV)

[redacted]

68. Aluminium and calcium have photoelectric work functions of $\phi_{Al} = 4.28$ eV and $\phi_{Ca} = 2.87$ eV, respectively.
   (i) Which metal requires higher frequency light to produce photoelectrons? Explain.
   (ii) Find out the minimum frequency that will produce photoelectrons from each surface.

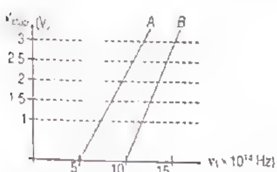
69. The work functions for the following metals are given, Na = 2.75 eV, K = 2.30 eV, Mo = 4.17 eV, Ni = 5.15 eV. Which of these metals will not give photoelectric emission for a radiation of wavelength 3300 Å from a He-Cd laser placed 1 m away from the photocell? What happens if the laser is brought nearer and placed 50 cm away? **NCERT**

70. In the study of a photoelectric effect, the graph between the stopping potential V and frequency ν of the incident radiation on two different metals P and Q is shown below



   (i) Which one of the two metals has higher threshold frequency?
   (ii) Determine the work function of the metal which has greater value.
   (iii) Find the maximum kinetic energy of electron emitted by light of frequency 8×10^{14} Hz for this metal. **Delhi 2017**

71. A student performs an experiment on photoelectric effect, using two materials A and B. A plot of V_{stop} versus ν is given in the figure.
   (i) Which material A or B has a higher work function?



   (ii) Given the electric charge of an electron $= 1.6 \times 10^{-19}$ C, find the value of h obtained from the experiment for both A and B.

Comment on whether it is consistent with the Einstein's theory. **NCERT Exemplar**

72. The work function of caesium metal is 2.14 eV. When light of frequency 6×10^{14} Hz is incident on the metal surface, photoemission of electrons occurs. What is the
   (i) maximum kinetic energy of the emitted electrons,
   (ii) stopping potential and
   (iii) maximum speed of the emitted photoelectrons? **NCERT**

73. When light with a frequency 547.5 THz illuminates a metallic surface, the most energetic photoelectrons have 1.260×10^{-19} J of kinetic energy. When light with a frequency of 738.8 THz is used instead, the most energetic photoelectrons have 2.480×10^{-19} J of kinetic energy. Using these experimental results, determine the approximate value of Planck's constant.

74. Monochromatic radiation of wavelength 640.2 nm (1 nm = 10^{-9} m) from a neon lamp irradiates a photosensitive material made of calcium or tungsten. The stopping voltage is measured to be 0.54 V. The source is replaced by an iron source and its 427.2 nm line irradiates the same photocell. Predict the new stopping voltage. **NCERT**

75. A mercury lamp is a convenient source for studying frequency dependence of photoelectric emission, since it gives a number of spectral lines ranging from the UV to the red end of the visible spectrum. In our experiment with rubidium photocell, the following lines from a mercury source were used

$\lambda_1 = 3650$ Å, $\lambda_2 = 4047$ Å, $\lambda_3 = 4358$ Å,
$\lambda_4 = 5461$ Å, $\lambda_5 = 6907$ Å

The stopping voltages respectively were measured to be

$V_{01} = 1.28$ V, $V_{02} = 0.95$ V,
$V_{03} = 0.74$ V, $V_{04} = 0.16$ V, $V_{05} = 0$

Determine the value of Planck's constant h, the threshold frequency and work function for the material. **NCERT**

76. What is the energy associated in joule with a photon of wavelength 4000 Å?

[redacted]

77. What is the energy of a photon in eV corresponding to the visible light of maximum wavelength?

78. The energy flux of sunlight reaching the surface of the earth is $1.388 \times 10^3\ W/m^2$. How many photons (nearly) per square metre are incident on the earth per second? Assume that the photons in the sunlight have an average wavelength of 550 nm. **NCERT**

79. There are two sources of light, each emitting with a power of 100 W. One emits X-rays of wavelength 1 nm and the other visible light at 500 nm. Find the ratio of number of photons of X-rays to the photons of visible light of the given wavelength. **NCERT Exemplar**

80. A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light which is incident on it.

The wavelength of the sodium light is 589 nm.

(i) What is the energy per photon associated with the sodium light?

(ii) At what rate are the photons delivered to the sphere? **NCERT**

81. How many photons per second does a 100 W bulb emit if its efficiency is 10% and wavelength of light emitted is 500 nm?

82. Light of intensity $10^{-5}\ Wm^{-2}$ falls on a sodium photocell of surface area $2\ cm^2$. Assuming that, the top 5 layers of sodium absorb the incident energy, estimate the time required for photoelectric emission in the wave picture of radiation. The work function of the metal is given to be about 2 eV. What is the implication of your answer?

Effective atomic area $= 10^{-20} m^2$. **NCERT**

HINTS AND SOLUTIONS

1. (d) Hallwachs and Lenard also observed that when ultraviolet light fell on the emitter plate, no electrons were emitted at all when the frequency of the incident light was smaller than a certain minimum value, called the threshold frequency. This minimum frequency depends on the nature of the material of the emitter plate.

2. (c) When the wavelength of incident light is $\lambda \le \lambda_0$, then the electrons will come out of the metal surface.

3. (d) Photocurrent varies linearly with intensity. The photocurrent is directly proportional to the number of photoelectrons emitted per second. This implies that, it is a straight line passing through origin.

4. (a) Given, work function = 2eV

Energy of incident photon = 3.4 eV

From Einstein's equation of photoelectric effect,

$$h\nu = h\nu_0 + k$$

$$3.4\ \text{eV} = 2\ \text{eV} + k$$

$$k = 3.4\ \text{eV} - 2.0\ \text{eV} = 1.4\ \text{eV}$$

5. (b) 3 : 4;

From Einstein's photoelectric equation, maximum kinetic energy of emitted electrons,

$$K_{max} = h(\nu - \nu_0)$$

where, h is Planck's constant,

ν is frequency of incident radiation

and ν_0 is threshold frequency of metal surface.

For metal A,

$$K_{(max)A} = h\left(\nu - \frac{\nu}{2}\right)$$

or $$K_{(max)A} = \frac{h\nu}{2} \quad \text{... (i)}$$

Similarly, for metal B,

$$K_{(max)B} = h\left(\nu - \frac{\nu}{3}\right)$$

or $$K_{(max)B} = \frac{2h\nu}{3} \quad \text{... (ii)}$$

So, from Eqs. (i) and (ii), the ratio of the maximum kinetic energy of electrons emitted from A to that from B is given as,

$$\frac{K_{(max)A}}{K_{(max)B}} = \frac{\frac{h\nu}{2}}{\frac{2h\nu}{3}}$$

$$= \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$

or $K_{(max)A} : K_{(max)B} = 3 : 4$

6. (d) When a beam of electrons of energy E_0 is incident on a metal surface kept in an evacuated chamber electrons can be emitted with maximum energy E_0 (due to elastic collision) and with any energy less than E_0, when part of incident energy of electron is used in liberating the electrons from the surface of metal.

7. (d) We know that, $E = h\nu$ and

$$E = mc^2 \quad \text{(Einstein mass energy equation)}$$

$\therefore\ mc^2 = h\nu \Rightarrow m = h\nu/c^2$

Moving mass, $m = \dfrac{(hc/\lambda)}{c^2} = \dfrac{h}{c\lambda}$

[redacted]

8. (b) Given in the question,
Energy of a photon, $E = 1\text{ MeV} \Rightarrow E = 10^6\text{ eV}$

Now, $hc = 1240\text{ eVnm}$

Now, $E = \frac{hc}{\lambda}$

$$\Rightarrow \lambda = \frac{hc}{E} = \frac{1240\text{ eVnm}}{10^6\text{ eV}} = 1.24 \times 10^{-3}\text{ nm}$$

9. (c) The photoelectric current in a photocell is related to the distance of source of light as

$$I \propto \frac{1}{r^2}$$

$$\therefore \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

Here, $r_1 = d, r_2 = \frac{d}{2}$ and $I_1 = I$

$$\therefore \frac{I}{I_2} = \frac{\left(\frac{d}{2}\right)^2}{(d)^2} = \frac{1}{4} \quad \text{or} \quad I_2 = 4I$$

10. (c) The photo electric current in a photocell is related to the distance of source of light as

$$I \propto \frac{1}{r^2}$$

$$\therefore \frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

Here $r_1 = d, \ r_2 = \frac{d}{3}, I_1 = I$

$$\therefore \frac{I}{I_2} = \frac{\left(\frac{d}{3}\right)^2}{(d)^2} = \frac{1}{9}$$

$$\Rightarrow I_2 = 9I$$

11. (a) Cathode ray particles when strike the electrons of glass atom, the electrons of glass atom are excited and move to higher energy levels. On de-excitation, they fall to their ground state and release energy. As energy levels are characteristics of glass, glow depends on glass.

12. (c) Sensitivity of a photoelectric materials greatly depends on its surface characteristics. When emitter plate is coated with a materials of low work function, photoemission occurs even at low frequency.

13. (a) We know that, intensity is energy per unit area per unit time

14. (a) The number of photoelectrons emitted per second is directly proportional to the intensity of incident radiation and kinetic energy of photoelectrons depends on frequency of incident radiation.

15. (b) Work function is the minimum energy required to eject the photoelectron from photosensitive metal. Hence for metal to be photosensitive, the work-function should be small.
Work function $= h\nu_0$
where, ν_0 is the threshold frequency.

16. (c) In a photon-particle collision such as photon-electron collision, the total energy and total momentum are conserved. However, the number of photons may not be conserved in a collision. The photon may be absorbed or a new photon may be created.

17. (c) Photocell is a technical application of the photoelectric effect. It is a device which converts light energy into electric energy. It is also called an electric eye. Photocell are used in the reproduction of sound in motion picture and in the television camera.

18. (i) (c) Energy of the incident photons,

$$E_i = 10.6\text{ eV} = 10.6 \times 1.6 \times 10^{-19}$$

$$E_i = 16.96 \times 10^{-19}\text{ J}$$

(ii) (b) Energy incident per unit area per unit time (intensity) = 2 J

$\therefore$ Number of photons incident on unit area in unit time

$$= \frac{2}{16.96 \times 10^{-19}} = 1.18 \times 10^{18}$$

Therefore, number of photons incident on given area $(1.0 \times 10^{-4}\text{ m}^2)$

$$= (1.18 \times 10^{18})(1.0 \times 10^{-4}) = 1.18 \times 10^{14}$$

(iii) (b) As, only 0.53% of incident photons emit photoelectrons

$\therefore$ Number of photoelectrons emitted per second (n),

$$n = \left(\frac{0.53}{100}\right)(1.18 \times 10^{14}) = 6.25 \times 10^{11}$$

(iv) (a) $K_{max} = E_i$ − work-function $= (10.6 - 5.6) = 5.0\text{ eV}$

(v) (d) $K_{min} = 0$, kinetic energy of photoelectrons varies from 0 $(KE)_{max}$. Hence, minimum possible KE of any photoelectron is zero.

19. **Intensity** It is the number of photons passing through an area in a given interval of time. Its SI unit is watt/steradian.

20. In photoelectric effect, we can observe that most electrons get scattered into the metal by absorbing a photon.
Thus, all the electrons that absorb a photon does not come out as photoelectron. Only a few comes out of metal whose energy becomes greater than the work function of metal.

21. The value of threshold frequency is more than 10^{14} Hz.

22. Since, work function is given as,

$$W_0 = h\nu_0$$

$$\Rightarrow W_0 \propto \nu_0$$

As work function of metal X is higher than metal Y, so metal X has higher threshold frequency than metal Y.

[redacted]

23. We know that,

$$\lambda_0 = \frac{hc}{\phi_0} \qquad \left[\because \phi_0 = \frac{hc}{\lambda_0}\right]$$

$$\therefore \qquad \lambda_0 \propto \frac{1}{\phi_0}$$

Hence, metal B has lower threshold wavelength.

24. The photocurrent increases linearly with the intensity of incident radiation.

25. Curves 1 and 2 correspond to similar materials, while curves 3 and 4 represent different materials, since the value of stopping potential for the pair of curves (1 and 2) and (3 and 4) are the same. For given frequency of the incident radiation, the stopping potential is independent of its intensity. So, the pairs of curves (1 and 3) and (2 and 4) correspond to different materials but same intensity of incident radiation.

26. Metal A has higher value of work function because the slopes of both materials are constant and the intercept of the line depends on work function.

27. As, $K_{max} = h\nu - W_0$

$$\therefore \qquad \nu = \frac{K_{max} + W_0}{h}$$

$\because W_{01} > W_{02}$ and K_{max} is same, hence $\nu_1 > \nu_2$.

28. We know that,

$$K_{max} = eV_0 = h(\nu - \nu_0)$$

Here, $\qquad \nu = \nu_0$

$\therefore \qquad eV_0 = h(\nu_0 - \nu_0)$

$\Rightarrow \qquad eV_0 = 0$

$\therefore \qquad V_0 = 0$

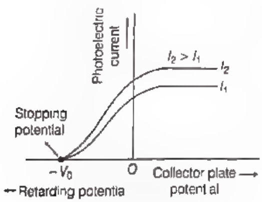
Hence, stopping potential becomes zero when frequency of incident radiation is equal to threshold frequency.

29. All the electrons in the photo-sensitive material do not belong to the highest level of energy. The energies of the free electrons in the material belongs to many different closely spaced levels. So, the energies of the photoelectrons emitted from the material are distributed over a certain range.

30. Since, $\qquad I \propto \dfrac{1}{r^2}$

So, $\qquad \dfrac{I_1}{I_2} = \left(\dfrac{r_2}{r_1}\right)^2$

31.



Variation of photoelectric current *versus* potential for different intensities.

32. According to first statement, when the materials which absorb photons of shorter wavelength has high energy of the incident photon on the material and low energy of emitted photon of longer wavelength.

But in second statement, the energy of the incident photon is low for the substances which has to absorb photons of larger wavelength and energy of emitted photon is high to emit light of shorter wavelength.

This means in this statement material has to supply the energy for the emission of photons. But this is not possible for a stable substances.

33. For a given frequency, intensity of light in the photon picture is determined by

$$I = \frac{\text{energy of photons}}{\text{area} \times \text{time}} = \frac{n \times h\nu}{A \times t}$$

where, n is the number of photons incident normally on cross-sectional area A in time t.

34. Refer to text on page 440.

35. The number of photons of beam $A = n_A$

The number of photons of beam $B = n_B$

According to the question, $n_A = 2n_B$

Let ν_A be the frequency of beam A and ν_B be the frequency of beam B.

$\therefore$ Intensity $\propto$ Energy of photons

$\Rightarrow \quad I \propto (h\nu) \times$ Number of photons

$$\therefore \qquad \frac{I_A}{I_B} = \frac{n_A \nu_A}{n_B \nu_B}$$

According to the question, $I_A = I_B$

$$\therefore \qquad n_A \nu_A = n_B \nu_B \quad \text{or} \quad \frac{\nu_A}{\nu_B} = \frac{n_B}{n_A} = \frac{1}{2}$$

So, $\qquad \nu_B = 2\nu_A$

36. The intensities for both the monochromatic radiations are same but their frequencies are different. It represents

(i) the number of electrons ejected in two cases are same because it depends on the number of incident photons.

(ii) As, $KE_{max} = h\nu - \phi_0 = hc/\lambda - \phi_0$

[Einstein's photoelectric equation]

$\therefore$ The KE_{max} of violet radiation will be more.

37. (i) Here, it is given that, an electron absorbs 2 photons each of frequency ν, then $\nu' = 2\nu$

where, ν' is the frequency of emitted electron.

Given, $\qquad E_{max} = h\nu - \phi_0$

Now, maximum energy for emitted electrons,

$$E'_{max} = h(2\nu) - \phi_0$$
$$= 2h\nu - \phi_0$$

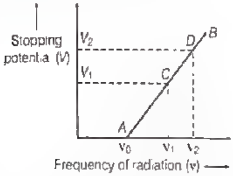
(ii) The probability of absorbing 2 photons by the same electron is very low

Hence, such emission will be negligible.

[redacted]

38. The variation of stopping potential with the frequency of radiation incident on a metal plate is a straight line AB as shown in the figure.

Take two points C and D on the graph.



The corresponding frequency of radiation is ν_1, ν_2 and stopping potential is V_1, V_2.

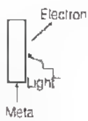
Then $\quad eV_1 = h\nu_1 - \phi_0$ and $eV_2 = h\nu_2 - \phi_0$

$\therefore \quad e(V_2 - V_1) = h(\nu_2 - \nu_1)$

or $$h = \frac{e(V_2 - V_1)}{\nu_2 - \nu_1}$$

Thus, Planck's constant can be determined.

39. During photoelectric emission, the momentum of incident photon is transferred to the metal. At microscopic level, atoms of a metal absorb the photon and its momentum is transferred mainly to the nucleus and electrons.



The excited electron is emitted. Therefore, the conservation of momentum is to be considered as the momentum of incident photon transferred to the nucleus and electrons.

40. Given, $\lambda = 412.5\text{ nm} = 412.5 \times 10^{-9}$ m

$$\therefore \quad E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{412.5 \times 10^{-9} \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= 3.01 \text{ eV}$$

From the given question, work function (ϕ) of the following metals are given as

Na → 1.92, K → 2.15

Ca → 3.20, Mo → 4.17

As the given energy is greater than the work function of Na and K only, hence these metals shows photoelectric emission.

41. Refer to text on page 438 (Photoelectric Effect and Wave Theory of Light)

42. From Einstein's photoelectric equation,

$$K_{max} = \frac{1}{2} m v_{max}^2 = h(\nu - \nu_0)$$

where, h = Planck's constant,

ν = frequency of incident light

and ν_0 = threshold frequency of the photosensitive surface.

So, for photoemission to takes place, $\nu > \nu_0$.

As the wavelength of light incident is same for all the three surfaces, so

(a) threshold frequency of surface A is higher than the frequency of incident light, as no emission takes place.

(b) threshold frequency of surface B is equal to the frequency of incident light, as photoelectrons are just emitted

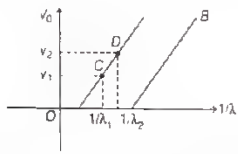
(c) threshold frequency of surface C is lower than the frequency of incident light, as the emitted photoelectrons have some kinetic energy.

$$\therefore \quad (\nu_0)_A > (\nu_0)_B > (\nu_0)_C$$

43. (i) Refer to the text on pages 435 to 437.

(ii) Refer to the text on page 438.

44. (a) The variation of stopping potential (V_0) for the photoelectron *versus* $\left(\frac{1}{\lambda}\right)$ graph is as shown below



Take any two points C and D on the graph as shown above.

According to Einstein's photoelectric equation, we can write, $$eV_1 = \frac{hc}{\lambda_1} - \phi_0 \quad \text{...(i)}$$

where, ϕ_0 is the work function of metal A.

and $$eV_2 = \frac{hc}{\lambda_2} - \phi_0 \quad \text{...(ii)}$$

Subtracting Eq. (i) from Eq. (ii), we get

$$\Rightarrow \quad e(V_2 - V_1) = hc\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

or $$h = \frac{e(V_2 - V_1)}{c\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)} = \frac{e(V_2 - V_1)\lambda_1\lambda_2}{c(\lambda_1 - \lambda_2)}$$

Thus, Planck's constant can be determined from graph.

Note Since, h is a constant, so it will be same for both metals A and B.

(b) Stopping potential (V_0) for the electrons emitted will not be affected by the increase in distance between light source and the metal surface A. This is because V_0 is independent of the intensity of the incident light but depends only upon the frequency (or wavelength) of incident light. So, increase in the given distance affects only the intensity of the light but not the frequency. Thus, V_0 remains same.

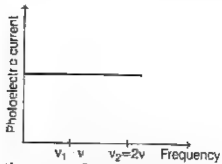
[redacted]

45. (i) Since, we know that, to eject an electron, a photon must have energy at least as great as work function (W_0) and thus the minimum or cut off frequency to eject an electron is $f_0 = \frac{W_0}{h}$.

If the incident light has the frequency below this cut off frequency, electrons are not ejected from the metal surface, so we have to increase the value of frequency, i.e. decrease the value of wavelength $\left(\text{as } \nu = \frac{c}{\lambda}\right)$.

(ii) (a) is the best explanation.

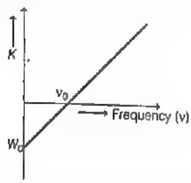
46. (i) Since, photoelectric current depends intensity of incident radiation and does not depend on the frequency of incident radiation. Therefore, when frequency of incident radiation is increased to double, then photoelectric current remains same. This is shown in the following graph



(ii) Kinetic energy of emitted photoelectrons,

$$K = \frac{1}{2}mv_{max}^2 = h\nu \Rightarrow K \propto \nu$$

Hence, on increasing the frequency of incident radiation to double, kinetic energy of emitted photoelectrons also will increase to double. This is shown below



where, ν_0 = threshold wavelength

47. For the graph, refer to text on page 437.

(i) From the graph for the same value of ν, stopping potential is more for material B

$$\text{As, } V = \frac{h}{e}(\nu - \nu_0)$$

$\therefore$ V is higher for lower value of ν_0. Here $\nu_B < \nu_A$, so $V_B > V_A$.

(ii) Slope of the graph is given by $\frac{h}{e}$ which is constant for all the materials. Hence, slope of the graph does not depend on the nature of the material used

48. **Cut-off voltage and threshold frequency**

Refer to text on pages 436 and 437.

Graph between stopping potential (V_0) and frequency (ν).

Refer to text on page 437.

49. For cut-off frequency, refer to text on page 437.

Given that threshold frequency of metal is f and frequency of light is $2f$. Using Einstein's equation for photoelectric effect, we can write

$$h(2f - f) = \frac{1}{2}mv_1^2 \quad \text{...(i)}$$

Similarly, for light having frequency $5f$, we have

$$h(5f - f) = \frac{1}{2}mv_2^2 \quad \text{...(ii)}$$

Using Eqs. (i) and (ii), we find

$$\frac{f}{4f} = \frac{v_1^2}{v_2^2}$$

$$\Rightarrow \quad \frac{v_1}{v_2} = \sqrt{\frac{1}{4}} \Rightarrow \frac{v_1}{v_2} = \frac{1}{2}$$

50. Refer to the text and graph on page 437.

51. (i) Refer to text on page 438.

(ii) Energy of incident photon is less than work function of P but just equal to that of Q.

(i) For Q,

$$\text{Work function, } \phi_0 = \frac{h\nu}{e}(\text{eV})$$

$$= \frac{6.6 \times 10^{-34} \times 10^{15}}{1.6 \times 10^{-19}}$$

$$= 4.1 \text{ eV}$$

52. (i) Refer to text on page 474.

(ii) Since, Einstein's photoelectric equation is given by

$$\text{KE}_{max} = \frac{1}{2}mv_{max}^2 = h\nu - h\nu_0 = eV_0$$

(a) For a given material, there exist a certain minimum frequency of the incident radiation, below which no emission of photoelectron takes place. This frequency is called threshold frequency (ν_0). Above threshold frequency, the maximum kinetic energy of the emitted photoelectron or equivalent stopping potential is independent of the intensity of the incident light but depends only upon the frequency of the incident light.

(b) If the collecting plate in the photoelectric apparatus is made at high negative potential, then most of the high energetic electrons get repelled back along the same path and the photoelectric current in the circuit becomes zero. So, for a particular frequency of incident radiation, the minimum negative potential for which the electric current becomes zero is called cut-off or stopping potential (V_0).

[redacted]

53. Einstein's photoelectric equations and its features

Refer to theory on pages 438 and 439

According to the photoelectric equation,

$$K_{max} = \frac{1}{2} mv_{max}^2 = h\nu - \phi_0$$

$$K_{max} = \frac{hc}{\lambda_1} - \phi_0 \quad ...(i)$$

Let the maximum kinetic energy for the incident radiation (of wavelength λ_2) be K'_{max}.

$$\Rightarrow \quad K'_{max} = \frac{hc}{\lambda_2} - \phi_0 \quad ...(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{hc}{\lambda_2} - \phi_0 = 2\left(\frac{hc}{\lambda_1} - \phi_0\right) \quad [\because K'_{max} = 2K_{max}]$$

$$\Rightarrow \quad \phi_0 = hc\left(\frac{2}{\lambda} - \frac{1}{\lambda_2}\right)$$

$$\Rightarrow \quad h\nu_0 = hc\left(\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$\frac{c}{\lambda} = c\left(\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda_0} = \left(\frac{2}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$\Rightarrow \quad \lambda_0 = \left(\frac{\lambda_1\lambda_2}{2\lambda_2 - \lambda_1}\right)$$

54. (a) Refer to text on page 438.

Refer to text on page 436.

55. (a) The energy of photoelectrons in a photocell is given by,

$$E = \frac{hc}{\lambda} - h\nu \Rightarrow E \propto \nu$$

So, if the frequency of light incident on the cathode is increased, the energy of photoelectrons increases linearly.

(b) As, photoelectric current/photocurrent of the photocell is independent of frequency of the incident light, till intensity remains constant. So, when the frequency of light incident on the cathode of photo cell is increased keeping other factors same, the photoelectric current remains the same.

56. Main implications of observations obtianed from various photoelectric experiments given as

(i) For a given material and a given frequency of incident radiation, the photoelectric current or number of photoelectrons ejected per second is directly proportional to the intensity of the incident light.

(ii) For a given material, there exists a certain minimum frequency of the incident radiation below which no emission of photoelectrons takes place. This frequency is called **threshold frequency.**

Above the threshold frequency, the maximum kinetic energy of the emitted photoelectrons or equivalent stopping potential is independent of the intensity of the incident light but depends upon only the frequency (or wavelength) of the incident light.

Photoelectric Effect and Wave Theory of Light

Huygens' wave theory of light could not explain the photoelectric emission due to the following main reasons

(i) According to the wave nature of light, the free electrons at the surface of the metal absorb the radiant energy continuously.

The greater the intensity of radiation, the greater should be the energy absorbed by each electron. The maximum kinetic energy of the photoelectrons on the surface is then expected to increase with increase in intensity.

But according to experimental facts, the maximum kinetic energy of ejected photoelectrons is independent of intensity of incident radiation.

(ii) According to wave theory of light, no matter what the frequency of radiation is, a sufficiently intense beam of radiation should be able to impart enough energy to the electrons, so that they exceed the minimum energy needed to escape from metal surface.

A threshold frequency, therefore should not exist which contradicts the experimental fact that, no photoelectric emission takes place below that threshold frequency, no matter whatsoever may be its intensity.

57. Given, cut-off voltage,

$$V_0 = 1.5\,V$$

Maximum kinetic energy is given by,

$$KE_{max} = eV_0 = 1.5\text{ eV}$$
$$= 1.5 \times 1.6 \times 10^{-19}$$
$$= 2.4 \times 10^{-19}\text{ J}$$

58. Given, $\phi_0 = 4.2$ eV

$$= 4.2 \times 1.6 \times 10^{-19}\text{ J}$$
$$= 6.72 \times 10^{-19}\text{ J}$$

and $\quad \lambda = 330\text{nm} = 330 \times 10^{-9}\text{m}$

Energy of incident photon, $E = \dfrac{hc}{\lambda}$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{330 \times 10^{-9}}$$
$$= 6.027 \times 10^{-19}\text{ J}$$

As energy of incident photon $E < \phi_0$, hence no photoelectric emission will take place.

59. We know that, $\quad h\nu = h\nu_0 + eV_0$

where, $\quad eV_0 = \frac{1}{2}mv^2_{max}$

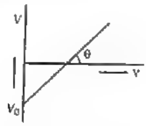
$$= 4\text{eV or } eV_0 = 4\text{eV}$$

$\therefore \quad V_0 = 4\text{V}$

[redacted]

60. Given, slope of graph,

$$\tan\theta = 4.12\times10^{-15}\ \text{V-s}$$



Charge on electron, $e = 1.6\times10^{-19}$ C

Slope of graph of cut off voltage *versus* frequency is

$\tan\theta = \dfrac{V}{\nu}$

We know that, $h\nu = eV$ or $\dfrac{V}{\nu} = \dfrac{h}{e}$

$\therefore \qquad \dfrac{h}{e} = 4.12\times10^{-15}$

$\Rightarrow \qquad h = 1.6\times10^{-19}\times4.12\times10^{-15}$

$= 6.592\times10^{-34}$ J-s

61. (i) Energy $= eV = h\nu$

or $\quad \nu = \dfrac{eV}{h} = \dfrac{1.6\times10^{-19}\times30\times10^{3}}{6.63\times10^{-34}} = 7.24\times10^{18}$ Hz

(ii) As, $\quad c = \nu\lambda$

$\therefore$ Wavelength, $\lambda = \dfrac{c}{\nu} = \dfrac{3\times10^{8}}{7.24\times10^{18}} = 0.0414$ nm

62. (i) As given in the question,

$\lambda_{min} = 0.45\ \text{Å} = 0.45\times10^{-10}$ m

The maximum energy of an X-ray photon is,

$$E_{max} = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

$$= \frac{6.63\times10^{-34}\times3\times10^{8}}{0.45\times10^{-16}}\ \text{J}$$

$$= \frac{6.63\times3\times10^{-16}}{0.45\times1.6\times10^{-19}}\ \text{eV}$$

$= 27.6\times10^{3}$ eV

$= 27.6$ keV

(ii) In X-ray tube, accelerating voltage provides the energy to the electrons which produces X-rays. For getting X-rays, photon of 27.6 keV is required such that the incident electrons must possess kinetic energy 27.61 keV.

Energy $= eV = E$, $eV = 27.6$ keV

$V = 27.6\ \text{kV} \approx 30\text{kV}$

So, the order of accelerating voltage is 30 kV.

63. Using the formula for kinetic energy,

Cut-off voltage, $V_0 = \dfrac{h(\nu-\nu_0)}{e}$

$$= \frac{6.63\times10^{-34}\,(8.2\times10^{14} - 3.3\times10^{14})}{1.6\times10^{-19}} = 2.03\ \text{V}$$

64. Given, $\lambda = 5000\text{Å} = 5\times10^{-7}$ m

and $\qquad \phi_0 = 1.2\text{eV} = 1.2\times1.6\times10^{-19}\text{J} = 1.92\times10^{-19}\text{J}$

We know that, $eV_0 = h\nu - h\nu_0 = \dfrac{hc}{\lambda} - \dfrac{hc}{\lambda_0}$

$= hc/\lambda - \phi_0 \qquad \left[\because \phi_0 = \dfrac{hc}{\lambda_0}\right]$

$\therefore \quad V_0 = \dfrac{hc}{e\lambda} - \dfrac{\phi_0}{e} = \dfrac{6.62\times10^{-34}\times3\times10^{8}}{1.6\times10^{-19}\times5\times10^{-7}} - \dfrac{1.92\times10^{-19}}{1.6\times10^{-19}}$

$= 2.4825 - 1.2 = 1.28$ V

65. Given,

$\nu = 7.21\times10^{14}$ Hz, $m = 9.1\times10^{-31}$ kg

$v_{max} = 6\times10^{5}$ m/s

Let ν_0 be the threshold frequency.

Use the formula for kinetic energy,

$\text{KE} = \dfrac{1}{2}mv_{max}^2 = h\nu - h\nu_0$

$= \dfrac{1}{2}\times9.1\times10^{-31}\times(6\times10^{5})^2 = 6.63\times10^{-34}(\nu-\nu_0)$

or $\nu - \nu_0 = \dfrac{36\times9.1\times10^{-21}}{2\times6.63\times10^{-34}} = 2.47\times10^{14}$ Hz

$\therefore \qquad \nu_0 = 4.74\times10^{14}$ Hz

66. Given, for the first condition, $\lambda = 600$nm

For the second condition, $\lambda' = 400$nm

$K'_{max} = 2K_{max}$

Here, $\qquad K'_{max} = \dfrac{hc}{\lambda} - \phi \Rightarrow 2K_{max} = \dfrac{hc}{\lambda'} - \phi_0$

$\Rightarrow \quad 2\left(\dfrac{1240}{600} - \phi\right) = \left(\dfrac{1240}{400} - \phi\right) \qquad [\because hc = 1240\ \text{eV-nm}]$

$\Rightarrow \qquad \phi = \dfrac{1240}{1200} = 1.03$ eV

67. Total energy of 2 γ-rays $= 10.2$ BeV $= 10.2\times10^{9}$ eV

$\therefore$ Energy of each γ-rays,

$E = \dfrac{1}{2}(10.2\times10^{9}\times1.6\times10^{-19})\ \text{J} = 8.16\times10^{-10}$ J

As energy of γ-rays, $E = h\nu = \dfrac{hc}{\lambda}$

$\therefore \lambda = \dfrac{hc}{E} = \dfrac{6.63\times10^{-34}\times3\times10^{8}}{8.16\times10^{-10}} = 2.43\times10^{-16}$ m

68. Given, $\qquad \phi_{Al} = 4.28\text{eV}, \phi_{cu} = 2.87\text{eV}$

Also, $\qquad \phi = h\nu_0$

$\therefore \qquad \phi_{Al} = 4.28\text{eV} = h\nu_{0Al}$

$\Rightarrow \nu_{0Al} = \dfrac{4.28\times1.6\times10^{-19}}{6.62\times10^{-34}} = 1.03\times10^{15}$Hz

Similarly, $\nu_{0cu} = \dfrac{2.87\times1.6\times10^{-19}}{6.62\times10^{-34}} = 6.93\times10^{14}$Hz

[redacted]

(i) Aluminium requires higher frequency of light to produce photoelectrons, i.e. 1.03×10^{15} Hz

(ii) Ca has minimum frequency, i.e. 6.93×10^{14} Hz that will produce photoelectrons from each surface.

69. Energy of the incident radiation of wavelength λ,

$$E=\frac{hc}{\lambda}=\frac{(6.63\times10^{-34})\times(3\times10^{8})}{3300\times10^{-10}\times1.6\times10^{-19}}$$

$= 3.76$ eV

This energy of the incident radiation is greater than the work function of Na and K but less than those of Mo and Ni. So, photoelectric emission will occur only in Na and K metals and not in Mo and Ni.

If the laser is brought closer, the intensity of incident radiation increases. This does not affect the result regarding Mo and Ni metals, while photoelectric current from Na and K will increase in proportion to intensity.

70. (i) Since, Q has greater negative intercept, it will have greater ϕ (work function) and hence higher threshold frequency.

(ii) To know work function of Q, we put $V=0$ in the following equation.

$$V=\frac{h\nu}{e}-\frac{\phi}{e}$$

$$\Rightarrow\quad 0=\frac{h\nu}{e}-\frac{\phi}{e}\Rightarrow\phi=h\nu$$

$$\therefore\quad \phi=6.6\times10^{-34}\times6\times10^{14}\text{ J}$$

$$=\frac{6.6\times6\times10^{-20}}{1.6\times10^{-19}}\text{ eV}=2.5\text{ eV}$$

(iii) From the equation, $\nu\lambda=c$

$$\Rightarrow\quad \lambda=\frac{c}{\nu}=\frac{3\times10^{8}}{8\times10^{14}}=\frac{30}{8}\times10^{-7}\text{m}$$

$$=\frac{30}{8}\times10^{3}\times10^{-10}\text{m}=\frac{30}{8}\times10^{3}\text{ Å}=3750\text{ Å}$$

$$\text{Energy}=\frac{12375}{\lambda(\text{Å})}=\frac{12375}{3750}\text{ eV}=3.3\text{ eV}$$

$\therefore$ Maximum KE of emitted electron $= 3.3-2.5$ eV $= 0.8$ eV

71. (i) Refer to Q. 56.

Thus, work function of B is higher than A.

(ii) For metal A, slope $=\frac{h}{e}=\frac{2}{(10-5)\times10^{14}}$

or $$h=\frac{2\times e}{5\times10^{14}}=\frac{2\times1.6\times10^{-19}}{5\times10^{14}}$$

$$=6.4\times10^{-34}\text{ J-s}$$

For metal B, slope $=\frac{h}{e}=\frac{2.5}{(15-10)\times10^{14}}$

or $$h=\frac{2.5\times e}{5\times10^{14}}=\frac{2.5\times1.6\times10^{-19}}{5\times10^{14}}=8\times10^{-34}\text{ J-s}$$

Since, the value of h from experiment for metals A and B is different. Hence, experiment is not consistent with theory.

72. Given, work function of caesium metal, $\phi_0=2.14$ eV

Frequency of light, $\nu=6\times10^{14}$ Hz

(i) Work function, $\phi_0=2.14$ eV, $\nu=6\times10^{14}$ Hz

$$\therefore K_{max}=h\nu-\phi_0$$

$$=6.63\times10^{-34}\times6\times10^{14}-2.14$$

$$=\frac{6.63\times6\times10^{-20}}{1.6\times10^{-19}}\text{ eV}-2.14$$

$$=2.48-2.14=0.34\text{ eV}$$

(ii) Let stopping potential be V_0.

We know that, $\quad KE_{max}=eV_0$

$\Rightarrow\quad 0.35\text{ eV}=eV_0$

$\therefore\quad V_0=0.35$ V

(iii) Maximum kinetic energy, $KE_{max}=\frac{1}{2}mv_{max}^2$

$$0.35\text{ eV}=\frac{1}{2}mv_{max}^2$$

(where, v_{max} is the maximum speed and m is the mass of electron)

$$\frac{0.35\times2\times1.6\times10^{-19}}{9.1\times10^{-31}}=v_{max}^2\qquad[\because e=1.6\times10^{-19}]$$

$$\Rightarrow\quad v_{max}^2=0.123\times10^{12}$$

$$\Rightarrow\quad v_{max}=350713.55\text{ m/s}$$

$$=350.7\text{ km/s}$$

73. Refer to Q. 59.

$h=6.377\times10^{-34}$ J-s

74. Here, for neon lamp, $\lambda=640.2$ nm $=640.2\times10^{-9}$ m

$$V_0=0.54\text{ V}$$

We know that, $eV_0=\frac{hc}{\lambda}-\phi_0$

$\therefore$ Work function, $\phi_0=\frac{hc}{\lambda}-eV_0$

$$=\frac{6.63\times10^{-34}\times3\times10^{8}}{640.2\times10^{-9}}-1.6\times10^{-19}\times0.54$$

$$=(3.1\times10^{-19}-0.864\times10^{-19})\text{ J}$$

$$=2.236\times10^{-19}\text{ J}=\frac{2.236\times10^{-19}}{1.6\times10^{-19}}\text{ eV}\simeq1.4\text{ eV}$$

For iron source, $\lambda=427.2$ nm $=427.2\times10^{-9}$ m

$$\therefore eV_0=\frac{hc}{\lambda}-\phi_0=\frac{6.63\times10^{-34}\times3\times10^{8}}{427.2\times10^{-9}}-2.236\times10^{-19}$$

$$=(4.656\times10^{-19}-2.236\times10^{-19})\text{ J}$$

$$=2.42\times10^{-19}\text{ J}$$

$\therefore$ Stopping potential, $V_0=\frac{2.42\times10^{-19}}{e}$

$$=\frac{2.42\times10^{-19}}{1.6\times10^{-19}}=1.51\text{ V}$$

[redacted]

75. Given, the following wavelengths from a mercury source were used

$$\lambda_1 = 3650\text{ Å} = 3650 \times 10^{-10}\text{ m}$$
$$\lambda_2 = 4047\text{ Å} = 4047 \times 10^{-10}\text{ m}$$
$$\lambda_3 = 4358\text{ Å} = 4358 \times 10^{-10}\text{ m}$$
$$\lambda_4 = 5461\text{ Å} = 5461 \times 10^{-10}\text{ m}$$
$$\lambda_5 = 6907\text{ Å} = 6907 \times 10^{-10}\text{ m}$$

The stopping voltages are as follows:

$$V_{01} = 1.28\text{ V}, V_{02} = 0.95\text{ V}, V_{03} = 0.74\text{ V}$$
$$V_{04} = 0.16\text{ V and } V_{05} = 0$$

Frequencies corresponding to wavelengths,

$$\nu_1 = \frac{c}{\lambda_1} = \frac{3 \times 10^8}{3650 \times 10^{-10}} = 8.219 \times 10^{14}\text{ Hz}$$

Similarly,

$$\nu_2 = 7.412 \times 10^{14}\text{ Hz}, \quad \nu_3 = 6.884 \times 10^{14}\text{ Hz}$$
$$\nu_4 = 5.493 \times 10^{14}\text{ Hz}, \quad \nu_5 = 4.343 \times 10^{14}\text{ Hz}$$

As we know that, $eV_0 = h\nu - \phi_0$

$$V_0 = \frac{h\nu}{e} - \frac{\phi_0}{e}$$

As the graph between V_0 and frequency ν is a straight line.

The slope of this graph gives the values of $\frac{h}{e}$.

$$\therefore \frac{h}{e} = \frac{V_{01} - V_{04}}{\nu_1 - \nu_4} = \frac{1.28 - 0.16}{(8.219 - 5.493) \times 10^{14}} = \frac{1.12}{2.726 \times 10^{14}}$$

$$h = \frac{1.12 \times 1.6 \times 10^{-19}}{2.726 \times 10^{14}} = 6.573 \times 10^{-34}\text{ J-s}$$

As, $\nu_{\text{average}} = 5 \times 10^{14}$ Hz [given]

$\therefore$ Work function, $\phi_0 = h\nu_0 = 6.573 \times 10^{-34} \times 5 \times 10^{14}$

$$= 32.865 \times 10^{-20}\text{ J} = 2.05\text{ eV}$$

76. We have, $E = h\nu \Rightarrow \frac{hc}{\lambda}$

$$\therefore E = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{4000 \times 10^{-10}} = 4.96 \times 10^{-19}\text{ J}$$

77. Maximum wavelength of visible light (i.e. of red light) is 7800 Å.

$\therefore$ Energy of red light, $E = \frac{hc}{\lambda} \cong \frac{12400\text{ (eV - Å)}}{7800\text{ (Å)}} = 1.6\text{ eV}$

78. Energy of a photon, $E = \frac{hc}{\lambda}$

$\therefore$ Number of photons incident per square metre per second,

$$n = \frac{P}{E} = \frac{P}{\frac{hc}{\lambda}} = \frac{P\lambda}{hc} = \frac{(1.388 \times 10^3) \times 550 \times 10^{-9}}{(6.63 \times 10^{-34}) \times (3 \times 10^8)}$$

$$= 3.84 \times 10^{21}\text{ photons/m}^2\text{-s}$$

79. Suppose wavelength of X-rays is λ_1 and the wavelength of visible light is λ_2.

Given, $P = 100\text{ W}$, $\lambda_1 = 1\text{ nm}$, $\lambda_2 = 500\text{ nm}$

Also, n_1 and n_2 represent number of photons of X-rays and visible light emitted from the two sources per second.

So, $\frac{E}{t} = P = n_1 \frac{hc}{\lambda_1} = n_2 \frac{hc}{\lambda_2}$

$$\Rightarrow \frac{n_1}{\lambda_1} = \frac{n_2}{\lambda_2} \Rightarrow \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{1}{500}$$

80. Refer to example 4 on page 475

(i) $E = 2.11$ eV (ii) $n = 3 \times 10^{20}$ photon / s

81. Here, $\lambda = 500\text{ nm} = 5 \times 10^{-7}\text{ m}$

Energy of one photon

$$= \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{5 \times 10^{-7}}$$
$$= 3.98 \times 10^{-19}\text{ J}$$

A bulb of 100 W supplied 100 J of energy per second.

$\therefore$ Energy released per second as visible photons

$$= \frac{100 \times 10}{100} = 10\text{ J}$$

$\therefore$ Number of photons emitted per second as visible light

$$= \frac{10}{3.98 \times 10^{-19}} = 2.5 \times 10^{19}$$

82. Here, $I = 10^{-5}\text{ Wm}^{-2}$, $A = 2 \times 10^{-4}\text{ m}^2$

$n = 5, t = ?,$

$\phi_0 = 2\text{ eV} = 2 \times 1.6 \times 10^{-19}\text{ J}$

Sodium has one conduction electron per atom and effective atomic area $= 10^{-20}\text{ m}^2$

Number of conduction electrons in five layers

$$= \frac{5 \times \text{Area of one layer}}{\text{Effective atomic area}} = \frac{5 \times 2 \times 10^{-4}}{10^{-20}} = 10^{17}$$

Incident power, $P = \text{Intensity} \times \text{Area}$

$$= 10^{-5} \times 2 \times 10^{-4} = 2 \times 10^{-9}\text{ W}$$

According to wave picture, the incident power is uniformly absorbed by all the electrons continuously. Hence, energy absorbed per second per electron

$$= \frac{\text{Incident power}}{\text{Number of electrons of five layers}}$$
$$= \frac{2 \times 10^{-9}}{10^{17}} = 2 \times 10^{-26}\text{ W}$$

$\therefore$ Time required for photoelectric emission will be,

$$t = \frac{\text{Energy required per electron for ejection}}{\text{Energy absorbed per second per atom}}$$
$$= \frac{2 \times 1.6 \times 10^{-19}}{2 \times 10^{-26}} = 1.6 \times 10^7\text{ s}$$

[redacted]

|TOPIC 2|

Matter Wave

Wave theory of electromagnetic radiations explained the phenomenon of interference, diffraction and polarisation of light.

On the other hand, quantum theory of electromagnetic radiations successfully explained the photoelectric effect, Compton effect, black body radiation, X-rays spectra, etc.

From photoelectric and Compton effects, it is clear that a particle (photon of radiation) is colliding against another particle (electron). It is due to this reason it was concluded that, in photoelectric effect and Compton effect, the radiation possesses particle nature.

It means radiation sometimes behaves as a wave and sometimes as a particle. Therefore, Louis Victor de-Broglie suggested that the particles like electrons, protons, neutrons, etc., have dual nature, i.e. they can have particle as well as wave nature.

Note Matter cannot exist both as a particle and as a wave simultaneously. At a particular instant of time, it is either the one or the other aspect, i.e. the two aspects are complementary to each other

WAVE NATURE OF PARTICLES: (DE-BROGLIE HYPOTHESIS)

According to de-Broglie, a wave is associated with moving material particle which controls the particle in every respect. The wave associated with moving material particle is called matter wave or de-Broglie wave whose wavelength is called de-Broglie wavelength which is given by $\lambda = \frac{h}{mv}$

where, m and v are the mass and velocity of the particle and h is Planck's constant.

According to Planck's quantum theory, the energy of the photon is given by

$$E = h\nu = \frac{hc}{\lambda} \quad \text{...(i)}$$

According to Einstein's theory, the energy of the photon is given by

$$E = mc^2 \quad \text{...(ii)}$$

Therefore, from Eqs. (i) and (ii), we get

$$\lambda = \frac{h}{mc} \text{ or } \boxed{\lambda = \frac{h}{p}}$$

where, $p = mc$ is momentum of a photon.

If a material particle of mass m is moving with velocity v, then momentum of the particle, $p = mv$.

According to de-Broglie hypothesis, the wavelength of wave associated with moving material particle becomes

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}}$$

which is the expression for de-Broglie wavelength.

From the above expression following observations we made

(i) The de-Broglie wavelength $\lambda \propto \frac{1}{v}$. If the particle moves faster, then the wavelength will be smaller and *vice-versa*.

(ii) If the particle is at rest ($v = 0$), then the de-Broglie wavelength is infinite ($\lambda = \infty$). Such a wave cannot be visualised.

(iii) The de-Broglie waves cannot be electromagnetic in nature because electromagnetic waves are produced by motion of charged particles.

(iv) The wavelength of a wave associated with moving particle defines a region of uncertainty, within which the whereabouts of the particle are unknown.

These facts lead to Heisenberg's uncertainty principle. According to this principle, it is not possible to measure both the position and momentum of a particle at the same time exactly There is always some uncertainty (Δx) in the specification of position and some uncertainty (Δp) in the specification of momentum. The product of Δx and Δp is of the order of $\hbar$, (with $\hbar = \frac{h}{2\pi}$).

i.e. $\Delta x \, \Delta p \approx \hbar = \frac{h}{2\pi}$.

Common Features of Matter Waves

Some common features of matter waves are as given below

(i) Matter waves can travel in vacuum and hence they are not mechanical waves.

(ii) Matter waves are probability waves, amplitude of which gives the probability of existence of the particle at the point.If at a point, the amplitude of the wave is A, then probability of the particle being found in a small volume dV around that point is $|A|^2 \, dV$.

[redacted]

EXAMPLE |1| An electron and a photon each have a wavelength 1.00 nm. Calculate

(i) their momenta,

(ii) the energy of the photon and

(iii) the kinetic energy of electron. **NCERT**

Sol. Given, $\lambda = 1 \text{ nm} = 1\times 10^{-9}$ m, $h = 6.63\times 10^{-34}$ J-s

$c = 3\times 10^8$ m/s, $p = ?$, $E = ?$, $K = ?$

(i) Momentum of the photon, $p_p = \dfrac{h}{\lambda}$

$$= \frac{6.63\times 10^{-34}}{1\times 10^{-9}} = 6.63\times 10^{-25} \text{ kg-m/s}$$

Momentum of the electron, $p_e = \dfrac{h}{\lambda}$

$$= \frac{6.63\times 10^{-34}}{1\times 10^{-9}} = 6.63\times 10^{-25} \text{ kg-m/s}$$

(ii) Energy of the photon,

$$E = \frac{hc}{\lambda} = \frac{6.63\times 10^{-34}\times 3\times 10^8}{1\times 10^{-9}} = 1.99\times 10^{-16} \text{ J}$$

(iii) Kinetic energy of the electron,

$$K = \frac{p^2}{2m_e} = \frac{(6.63\times 10^{-25})^2}{2\times 9.1\times 10^{-31}} = 2.41\times 10^{-19} \text{ J}$$

EXAMPLE |2| A proton and an electron have same de-Broglie wavelength. Which of them moves fast and which possesses more kinetic energy? Justify your answer.

Sol. Kinetic energy of particle of mass m having momentum p is given by

$$K = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mK}$$

The de-Broglie wavelength, $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mK}}$

$$\therefore \quad p = \frac{h}{\lambda} \quad \text{...(i)}$$

$$\text{and} \quad K = \frac{h^2}{2m\lambda^2} \quad \text{...(ii)}$$

If λ is constant, then from Eq. (i), we get

p = constant, i.e. $m_p v_p = m_e v_e$

or $\dfrac{v_p}{v_e} = \dfrac{m_e}{m_p} < 1$

or $v_p < v_e$

If λ is constant, then from Eq. (ii), $K \propto \dfrac{1}{m}$

$\therefore \dfrac{K_p}{K_e} = \dfrac{m_e}{m_p} < 1$ or $K_p < K_e$.

It means that the velocity of electron is greater than that of proton. Kinetic energy of electron is greater than that of proton.

Relation between de-Broglie Wavelength (λ) and Temperature (T)

From kinetic theory of matter, the average kinetic energy of a particle at a given temperature T kelvin is given by

$$K = \frac{3}{2}kT$$

where, k = Boltzmann constant.

If a particle of mass m is moving with velocity v, then its kinetic energy is,

$$K = \frac{1}{2}mv^2$$

Momentum of particle is

$$p = mv = \sqrt{2mK}$$

$$= \sqrt{2m\times \frac{3}{2}kT} = \sqrt{3mkT}$$

$$\Rightarrow \quad \boxed{\text{de-Broglie wavelength, } \lambda = \frac{h}{p} = \frac{h}{\sqrt{3mkT}}}$$

EXAMPLE |3| Find de-Broglie wavelength of neutron at 127° C. Given, mass of neutron $= 1.66\times 10^{-27}$ kg, Boltzmann constant, $k = 1.38\times 10^{-23}$ J mol^{-1} K^{-1}, and Planck's constant, $h = 6.63\times 10^{-34}$ J-s.

Sol. Here,

$T = 127°\text{C} = 127 + 273 = 400$ K

Energy of neutron at 127° C,

$$E = \frac{3}{2}kT = \frac{3}{2}\times 1.38\times 10^{-23}\times 400$$

$$= 8.28\times 10^{-21} \text{ J}$$

$$\therefore \quad \lambda = \frac{h}{\sqrt{2mE}}$$

$$= \frac{6.63\times 10^{-34}}{\sqrt{2\times 1.66\times 10^{-27}\times 8.28\times 10^{-21}}}$$

$$= 1.264\times 10^{-10} \text{ m}$$

$$= 1.264 \text{ Å}$$

de-Broglie Wavelength of an Electron

Let an electron of charge e having mass m be accelerated from rest through a potential difference V, then

Gain in kinetic energy of an electron $= \dfrac{1}{2}mv^2$

Work done on the electron $= eV$

$$\therefore \quad \frac{1}{2}mv^2 = eV$$

[redacted]

$$\Rightarrow \qquad v = \sqrt{\frac{2eV}{m}}$$

Momentum is given by $p = mv = \sqrt{2eVm}$

The wavelength associated with moving charge is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} \qquad \dots \text{(i)}$$

If accelerated charge is electron, then $\lambda = \dfrac{h}{\sqrt{2eVm_e}}$

where, m_e = mass of electron.

Substituting the numerical values of h, m_e and e in Eq. (i) we get

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9 \times 10^{-31} \times 1.6 \times 10^{-19} \times V}}$$

$$= \frac{12.27}{\sqrt{V}} \times 10^{-10}\text{m}$$

$$= \frac{12.27}{\sqrt{V}} \text{Å}$$

$$= \frac{1.227}{\sqrt{V}} \text{nm}$$

EXAMPLE |4| Determine the de-Broglie wavelength associated with an electron, accelerated through a potential difference of 100 V.

Sol. Given, potential difference $(V) = 100$ V

$\therefore$ de-Broglie wavelength $(\lambda) = \dfrac{12.27}{\sqrt{V}} = \dfrac{12.27}{\sqrt{100}} = 1.227$ Å

In this case, the wavelength associated with an electron is of the order of wavelength of X-rays.

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. The de-Broglie wave of a moving particle does not depend on

(a) mass (b) charge
(c) velocity (d) momentum

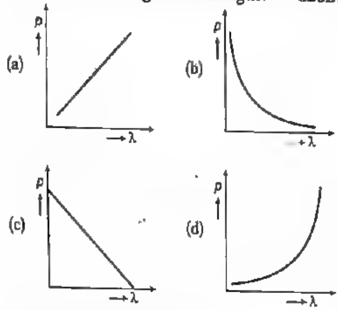
2. The de-Broglie wavelength of a particle of KE, K is λ. What will be the wavelength of the particle, if it's kinetic energy is $\dfrac{K}{9}$?

(a) λ (b) 2λ
(c) 3λ (d) 4λ

3. A proton, a neutron, an electron and an α-particle have same energy. Then, their de-Broglie wavelengths compare as

(a) $\lambda_p = \lambda_n > \lambda_e > \lambda_\alpha$ (b) $\lambda_\alpha < \lambda_p = \lambda_n > \lambda_e$
(c) $\lambda_e < \lambda_p = \lambda_n > \lambda_\alpha$ (d) $\lambda_e = \lambda_p = \lambda_n = \lambda_\alpha$

4. Which of the following figures represent the variation of particle momentum and the associated de-Broglie wavelength? **CBSE 2020**



5. The kinetic energy of a proton and that of an α-particle are 4 eV and 1 eV, respectively. The ratio of the de-Broglie wavelengths associated with them, will be

(a) 2 : 1 (b) 1 : 1
(c) 1 : 2 (d) 4 : 1

VERY SHORT ANSWER Type Questions

6. What consideration led de-Broglie to suggest that material particles can also show wave property?

7. Are the matter waves electromagnetic in nature?

8. Show graphically the variation of de-Broglie wavelength λ with the potential V through which an electron is accelerated from rest. **Delhi 2011**

9. Write the expression for the de-Broglie wavelength associated with a charged particle having charge q and mass m, when it is accelerated by a potential V. **Delhi 2013**

10. A photon and an electron have the same de-Broglie wavelength, which one has higher total energy?

11. A proton and an electron have same kinetic energy. Which one has greater de-Broglie wavelength and why? **All India 2012**

[redacted]

12. A photon and a proton have the same de-Broglie wavelength λ. Prove that the energy of the photon is $(2m\lambda c/h)$ times the kinetic energy of the proton. CBSE 2019

13. A particle with rest mass m_0 is moving with velocity c. What is the de-Broglie wavelength associated with it?

14. Plot a graph showing variation of de-Broglie wavelength (λ) associated with a charged particle of mass m, *versus* $\frac{1}{\sqrt{V}}$, where V is the potential difference through which the particle is accelerated. How does this graph give us the information regarding the magnitude of the charge of the particle?

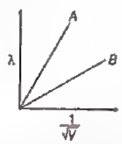
SHORT ANSWER Type Questions

15. Why is the wave nature of matter not more apparent to our daily observations?

16. Show that the wavelength of electromagnetic radiation is equal to the de-Broglie wavelength of its quantum (photon). NCERT

17. A proton and an α-particle are accelerated through the same potential. Which one of the two has
    (i) greater value of de-Broglie wavelength associated with it and
    (ii) less kinetic energy?
    Give reasons to justify your answer. Delhi 2014

18. The two lines marked A and B in the given figure, show a plot of de-Broglie wavelength λ *versus* $\frac{1}{\sqrt{V}}$, where V is the accelerating potential for two nuclei ${}_1^2H$ and ${}_1^3H$.



    (i) What does the slope of the lines represent?
    (ii) Identify, which of the lines corresponded to these nuclei. All India 2010

19. Assuming an electron is confined to a 1 nm wide region, find the uncertainty in momentum using Heisenberg uncertainty principle ($\Delta x \times \Delta p \approx h$). You can assume the uncertainty in position Δx as 1 nm. Assuming $p \approx \Delta p$, find the energy of the electron in eV. NCERT Exemplar

LONG ANSWER Type I Questions

20. An electron, α-particle and a proton have the same de-Broglie wavelengths. Which of these particle has
    (i) minimum kinetic energy?
    (ii) maximum kinetic energy and why?
    In what way has the wave nature of electron beam exploited in electron microscope?

21. Electrons are emitted from the cathode of a photocell of negligible work function, when photons of wavelength λ are incident on it. Derive the expression for the de-Broglie wavelength of the electrons emitted on terms of the wavelength of the incident light. All India 2017 C

22. (a) Explain de-Broglie argument to propose his hypothesis. Show that de-Broglie wavelength of photon equals electromagnetic radiation.
    (b) If deuterons and alpha particle are accelerated through same potential, find the ratio of the associated de-Broglie wavelengths of two.

NUMERICAL PROBLEMS

23. A particle is moving three times as fast as an electron. The ratio of the de-Broglie wavelength of the particle to that of the electron is 1.813×10^{-4}. Calculate the particle's mass and identify the particle. All India 2011

24. de-Broglie postulated that the relationship, $\lambda = \frac{h}{p}$ is valid for relativistic particles. Find out the de-Broglie wavelength for an (relativistic) electron whose kinetic energy is 3 MeV.

25. The wavelength of light from the spectral emission line of sodium is 589 nm. Find the kinetic energy at which
    (i) an electron and
    (ii) a neutron would have the same de-Broglie wavelength? NCERT

26. What is the
    (i) momentum (ii) speed and
    (iii) de-Broglie wavelength of an electron with kinetic energy of 120 eV? NCERT

[redacted]

27. (i) Determine the de-Broglie wavelength of a proton whose kinetic energy is equal to the rest mass energy of an electron. Mass of proton is 1836 times that of electron.
    (ii) In which region of electromagnetic spectrum does this wavelength lie? **All India 2011**

28. What is the de-Broglie wavelength of
    (i) a bullet of mass 0.040 kg travelling at the speed of 1.0 km/s
    (ii) a ball of mass 0.060 kg moving at a speed of 1.0 m/s
    (iii) a dust particle of mass 1.0×10^{-9} kg drifting with a speed of 2.2 m/s? **NCERT**

29. Obtain the de-Broglie wavelength of an electron of kinetic energy 100 eV. Mass of electron $= 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C, $h = 6.63 \times 10^{-34}$ J-s.

30. The wavelength of light from the spectral emission line of sodium is 590 nm. Find the kinetic energy at which the electron would have the same de-Broglie wavelength. **CBSE 2019**

31. (i) For what kinetic energy of a neutron will associated de-Broglie wavelength be 1.40×10^{-10} m?
    (ii) Also, find the de-Broglie wavelength of a neutron, in thermal equilibrium with matter, having an average kinetic energy of $(3/2)\,kT$ and temperature is 300 K. **NCERT**

32. Calculate the
    (i) momentum and
    (ii) de-Broglie wavelength of the electrons accelerated through a potential difference of 56 V. **NCERT**

33. Find the ratio of the de-Broglie wavelength, associated with protons, accelerated through a potential of 128 V and α-particles, accelerated through a potential of 64 V. **Delhi 2010C**

34. A proton and an α-particle have the same de-Broglie wavelength. Determine the ratio of
    (i) their accelerating potentials
    (ii) their speeds. **All India 2015**

35. An electron microscope uses electrons accelerated by a voltage of 50 kV. Determine the de-Broglie wavelength associated with the electrons. Taking other factors, such as numerical aperture, etc., to be same, how does the resolving power of an electron microscope compare with that of an optical microscope which uses yellow light? **NCERT, All India 2014**

36. Crystal diffraction experiments can be performed using X-rays or electrons accelerated through appropriate voltage. Which probe has greater energy? (For quantitative comparison, take the wavelength of the probe equal to 1 Å, which is of the order of interatomic spacing in the lattice.) $(m_e = 9.11 \times 10^{-31}$ kg$)$ **NCERT**

37. An electron is accelerated through a potential difference of 64 V. What is the de-Broglie wavelength associated with it? To which part of the electromagnetic spectrum does this value of wavelength correspond?

38. Compute the typical de-Broglie wavelength of an electron in a metal at 27°C and compare it with the mean separation between two electrons in a metal which is given to be about 2×10^{-10} m. **NCERT**

39. Find the typical de-Broglie wavelength associated with a He atom in helium gas at room temperature (27° C) and 1 atm pressure and compare it with the mean separation between two atoms under these conditions. **NCERT**

40. The de-Broglie wavelength associated with an electron accelerated through a potential difference V is λ. What will be its wavelength when the accelerating potential is increased to 4 V?

41. An electron gun with its collector at a potential of 100 V fires out electrons in a spherical bulb containing hydrogen gas at low pressure ($\sim 10^{-2}$ mm of Hg). A magnetic field of 2.83×10^{-4} T curves the path of the electrons in a circular orbit of radius 12.0 cm. (The path can be viewed because the gas ions in the path focus, the beam by attracting electrons and emitting light by electron capture, this method is known as the fine beam tube method.) Determine e/m from the data. **NCERT**

42. (i) Estimate the speed with which electrons emitted from a heated emitter of an evacuated tube impinge on the collector maintained at a potential difference of 500 V with respect to the emitter. Ignore the small initial speeds of the electrons.

[redacted]

The specific charge of the electron, i.e. its e/m is given to be 1.76×10^{11} C/kg.

(ii) Use the same formula you employ in (i) to obtain electron speed for a collector potential of 10 MV. Do you see what is wrong? In what way is the formula to be modified? NCERT

HINTS AND SOLUTIONS

1. (b) de-Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{mv}$

So, the de-Broglie wavelength does not depend on charge.

2. (c) de-Broglie wavelength, $\lambda = \frac{h}{\sqrt{2mK}}$ ...(i)

When the KE is $\frac{K}{9}$, then

$$\lambda' = \frac{h}{\sqrt{2m\left(\frac{K}{9}\right)}} = \frac{3h}{\sqrt{2mK}} = 3\lambda \quad \text{[using Eq. (i)]}$$

3. (b) We know that the relation between λ and K is given by

$$\lambda = \frac{h}{\sqrt{2mk}}$$

or $$\lambda \propto \frac{1}{\sqrt{m}}$$

Since, $m_p = m_n$, hence $\lambda_p = \lambda_n$

As, $m_\alpha > m_p$, therefore $\lambda_\alpha < \lambda_p$

As, $m_e < m_n$, therefore $\lambda_e > \lambda_n$

Hence, $\lambda_\alpha < \lambda_p = \lambda_n < \lambda_e$

4. (b) The de-Broglie wavelength is given by

$\lambda = h/p \Rightarrow p\lambda = h$

This equation is in the form of $yx = c$, which is the equation of a rectangular hyperbola. Hence, the graph given in option (b) is the correct one.

5. (b) The de-Broglie wavelength associated with a particle is given by

$$\lambda = \frac{h}{\sqrt{2mK}}$$

Given, $K_p = 4\text{eV}$ and $K_\alpha = 1\text{eV}$

$$\therefore \quad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha K_\alpha}{m_p K_p}} = \sqrt{\frac{4}{1} \times \frac{1}{4}} \quad \left[\because \frac{m_\alpha}{m_p} = 4\right]$$

$$= 1$$

or $\lambda_p : \lambda_\alpha = 1:1$

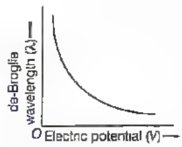
6. The following considerations led de-Broglie to suggest that material particles can also show wave property

(i) The Einstein's mass-energy relationship $E = mc^2$, i.e. matter can be converted into energy and *vice-versa*.

(ii) Wave nature loves symmetry, hence from symmetry consideration particles like electrons, protons should exhibit wave nature when in motion.

7. No, matter waves are not electromagnetic in nature, because electromagnetic waves are only associated with accelerated charged particles, but de-Broglie wavelength $\lambda = \frac{h}{p}$, i.e. associated with momentum

8. We know that, $\lambda \propto \frac{1}{\sqrt{V}}$



$$\Rightarrow \quad \lambda^2 V = \text{constant} \quad \left[\because \text{constant} = \frac{1}{\sqrt{2me}}\right]$$

9. de-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

10. Total energy of an electron, $E_e = mc^2$

Total energy of a photon, $E_p = \frac{hc}{\lambda}$

de-Broglie wavelength of electron of mass m moving with velocity v,

$$\lambda = \frac{h}{mv}$$

$$\Rightarrow \quad m = \frac{h}{\lambda v}$$

$\therefore$ Energy of an electron, $E_e = mc^2 = \frac{hc^2}{\lambda v}$

$$\therefore \quad \frac{E_e}{E_p} = \frac{\frac{hc^2}{\lambda v}}{\frac{hc}{\lambda}} = \frac{c}{v}$$

As $c >> v$, therefore the total energy of electron is more than the total energy of photon.

11. de-Broglie wavelength,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}, \text{ where } K = \text{kinetic energy}$$

For given KE, $\lambda \propto \frac{1}{\sqrt{m}}$

[redacted]

Since, electrons have smaller mass, i.e. $\lambda_e > \lambda_p$.
For given kinetic energy, electrons have greater de-Broglie wavelength as these have smaller mass.

12. Energy of photon, $E_p = \dfrac{hc}{\lambda}$

Energy of electron (moving particle),

$$E_e = \frac{1}{2}\frac{p^2}{m}$$

de-Broglie wavelength associated with the moving particle is

$$\lambda = h/p \text{ or } p = h/\lambda$$

$$E_e = \frac{1}{2}\frac{(h/\lambda)^2}{m} = \frac{1}{2}\frac{h^2}{\lambda^2 m}$$

$$\therefore \quad \frac{E_p}{E_e} = \frac{hc/\lambda}{\frac{1}{2}\frac{h^2}{\lambda^2 m}} = \frac{2m\lambda c}{h}$$

13. de-Broglie wavelength,

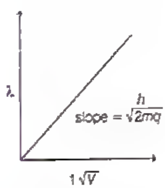
$$\lambda = \frac{h}{mv} = \frac{h\sqrt{1-v^2/c^2}}{m_0 v}$$

$$= \frac{h\sqrt{1-c^2/c^2}}{m_0 c} = 0 \qquad [\because v = c]$$

14. The de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mqV}}$$

$$\Rightarrow \quad \lambda \propto \frac{1}{\sqrt{V}}$$



Thus, it gives a straight line graph.

$$\lambda\sqrt{V} = \frac{h}{\sqrt{2mq}} = \text{slope of graph}$$

Knowing the mass of particle (m) and slope of graph, we can calculate charge (q) on a particle.

15. The de-Broglie wavelength associated with a body of mass m, moving with velocity v is given by $\lambda = \dfrac{h}{mv}$.

Since, the mass of the objects used in our daily life is very large, hence the de-Broglie wavelength associated with them is quite small and is not visible. Hence, the wave nature of matter is not more apparent to our daily observations.

16. The momentum of an electromagnetic wave of frequency ν, wavelength λ is given by

$$p = \frac{h\nu}{c} = \frac{h}{\lambda}$$

or $$\lambda = \frac{h}{p}$$

de-Broglie wavelength of photon,

$$\lambda = \frac{h}{p}$$

Thus, wavelength of electromagnetic radiation is equal to the de-Broglie wavelength.

17. (i) The de-Broglie wavelength of a particle is given

$$\lambda = \frac{h}{\sqrt{2mV_0 q}}$$

So, potential V_0 is same.
Since, α-particle and proton both are accelerated through the same.

$$\therefore \quad \lambda \propto \frac{1}{\sqrt{mq}}$$

or $$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{m_p q_p}{m_\alpha q_\alpha}}$$

As, charge on α-particle = 2 × charge on proton

$$q_\alpha = 2q_p \Rightarrow \frac{q_p}{q_\alpha} = \frac{1}{2}$$

Mass of α-particle = 4 × mass of proton

$$m_\alpha = 4 \times m_p \Rightarrow \frac{m_p}{m_\alpha} = \frac{1}{4}$$

$$\therefore \quad \frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{1}{4}\cdot\frac{1}{2}} = \frac{1}{2\sqrt{2}}$$

$$\Rightarrow \quad \lambda_p = 2\sqrt{2}\lambda_\alpha$$

i.e. proton has greater de-Broglie wavelength than that of α-particle.

(ii) KE ∝ q (for same accelerating potential)
The charge of an α-particle is more as compared to a proton. So, it will have a greater value of KE. Hence, proton will have lesser KE.

18. de-Broglie wavelength of accelerating charged particle is given by

$$\lambda = \frac{h}{\sqrt{2mqV}} \Rightarrow \lambda\sqrt{V} = \frac{h}{\sqrt{2mq}} = \text{constant}$$

(i) The slope of the lines represents $\dfrac{h}{\sqrt{2mq}}$

where, h = Planck's constant, q = charge and m = mass of charged particle.

(ii) $_1H^2$ and $_1H^3$ carry same charge (as they have same atomic number).

$$\therefore \quad \lambda\sqrt{V} \propto \frac{1}{\sqrt{m}}$$

The lighter mass, i.e. $_1H^2$ is represented by line of greater slope, i.e. A and similarly, $_1H^3$ by line B.

[redacted]

19. Here, $\Delta x = 1\ \text{nm} = 10^{-9}$ m, $\Delta p = ?$ As, $\Delta x \Delta p = \hbar$

$$\therefore \quad \Delta p = \frac{\hbar}{\Delta x} = \frac{h}{2\pi\Delta x} = \frac{6.6 \times 10^{-34}\ \text{J-s}}{2 \times (22/7)(10^{-9})\ \text{m}}$$

$$= 1.05 \times 10^{-25}\ \text{kg-m/s}$$

$$\therefore \quad \text{Energy}\,(E) = \frac{p^2}{2m} = \frac{(\Delta p)^2}{2m} \qquad [\because p \approx \Delta p]$$

$$= \frac{(1.05 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}}\ \text{J}$$

$$= \frac{(1.05 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19}}\ \text{eV}$$

$$= 3.8 \times 10^{-2}\ \text{eV}$$

20. de-Broglie matter wave equation,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} \qquad \left[\because K = \frac{p^2}{2m}\right]$$

where, K is kinetic energy and m is mass of particle.

$$K = \frac{h^2}{2m\lambda^2} \qquad \text{[for same wavelength } \lambda]$$

$$\Rightarrow \quad K \propto \frac{1}{m} \Rightarrow K_e : K_\alpha : K_p = \frac{1}{m_e} : \frac{1}{m_\alpha} : \frac{1}{m_p}$$

where, m_e, m_p and m_α are masses of electron, proton and α-particle, respectively.

Also, K_e, K_α and K_p are their respective kinetic energies.

$\because \quad m_\alpha > m_p > m_e$

$\Rightarrow \quad m_\alpha m_p > m_e m_\alpha > m_e m_p$

$\Rightarrow \quad K_e > K_p > K_\alpha$

(i) α-particle possesses minimum kinetic energy

(ii) Electron has maximum kinetic energy.

The magnifying power of an electron microscope is inversely related to wavelength of radiation used. Smaller wavelength of electron beam in comparison to visible light increases the magnifying power of microscope.

21. We know that, $\dfrac{hc}{\lambda} = \dfrac{hc}{\lambda_0} + \dfrac{1}{2}mv^2$

Neglecting the work function, we get

$$\frac{hc}{\lambda} = \frac{1}{2}mv^2$$

de-Broglie wavelength is given by, $\lambda_e = \dfrac{h}{mv}$

$$\therefore \quad \lambda_e = \frac{h\sqrt{\lambda}}{\sqrt{2mhc}} = \sqrt{\frac{h\lambda}{2mc}}$$

22. (a) **de-Broglie Hypothesis** According to de-Broglie, a wave is associated with moving material particle which controls the particle in every respect. The wave associated with moving material particle is called **matter wave or de-Broglie** wave whose wavelength is called de-Broglie wavelength which is given by

$$\lambda = \frac{h}{mv}$$

where, m is mass of the particle, v is velocity of the particle and h is Planck's constant.

This means the mass and radiation are symmetrical in nature.

According to de-Broglie equation, wavelength,

$$\lambda = \frac{h}{p}$$

where, p = momentum of photon,

$$= mc = \frac{h\nu}{c}$$

$$\therefore \quad \lambda = \frac{hc}{h\nu} = \frac{c}{\nu}$$

which is equal to the wavelength of an electromagnetic radiation.

(b) The de-Brogile wavelength of a particle is given by

$$\lambda = \frac{h}{\sqrt{2mV_0 q}}$$

Since, V_0 is same, so $\lambda \propto \dfrac{1}{\sqrt{mq}}$

For deuteron, $q_d = +e$, $m_d = 2m_p$

For α-particle, $q_\alpha = +2e$, $m_\alpha = 4m_p$

$$\therefore \quad \frac{\lambda_d}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{q_d m_d}} = \sqrt{\frac{2e \times 4m_p}{e \times 2m_p}}$$

$$= 2:1$$

23. Given, $v_{\text{particle}} = 3\,v_{\text{electron}}$ ...(i)

and $\lambda_{\text{particle}} = 1.813 \times 10^{-4}\,\lambda_{\text{electron}}$

As, $\lambda = \dfrac{h}{mv}$ [de-Broglie equation]

$$\Rightarrow \quad \frac{m_{\text{particle}}}{m_{\text{electron}}} = \frac{\lambda_{\text{electron}} \times v_{\text{electron}}}{\lambda_{\text{particle}} \times v_{\text{particle}}}$$

$$= \frac{\lambda_{\text{electron}} \times v_{\text{electron}}}{1.813 \times 10^{-4} \times \lambda_{\text{electron}} \times 3v_{\text{electron}}}$$

$$\therefore \quad m_{\text{particle}} = 1839\ m_{\text{electron}} \qquad \text{[given][from Eq. (i)]}$$

$$= 1839 \times 9.1 \times 10^{-31}$$

$$= 1.673 \times 10^{-27}\ \text{kg}$$

$\therefore$ Particle is either a proton or a neutron.

24. de-Broglie wavelength

$$\lambda = \frac{h}{\sqrt{2mK}}$$

Substituting the given values, we get

$$\lambda = 3.58 \times 10^{-3}\text{m}$$

25. Given, wavelength of light,

$$= 589\ \text{nm} = 589 \times 10^{-9}\ \text{m}$$

Mass of electron, $m_e = 9.1 \times 10^{-31}$ kg

Mass of neutron, $m_n = 1.675 \times 10^{-27}$ kg

Planck's constant, $h = 6.63 \times 10^{-34}$ J-s

[redacted]

(i) Using formula, $\lambda = \frac{h}{\sqrt{2mK}}$

Kinetic energy of electron,

$$K_e = \frac{h^2}{2\lambda^2 m_e} = \frac{(6.63\times 10^{-34})^2}{2\times(589\times 10^{-9})^2\times 9.1\times 10^{-31}}$$

$$= 6.96\times 10^{-25}\text{ J}$$

(ii) Kinetic energy of neutron,

$$K_n = \frac{h^2}{2\lambda^2 m_n} = \frac{(6.63\times 10^{-34})^2}{2\times(589\times 10^{-9})^2\times 1.675\times 10^{-27}}$$

$$= 3.81\times 10^{-28}\text{ J}$$

26. Given, kinetic energy = KE = 120 eV

(i) Momentum, $p = \sqrt{2\,eVm} = \sqrt{2\,\text{KE}\cdot m}$ $\quad [\because \text{KE} = \text{eV}]$

$$= \sqrt{2\times 120\times 1.6\times 10^{-19}\times 9.1\times 10^{-31}}$$

$$= 5.91\times 10^{-24}\text{ kg-m/s}$$

(ii) We know that momentum, $p = mv$

or $\quad v = \frac{p}{m} = \frac{5.91\times 10^{-24}}{9.1\times 10^{-31}}$

$$= 6.5\times 10^{6}\text{ m/s}$$

(iii) de-Broglie wavelength associated with electron,

$$\lambda = \frac{12.27}{\sqrt{V}}\text{ Å} = \frac{12.27}{\sqrt{120}}\text{ Å}$$

$$= 0.112\times 10^{-9}\text{ m} = 0.112\text{ nm}$$

27. (i) de-Broglie matter wave equation is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

$\because \quad p = \sqrt{2mK}$

where, m = mass of proton and

K = kinetic energy of proton.

According to the question, kinetic energy of proton,

$K = m_e c^2$ [using Einstein's mass-energy relation]

$$\Rightarrow \quad \lambda = \frac{h}{\sqrt{2m(m_e c^2)}}$$

$$= \frac{h}{\sqrt{2}c\cdot\sqrt{m\cdot m_e}}$$

$$= \frac{h}{\sqrt{2}\,c\cdot m_e\sqrt{1836}} \quad [\because m = 1836\,m_e]$$

$$= \frac{6.63\times 10^{-34}}{1.414\times(3\times 10^{8})\times 9.1\times 10^{-31}\times 42.8}$$

$$= 4\times 10^{-14}\text{ m}$$

(ii) This region of electromagnetic spectrum is γ-ray.

28. (i) Given, mass of bullet, $m = 0.040$ kg

Speed of bullet, $v = 1000$ m/s

de-Broglie wavelength, $\lambda = \frac{h}{mv} = \frac{6.63\times 10^{-34}}{0.040\times 1\times 10^{3}}$

$$= 1.66\times 10^{-35}\text{ m}$$

(ii) Mass of the ball, $m = 0.060$ kg and speed of the ball, $v = 1$ m/s

$$\lambda = \frac{h}{mv} = \frac{6.63\times 10^{-34}}{0.060\times 1}$$

$$= 1.1\times 10^{-32}\text{ m}$$

(iii) Mass of a dust particle, $m = 1\times 10^{-9}$ kg and speed of the dust particle, $v = 2.2$ m/s

$$\lambda = \frac{h}{mv} = \frac{6.63\times 10^{-34}}{1\times 10^{-9}\times 2.2}$$

$$= 3.0\times 10^{-25}\text{ m}$$

29. Refer to Q.28 on page 461, $\lambda = 1.2\times 10^{-10}$ m

30. Given, $\lambda = 590\text{ nm} = 590\times 10^{-9}$ m

de-Broglie wavelength $(\lambda) = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

where, m is the mass of electron.

Thus, $590\times 10^{-9} = \frac{6.626\times 10^{-34}}{\sqrt{2\times 9.11\times 10^{-31}\times E}}$

$$\Rightarrow \sqrt{2\times 9.11\times 10^{-31}\times E} = \frac{6.626\times 10^{-34}}{590\times 10^{-9}}$$

$$= 0.0112\times 10^{-25}$$

$$\Rightarrow \quad 2\times 9.11\times 10^{-31}\times E = (0.0112\times 10^{-25})^2$$

$$\Rightarrow \quad E = \frac{0.00012544\times 10^{-50}}{2\times 9.11\times 10^{-31}}$$

$$= 6.884\times 10^{-25}\text{ J}$$

31. (i) Refer to Q. 26 on page 460.

KE $= 6.714\times 10^{-21}$ J, using $K = \frac{h^2}{2m\lambda^2}$

(ii) Kinetic energy associated with temperature,

$$\text{KE} = \frac{3}{2}kT = \frac{3}{2}(1.38\times 10^{-23})\times 300$$

$$= 6.21\times 10^{-21}\text{ J}$$

[$\because$ absolute temperature, $T = 300$ K and Boltzmann's constant, $k = 1.38\times 10^{-23}$ J/K]

KE $= 6.21\times 10^{-21}$ J

de-Broglie wavelength associated with kinetic energy,

$$\lambda = \frac{h}{\sqrt{2m_n\,\text{KE}}} = \frac{6.63\times 10^{-34}}{\sqrt{2\times 1.675\times 10^{-27}\times 6.21\times 10^{-21}}}$$

$$= 1.45\times 10^{-10}\text{ m} = 1.45\text{ Å}$$

32. Given, potential difference, $V = 56$ V

(i) Use the formula for kinetic energy,

$$eV = \frac{1}{2}mv^2 \Rightarrow \frac{2\,eV}{m} = v^2$$

$$\Rightarrow \quad v = \sqrt{\frac{2eV}{m}}$$

[redacted]

where, m is mass and v is velocity of electron.
Momentum associated with accelerated electron,

$$p = mv = m\sqrt{\frac{2eV}{m}} = \sqrt{2\,eVm}$$

$$= \sqrt{2 \times 1.6 \times 10^{-19} \times 56 \times 9 \times 10^{-31}}$$

$$= 4.02 \times 10^{-24} \text{ kg-m/s}$$

(ii) $\lambda = 0.164$ nm; refer to example 4 on page 459.

33. de-Broglie wavelength is given by

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}} \qquad [\because K = qV]$$

$$\Rightarrow \quad \lambda \propto \frac{1}{\sqrt{mqV}}$$

Ratio of de-Broglie wavelengths of proton and α particle is given by

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha V_\alpha}{m_p q_p V_p}} = \sqrt{\left(\frac{m_\alpha}{m_p}\right)\left(\frac{q_\alpha}{q_p}\right)\left(\frac{V_\alpha}{V_p}\right)}$$

Here, $\frac{m_\alpha}{m_p} = 4$, $\frac{q_\alpha}{q_p} = 2$,

$$\Rightarrow \quad \frac{V_\alpha}{V_p} = \frac{64}{128} = \frac{1}{2}$$

[$\because$ α-particle is 4 times heavier than proton and it has double the charge than that of proton]

$$\Rightarrow \quad \frac{\lambda_p}{\lambda_\alpha} = \sqrt{4 \times 2 \times \frac{1}{2}} = 2$$

$$\Rightarrow \quad \lambda_p : \lambda_\alpha = 2:1$$

34. (i) The de-Broglie wavelength of a particle is given by

$$\lambda = \frac{12.27}{\sqrt{V}} \text{Å}$$

[where, V is the accelerating potential of the particle]

$$\because \quad \lambda_p = \lambda_\alpha \qquad \text{[given]}$$

$$\Rightarrow \quad \frac{12.27}{\sqrt{V_p}} = \frac{12.27}{\sqrt{V_\alpha}}$$

$$\Rightarrow \quad \frac{V_p}{V_\alpha} = 1$$

(ii) The de-Broglie wavelength of a particle is given by

$$\lambda = \frac{h}{mv}$$

$$\therefore \quad \lambda_p = \frac{h}{m_p \cdot v_p} \text{ and } \lambda_\alpha = \frac{h}{m_\alpha v_\alpha}$$

We know that, $m_\alpha = 4\,m_p$

$$\because \quad \lambda_p = \lambda_\alpha \qquad \text{[given]}$$

$$\therefore \quad \frac{h}{m_p \cdot v_p} = \frac{h}{4\,m_p \cdot v_\alpha}$$

$$\Rightarrow \quad \frac{v_p}{v_\alpha} = 4$$

35. $\lambda = 0.0548$Å; refer to Example 4 on page 459.

Resolving power of a microscope, $R = \frac{2\mu \sin\theta}{\lambda}$

From the formula, it is clear that, if other factors remain same, then resolving power is inversely proportional to wavelength of the radiation used. Wavelength of moving electron is very small as compared to that of yellow light, so it has greater resolving power than optical microscope.

36. Given, wavelength of X-rays, $\lambda = 1\text{ Å} = 10^{-10}$ m

Mass of electron, $m_e = 9.11 \times 10^{-31}$ kg

As, $\lambda = \frac{h}{\sqrt{2m\text{KE}}}$ or $\text{KE} = \frac{h^2}{2\lambda^2 m}$

$$= \frac{(6.63 \times 10^{-34})^2}{2 \times (10^{-10})^2 \times 9.11 \times 10^{-31} \times 1.6 \times 10^{-19}}$$

$$= 150.78 \text{ eV}$$

$$\therefore \text{Energy of photon} = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1 \times 10^{-10} \times 1.6 \times 10^{-19}}$$

$$= 12.4 \times 10^3 \text{ eV}$$

Thus, for the same wavelength, a X ray photon has greater kinetic energy than an electron

37. Given that, $V = 64$V

Now, from the de-Broglie equation,

$$\lambda = \frac{12.27}{\sqrt{V}}\text{Å} = \frac{12.27\text{Å}}{\sqrt{64}} \qquad [\because V = 64\text{V}]$$

$$= \frac{12.27}{8}\text{Å}$$

$$= 0.153 \text{ nm}$$

This wavelength belongs to the X-ray part of the electromagnetic radiation.

38. Given, temperature,

$$T = 27\,°\text{C} = 27 + 273 = 300 \text{ K}$$

Separation, between two electrons,

$$r = 2 \times 10^{-10} \text{ m}$$

Momentum $p = \sqrt{3m\,kT}$

$$= \sqrt{3 \times 9.11 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}$$

$$[\because k = 1.38 \times 10^{-23} \text{ J/K}]$$

$$= 1.06 \times 10^{-25} \text{ kg m/s}$$

de-Broglie wavelength, $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{1.06 \times 10^{-25}}$

$$= 62.6 \times 10^{-10} \text{ m}$$

Mean separation, $r = 2 \times 10^{-10}$ m

$$\therefore \quad \frac{\lambda}{r} = \frac{62.6 \times 10^{-10}}{2 \times 10^{-10}} = 31.3$$

We can see that de-Broglie wavelength is much greater than the electron separation.

[redacted]

39. Mass of helium atom,

$$m = \frac{\text{Atomic weight}}{\text{Avogadro's number}} = \frac{4 \times 10^3}{6 \times 10^{23}} \text{ g}$$

Boltzmann constant, $k = 1.38 \times 10^{-23}$ J mol^{-1} K^{-1}

de-Broglie wavelength, $\lambda = \frac{h}{\sqrt{3\,mkT}}$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{3 \times \frac{4 \times 10^3}{6 \times 10^{23}} \times 1.38 \times 10^{-23} \times 300}} \quad \left[\begin{array}{l} \because\ T = 27°\text{ C} \\ = (27 + 273)\text{K} \\ = 300\text{ K} \end{array}\right]$$

$= 0.73 \times 10^{-10}$ m

Now, $\quad pV = RT = kNT$

or $\quad \frac{V}{N} = \frac{kT}{p}$

Mean separation, $r = \left(\frac{V}{N}\right)^{1/3} = \left(\frac{kT}{p}\right)^{1/3}$

$$= \left[\frac{1.38 \times 10^{-23} \times 300}{1.01 \times 10^5}\right]^{1/3}$$

$= 3.4 \times 10^{-9}$ m

$\therefore \quad \frac{\lambda}{r} = \frac{0.73 \times 10^{-10}}{3.4 \times 10^{-9}} = 0.021$

40. $\frac{\lambda}{2}$; refer to example 4 on page 459.

41. Given, potential at anode, $V = 100$ V

Magnetic field, $B = 2.83 \times 10^{-4}$ T

Radius of circular path, $r = 12$ cm $= 0.12$ m

Kinetic energy, $\text{KE} = \frac{1}{2}mv^2 \quad \left[\begin{array}{l} \because\ m_e = 9.1 \times 10^{-31}\text{ kg} \\ \text{and } e = 1.6 \times 10^{-19}\text{ C} \end{array}\right]$

So, $\quad eV = \frac{1}{2}mv^2$

$\Rightarrow 1.6 \times 10^{-19} \times 100 = \frac{1}{2} \times 9.1 \times 10^{-31} \times v^2$

$\Rightarrow \quad v^2 = \frac{2 \times 1.6 \times 10^{-17}}{9.1 \times 10^{-31}}$

$= 5.93 \times 10^6$ m/s

As, the angle between v and B is 90°.

The magnetic force ($F_m = evB$) is balanced by the centripetal force.

i.e. $\quad evB = \frac{mv^2}{r}$

or $\quad \frac{e}{m} = \frac{v}{Br} = \frac{5.93 \times 10^6}{2.83 \times 10^{-4} \times 0.12}$

$\therefore$ Specific charge of an electron,

$$\frac{e}{m} = 1.74 \times 10^{11} \text{ C/kg}$$

42. (i) Given, potential difference, $V = 500$ V

Specific charge of the electron,

$$e/m = 1.76 \times 10^{11} \text{ C/kg}$$

Kinetic energy of an electron,

$$\text{KE} = \frac{1}{2}mv^2 = eV$$

$$\Rightarrow \quad v = \sqrt{\frac{e}{m} \times 2V} \quad \text{...(i)}$$

$= \sqrt{1.76 \times 10^{11} \times 2 \times 500}$

$= 1.326 \times 10^7$ m/s

(ii) Potential, $V = 10$ MV $= 10^7$ V

Again from Eq. (i), $v = \sqrt{\frac{2e}{m}V}$

$= \sqrt{2 \times 1.76 \times 10^{11} \times 10^7}$

$= 1.8762 \times 10^9$ m/s

This speed is greater than the speed of light, which is not possible. As v approaches to c, then mass

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

[redacted]

SUMMARY

- **Electron Emission** The phenomenon of emission of electrons from metal surface is called electron emission
- **Work Function** It is the minimum energy required by an electron to just escape from metal surface, so as to overcome the attractive pull of the ions
- **Photoelectric Effect** It is the phenomenon of emission of electrons from the metal surface, when the radiations of suitable frequency falls on it
- **Hertz's Observation** He observed that high voltage sparks across the detector loop were enhanced when the emitter plate was illuminated by UV light from an arc lamp
- **Hallwachs and Lenard's Observations** They also observed that UV light falls on the emitter plate no electrons were emitted at all when the frequency of incident light was smaller than a certain minimum value is called threshold frequency
- **Effect of Intensity of Light on Photoelectric Current** For a fixed frequency of incident radiation photoelectric current increases linearly with increase in intensity of incident light
- **Effect of Potential on Photoelectric Current** For a fixed frequency and intensity of incident light photoelectric current increases with increase in potential applied to the collector
- **Effect of Incident Photon Energy and Kinetic Energy**

$$K_{max} = \frac{1}{2}mv^2_{max} = (h\nu - \phi_0)$$

This equation is called Einstein's photoelectric equation

- **Relation between Stopping Potential and Threshold Wavelength** The relation between stopping potential and threshold wavelength is $eV_0 = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$
- **Particle Nature of Light Photon** Photoelectric effect gave the evidence that light consists of packets of energy and these packets of energy are called light quanta, that are associated with photons.
- **Characteristic Properties of Photon**

  Photons has zero rest mass.

  Photons travel in a straight line.

  Photons may show diffraction under given conditions

  The inertial mass of a photon is $m = \frac{hc}{\lambda}$.
- **Photocell** It is a device which converts light energy into electrical energy
- **Dual Nature of Radiation** Wave theory of electromagnetic radiations explained the phenomenon of interference, diffraction etc, whereas quantum theory successfully explained the photoelectric effect Compton effect etc. So, Louis de-Broglie suggested that the particles like electrons, protons etc. have dual nature of radiation
- **Wave Nature of Particles (de-Broglie Hypothesis)** According to de-Broglie, a wave is associated with moving material particle which control the particle in every respect The wave associated with moving material particle is called matter wave It is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

- **Relation between de-Broglie Wavelength and Temperature** It is given by

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

- **de-Broglie Wavelength of an Electron** It is given by $\lambda = \frac{h}{\sqrt{2eVm}}$

[redacted]

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. Work-function is
   (a) maximum possible energy acquired by an electron
   (b) energy of electrons in valence shell
   (c) minimum energy required by an electron to move out of metal surface
   (d) maximum energy which is given to electron to move it out of metal surface

2. The work function of platinum is 6.35 eV. The threshold frequency of platinum is
   (a) 15.32×10^{14} Hz
   (b) 15.32×10^{16} Hz
   (c) 15.32×10^{19} Hz
   (d) 15.32×10^{18} Hz

3. With the increase in potential difference of emitter and collector, the photoelectric current
   (a) increases
   (b) decreases
   (c) remains constant
   (d) increases initially and then become constant

4. The photoelectric threshold frequency of a metal is ν. When light of frequency 6ν is incident on the metal, the maximum kinetic energy of the emitted photo electron is
   (a) $4h\nu$ (b) $5h\nu$
   (c) $3h\nu$ (d) $(3/2)h\nu$

5. Light of wavelengths λ_A and λ_B falls on two identical metal plates A and B respectively. The maximum kinetic energy of photoelectrons is K_A and K_B respectively, then which one of the following relations is true? $(\lambda_A = 2\lambda_B)$
   (a) $K_A < \frac{K_B}{2}$ (b) $2K_A = K_B$
   (c) $K_A = 2K_B$ (d) $K_A > 2K_B$

6. All photons present in a light beam of single frequency have
   (a) same frequency but different momentum
   (b) same momentum but different frequency
   (c) different frequency and different momentum
   (d) same frequency and same momentum

7. The linear momentum of a 6 MeV photon is
   (a) 0.01 eV sm^{-1}
   (b) 0.02 eV s m^{-1}
   (c) 0.03 eV s m^{-1}
   (d) 0.04 eV s m^{-1}

8. A photocell converts
   (a) change in current into change in light intensity
   (b) change in intensity of light into change in current
   (c) change in current into change in voltage
   (d) change in intensity into change in potential difference

9. The de-Broglie wavelength (λ) of equal mass particles depends upon the mass in the following way
   (a) $\lambda \propto m$ (b) $\lambda \propto m^{1/2}$
   (c) $\lambda \propto m^{-1}$ (d) $\lambda \propto m^{-1/2}$

VERY SHORT ANSWER Type Questions

10. Define the term stopping potential in relation to photoelectric effect.

11. Show graphically the variation of photoelectric current with frequency of the incident photons.

12. Two metals M_1 and M_2 have work functions 2 eV and 4 eV, respectively. Which of the two has a higher threshold wavelength for photoelectric emission?

13. The frequency ν of incident radiation is greater than threshold frequency ν_0 in a photocell. How will the stopping potential vary, if frequency ν is increased, keeping other factors constant.

[redacted]

14. The de-Broglie wavelength associated with an electron accelerated through a potential difference V is λ. What will be its wavelength when the accelerating potential is increased to 5 V?

SHORT ANSWER Type Questions

15. What are the energies of photons at the (i) violet and (ii) red ends of the visible spectrum? The wavelength of light is about 390 nm for violet and about 760 nm for red.

16. An electron is accelerated through a potential difference of 250 V. What is the de-Broglie wavelength associated with it? To which part of electromagnetic spectrum does this wavelength correspond?

17. The de-Broglie wavelength of a body moving with speed v is λ. On its way, it losses some of its mass and gains twice the speed. Kinetic energy also increases to twice of its initial value. What will be the new value of de-Broglie wavelength?

18. For what kinetic energy of a neutron, will the associated de-Broglie wavelength be 2.64×10^{-10} m?

19. The de-Broglie wavelength of a particle of kinetic energy K is λ. What would be the wavelength of the particle, if its kinetic energy were $\frac{K}{4}$?

20. Ultraviolet light of wavelength 200 nm is incident on polished surface of iron. Work function of the surface is 4.71 eV. Calculate its stopping potential.

LONG ANSWER Type I Questions

21. Write Einstein's photoelectric equation relating the maximum kinetic energy of the emitted electron to the frequency of the radiation incident on a photosensitive surface. State clearly, the basic elementary process involved in photoelectric effect.

22. Define the terms threshold frequency and stopping potential in the study of photoelectric emission. Explain briefly the reasons, why wave theory of light is not able to explain the observed features in photoelectric effect?

23. Light of wavelength 2500 Å falls on a metal surface of work function 3.5 eV. What is the kinetic energy (in eV) of
    (i) the fastest and
    (ii) the slowest electrons emitted from the surface? If the same light falls on another surface of work function 5.5 eV, what will be the energy of emitted electrons?

24. Light of wavelength 2000 Å falls on a metal surface of work function 4.2 eV.
    (i) What is the kinetic energy (in eV) of the fastest electrons emitted from the surface?
    (ii) What will be the change in the energy of the emitted electrons, if the intensity of light with same wavelength is doubled?
    (iii) If the same light falls on another surface of work function 6.5 eV, what will be the energy of emitted electrons? CBSE SQP (Term-II)

ANSWERS

1. (c) 2. (a) 3. (d) 4. (b) 5. (a)
6. (d) 7. (b) 8. (b) 9. (c)

10. For a particular frequency of incident radiation, the minimum negative (retarding) potential V_0 given to plate A for which the photoelectric current becomes zero, is called cut-off or stopping potential.

11. Refer to plot on page 437.

12. We know that, $E_0 = h\nu_0 = \frac{hc}{\lambda_0}$

Thus, λ_0 or threshold wavelength is inversely proportional to the energy or work-function. So, metal M_1 has higher threshold wavelength for photoelectric emission.

13. We know that, $\frac{1}{2}mv^2_{max} = eV_0 = h(\nu - \nu_0)$

Here, the frequency of the incident radiation is greater than the threshold frequency. Therefore, the value of stopping potential (V_0) increases with increase in frequency (ν) of the incident radiation and KE will also increases.

14. Refer to Q. 3 on page 461.

$$\lambda' = \frac{\lambda}{\sqrt{5}}$$

[redacted]

15. We know that, $E = \frac{hc}{\lambda}$ joule

or $E = \frac{hc}{\lambda \times 1.6 \times 10^{-19}}$ eV

$\therefore \quad \lambda_V = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{390 \times 10^{-9} \times 1.6 \times 10^{-19}} = 3.17\,\text{eV}$

$\therefore \quad \lambda_R = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{760 \times 10^{-9} \times 1.6 \times 10^{-19}} = 1.63\,\text{eV}$

16. Refer to Q. 37 on page 461. [Ans. 49.5 Å]

17. Hint $\lambda = \frac{h}{\sqrt{2mK}}$

de-Broglie wavelength remains same.

18. Refer to the Q. 31 (i) on page 461.

19. As we know, de-Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$

Hence, $K_1 = \frac{h^2}{2m\,\lambda_1^2}$ ...(i)

If according to the question, $K_2 = \frac{K_1}{4}$

$$K_2 = \frac{h^2}{2\lambda_2^2}$$

$$\frac{K_1}{4} = \frac{h^2}{2m\lambda_2^2} \quad \text{..(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\frac{K_1}{4}}{K_1} = \frac{h^2}{2m\lambda_2^2} \times \frac{2m\lambda_1^2}{h^2}$$

$$\frac{1}{4} = \frac{\lambda_1^2}{\lambda_2^2}$$

$$\Rightarrow \quad \frac{\lambda_1}{\lambda_2} = \frac{1}{2}$$

Hence, wavelength of the particle double the wavelength when kinetic energy is $\frac{1}{4}$ th.

20. Given, $\lambda = 200\,\text{nm} = 200 \times 10^{-9}\,\text{m}$

$$KE_{max} = h\nu - \phi$$
$$KE_{max} = eV_0$$
$$eV_0 = h\nu - \phi$$
$$eV_0 = \frac{hc}{\lambda} - \phi$$

$$\therefore 1.6 \times 10^{-19} V_0 = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{200 \times 10^{-9}} - 4.71 \times 1.6 \times 10^{-19}$$

$$V_0 = 6.19 - 4.17 = 1.48 = 1.50\,\text{V}$$

21. Refer to the text on page 438.

22. Refer to the text on pages 436, 437 and 438.

23. Refer to Q. 72 (iii) on page 447.

24. Wavelength of light, $\lambda = 2000\,\text{Å} = 2 \times 10^{-7}$ m

Work function, $\phi_0 = 4.2$ eV

$h = 6.63 \times 10^{-34}$ J-s

(i) Using Einstein's photoelectric equation, kinetic energy of fastest electron,

$$K_{max} = h\nu - \phi_0 = h\frac{c}{\lambda} - \phi_0$$
$$= \left(\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2 \times 10^{-7} \times 1.6 \times 10^{-19}} - 4.2\right) \text{eV}$$
$$= 6.2\,\text{eV} - 4.2\,\text{eV}$$
$$= 2.0\,\text{eV}$$

(ii) Since the energy of emitted electron does not depend upon intensity of incident light, hence the energy of emitted electrons remains unchanged.

(iii) For this surface, electrons will not be emitted as the energy of incident light (6.2 eV) is less than the work function ($\phi' = 6.5$ eV) of the surface.

[redacted]

12

All elements consist of very small invisible particles are called **atoms**. Atoms of same element are same and atoms of different elements are different.

Every atom is a sphere of radius of the order 10^{-10}m, in which entire mass is uniformly distributed and negative charged electrons revolve around the nucleus.

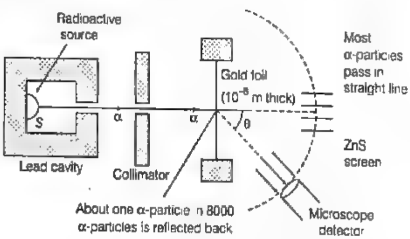
ATOMS

The first model of atom was proposed by JJ Thomson in 1898 called **plum pudding model** of the atom. Later, Rutherford worked on it and named this model as Rutherford's planetary model of atom in 1911. In 1913, Niels Bohr worked on the model named as Bohr model of H-atom.

CHAPTER CHECKLIST
- α-Particles Scattering Experiment by Rutherford
- Rutherford's Model of Atom
- Electron Orbits
- Bohr's Model of Hydrogen Atom
- Hydrogen Spectrum or Line Spectra of Hydrogen Atom

α-PARTICLES SCATTERING EXPERIMENT BY RUTHERFORD

This experiment was suggested by Rutherford in 1911 as given in the figure below



Experimental arrangement for Rutherford's theory

In this experiment, H Geiger and E Marsden took $^{214}_{83}$Bi as a source for α-particles. A collimated beam of α-particles of energy 5.5 MeV was allowed to fall on 2.1×10^{-7} m thick gold foil. The α-particles were observed through a rotatable detector consisting of a zinc sulphide screen and microscope and it was found that α-particles got scattered. These scattered α particles produced scintillations on the zinc sulphide screen. Now, these scintillations were counted at different angles from the direction of incident beam.

[redacted]

Observations

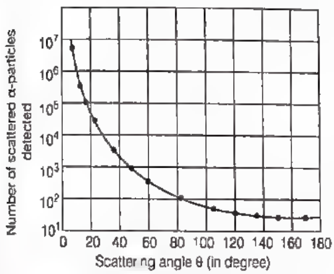
Rutherford made the following observations from his experiment that are given below

(i) Most of the α-particles passed through the gold foil without any appreciable deflection.

(ii) Only about 0.14% of the incident α-particles scattered by more than 1°.

(iii) About one α-particle in every 8000 α-particles deflected by more than 90°.

The total number of α-particles (N) scattered through an angle (θ) is as shown in the below figure



Experimental data points (shown by dots) on scattering of α-particles by a thin foil at different angles

(iv) The number of α-particles scattered per unit area $N(\theta)$ at scattering angle θ varies inversely as $\sin^4 \theta/2$.

$$\boxed{N(\theta) \propto \frac{1}{\sin^4 \theta/2}}$$

(v) The force between α-particles and nucleus is given by

$$\boxed{F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(2e)(Ze)}{r^2}}$$

where, r is the distance between the α-particles and the nucleus. This force is directed along the line joining the α-particle and the nucleus. The magnitude and direction of this force on α-particle continuously changes as it approaches the nucleus and recedes away from it.

Conclusions

On the basis of his experiment, Rutherford concluded that

(i) Atom has a lot of empty space and practically the entire mass of the atom is confined to an extremely small central core called nucleus, whose size is of the order from 10^{-15} m to 10^{-14} m.

(ii) Scattering of α-particles (positively charged) is due to the Coulomb's law for electrostatic force of repulsion between the positive charge of nucleus and α-particles.

(iii) Distance between electron and nucleus is from 10^4 to 10^5 times the size of the nucleus itself.

(iv) More is the distance of the velocity vector of an α-particle from the central line of the nucleus, lesser is the angle of scattering.

EXAMPLE |1| The number of α-particles scattered at an angle of 90° is 100 per minute. What will be the number of α-particles, when it is scattered at an angle of 60°?

Sol. Number of α-particles scattered at an angle of θ is given by

$$N \propto \frac{1}{\sin^4 \theta/2} \Rightarrow \frac{N_1}{N_2} = \left(\frac{\sin\frac{\theta_2}{2}}{\sin\frac{\theta_1}{2}}\right)^4$$

$$\Rightarrow \frac{100}{N_2} = \left(\frac{\sin 30°}{\sin 45°}\right)^4 \Rightarrow \frac{100}{N_2} = \frac{4}{16} \Rightarrow N_2 = 400$$

RUTHERFORD'S MODEL OF ATOM

The essential features of Rutherford's nuclear model of the atom or planetary model of the atom are as follows

(i) Every atom consists of a central core, called the atomic nucleus, in which the entire positive charge and almost entire mass of the atom is concentrated.

(ii) The size of nucleus is of the order of 10^{-15} m, which is very small as compared to the size of the atom which is of the order of 10^{-10} m.

(iii) The atomic nucleus is surrounded by certain number of electrons. As atom on the whole is electrically neutral, the total negative charge of electrons surrounding the nucleus is equal to total positive charge on the nucleus.

(iv) These electrons revolve around the nucleus in various circular orbits as the planets do around the sun.

The centripetal force required by electrons for revolution is provided by the electrostatic force of attraction between the electrons and nucleus.

Distance of Closest Approach

As the α-particle approaches the nucleus, the electrostatic force of repulsion due to nucleus increases and the kinetic energy of α-particle goes on converting into the electrostatic potential energy.

At a certain distance r_0 from the nucleus, whole of the KE of α-particle converts into electrostatic potential energy and α-particles cannot go further close to nucleus, this distance (r_0) is called **distance of closest approach**.

[redacted]

At distance of closest approach,

KE of α-particle = Electrostatic potential energy

$$K = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(Ze)(2e)}{r_0}$$

[∵ charge on α-particle is $+2e$ and charge on nucleus is Ze, where Z is atomic number]

$$\therefore \quad r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{K} \text{ or } \boxed{r_0 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2Ze^2}{\left(\frac{1}{2}mv^2\right)}}$$

where, m = mass of α-particle and v = initial velocity of α-particle.

From the formula, it is clear that distance of closest approach of α-particle to the nucleus depends on the kinetic energy of α-particle.

EXAMPLE |2| In a head on collision between an α-particle and gold nucleus, the closest distance of approach is 4×10^{-14} m. Calculate the initial kinetic energy of α-particle.

Sol. Here, closest distance of approach, $r_0 = 4 \times 10^{-14}$ m, atomic number, $Z = 79$, $KE_i = ?$

$$\therefore \; KE_i \text{ of α-particle} = \frac{Ze(2e)}{4\pi\varepsilon_0 r_0} = \frac{2Ze^2}{4\pi\varepsilon_0 r_0}$$

$$= \frac{2 \times 79 \times (1.6 \times 10^{-19})^2 \times 9 \times 10^9}{4 \times 10^{-14}}$$

$$= 9.1 \times 10^{-13}\text{ J}$$

Angle of Scattering (θ)

Angle by which α-particle gets deviated from its original path around the nucleus is called angle of scattering.

Impact Parameter (b)

Perpendicular distance of the velocity vector of α-particle from the central line of the nucleus of the atom is called impact parameter. Mathematically, it is expressed as

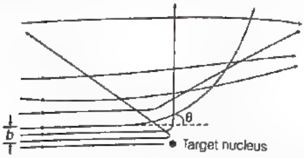
$$\boxed{b = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2 \cot\frac{\theta}{2}}{\text{KE}}}$$

where, b = impact parameter

θ = angle of scattering

KE = kinetic energy of α-particle $= \frac{1}{2}mv^2$.

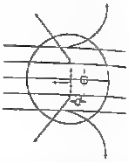
In case of head on collision, the impact parameter is minimum and the α-particle rebounds back (θ = π). For a large impact parameter, the α-particle goes nearly undeviated and has a small deflection (θ = 0°).



Trajectory of α-particles in the Coulombic field of a target nucleus. The impact parameter b and scattering angle θ are also depicted

Alpha-Particle Trajectory

The α-particles which pass through the atom at a large distance from the nucleus experience a small electrostatic force of repulsion due to the nucleus and hence, undergo a very small deflection. The α-particles which pass through the atom at a close distance from the nucleus suffer a large deflection. The α-particles which travel towards the nucleus directly, slow down and ultimately comes to rest and then after being deflected through 180° retrace their path.



Trajectory of α-particles close to an atom

ELECTRON ORBITS

The Rutherford nuclear model of the atom pictures the atom as an electrically neutral sphere consisting of a very small, massive and positively charged nucleus at the centre surrounded by the revolving electrons in their respective dynamically stable orbits.

The electrostatic force of attraction F_e between the revolving electrons and the nucleus provides the requisite centripetal force (F_c) to keep them in their orbits. Thus, for a dynamically stable orbit in a H-atom,

$$F_c = F_e$$

$$\Rightarrow \quad \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r^2} \qquad [\because Z = 1]$$

Thus, the relation between the orbit radius and the electron velocity is

$$r = \frac{e^2}{4\pi\varepsilon_0 mv^2}$$

The kinetic energy (K) and electrostatic potential energy (U) of the electron in H-atom are

[redacted]

$$K = \frac{1}{2}mv^2 = \frac{e^2}{8\pi\varepsilon_0 r} \qquad \left[\because mv^2 = \frac{e^2}{4\pi\varepsilon_0 r}\right]$$

and $$U = -\frac{e^2}{4\pi\varepsilon_0 r}$$

(the negative sign in U signifies that the electrostatic force is in the $-r$ direction or attractive in nature.)

Thus, the total mechanical energy E of the electron in a H-atom is

$$E = K + U = \frac{e^2}{8\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$\boxed{E = -\frac{e^2}{8\pi\varepsilon_0 r}}$$

The total energy of the electron is negative. This implies the fact that the electron is bound to the nucleus. If E is positive, then an electron will not follow a closed orbit around the nucleus and it would leave the atom.

EXAMPLE |3| It is found experimentally that 13.6 eV energy is required to separate a H-atom into a proton and an electron. Compute the orbital radius and velocity of the electron in a H-atom.

Sol. Total energy of the electron in H-atom,

$$TE = -13.6 \text{ eV} = -13.6 \times 1.6 \times 10^{-19} \text{ J}$$
$$= -2.2 \times 10^{-18} \text{ J}$$

Total energy is

$$TE = \frac{-e^2}{8\pi\varepsilon_0 r} \Rightarrow r = \frac{-e^2}{8\pi\varepsilon_0 TE}$$
$$= \frac{-9 \times 10^9 \times (1.6 \times 10^{-19})^2}{2 \times (-2.2 \times 10^{-18})}$$
$$= 5.3 \times 10^{-11} \text{ m}$$

$\therefore$ Velocity of the revolving electron, $v = \frac{e}{\sqrt{4\pi\varepsilon_0 mr}}$

$$= \frac{1.6 \times 10^{-19}}{\sqrt{4 \times 3.14 \times 8.85 \times 10^{-12} \times 9.1 \times 10^{-31} \times 5.3 \times 10^{-11}}}$$
$$= 2.2 \times 10^6 \text{ m/s}$$

Drawbacks of Rutherford's Model

Rutherford's model suffers two major drawbacks

Regarding Stability of Atom

Electrons revolving around the nucleus have centripetal acceleration. According to classical electromagnetic theory, the electrons must radiate energy in the form of electromagnetic wave.

Due to this continuous loss of energy of the electrons, the radii of their orbits should be continuously decreasing and ultimately the electrons should fall in the nucleus. Thus, atom cannot remain stable.

Regarding Explanation of Line Spectrum

Due to continuous decrease in radii of electron's orbit, the frequency of revolution of electron will also change. According to classical theory of electromagnetism, frequency of EM wave emitted by electron is equal to frequency of revolution of electron.

So, due to continuous change in frequency of revolution of electron, it will radiate EM waves of all frequencies, i.e. the spectrum of these waves will be continuous in nature. But, this is not the case, experimentally we get line spectrum. Rutherford model was unable to explain line spectrum.

BOHR'S MODEL OF HYDROGEN ATOM

Bohr combined classical and early quantum concepts and gave his theory in the form of three postulates

These three postulates are as follows

(i) **Bohr's first postulate** was that an electron in an atom could revolve in certain stable orbits without the emission of radiant energy, contrary to the predictions of electromagnetic theory. According to this postulate, each atom has certain definite stable states in which it can exist and each possible state has definite total energy. These are called the **stationary states** of the atom.

(ii) **Bohr's second postulate** states that the electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $h/2\pi$, where h is the Planck's constant $(= 6.63 \times 10^{-34} \text{ J-s})$.

Thus, the angular momentum (L) of the orbiting electron is quantised,

i.e. $$L = \frac{nh}{2\pi}$$

As, angular momentum of electron = mvr

$\therefore$ For any permitted (stationary) orbit

$$\boxed{mvr = \frac{nh}{2\pi}}$$

where, n = any positive integer 1, 2, 3, ...

It is also called **principal quantum number.**

[redacted]

(iii) Bohr's third postulate states that an electron might make a transition from one of its specified non-radiating orbits to another of lower energy. When it does so, a photon is emitted having energy equal to the energy difference between the initial and final states.

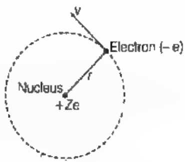
The frequency of the emitted photon is given by

$$h\nu = E_i - E_f$$

where, E_i and E_f are the energies of the initial and final states and $E_i > E_f$.

Bohr's Theory

Bohr's model is valid for all one-electron atoms or ions which consists of a tiny positively charged nucleus and an electron revolving in a stable circular orbit around the nucleus. These one-electron atoms or ions can be called hydrogen like atoms. For example, singly ionised helium (He^+) and doubly ionised lithium (Li^{2+})



Let e, m and v be respectively the charge, mass and velocity of the electron and r be the radius of the orbit. The positive charge on the nucleus is Ze, where Z is the atomic number (in case of H-atom, $Z = 1$). As, the centripetal force is provided by the electrostatic force of attraction, we have

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(Ze) \times e}{r^2}$$

$$\Rightarrow \quad mv^2 = \frac{Ze^2}{4\pi\varepsilon_0 r} \quad \text{...(i)}$$

From the second postulate, the angular momentum of the electron is

$$mvr = n\frac{h}{2\pi} \quad \text{...(ii)}$$

where, n (= 1, 2, 3, ...) is principal quantum number.

From Eqs. (i) and (ii), we get

$$\boxed{r = n^2 \frac{h^2\varepsilon_0}{\pi m Z e^2}} \quad \text{...(iii)}$$

This is the equation for the radii of the permitted orbits. According to this equation,

$$r_n \propto n^2$$

Since, $n = 1, 2, 3, \ldots$ it follows that the radii of the permitted orbits increase in the ratio 1 : 4 : 9 : 16 : ..., from the first orbit. Clearly, the stationary orbits are not equally spaced.

Bohr Radius

The radius of the first orbit ($n = 1$) of H-atom ($Z = 1$) will be

$$\boxed{r_1 = \frac{h^2\varepsilon_0}{\pi m e^2}}$$

This is called Bohr radius and its value is 0.53 Å. Since, $r \propto n^2$, the radius of the second orbit of H-atom will be (4×0.53) Å and that of the third orbit (9×0.53) Å.

Velocity of Electron in Stationary Orbits

We can obtain formula for the velocity of electron in permitted orbits. From Eq. (ii), we have

$$v = n\frac{h}{2\pi mr}$$

Putting the value of r from Eq. (iii), we get

$$v = \frac{Ze^2}{2h\varepsilon_0} \cdot \frac{1}{n}$$

where, principal quantum number, $n = 1, 2, 3, \ldots$

Thus, $\quad v \propto \frac{1}{n}$

This shows that the velocity of electron is maximum in the lowest orbit ($n = 1$) and goes on decreasing in higher orbits. The velocity of electron in the first orbit ($n = 1$) of H-atom ($Z = 1$) is

$$\boxed{v_1 = \frac{e^2}{2h\varepsilon_0} = \frac{c}{137}} \quad [\because c = 3 \times 10^8 \text{ m/s}]$$

Frequency of Electron in a Stationary Orbit

It is the number of revolutions completed per second by the electron in a stationary orbit around the nucleus.

It is represented by ν.

From $\quad v = r\omega$

$$= r(2\pi\nu) \quad [\because \omega = 2\pi\nu]$$

$$\therefore \quad \nu = \frac{v}{2\pi r}$$

Putting the values of v and r in above equation, we get

$$\nu = \frac{1}{2\pi r} \cdot \frac{Ze^2}{2h\varepsilon_0} \cdot \frac{1}{n}$$

$$\Rightarrow \quad \nu = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Ze^2}{nhr}$$

$$\boxed{\nu = \frac{kZe^2}{nhr}} \quad \left[\because k = \frac{1}{4\pi\varepsilon_0}\right]$$

[redacted]

Energy of Electron in Stationary Orbits

The energy E of an electron in an orbit is the sum of kinetic and potential energies.

Using Eq. (i) in Bohr's theory $mv^2 = \frac{Ze^2}{4\pi\varepsilon_0 r}$

The kinetic energy of the electron is

$$KE = \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r}$$

Substituting for r from Eq. (iii), we get kinetic energy of the electron in the nth orbit

$$\boxed{KE = \frac{mZ^2e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n^2}\right)}$$

In terms of Rydberg constant R, its simplified form is

$$KE = \frac{Rhc}{n^2} \qquad \left[\because R = \frac{me^4}{8\varepsilon_0^2 ch^3}\right]$$

The potential energy of the electron in an orbit of radius r due to the electrostatic attraction by the nucleus is given by

$$PE = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(Ze)(-e)}{r} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze^2}{r}$$

In terms of Rydberg constant R, its simplified form is

$$PE = -\frac{2Rhc}{n^2}$$

The total energy of the electron is

$$E = KE + PE = \frac{Ze^2}{8\pi\varepsilon_0 r} - \frac{Ze^2}{4\pi\varepsilon_0 r}$$

$$= -\frac{Ze^2}{8\pi\varepsilon_0 r} = -\frac{Rhc}{n^2}$$

Substituting for r from Eq. (iii), we get

$$\boxed{E = -\frac{mZ^2e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n^2}\right)}$$

where, $n = 1, 2, 3, \ldots$. This is the expression for the energy of the electron in the nth orbit.

For hydrogen atom $Z = 1$, substituting the standard values, we get $E_n = \frac{-13.6}{n^2}$ eV. Negative energy of the electron shows that the electron is bound to the nucleus and is not free to leave it.

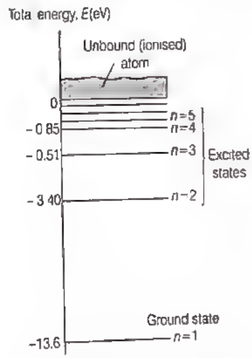
This topic is not included into the syllabus but essential to understand line spectrum of hydrogen atom.

Energy Levels

The energy of an atom is the least, when its electron is revolving in an orbit closest to the nucleus, i.e. for which $n = 1$. For $n = 2, 3, \ldots$ the absolute value of energy E is smaller, so the energy is progressively larger in outer orbits.

The lowest state of the atom is called the **ground state**, this state has lowest energy. The energy of this state is -13.6 eV. Therefore, the minimum energy required to free the electron from the ground state of the H-atom is -13.6 eV. It is called ionisation energy of the H-atom.

At room temperature, most of the H-atoms are in ground state. When an atom receives some energy (i.e. by electron collisions), the atom may acquire sufficient energy to raise electron to higher energy state. In this condition, the atom is said to be in excited state. From the excited state, the electron can fall back to a state of lower energy, emitting a photon equal to the energy difference of the orbit.



Energy level diagram for hydrogen atom

Suppose in the excited atom, an electron jumps from some higher energy state n_2 to a lower energy state n_1.

The energy difference between these states is

$$E_2 - E_1 = \frac{mZ^2e^4}{8\varepsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

According to Bohr's third postulate, the frequency ν of the emitted electromagnetic wave (photon) is

$$\nu = \frac{E_2 - E_1}{h} = \frac{mZ^2e^4}{8\varepsilon_0^2 h^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

[redacted]

The corresponding wavelength λ of the emitted radiation is given by

$$\boxed{\frac{1}{\lambda}=\frac{\nu}{c}=\frac{mZ^2e^4}{8\varepsilon_0^2ch^3}\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)}$$

$\frac{1}{\lambda}$ is called wave number (number of waves per unit length).

In the last equation, the quantity $\frac{me^4}{8\varepsilon_0^2ch^3}$ is a constant known as Rydberg constant R and its value is $1.097\times10^7\ m^{-1}$.

i.e. $$\frac{me^4}{8\varepsilon_0^2ch^3}=R.$$

Thus, $$\boxed{\frac{1}{\lambda}=Z^2R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)}$$

This is Bohr's formula for hydrogen and hydrogen like atoms (He^+, Li^{2+},...).

For hydrogen atom ($Z = 1$), we have

$$\boxed{\frac{1}{\lambda}=R\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)}$$

EXAMPLE |4|

(i) The radius of the innermost electron orbit of a hydrogen atom is 5.3×10^{-11} m. Calculate its radius in $n=2$ orbit.

(ii) The total energy of an electron in the second excited state of the hydrogen atom is −1.51 eV. Find out its
(a) kinetic energy and
(b) potential energy in this state. **Delhi 2014**

Sol. (i) Given, Bohr radius, $r_1=5.3\times10^{-11}$m

We know that, $r_n=n^2r_1$

Let r_2 be radius of the orbit for $n=2$.

$\therefore$ $r_2=(2)^2\times5.3\times10^{-11}$
$=2.12\times10^{-10}$m

(ii) Given, total energy of an electron in second excited state,
$E=-1.51$ eV

(a) Kinetic energy of electron is equal to negative of the total energy.
$\therefore$ $K=-E=-(-1.51)$
$=1.51$ eV

(b) Potential energy of electron is equal to negative of twice of its kinetic energy.
$U=-2K=-2\times1.51=-3.02$ eV

Hydrogen Spectrum or Line Spectra of Hydrogen Atom

Hydrogen spectrum consists of discrete bright lines in a dark background and it is specifically known as hydrogen emission spectrum. There is one more type of hydrogen spectrum that exists where we get dark lines on the bright background, it is known as absorption spectrum.

Balmer found an empirical formula by the observation of a small part of this spectrum and it is represented by

$$\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n^2}\right),\ \text{where } n=3,4,5,\ldots$$

where, R is a constant called Rydberg constant and its value is $1.097\times10^7\ m^{-1}$.

So, $\frac{1}{\lambda}=1.522\times10^6\ m^{-1}=656.3$ nm for $n=3$

Other series of spectra for hydrogen were subsequently discovered and known by the name of their discoverers. The lines of Balmer series are found in the visible part of the spectrum. Other series were found in the invisible parts of the spectrum.

e.g. Lyman series in the ultraviolet region and Paschen, Brackett and Pfund in the infrared region.

The wavelengths of line in these series can be expressed by the following formulae

(i) For Lyman series
$$\frac{1}{\lambda}=R\left(\frac{1}{1^2}-\frac{1}{n^2}\right),\ \text{where } n=2,3,4,\ldots$$

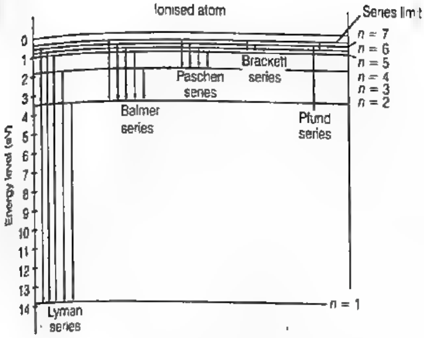
(ii) For Balmer series
$$\frac{1}{\lambda}=R\left(\frac{1}{2^2}-\frac{1}{n^2}\right),\ \text{where } n=3,4,5,\ldots$$

(iii) For Paschen series
$$\frac{1}{\lambda}=R\left(\frac{1}{3^2}-\frac{1}{n^2}\right),\ \text{where } n=4,5,6,\ldots$$

(iv) For Brackett series
$$\frac{1}{\lambda}=R\left(\frac{1}{4^2}-\frac{1}{n^2}\right),\ \text{where } n=5,6,7,\ldots$$

(v) For Pfund series
$$\frac{1}{\lambda}=R\left(\frac{1}{5^2}-\frac{1}{n^2}\right),\ \text{where } n=6,7,8,\ldots$$

[redacted]



Line spectra of the H-atom

> ***Balmer Series In Emission Spectrum Of Hydrogen***
>
> In Balmer series, the line with the longest wavelength (656.3 nm) is red and is called H_α. Next line with wavelength 486.1 nm is blue-green and is called H_β, the third line with 434.1 nm is violet and is called H_γ and so on. As the wavelength decreases, the lines are weaker in intensity and appear closer together.

Explanation

The different series of hydrogen spectrum can be explained by Bohr's theory. According to Bohr's theory, if the ionised state of hydrogen atom be taken as zero energy level, then the energies of the different energy levels of the atom can be expressed by the following formula.

$$E_n = -\frac{Rhc}{n^2}, \text{ where } n = 1, 2, 3, \ldots$$

where, R is Rydberg constant and h is Planck constant. The integer n is called principal quantum number.

When the atom gets energy from outside, its electron goes from the lowest energy level to some higher energy level. But it returns from there, within 10^{-8} s, to the lowest energy level directly or through other lower energy levels. While returning back, the atom emits photons.

EXAMPLE |5| The energy of the electron in the ground state of hydrogen is − 13.6 eV. Calculate the energy of the photon that would be emitted, if electron was to make a transition corresponding to the emission of the first line of the Lyman series of the H-atom.

Sol. Here, energy of e^- in ground state of H-atom = −13.6 eV

i.e. $E_1 = -13.6$ eV

For $n = 2$, $E_2 = -3.4$ eV $\left[\because E_n = -\frac{13.6}{n^2} \text{ eV}\right]$

The energy of photon corresponding to the first line is given by $E = E_2 - E_1$

$\therefore$ $E = [-3.4 - (-13.6)]$ eV $= 10.2$ eV

EXAMPLE |6| In H-atom, a transition takes place from $n = 3$ to $n = 2$ orbit. Calculate the wavelength of the emitted photon, will the photon be visible? To which spectral series will this photon belong?

(Take, $R = 1.097 \times 10^7 \text{ m}^{-1}$)

Sol. The wavelength of the emitted photon is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

When the transition takes place from $n = 3$ to $n = 2$, then

$$\frac{1}{\lambda} = (1.097 \times 10^7)\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.097 \times 10^7 \times \frac{5}{36}$$

$$\therefore \quad \lambda = \frac{36}{1.097 \times 10^7 \times 5} = 6.563 \times 10^{-7} \text{ m} = 6563 \text{ Å}$$

Since, λ falls in the visible (red) part of the spectrum, hence the photon will be visible. This photon is the first member of the Balmer series.

de-Broglie's Comment on Bohr's Second Postulate

According to de-Broglie, a stationary orbit is that which contains an integral number of de-Broglie standing waves associated with the revolving electron.

For an electron revolving in nth circular orbit of radius r_n, total distance covered = circumference of the orbit = $2\pi r_n$.

$\therefore$ For the permissible orbit, $2\pi r_n = n\lambda$

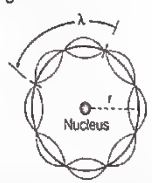
According to de-Broglie wavelength, $\lambda = \frac{h}{mv_n}$

where, v_n is speed of electron revolving in nth orbit.

$$\therefore \quad 2\pi r_n = \frac{nh}{mv_n}$$

or $$mv_n r_n = \frac{nh}{2\pi} = n(h/2\pi)$$

i.e. angular momentum of electron revolving in nth orbit must be an integral multiple of $h/2\pi$, which is the quantum condition proposed by Bohr in his second postulate.



A standing wave is shown on a circular orbit

EXAMPLE |7| When an electron in hydrogen atom jumps from the third excited state to the ground state, how would the de-Broglie wavelength associated with the electron change? Justify your answer. **Delhi 2015**

Sol. We know that, $\lambda = \frac{h}{p} = \frac{h}{mv}$ or $mv = \frac{h}{\lambda}$

[redacted]

or $\quad mvr = \frac{hr}{\lambda} = \frac{nh}{2\pi}$ or $\lambda = \frac{2\pi}{nh} \times hr = \frac{2\pi r}{n}$

As, $\quad r \propto n^2 \Rightarrow \lambda \propto \frac{1}{n}(n^2) = n$

Thus, we can say that, $\frac{\lambda_4}{\lambda_1} = \frac{4}{1}$ or $\lambda_1 = \frac{\lambda_4}{4}$

Thus, wavelength decreases 4 times as an electron jumps from third excited state to the ground state.

Limitations of Bohr's Model

The limitations of Bohr's model are as follows

(i) This model is applicable only to a simple atom like hydrogen having $Z = 1$. This theory fails, if $Z > 1$.

(ii) It does not explain the fine structure of spectral lines in H-atom.

(iii) This model does not explain why orbits of electrons are taken as circular whereas elliptical orbits are also possible.

Orbital Picture of Electron in an Atom

With the development of quantum mechanics, we have a better understanding of structure of atom. The Schrodinger wave equation gives information about the probability of finding an electron in various regions around the nucleus, which is known as orbital. This function only depends on the coordinates of the electron.

CHAPTER PRACTICE (SOLVED)

OBJECTIVE Type Questions

1. For scattering of α-particles, Rutherford's suggested that

(a) mass of atom and its positive charge were concentrated at centre of atom
(b) only mass of atom is concentrated at centre of atom
(c) only positive charge of atom is concentrated at centre of atom
(d) mass of atom is uniformly distributed throughout its volume

2. In the α-particle scattering experiment, the shape of the trajectory of the scattered α-particles depend upon [CBSE 2020]

(a) only on impact parameter
(b) only on the source of α-particles
(c) Both impact parameter and source of α-particles
(d) impact parameter and the screen material of the detector

3. The angular momentum of an electron in hydrogen atom in ground state is

(a) $\frac{h}{\pi}$ (b) $\frac{h}{2\pi}$ (c) $\frac{2\pi}{h}$ (d) $\frac{\pi}{h}$

4. If the orbital radius of the electron in a hydrogen atom is 4.7×10^{-11} m. Compute the kinetic energy of the electron in hydrogen atom.

(a) 15.3 eV (b) − 15.3 eV (c) 13.6 eV (d) −13.6 eV

5. A set of atoms in an excited state decays NCERT Exemplar

(a) in general to any of the states with lower energy
(b) into a lower state only when excited by an external electric field
(c) all together simultaneously into a lower state
(d) to emit photons only when they collide

6. In Pfund series, ratio of maximum to minimum wavelength of emitted spectral lines is

(a) $\frac{\lambda_{max}}{\lambda_{min}} = \frac{4}{3}$ (b) $\frac{\lambda_{max}}{\lambda_{min}} = \frac{9}{5}$
(c) $\frac{\lambda_{max}}{\lambda_{min}} = \frac{16}{7}$ (d) $\frac{\lambda_{max}}{\lambda_{min}} = \frac{36}{11}$

7. Paschen series of atomic spectrum of hydrogen gas lies in CBSE All India 2020

(a) infrared region
(b) ultraviolet region
(c) visible region
(d) partly in ultraviolet and visible region

Direction (Q. Nos. 8-12) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

8. Assertion Large angle of scattering of alpha particles led to the discovery of atomic nucleus.

Reason Entire positive charge of atom is concentrated in the central core.

9. Assertion Atom as a whole is electrically neutral.

Reason Atom contains equal amount of positive and negative charges.

[redacted]

10. **Assertion** The total energy of an electron revolving in any stationary orbit is negative.
Reason Energy can have positive or negative values.

11. **Assertion** Atoms of each element are stable and emit characteristic spectrum.
Reason The spectrum provides useful information about the atomic structure.

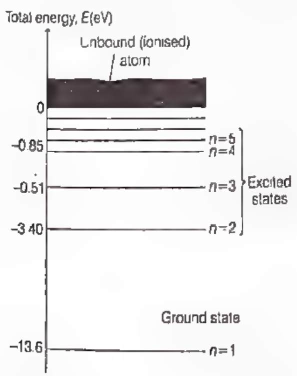
12. **Assertion** Bohr's postulate states that the stationary orbits are those for which the angular momentum is some integral multiple of $\frac{h}{2\pi}$.
Reason Linear momentum of the electron in the atom is quantised.

Directions (Q.Nos. 13-14) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

13. **Excited State of Atom**

At room temperature, most of the H-atoms are in ground state. When an atom receives some energy (i.e. by electron collisions), the atom may acquire sufficient energy to raise electron to higher energy state. In this condition, the atom is said to be in excited state.

From the excited state, the electron can fall back to a state of lower energy, emitting a photon equal to the energy difference of the orbit.



In a mixture of H–He$^+$ gas (He$^+$ is single ionized He atom), H-atoms and He$^+$ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He$^+$ ions (by collisions).

(i) The quantum number n of the state finally populated in He$^+$ ions is
(a) 2 (b) 3
(c) 4 (d) 5

(ii) The wavelength of light emitted in the visible region by He$^+$ ions after collisions with H-atoms is
(a) 6.5×10^{-7} m (b) 5.6×10^{-7} m
(c) 4.8×10^{-7} m (d) 4.0×10^{-7} m

(iii) The ratio of kinetic energy of the electrons for the H-atom to that of He$^+$ ion for $n=2$ is
(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) 1 (d) 2

(iv) The radius of the ground state orbit of H-atom is
(a) $\frac{\varepsilon_0}{h\pi me^2}$ (b) $\frac{h^2\varepsilon_0}{\pi me^2}$ (c) $\frac{\pi me^2}{h}$ (d) $\frac{2\pi h\varepsilon_0}{me^2}$

(v) Angular momentum of an electron in H-atom in first excited state is
(a) $\frac{h}{\pi}$ (b) $\frac{h}{2\pi}$ (c) $\frac{2\pi}{h}$ (d) $\frac{\pi}{h}$

14. **α-Particle Scattering Experiment**

In this experiment, H. Geiger and E. Marsden took radioactive source ($^{214}_{83}$Bi) for α-particles.

A collimated beam of α-particles of energy 5.5 MeV was allowed to fall on 2.1×10^{-7} m thick gold foil. The α-particles were observed through a rotatable detector consisting of a zinc sulphide screen & microscope and it was found that α-particles got scattered. These scattered α-particles produced scintillations on the zinc sulphide screen.

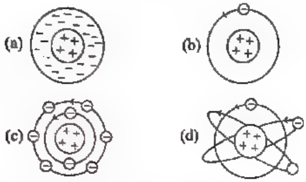
Observations of this experiment are as follows

I. Many of the α-particles pass through the foil without deflection.
II. Only about 0.14% of the incident α-particles scattered by more than 1°.
III. Only about one α-particle in every 8000 α-particles deflected by more than 90°.

Based on these observation, they were able to proposed a nuclear model of atom, are called planetary model, in which entire positive charge and most of the mass of atom is concentrated in a small volume called the nucleus with electron revolving around the nucleus as planets revolve around the sun.

[redacted]

(i) Rutherford's atomic model can be visualised as



(ii) Gold foil used in Geiger-Marsden experiment is about 10^{-8} m thick. This ensures
 (a) gold foil's gravitational pull is small or possible
 (b) gold foil is deflected when α-particle stream is not incident centrally over it
 (c) gold foil provides no resistance to passage of α-particles
 (d) most α-particle will not suffer more than 1° scattering during passage through gold foil

(iii) In Geiger-Marsden experiment, detection of α-particles scattered at a particular angle is done by
 (a) counting flashes produced by α-particles on a ZnS coated screen
 (b) counting spots produced on a photographic film
 (c) using a galvanometer detector
 (d) using a Geiger-counter

(iv) Atoms consist of a positively charged nucleus is obviously from the following observation of Geiger-Marsden experiment
 (a) most of α-particles pass straight through the gold foil
 (b) many of α-particles are scattered through the acute angles
 (c) very large number of α-particles are deflected by large angles
 (d) None of the above

(v) The fact that only a small fraction of the number of incident particles rebound back in Rutherford scattering indicates that
 (a) number of α-particles undergoing head-on collision is small
 (b) mass of the atom is concentrated in a small volume
 (c) mass of the atom is concentrated in a large volume
 (d) Both (a) and (b)

VERY SHORT ANSWER Type Questions

15. Why is the classical (Rutherford) model for an atom of electron orbiting around the nucleus not able to explain the atomic structure? **Delhi 2012**

16. What is the ratio of radii of the orbits corresponding to first excited state and ground state in a H-atom? **Delhi 2010**

17. Consider two different H-atoms. The electron in each atom is in an excited state.

Is it possible for the electrons to have different energies, but the same orbital angular momentum according to the Bohr's model? **NCERT Exemplar**

18. What is the value of angular momentum of electron in the second orbit of Bohr's model of hydrogen atom? **CBSE SQP (Term-I)**

19. When H_α/-line of the Balmer series in the emission spectrum of H-atom is obtained? **Delhi 2013C**

20. Imagine removing one electron from He^4 and He^3. Their energy levels, as worked out on the basis of Bohr's model will be very close. Explain, why? **NCERT Exemplar**

> **Hints:** Niels Bohr proposed a model for hydrogenic (single electron) atoms in order to explain the stability of atoms.

SHORT ANSWER Type Questions

21. Define the distance of closest approach. An α-particle of kinetic energy K is bombarded on a thin gold foil. The distance of the closest approach is r. What will be the distance of closest approach for an α-particle of double the kinetic energy? **All India 2016**

22. An α-particle moving with initial kinetic energy K towards a nucleus of atomic number Z approaches a distance d at which it reverses its direction. Obtain the expression for the distance of closest approach d in terms of the kinetic energy of α-particle K. **Compt. 2016**

23. Using Rutherford's model of the atom, derive the expression for the total energy of the electron in H-atom. What is the significance of total negative energy possessed by the electron? **All India 2014**

[redacted]

24. Explain in brief, why Rutherford's model cannot account for the stability of an atom? **Delhi 2010**

25. Write shortcomings of Rutherford atomic model. Explain, how these were overcome by the postulates of Bohr's atomic model. **CBSE 2020**

26. State Bohr's postulate of hydrogen atom that gives the relationship for the frequency of emitted photon in a transition. **Foreign 2016**

27. Use Bohr's model of hydrogen atom to obtain the relationship between the angular momentum and the magnetic moment of the revolving electron. **CBSE 2020**

28. Show that the radius of the orbit in hydrogen atom varies as n^2, where n is the principal quantum number of the atom. **All India 2015**

29. Using Bohr's postulates of the atomic model, derive the expression for radius of nth electron orbit. Hence, obtain the expression for Bohr's radius. **All India 2014, Delhi 2010**

30. How is the stability of hydrogen atom in Bohr model explained by de-Broglie's hypothesis? **CBSE 2019**

31. Would the Bohr's formula for the H-atom remains unchanged, if proton had a charge $(+4/3)\,e$, and electron had a charge $(-3/4)e$, where, $e = 1.6 \times 10^{-19}$ C. Give reasons for your answer. **NCERT Exemplar**

32. Consider two different hydrogen atoms. The electron in each atom is in an excited state. Is it possible for the electrons to have different energies but same orbital angular momentum according to the Bohr model? Justify your answer. **CBSE SQP (Term-II)**

33. Positronium is just like a H-atom with the proton replaced by the positively charged anti-particle of the electron (called the positron which is as massive as the electron). What would be the ground state energy of positronium? **NCERT Exemplar**

34. How many different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to states with principal quantum number n?

35. State Bohr's quantisation condition of angular momentum. Calculate the shortest wavelength of the Brackett series and state to which part of the electromagnetic spectrum does it belong. **CBSE 2019**

36. Calculate the orbital period of the electron in the first excited state of hydrogen atom. **CBSE 2019**

LONG ANSWER Type I Questions

37. Draw a plot of α-particle scattering by a thin foil of gold to show the variation of the number of the scattered particles with scattering angle. Describe briefly how the large angle scattering explains the existence of the nucleus inside the atom. Explain with the help of impact parameter picture, how Rutherford scattering serves a powerful way to determine an upper limit on the size of the nucleus. **CBSE All India 2019**

38. Using the relevant Bohr's postulates, derive the expression for the
    (i) velocity of the electron in the nth orbit
    (ii) radius of the nth orbit of the electron in H-atom. **Delhi 2010**

39. Using the postulates of Bohr's model of H-atom, obtain an expression for the frequency of radiation emitted when the atom makes a transition from the higher energy state with quantum number n_i to the lower energy state with quantum number $n_f (n_f < n_i)$. **Foreign 2011**

40. Using Bohr's postulates for H-atom, show that the total energy (E) of the electron in the stationary states can be expressed as the sum of kinetic energy (K) and potential energy (U), where $K = -2U$. Hence, deduce the expression for the total energy in the nth energy level of hydrogen atom. **Foreign 2012**

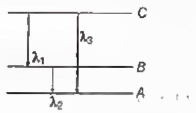
41. (i) Using Bohr's second postulate of quantisation of orbital angular momentum, show that the circumference of the electron in the nth orbital state in H-atom is n times the de-Broglie wavelength associated with it.
    (ii) The electron in H-atom is initially in the third excited state. What is the maximum number of spectral lines which can be emitted when it finally moves to the ground state? **Delhi 2012**

[redacted]

42. Using Bohr's postulates, obtain an expression for the total energy of the electron in the stationary states of the H-atom. Hence, draw the energy level diagram showing how the line spectral corresponding to Balmer series occur due to transition between energy levels. **Delhi 2013**

43. (i) State Bohr's quantisation condition for defining stationary orbits. How does de-Broglie's hypothesis explain the stationary orbits?

(ii) Find the relation between the three wavelengths λ_1, λ_2 and λ_3 from the energy level diagram shown below.



Delhi 2016

44. Assume that there is no repulsive force between the electrons in an atom but the force between positive and negative charges is given by Coulomb's law as usual. Under such circumstances, calculate the ground state energy of a He-atom. **NCERT Exemplar**

45. (a) State Bohr's postulate to define stable orbits in hydrogen atom. How does de-Broglie's hypothesis explain the stability of these orbits?

(b) A hydrogen atom initially in the ground state absorbs a photon which excites it to the $n = 4$ level. Estimate the frequency of the photon. **CBSE 2018**

LONG ANSWER Type II Questions

46. Obtain an expression for the frequency of radiation emitted when a hydrogen atom de-excites from level n to level $(n-1)$. For large n, show that this frequency equals to the classical frequency of revolution of the electron in the orbit. **NCERT**

47. (a) State the postulates of Bohr's model of hydrogen atom and derive the expression for Bohr radius.

(b) Find the ratio of the longest and the shortest wavelengths amongst the spectral lines of Balmer series in the spectrum of hydrogen atom. **CBSE 2020**

48. Using Bohr's postulates, derive an expression for the frequency of radiation emitted when electron in H-atom undergoes transition from higher energy state quantum number (n_i) to the lower energy state (n_f).
When electron in H-atom jumps from energy state $n_i = 4$ to $n_f = 3, 2, 1$. Identify the spectral series to which the emission lines belong.

NUMERICAL PROBLEMS

49. Calculate the de-Broglie wavelength associated with the electron in the second excited state of hydrogen atom. The ground state energy of the hydrogen atom is 13.6 eV. **CBSE 2020**

50. A 12.5 eV electron beam is used to excite a gaseous hydrogen atom at room temperature. Determine the wavelengths and the corresponding series of the lines emitted. **All India 2017, 16**

51. The number of α-particles scattered at 90° is 50 per minute. What will be the number of α-particles, when it is scattered at an angle of 120°?

52. The ground state energy of H-atom is -13.6 eV. What are the kinetic and potential energies of electron in this state? **NCERT, All India 2014 C, All India 2010**

53. Find the ratio of energies of photons produced due to transition of an electron of H-atom from its

(i) second permitted energy level to the first level and

(ii) the highest permitted energy level to the first permitted level. **All India 2010**

54. The gravitational attraction between electron and proton in a H-atom is weaker than the Coulombic attraction by a factor of about 10^{-40}. Estimate the radius of the first Bohr orbit of a H-atom, if the electron and proton were bound by gravitational attraction. **NCERT**

55. In accordance with the Bohr's model, find the quantum number that characterises in the earth's revolution around the sun in an orbit of radius 1.5×10^{11} m with orbital speed 3×10^4 m/s. (Mass of the earth $= 6 \times 10^{24}$ kg) **NCERT**

[redacted]

56. The radius of the innermost electron orbit of a H-atom is 5.3×10^{-11} m. What are the radii of the $n = 2$ and $n = 3$ orbits? **NCERT**

57. In Bohr's model of H-atom, the radius of the first electron orbit is 0.53 Å. What will be the radius of the third orbit and the first orbit of singly ionised helium atom?

58. In the ground state of H-atom, its Bohr radius is given as 5.3×10^{-11} m. The atom is excited such that the radius becomes 21.2×10^{-11} m. Find (i) the value of the principal quantum number and (ii) the total energy of the atom in this excited state. **Delhi 2013C**

59. If the average life time of an excited state of hydrogen is of the order of 10^{-8} s. Estimate how many orbits an electron makes when it is in the state $n = 2$ and before it suffers a transition to state $n = 1$ (Bohr radius, $a_0 = 5.3 \times 10^{11}$ m)?

60. A H-atom initially in the ground level absorbs a photon, which excites it to the $n = 4$ level. Determine the wavelength and frequency of photon. **NCERT**

61. The short wavelength limit for the Lyman series of the hydrogen spectrum is 913.4 Å. Calculate the short wavelength limit for Balmer series of the hydrogen spectrum **Delhi 2016**

62. The ground state energy of hydrogen atom is −13.6 eV. If an electron makes a transition from an energy level − 1.51 eV to − 3.4 eV, then calculate the wavelength of the spectral line emitted and name the series of hydrogen spectrum to which it belongs. **Delhi 2016**

63. A photon emitted during the de-excitation of electron from a state n to the first excited state in a hydrogen atom, irradiates a metallic cathode of work function 2eV, in a photocell, with a stopping potential of 0.55 V. Obtain the value of the quantum number of the state n. **CBSE 2019**

64. A hydrogen atom in the ground state is excited by an electron beam of 12.5 eV energy. Find out the maximum number of lines emitted by the atom from its excited state. **CBSE 2019**

65. (a) Draw the energy level diagram for the line spectra representing Lyman series and Balmer series in the spectrum of hydrogen atom.

(b) Using the Rydberg formula for the spectrum of hydrogen atom, calculate the largest and shortest wavelengths of the emission lines of the Balmer series in the spectrum of hydrogen atom. **CBSE 2019**

(Use the value of Rydberg constant, $R = 1.1 \times 10^7 \text{ m}^{-1}$).

66. Calculate the de-Broglie wavelength associated with the electron revolving in the first excited state of hydrogen atom. The ground state energy of the hydrogen atom is − 13.6 eV. **CBSE 2020**

67. An electron jumps from fourth to first orbit in an atom. How many maximum number of spectral lines can be emitted by the atom? To which series these lines correspond? **Foreign 2016**

68. What is the minimum energy that must be given to a H-atom in ground state so that it can emit an H_γ-line in Balmer series? If the angular momentum of the system is conserved, what would be the angular momentum of such H_γ photon? **NCERT Exemplar**

69. Find the quantum number n corresponding to the excited state of He^+ ion, if on transition to the ground state that ion emits two photons in succession with wavelength 1026.7Å and 304 Å. (Take, $R = 1.097 \times 10^7$ per m)

70. Calculate the shortest wavelength in the Balmer series of hydrogen atom. In which region (infrared, visible, ultraviolet) of hydrogen spectrum does this wavelength lie? **Delhi 2015, All India 2016**

71. Find the ratio between the wavelengths of the 'most energetic' spectral lines in the Balmer and Paschen series of the hydrogen spectrum. **Compt. 2016**

72. What is the shortest wavelength present in the Paschen series of spectral lines? **NCERT**

73. (i) In H-atom, an electron undergoes transition from second excited state to the first excited state and then to the ground state. Identify the spectral series to which these transitions belong.

(ii) Find out the ratio of the wavelengths of the emitted radiations in the two cases. **Delhi 2012**

74. Find out the wavelength of the electron orbiting in the ground state of hydrogen atom. **Delhi 2016**

[redacted]

75. Find the wavelength of the electron orbiting in the first excited state in hydrogen atom. **All India 2016**

75. Use de-Broglie's hypothesis to write the relation for the nth radius of Bohr orbit in terms of Bohr's quantisation condition of orbital angular momentum. **Foreign 2016**

77. The ground state energy of a H-atom is -13.6 eV. If an electron makes a transition from an energy level -0.85 eV to -1.51 eV, then calculate the wavelength of the spectral line emitted. To which series of hydrogen spectrum does this wavelength belong? **All India 2012**

78. The total energy of an electron in the first excited state of the H-atom is about -3.4 eV.
   (i) What is the kinetic energy of the electron in this state?
   (ii) What is the potential energy of the electron in this state?
   (iii) Which of the answers above would change, if the choice of the zero of potential energy is changed? **NCERT**

79. Obtain the first Bohr radius and the ground state energy of a muonic H-atom (i.e. an atom in which a negatively charged muon (μ) of mass about 207 m_e (orbit around a proton). **NCERT**

80. State any two postulates of Bohr's theory of H-atom. What is the maximum possible number of spectral lines when the H-atom is in its second excited state? Justify your answer. Calculate the ratio of the maximum and minimum wavelengths of the radiations emitted in this process. **All India 2010]**

81. A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. Upto which energy level the H-atoms would be excited? Calculate the wavelengths of the first member of Lyman and first member of Balmer series. **All India 2014]**

82. (i) Using the Bohr's model, calculate the speed of the electron in a H-atom in the $n = 1, 2$ and 3 levels.
   (ii) Calculate the orbital period in each of these levels. **NCERT**

HINTS AND SOLUTIONS

1. (a) In Rutherford's nuclear model of the atom, the entire positive charge and most of the mass of the atom are concentrated in the nucleus with the electrons some distance away.

2. (a) In α-particle scattering experiment, the shape of the trajectory depends on the impact parameter only.

3. (b) From the formula of angular momentum and Bohr's assumption, $mvr = n(h/2\pi)$

   Here, $n = 1 \Rightarrow \quad mvr = h/2\pi$

4. (a) $K = \dfrac{e^2}{8\pi\varepsilon_0 r} = \dfrac{(9\times10^9 \text{ Nm}^2/\text{C}^2)(1.6\times10^{-19}\text{ C})^2}{(2)(4.7\times10^{-11}\text{ m})}$

   $= 2.45\times10^{-18}$ J

   $= 15.3$ eV

5. (a) A set of atoms in an excited state decays in general to any of the states with lower energy.

6. (d) In Pfund series,

   $$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right);\ n = 6, 7, \cdots$$

   Maximum wavelength is given by

   In transition $6 \to 5$

   $$\frac{1}{\lambda_{max}} = R\left(\frac{1}{5^2} - \frac{1}{6^2}\right)$$

   Minimum wavelength is given by

   In transition $\infty \to 5$

   $$\frac{1}{\lambda_{min}} = R\left(\frac{1}{5^2} - \frac{1}{\infty}\right)$$

   So, ratio is $\frac{36}{11}$.

7. (a) Paschen series of hydrogen gas lies in infrared region.

8. (a) α-particle is positively charged, so is the nucleus, so the large angle of scattering of α-particle shows that the nucleus is positively charged and concentrated in the central core.

9. (a)

10. (b)

11. (b)

12. (c)

13. (i) (c) $E_n = \dfrac{-13.6}{n^2}(Z^2)$

   In first excited state, $E_{H_2} = 3.4$ eV

[redacted]

and $E_{He} = -13.6$ eV

So, H_2 atom gives excitation energy

$(13.6 - 3.4 = 10.2 \text{ eV})$ to helium atom.

Now energy of He ion $= -13.6 + 10.2 = -3.4$ eV

$$\text{Again, } E = \frac{-13.6}{n^2} \times Z^2$$

$$\Rightarrow -3.4 = \frac{-13.6}{n^2} \times (2)^2 \Rightarrow n = 4$$

(ii) (c) $\frac{1}{\lambda} = \frac{13.6 Z^2}{hc}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

Here $n_1 = 3$ and $n_2 = 4 \Rightarrow \lambda = 4.8 \times 10^{-7}$ m

(iii) (a) Kinetic energy, $K \propto \frac{Z^2}{n^2}$

$$\frac{K_{H_2}}{K_{He}} = \left(\frac{Z_{H_2}}{Z_{He}}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

(iv) (b) Radius of the permitted orbit is $r = \frac{n^2 h^2 \varepsilon_0}{\pi m Z e^2}$

For hydrogen atom in ground state, i.e.,

$$n = 1, Z = 1 \Rightarrow r = \frac{h^2 \varepsilon_0}{\pi m e^2}$$

(v) (a) Angular momentum for hydrogen atom,

$$L = \frac{nh}{2\pi}$$

For first excited state, $n = 2$

$$\Rightarrow \quad L = \frac{h}{\pi}$$

14. (i) (d) Rutherford's atom had a positively charged centre and electrons were revolving outside it. It is also called the planetary model of the atom as in option (d).

(ii) (d) As the gold foil is very thin, it can be assumed that α-particles will suffer not more than one scattering during their passage through it. Therefore, computation of the trajectory of an α-particle scattered by a single nucleus is enough.

(iii) (a) The scattered α-particles were observed through a rotatable detector consisting of zinc sulphide screen and a microscope. The scattered α-particles on striking the screen produced brief light flashes or scintillations. These flashes may be viewed through a microscope and the distribution of the number of scattered particles may be studied as a function of angle of scattering.

(iv) (a) In Rutherford's nuclear model of the atom, the entire positive charge and most of the mass of the atom are concentrated in the nucleus with the electrons some distance away. It is obvious from the observation of Geiger Marsden experiment that most of the α-particles pass straight through the gold foil.

(v) (d) In case of head-on-collision, the impact parameter is minimum and the α-particle rebounds back. So, the fact that only a small fraction of the number of incident particles rebound back indicates that the number of α-particles undergoing head-on collision is small.

This in turn implies that the mass of the atom is concentrated in a small volume.

Hence, options (a) and (b) are correct.

15. (i) Rutherford's model did not explain the stability of nucleus.

(ii) It does not explain the line spectrum of hydrogen atom.

16. For first excited state, $n = 2$

Ground state occurs for $n = 1$

Since, $r_n = r_0 n^2$ and $r_n \propto n^2$

$$\Rightarrow \quad \frac{r_2}{r_1} = \left(\frac{n_2}{n_1}\right)^2 = \left(\frac{2}{1}\right)^2$$

So, $r_2 : r_1 = 4 : 1$, where r_2 and r_1 are radii corresponding to first excited state and ground state of the atom.

17. No, it is not possible for the electron to have different energies because according to Bohr's model,

$$E_n = \frac{-13.6}{n^2}$$

The electrons which have different energies, have different values of n.

Angular momentum, $mvr = \frac{nh}{2\pi}$, so as n changes angular momentum changes.

18. The angular momentum of electron revolving in nth orbit of hydrogen atom is

$$L = mvr = \frac{nh}{2\pi}$$

For second orbit, $n = 2$

$$\therefore \quad L = \frac{2h}{2\pi} = \frac{h}{\pi}$$

19. H_α-line of the Balmer series in the emission spectrum of H-atom is obtained in visible region.

20. On removing one electron from He^4 and He^3, the energy levels, as worked out on the basis of Bohr's model will be very close as both the nuclei are very heavy as compared to electron mass.

Also, after removing one electron, He^4 and He^3 atoms contain one electron and are hydrogen like atoms.

21. Refer to text on pages 473 and 474.

The distance of closest approach is given by

$$\frac{1}{4\pi\varepsilon_0} \cdot \frac{2e \times Ze}{r} = K \qquad \text{...(i)}$$

i.e. $\quad r \propto \frac{1}{K}$

Let r_0 be the new distance of closest approach for a twice energetic α-particle.

$$\frac{r_0}{r} = \frac{K}{2K} = \frac{1}{2} \Rightarrow r_0 = \frac{r}{2}$$

[redacted]

22. Refer to text on pages 473 and 474.

23. Refer to text on pages 474 and 475.

24. Refer to text on page 475.

25. Refer to text on pages 474, 475 and 476.

26. An atom can emit or absorb radiation in the form of discrete energy photons only when an electron jumps from a higher to a lower orbit or from a lower to a higher orbit, respectively.

Frequency condition $h\nu = E_i - E_f$

where, ν is frequency of radiation emitted, E_i and E_f are the energies associated with stationary orbits of principal quantum numbers n_i and n_f respectively (where $n_i > n_f$).

27. Refer to text on page 475.

28. Refer to text on page 476

29. Refer to text on page 476

30. Refer to text on page 479. (de-Broglie's Comment on Bohr's Second Postulate)

31. According to Bohr's theory, centripetal force required by the electron for its motion around the nucleus = Electric force between the proton and electron.

$$\Rightarrow \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(q_p)(q_e)}{r^2} \quad \text{[from Coulomb's law]}$$

where, r = atomic radius, q_p = charge of proton = $+e$

q_e = charge of electron = $-e$

$$\Rightarrow \frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(e)(-e)}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{-e^2}{r^2}$$

Now, given charge on proton, $q_p = +\frac{4}{3}e$

Charge on electron, $q_e = -\frac{3}{4}e$

Putting the new value (keeping after factors unchanged),

$$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\left(\frac{4}{3}e\right)\left(-\frac{3}{4}e\right)}{r^2}$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{-e^2}{r^2}$$

i.e. Bohr's formula remain unchanged.

32. No; Because according to Bohr's model, energy of electron in nth orbit of H-atom, $E_n = -\frac{13.6}{n^2}$

Hence, electrons having different energies belong to different energy levels, i.e. different values of n.

Therefore, their angular momentum will be different due to different values of n as

Angular momentum, $L = mvr = \frac{nh}{2\pi}$

33. The reduced mass m of two particle system of masses m_1 and m_2 is given by $\frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2}$.

The total energy of the electron in the stationary states of the hydrogen atom is given by $E_n = -\frac{me^4}{8n^2\varepsilon_0^2 h^2}$, where signs are as usual but m is the reduced mass of electron and proton. Also, the total energy of the electron in the ground state of the hydrogen atom is -13.6 eV. For H-atom, reduced mass is m_e whereas the positronium, the reduced mass is $m = \frac{m_e}{2}$. Hence, the total energy of the electron in the ground state of the positronium atom is $\frac{-13.6 \text{ eV}}{2} = -6.8$ eV.

34. From the nth state, the atom may go to $(n-1)$ th state, ..., 2nd state or 1st state. So, there are $(n-1)$ possible transitions starting from the nth state.

The atoms reaching $(n-1)$th state may make $n-2$ different transitions. Similarly, for other lower states, the total number of possible transitions is

$$(n-1) + (n-2) + (n-3) + \cdots + 2 + 1 = \frac{n(n-1)}{2}$$

35. Refer to text on pages 507 and 508. (Bohr's Model of Hydrogen Atom)

For Brackett-series, $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$, where

$n = 5, 6, 7, \ldots$

For shortest wavelength, $n = 5$

$$\Rightarrow \frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{16} - \frac{1}{25}\right)$$

$$= 1.097 \times 10^7 \times \frac{9}{16 \times 25} = 0.0246 \times 10^7$$

$$\Rightarrow \lambda = 40.514 \times 10^{-7} = 4051 \text{ nm}$$

It lies in infrared region of electromagnetic spectrum.

36. The velocity of electron,

$$v = \frac{1}{n} \frac{Ze^2}{2h\varepsilon_0}$$

Here, $Z = 1$, $e = 1.6 \times 10^{-19}$ C,

$\varepsilon_0 = 8.85 \times 10^{-12}$ NC^{2}m^{-2},

$h = 6.62 \times 10^{-34}$ J-s and $n = 2$

(in 1st excited state)

$$\Rightarrow v_2 = \frac{1 \times (1.6 \times 10^{-19})^2}{2 \times 2 \times (6.62 \times 10^{-34}) \times (8.85 \times 10^{-12})}$$

$$= 1.09 \times 10^5 \text{ m/s}$$

Radius of orbit, $r_2 = \frac{n^2 h^2 \varepsilon_0}{\pi m e^2}$

Here, $m = 9.1 \times 10^{-31}$ kg

$$\Rightarrow r_2 = \frac{(2)^2 \times (6.62 \times 10^{-34})^2 \times (8.85 \times 10^{-12})}{3.14 \times (9.1 \times 10^{-31}) \times (1.6 \times 10^{-19})^2}$$

$$= 2.12 \times 10^{-10} \text{ m}$$

[redacted]

Time period or orbital period,

$$T = \frac{2\pi r_2}{v_2}$$

$$= \frac{2 \times 3.14 \times 2.12 \times 10^{-10}}{1.09 \times 10^6}$$

$$= 1.22 \times 10^{-15}\ \text{s}$$

37. Refer to text on page 473. (Observations)

Refer to text on page 474. (Impact parameter)

38. Refer to text on page 476.

39. Refer to text on page 477.

40. Refer to text on page 477.

41. (i) Bohr's second postulate states that the electron revolves around the nucleus in certain privileged orbit which satisfy certain quantum condition that angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$, where h is Planck's constant

i.e. $$L = mvr = \frac{nh}{2\pi}$$

where, m = mass of electron, v = velocity of electron and r = radius of orbit of electron.

$$\Rightarrow \quad 2\pi r = n\left(\frac{h}{mv}\right)$$

$\therefore$ Circumference of electron in n th orbit

$= n \times$ de-Broglie wavelength associated with electron. $\left[\because \lambda = \frac{h}{mv}\right]$

(ii) Given, the electron in H-atom is initially in third excited state.

$\therefore \quad n = 4$

And the total number of spectral lines of an atom that can exist is given by the relation

$$= \frac{n(n-1)}{2}$$

Here, $n = 4$

So, number of spectral lines

$$= \frac{4(4-1)}{2} = \frac{4 \times 3}{2} = 6$$

Hence, when a H-atom moves from third excited state to ground state, it emits six spectral lines.

42. Refer to text on page 477.

In H-atom when an electron jumps from the orbit n_i to orbit n_f, the wavelength of the emitted radiation is given by

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

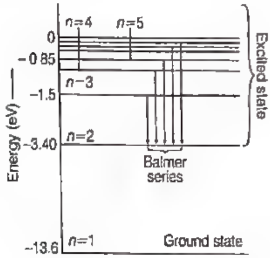
where, R = Rydberg constant = $1.097 \times 10^7\ \text{m}^{-1}$

For Balmer series, $n_f = 2$ and $n_i = 3, 4, 5, \ldots$

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right)$$

where, $n_i = 3, 4, 5, \ldots$

These spectral lines lie in the visible region.



43. (i) According to Bohr's principle, electrons revolve in a stationary orbit of which energy and momentum are fixed. The momentum of electrons in the fixed orbit is given by $\frac{nh}{2\pi}$ (where, n = number of orbits).

According to de-Broglie's hypothesis, the electron is associated with wave character. Hence, a circular orbit can be taken to be a stationary energy state only if it contains an integral number of de-Broglie wavelengths, i.e. $2\pi r = n\lambda$

(ii) According to question,

$$E_B - E_C = \frac{hc}{\lambda_1} \quad \ldots \text{(i)}$$

$$E_A - E_B = \frac{hc}{\lambda_2} \quad \ldots \text{(ii)}$$

$$E_C - E_A = \frac{-hc}{\lambda_3} \quad \ldots \text{(iii)}$$

On adding Eqs. (i), (ii) and (iii), we get

$$E_B - E_C + E_A - E_B + E_C - E_A = hc\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_3}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda_3} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \Rightarrow \lambda_3 = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$$

This is the required expression.

44. $\therefore$ The total energy of the electron in the nth stationary state of hydrogen like atom of atomic number Z is given by

$$E_n = Z^2\left(\frac{-13.6\ \text{eV}}{n^2}\right)$$

For a He-nucleus with charge $2e$ and electrons of charge $-e$, the energy level in ground state is

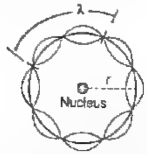
$$E_n = Z^2\left(\frac{-13.6\ \text{eV}}{n^2}\right) = 2^2\left(\frac{-13.6\ \text{eV}}{1^2}\right)$$

$$= -54.4\ \text{eV}$$

Thus, the ground state will have two electrons each of energy E and the total ground state energy would be $-(4 \times 13.6)\ \text{eV} = -54.4\ \text{eV}$

[redacted]

45. (a) Bohr's second postulate defines the stable orbits. This postulate states that the electron revolves around the nucleus only in those orbits for which the angular momentum is some integral multiple of $h/2\pi$, where h is the Planck's constant $(= 6.63 \times 10^{-34}$ J-s).



A standing wave is shown on a circular orbit

According to de-Broglie wavelength of moving electron $\lambda = \dfrac{h}{mv_n}$

where, v_n is speed of electron revolving in nth orbit.

As, $\quad 2\pi r_n = n\lambda$ [from figure]

$\therefore \quad 2\pi r_n = \dfrac{nh}{mv_n}$

or $\quad mv_n r_n = \dfrac{nh}{2\pi} = n(h/2\pi)$

i.e. Angular momentum of electron revolving in nth orbit must be an integral multiple of $h/2\pi$, which is the quantum condition proposed by Bohr in his second postulate

(b) We know that, energy of electron in nth orbit is

$$E_n = -\frac{13.6}{n^2}\text{ eV}$$

For $\quad n = 1$,

$$E_1 = -13.6\text{ eV}$$

Similarly, for $n = 4$,

$$E_4 = -\frac{13.6}{(4)^2}\text{ eV}$$

$\therefore$ Energy difference, $\Delta E = E_4 - E_1$

$$= \left[-\frac{13.6}{16} - (-13.6)\right]\text{eV} \quad \text{... (i)}$$

Also, energy of photon is

$$\Delta E = h\nu$$

$$\Rightarrow \quad \nu = \frac{\Delta E}{h} \quad \text{... (ii)}$$

From Eqs. (i) and (ii), we get

$$\nu = \left(-\frac{13.6}{16} + 13.6\right) \times \frac{1.6\times 10^{-19}}{6.63\times 10^{-34}}$$

$\therefore \quad \nu = 3.1 \times 10^{15}$ Hz

46. Let ν be the frequency when a hydrogen atom jumps from level n to $(n-1)$.

i.e. $\quad n_1 = (n-1)$

$\quad n_2 = n$

Energy, $\quad E = h\nu = E_2 - E_1$

$$\Rightarrow \quad \nu = \frac{1}{2}\cdot\frac{mc^2\alpha^2}{h} \times \left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

$$= \frac{mc^2\alpha^2}{2h}\left[\frac{n^2-(n-1)^2}{n^2(n-1)^2}\right]$$

$$= \frac{mc^2\alpha^2[(n+n-1)(n-n+1)]}{2hn^2(n-1)^2}$$

$$= \frac{mc^2\alpha^2(2n-1)}{2hn^2(n-1)^2}$$

For large values of n, $(2n-1 = 2n)$, $(n-1 = n)$, we have

$$\nu = \frac{mc^2\alpha^2\, 2n}{2hn^2n^2} = \frac{mc^2\alpha^2}{hn^3} \qquad \left[\because \alpha = \frac{2\pi Ke^2}{cn}\right]$$

$$= \frac{mc^2}{hn^3}\cdot\frac{4\pi^2K^2e^4}{c^2n^2}$$

$$= \frac{4\pi^2K^2me^4}{hn^5} \quad \text{...(i)}$$

In Bohr's atomic model, velocity of nth orbit, $v = \dfrac{hn}{2\pi mr}$

and radius, $r = \dfrac{n^2h^2}{4\pi^2 mKe^2}$

Thus, frequency of oscillation

$$\nu = \frac{v}{2\pi r} = \frac{nh}{2\pi mr}\left(\frac{4\pi^2mKe^2}{2\pi n^2h^2}\right)$$

$$= \frac{Ke^2}{nhr} = \frac{Ke^2}{nh}\left(\frac{4\pi^2\, mKe^2}{n^2h^2}\right) = \frac{4\pi^2mK^2e^4}{n^3h^3}$$

It is same as Eq. (i).

So, we can say that for large values of n, the classical frequency of revolution of electron in nth orbit is same as the frequency of radiation emitted when hydrogen atom de-excites from level n to level $(n-1)$.

47. (a) Refer to text on pages 475 and 476.

(b) The wavelength of the lines in Balmer series is expressed by the formula,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

For shortest wavelength of the spectral line emitted in Balmer series is given by

$$\frac{1}{\lambda_s} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right) \qquad (\because n \to \infty)$$

$$= \frac{10^7}{4} \qquad [\because R = 10^7]$$

$$\Rightarrow \quad \lambda_s = \frac{4}{10^7} = 4 \times 10^{-7}\text{ m} = 4000\text{ Å}$$

For longest wavelength in Balmer series,

$$\frac{1}{\lambda_l} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \qquad (\because n = 3)$$

[redacted]

$\Rightarrow \quad \frac{1}{\lambda_l} = \frac{5\times10^7}{36}$

or $\quad \lambda_l = 7.2\times10^{-7}\,m = 7200\,Å$

$\therefore \quad \frac{\lambda_l}{\lambda_s} = \frac{7200}{4000} = \frac{9}{5}$

48. Refer to text on pages 477 and 478

As, $\nu = Rc\times\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right)$

Here, higher state is $n_i = 4$ and lower state is $n_f = 3, 2, 1$

For the transition,

$n_i = 4$ to $n_f = 3 :\rightarrow$ Paschen series

$n_i = 4$ to $n_f = 2 :\rightarrow$ Balmer series

$n_i = 4$ to $n_f = 1 :\rightarrow$ Lyman series

49. Energy in second excited state,

$$E_2 = -\frac{13.6}{(3)^2}\,eV$$

$$= -\frac{13.6}{9} = -1.51\,eV$$

Energy in ground state,

$$E_0 = -13.6\,eV$$

$$\Delta E = E_2 - E_0$$
$$= 1.51-(-13.6)$$
$$= -1.51+13.6$$
$$= 12.09\,eV$$

Wavelength, $\lambda = \frac{12375}{\Delta E}\,Å - \frac{12375}{12.09}\,Å = 1023\,Å$

50. Energy of electron beam,

$$E = 12.5\,eV$$
$$= 12.5\times1.6\times10^{-19}\,J$$

Planck constant, $h = 6.63\times10^{-34}$ J-s

Velocity of light, $c = 3\times10^8$ m/s

Using the relation, $E = \frac{hc}{\lambda}$

$$\Rightarrow \quad \lambda = \frac{6.63\times10^{-34}\times3\times10^8}{12.5\times1.6\times10^{-19}}$$
$$= 0.993\times10^{-7}\,m$$
$$= 993\times10^{-10}\,m$$
$$= 993\,Å$$

So, wavelength falls in the range of Lyman series from 912Å to 1216Å.

51. Approx. 22 per minute, refer to Example 1 on page 473.

52. Refer to Example 4 part (ii) on page 478.

53. (i) 10.2 eV; refer to Example 5 on page 479.

(ii) The highest permitted energy level to the first permitted level,

$$\Delta E = E_\infty - E_1 = 0-(-13.6) = 13.6\,eV$$

Ratio of energies of photon

$$= \frac{10.2}{13.6} = \frac{3}{4} = 3:4$$

54. As, we know that the radius of first Bohr orbit of H-atom is $\quad r_0 = \frac{4\pi\varepsilon_0\left(\frac{h}{2\pi}\right)^2}{m_e e^2}$

Let us consider that the atom is bound by the gravitational force $= \frac{Gm_p m_e}{r^2}$

We replace $\frac{e^2}{4\pi\varepsilon_0}$ by $Gm_p m_e$. In that case, radius of first orbit (Bohr) of H-atom would be

$$r_0 = \frac{\left(\frac{h}{2\pi}\right)^2}{Gm_p\cdot m_e^2}$$

By substituting the standard values, we get

$$r_0 = \frac{\left(\frac{6.63\times10^{-34}}{2\times3.14}\right)^2}{6.6\times10^{-11}\times1.67\times10^{-27}\times(9.1\times10^{-31})^2}$$
$$= 1.2\times10^{29}\,m$$

55. Given, radius of the orbit of the earth around the sun,

$$r = 1.5\times10^{11}\,m$$

Orbital speed of the earth, $v = 3\times10^4$ m/s

Mass of the earth, $m = 6\times10^{24}$ kg

According to Bohr's model, angular momentum is quantised and given as, $mvr = \frac{nh}{2p}$

where, h = Planck constant $= 6.63\times10^{-34}$ J-s

n = quantum number.

$$\therefore\ n = \frac{mvr\,2\pi}{h} = \frac{2\pi\times6\times10^{24}\times3\times10^4\times1.5\times10^{11}}{6.63\times10^{-34}}$$
$$= 25.61\times10^{73}$$
$$= 2.6\times10^{74}$$

Hence, the quantum number that characterises the earth's revolution is 2.6×10^{74}.

56. 2.12×10^{-10} m and 4.47×10^{-10} m; refer to example 4 (i) on page 510.

57. Radius of the nth Bohr orbit, $r = \frac{n^2h^2\varepsilon_0}{\pi mZe^2}$

Again, $\quad r \propto \frac{1}{Z}$

$$\therefore \quad \frac{r_{He^+}}{r_H} = \frac{Z_H}{Z_{He^+}}$$

For hydrogen, $Z = 1$ and for helium, $Z = 2$

[redacted]

$\therefore \quad \frac{r_{He^+}}{r_H} = \frac{1}{2}$

$\Rightarrow \quad r_{He^+} = \frac{1}{2} r_H$

$= \frac{0.53}{2} = 0.265 \text{ Å}$

For radius of third orbit, i.e. for $n = 3$

$r_3 = (3)^2 \times 0.265 \text{ Å}$

$= 9 \times 0.265 \text{ Å}$

$= 2.38 \text{ Å}$

58. (i) We know that, $r \propto n^2$

$$\frac{r_1}{r_2} = \frac{n_1^2}{n_2^2}$$

$$\Rightarrow \quad \frac{1}{n_2^2} = \frac{5.3 \times 10^{-11}}{21.2 \times 10^{-11}}$$

$$n_2^2 = 4$$

$$n_2 = 2$$

(ii) We know that, $E = \frac{-13.6}{n^2} = \frac{-13.6}{4}$

$= -3.4 \text{ eV}$

59. Angular momentum of an electron in nth orbit $= \frac{nh}{2\pi}$

By Bohr's hypothesis, we have

$$mvr = \frac{nh}{2\pi} \Rightarrow v = \frac{nh}{2\pi mr}$$

Time period to complete a revolution in an orbit,

$$T = \frac{2\pi r}{v} = \frac{2\pi r(2\pi mr)}{nh} = \frac{4\pi^2 mr^2}{nh}$$

Since, radius of the orbit is proportional to n^2, hence

$$r \propto n^2$$

$$\Rightarrow \quad r = a_0 n^2$$

$$\therefore \quad T = \frac{4\pi^2 m a_0^2 n^4}{nh} = \frac{4\pi^2 m a_0^2 n^3}{h}$$

$\therefore$ Number of orbits completed in 10^{-8} s

$$= \frac{10^{-8}}{T} = \frac{10^{-8} \times h}{4\pi^2 m a_0^2 n^3}$$

$$= \frac{10^{-8} \times 6.6 \times 10^{-34}}{4 \times (3.14)^2 \times 9.1 \times 10^{-31} \times (5.3 \times 10^{-11})^2 \times (2)^3}$$

$$= 8 \times 10^6$$

60. To find the wavelength and frequency of photon, use the relation of energy of electron in hydrogen atom.

$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

$$\because \quad E_n = \frac{-13.6}{n^2} \text{eV}$$

$$\Rightarrow \quad E_1 = \frac{-13.6}{1^2} \text{ eV}$$

$$= -13.6 \text{ eV}$$

$$E_4 = \frac{-13.6}{4^2} \text{ eV}$$

$$= -\frac{13.6}{16} \text{eV} = -0.85 \text{ eV}$$

$$\Delta E = E_4 - E_1 = -0.85 - (-13.6)$$

$$= 12.75 \text{ eV}$$

$$\Delta E = 12.75 \text{eV} = 12.75 \times 1.6 \times 10^{-19}$$

$$= 20.4 \times 10^{-19} \text{ J}$$

$$\because \quad \Delta E = h\nu \Rightarrow \nu = \frac{\Delta E}{h} = \frac{20.4 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$= 3.1 \times 10^{15} \text{ Hz}$$

Wavelength of photon, $\lambda = \frac{c}{\nu} = \frac{3 \times 10^8}{3.1 \times 10^{15}}$

$= 9.74 \times 10^{-8}$ m

Thus, the wavelength is 9.74×10^{-8} m and frequency is 3.1×10^{15} Hz.

61. Lyman series, $n = 2, 3, 4 \ldots$ to $n = 1$

For short wavelength, $n = \infty$ to $n = 1$

Energy, $\quad E = \frac{12375}{\lambda(\text{Å})} = \frac{12375}{913.4} \text{eV}$

$= 13.54$ eV

Also, energy of nth orbit, $E = \frac{13.54}{n^2}$

So, energy of $n = 1$, energy level $= 13.54$ eV

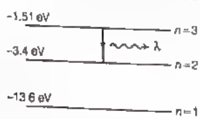
Energy of $n = 2$, energy level

$$= \frac{13.54}{2^2} = 3.387 \text{ eV}$$

So, short wavelength of Balmer series $= \frac{12375}{3.387}$

$= 3653 \text{ Å}$

62. Energy levels of H-atom are as shown below



Wavelength of spectral line emitted,

$$\lambda = \frac{hc}{\Delta E}$$

Taking, $\quad hc = 1240$ eV-nm,

We have, $\quad \Delta E = -1.51 - (-3.4)$

$= 1.89$ eV

$$\therefore \quad \lambda = \frac{1240}{1.89}$$

≈ 656 nm

This belongs to Balmer spectral series.

[redacted]

63. Here, $\phi = 2\,\text{eV}$, $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$

$$E = \frac{hc}{\lambda} = hcR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = \phi + \text{KE}$$

Also, $\text{KE} = eV_0$

$n_1 = 2, n_2 = n$

$$hcR\left(\frac{1}{4} - \frac{1}{n^2}\right) = 2 \times 1.6 \times 10^{-19} + 1.6 \times 10^{-19} \times 0.55$$

$$\Rightarrow 6.62 \times 10^{-34} \times 3 \times 10^8 \times 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{n^2}\right)$$

$$= (3.2 + 0.88) \times 10^{-19}$$

$$\Rightarrow 21.786 \times 10^{-19}\left(\frac{1}{4} - \frac{1}{n^2}\right) = 4.08 \times 10^{-19}$$

$$\frac{1}{4} - \frac{1}{n^2} = 0.187 \Rightarrow n = 4$$

64. The energy absorbed by it is

$-13.6 + 12.5 = -1.1\,\text{eV}$

The energy, $E_n = -\dfrac{13.6}{n^2}$

$$\Rightarrow \quad n^2 = \frac{-13.6}{-1.1} = 12.36$$

$$\Rightarrow \quad n = 3$$

Thus, number of transitions = 3

65. (a) Refer to diagram on page 511.
(Line Spectrum of the H-atom)

(b) For largest wavelength, $n = \infty$

$$\frac{1}{\lambda_m} = R\left(\frac{1}{2^2} - \frac{1}{\infty}\right)$$

$$\frac{1}{\lambda_m} = \frac{1.1 \times 10^7}{4}$$

$$\Rightarrow \quad \lambda_m = \frac{4}{1.1} \times 10^{-7}$$

$$= 3.636 \times 10^{-7}\,\text{m}$$

$$= 3636\,\text{Å}$$

For shortest wavelength,

$n = 3$

$$\frac{1}{\lambda_s} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= 1.1 \times 10^{-7} \times \frac{5}{36}$$

$$\Rightarrow \quad \lambda_s = \frac{36}{5.5} \times 10^{-7}$$

$$= 6.545 \times 10^{-7}\,\text{m}$$

$$= 6545\,\text{Å}$$

66. Energy stored in first excited state,

$$E_1 = \frac{-13.6}{(2)^2}\,\text{eV}$$

$$= \frac{-13.6}{4} = -3.4\,\text{eV}$$

Energy in ground state, $E_0 = -13.6\,\text{eV}$

$$\Delta E = E_0 - E_1 = -13.6 - (-3.4)$$

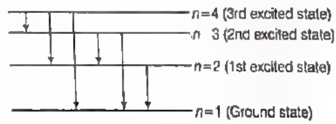
$$= -10.2\,\text{eV}$$

$$\text{Wave length}\,(\lambda) = \frac{12375\text{Å}}{\Delta\varepsilon} = \frac{12375}{10.2}\text{Å}$$

$$= 1213.23\,\text{Å}$$

67. Number of spectral lines obtained due to transition of electron from $n = 4$ (3rd excited state) to $n = 1$ (ground state) is

$$N = \frac{(4)(4-1)}{2} = 6$$



These lines correspond to Lyman series.

68. The third line in Balmer series in the spectrum of hydrogen atom is H_γ. H_γ in Balmer series corresponds to transition $n = 5$ to $n = 2$. So, the electron in ground state, i.e. from $n = 1$ must first be placed in state $n = 5$.

Energy required for the transition from $n = 2$ to $n = 5$ is given by

$$= E_1 - E_5 = 13.6 - 0.54 = 13.06\,\text{eV}$$

Since, angular momentum is conserved.

Angular momentum corresponding to H_γ photon = Change in angular momentum of electron

$$= L_5 - L_2 = 5\hbar - 2\hbar = 3\hbar = 3 \times 1.06 \times 10^{-34}$$

$$= 3.18 \times 10^{-34}\,\text{kg-m}^2/\text{s}$$

69. Given, $\dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2} = RZ^2\left(1 - \dfrac{1}{n^2}\right)$

$$\Rightarrow \quad \frac{1}{n^2} = 1 - \left[\frac{\lambda_1 + \lambda_2}{\lambda_1 \lambda_2} \times \frac{1}{RZ^2}\right]$$

$$= 1 - \left[\frac{1026.7 + 304}{1026.7 \times 304} \times \frac{1}{4 \times 1.097 \times 10^7}\right]$$

$$\Rightarrow \quad \frac{1}{n^2} = 0.0275$$

$$\Rightarrow \quad n = 6.03$$

Hence, the principal quantum number is 6.

[redacted]

70. Since, we know that for Balmer series,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n_2^2}\right),\ n_2 = 3, 4, 5, \ldots$$

For shortest wavelength in Balmer series, the spectral series is given by

$$n_1 = 2,\ n_2 = \infty$$

$$\Rightarrow \quad \frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{\infty^2}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda} = R \times \frac{1}{4} \Rightarrow \frac{1}{\lambda} = \frac{R}{4}$$

$$\Rightarrow \quad \lambda = \frac{4}{R} = \frac{4}{1.097 \times 10^7}$$

$$[\because R = 1.097 \times 10^7\ \text{m}^{-1}]$$

$$\Rightarrow \quad \lambda = 3.64 \times 10^{-7}\ \text{m}$$

The lines of Balmer series are found in the visible part of the spectrum.

71. For Balmer series, $\frac{1}{\lambda_B} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$

For highest energy $n \to \infty \Rightarrow \lambda_B = \frac{4}{R}$

$$\Rightarrow \quad \frac{1}{\lambda_B} = \frac{R}{2^2} = \frac{R}{4}$$

For Paschen series, $\frac{1}{\lambda_P} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$

For highest energy,

$$n \to \infty \Rightarrow \lambda_P = \frac{9}{R}$$

$$\Rightarrow \quad \lambda_B : \lambda_P = \frac{4}{R} : \frac{9}{R} \Rightarrow 4 : 9$$

72. Using formula for Paschen series,

$$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right],\ n_2 = 4, 5, 6, \ldots$$

For shortest wavelength, $n_2 = \infty$

$$\therefore \quad \frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{\infty^2}\right] = \frac{R}{9}$$

$$\text{or} \quad \lambda = \frac{9}{R} = 820.4\ \text{nm}$$

73. (i) An electron undergoes transition from second excited state to the first excited state which corresponds to Balmer series and then to the ground state which corresponds to Lyman series.

(ii) The wavelength of the emitted radiations in the two cases.

We know that, $\lambda = \frac{hc}{\Delta E}$

From $n_3 \to n_2$,

$$\lambda_1 = \frac{hc}{E_3 - E_2} = \frac{hc}{(-1.5)-(-3.4)} = \frac{hc}{1.9}$$

From $n_2 \to n_1$,

$$\lambda_2 = \frac{hc}{E_2 - E_1} = \frac{hc}{(-3.4)-(-13.6)} = \frac{hc}{10.20}$$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} = \frac{10.20}{1.9} = 5.3$$

74. For an electron revolving in nth orbit of radius r_0, then we have, $n\lambda = 2\pi r_0$

For electron orbiting in ground state, $n = 1$.

$$1\lambda = 2\pi r_0$$
$$= 2\pi \times 0.5\,\text{Å} \qquad [\because r_0 = 0.5\,\text{Å}]$$
$$= \pi\,\text{Å}$$
$$\text{or} \quad \lambda = 3.14\,\text{Å}$$

75. For electron in first excited state, i.e. $n = 2$.

So, if λ be its wavelength (de-Broglie), then we have

$$n\lambda = 2\pi r_n$$

where, r_n is the radius of second orbit.

$$r_n = 0.5 \times n^2\ (\text{in Å})$$
$$= 0.5 \times 4 = 2\,\text{Å}$$
$$\therefore \quad 2 \times \lambda = 2 \times \pi \times 2\,\text{Å}$$
$$\Rightarrow \quad \lambda = 2\pi\,(\text{Å}) = 6.28\,\text{Å}$$

76. According to Bohr's postulates,

$$mvr = \frac{nh}{2\pi} \quad \ldots(\text{i})$$

(where, mvr = angular momentum of an electron and n is an integer).

Thus, the centripetal force, $\frac{mv^2}{r}$ (experienced by the electron) is due to the electrostatic attraction, $\frac{kZe^2}{r^2}$

where, Z = Atomic number of the atom.

$$\text{Therefore,} \quad \frac{mv^2}{r} = \frac{kZe^2}{r^2}$$

Substituting the value of v^2 from Eq. (i), we obtain

$$\frac{m}{r} \cdot \frac{n^2h^2}{4\pi^2m^2r^2} = \frac{kZe^2}{r^2}$$

$$\Rightarrow \quad r = \frac{n^2h^2}{4\pi^2mkZe^2}$$

[redacted]

The relation for the nth radius of Bohr orbit in terms of Bohr's quantisation condition of orbital angular momentum $= \dfrac{n^2h^2}{4\pi^2 mkZe^2}$

77. 18751 Å; refer to Q. 62 on page 485.
The wavelength belongs to Paschen series of hydrogen spectrum.

78. (i) 3.4 eV and (ii) – 6.8 eV; refer to Example 4 part (ii) on page 478.
(iii) The potential energy of a system depends on the reference point taken. Here, the potential energy of the reference point is taken as zero. If the reference point is changed, then the value of the potential energy of the system also changes. Since, total energy is the sum of kinetic and potential energies, total energy of the system will also change.

79. Muonic hydrogen is the atom in which a negatively charged muon of mass about 207 m_e revolves around a proton.

In Bohr's atomic model, $r \propto \dfrac{1}{m}$

$$\frac{r_{\text{muon}}}{r_{\text{electron}}} = \frac{m_e}{m_\mu} = \frac{m_e}{207\, m_e} = \frac{1}{207} \qquad [\because m_\mu = 207 m_e]$$

Here, r_e is radius of orbit of electron in H-atom = 0.53 Å

$$r_\mu = \frac{r_e}{207} = \frac{0.53 \times 10^{-10}}{207} = 2.56 \times 10^{-13}\ \text{m}$$

Again in Bohr's atomic model,

$$E \propto m$$

$$\therefore \quad \frac{E_\mu}{E_e} = \frac{m_\mu}{m_e} = \frac{207\, m_e}{m_e}$$

$$\Rightarrow \quad E_\mu = 207\, E_e$$

For ground state, energy of electron in H-atom,

$$E_e = -13.6\ \text{eV}$$

$$\therefore \quad E_\mu = 207\,(-13.6) = -2815.2\ \text{eV} = -2.8152\ \text{keV}$$

80. Refer to text on pages 475 and 476.
In second excited state, i.e. $n = 3$, three spectral lines namely Lyman series and Balmer series can be obtained corresponding to transition of electron from $n = 3$ to $n = 1$ and $n = 3$ to $n = 2$, respectively and $n = 2$ to $n = 1$.
For Lyman series (minimum wavelength) $n = 3$ to $n = 1$,

$$\frac{1}{\lambda_{\min}} = R\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 8R/9 \qquad \text{...(i)}$$

For Balmer series (maximum wavelength)

$n = 3$ to $n = 2$,

$$\frac{1}{\lambda_{\max}} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{9-4}{36} = \frac{5}{36}R$$

$$\Rightarrow \quad \frac{1}{\lambda_{\max}} = \frac{5R}{36} \qquad \text{...(ii)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{\lambda_{\max}}{\lambda_{\min}} = \frac{\frac{8R}{9}}{\frac{5R}{36}} = \frac{8R}{9} \times \frac{36}{5R} = \frac{32}{5}$$

$$\lambda_{\max} : \lambda_{\min} = 32 : 5$$

81. The energies of gaseous hydrogen at room temperature are as given below

$$E_1 = -13.6\ \text{eV},\ E_2 = -3.4\ \text{eV}$$
$$E_3 = -1.51\ \text{eV},\ E_4 = -0.85\ \text{eV}$$
$$E_3 - E_1 = -1.51 - (-13.6) = 12.09\ \text{eV}$$

and $E_4 - E_1 = -0.85 - (-13.6) = 12.75$ eV

As, both the values do not match the given value, but it is nearest to $E_4 - E_1$.
$\therefore$ Upto $E_4 - E_1$ energy level, the H-atoms would be excited.

Lyman series, $\dfrac{1}{\lambda} = R\left[\dfrac{1}{1^2} - \dfrac{1}{n^2}\right]$

For first member, $n = 2$

$$\therefore \quad \frac{1}{\lambda_1} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = 1.097 \times 10^7 \left[\frac{4-1}{4}\right]$$

$$\Rightarrow \quad \lambda_1 = 1.215 \times 10^{-7}\ \text{m}$$

Balmer series, $\dfrac{1}{\lambda} = R\left[\dfrac{1}{2^2} - \dfrac{1}{n^2}\right]$

For first member, $n = 3$

$$\therefore \quad \frac{1}{\lambda_1} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = 1.097 \times 10^7 \left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\Rightarrow \quad \lambda_1 = 6.56 \times 10^{-7}\ \text{m}$$

82. (i) Let v_1 be the orbital speed of the electron in a H-atom in the ground state level, $n_1 = 1$. For charge (e) of an electron, v_1 is given by the relation,

$$v_1 = \frac{e^2}{n_1 4\pi \varepsilon_0 \left(\frac{h}{2\pi}\right)} = \frac{e^2}{2\varepsilon_0 h}$$

where, e = charge on an electron
$= 1.6 \times 10^{-19}$ C
ε_0 = permittivity of free space

[redacted]

$= 8.85\times 10^{-12}\ N^{-1}C^2m^{-2}$

h = Planck constant $= 6.63\times 10^{-34}$ J-s

$$\therefore \quad v_1 = \frac{(1.6\times10^{-19})^2}{2\times 8.85\times10^{-12}\times 6.63\times10^{-34}}$$

$= 0.0218\times10^{8} = 2.18\times10^{6}$ m/s

We know that, $v_n = v_1/n$

For level $n_2 = 2$, we can write the relation for the corresponding orbital speed as,

$$v_2 = \frac{v_1}{2} = \frac{2.18\times 10^6}{2}$$

$= 1.09\times10^6$ m/s

and for level $n_3 = 3$, we can write the relation for the corresponding orbital speed as

$$v_3 = \frac{v_1}{3} = \frac{2.18\times10^6}{3} = 7.27\times10^5 \text{ m/s}$$

Hence, the speed of the electron in a H-atom in $n=1$, $n=2$ and $n=3$ is 2.18×10^6 m/s, 1.09×10^6 m/s and 7.27×10^5 m/s, respectively.

(ii) Let T_1 be the orbital period of the electron and is given by $T = \frac{2\pi r}{v}$

where, r = radius of the orbit $= \frac{n^2h^2\varepsilon_0}{\pi me^2}$

h = Planck constant $= 6.63\times10^{-34}$ J-s

e = charge on an electron $= 1.6\times10^{-19}$ C

ε_0 = permittivity of free space

$= 8.85\times 10^{-12}\ N^{-1}C^2m^{-2}$

m = mass of an electron $= 9.1\times 10^{-31}$ kg

For $n = 1$,

$$T_1 = \frac{2\pi r_1}{v_1} = \frac{2\times 3.14\times 0.53\times 10^{-10}}{2.18\times 10^6}$$

$= 1.52\times 10^{-16}$ s

As, $T_n = n^3 T_1$

Then, $T_2 = (2)^3 T_1 = 8\times 1.52\times 10^{-16}$

$= 1.22\times10^{-15}$ s

$T_3 = (3)^3 T_1 = 27\times 1.52\times 10^{-16}$

$= 4.10\times 10^{-15}$ s

Then, the orbital period in each of these levels is 1.52×10^{-16} s, 1.22×10^{-15} s and 4.10×10^{-15} s, respectively.

[redacted]

SUMMARY

- **α-Particle Scattering Experiment by Rutherford** In this experiment, a collimated beam of α-particles of energy 5.5 MeV was allowed to fall on 2.1×10^{-7} m thick gold foil. The α-particles were observed through a rotatable detector consisting of zinc sulphide screen and microscope and it was found that α-particles got scattered, which produce scintillations on zinc sulphide screen.
- **Rutherford's Model of Atom** According to this model, every atom consists of a central core called the nucleus of an atom and the size of the nucleus is of the order of 10^{-15} m. The atomic nucleus is surrounded by certain number of electrons and they revolve around the nucleus in various circular orbits.
- **Electron Orbits** The total mechanical energy E of electron in a hydrogen atom is $E = \dfrac{-e^2}{8\pi\varepsilon_0 r}$.
- **Drawbacks of Rutherford's Model** It could not explain
  (i) stability of atom
  (ii) line spectrum.
- **Distance of Closest Approach** At a certain distance r_0 from the nucleus, whole of the kinetic energy of α-particles cannot go further closed to the nucleus.
- **Scattering Angle** Angle by which α-particles get deviated from its original path around the nucleus is called angle of scattering.
- **Impact Parameter** Perpendicular distance of the velocity vector of the α-particles from the central line of the nucleus of an atom is called impact parameter.
- **Bohr's Model of Hydrogen Atom** Bohr's combined classical and early quantum concepts and gave his theory in the form of three postulates.
  First postulate state that an electron could revolve in a certain stable orbits without the emission of radiant energy.
  Second postulate tells that $mvr = \dfrac{nh}{2\pi}$.
  Third postulate tells that $h\nu = E_2 - E_1$.
- **Radii of Bohr's Stationary Orbits**
  $$r = \frac{n^2h^2}{4\pi^2 mke^2}$$
  $$\Rightarrow \quad r \propto n^2$$
- **Velocity of Electrons in Bohr's Stationary Orbits**
  $$v = \frac{2\pi Zke^2}{nh}$$
- **Frequency of Electrons in Bohr's Stationary Orbits**
  $$\nu = \frac{kZe^2}{nhr} \Rightarrow \nu \propto \frac{1}{n}$$
- **Total Energy of Electrons in Bohr's Stationary Orbits**
  $$E = \frac{-me^4Z^2}{8n^2\varepsilon_0^2h^2}$$
- **Hydrogen Spectrum** It consists of discrete bright lines a dark background and is known as hydrogen emission spectrum.
  There is one more type of hydrogen spectrum exists, where we get dark lines on the bright background, it is known as absorption spectrum.
- **Formulae for the Spectral Series of Hydrogen**
  Lyman series $\dfrac{1}{\lambda} = R\left(\dfrac{1}{1^2} - \dfrac{1}{n^2}\right)$, where $n = 2, 3, 4, \ldots$
  Balmer series $\dfrac{1}{\lambda} = R\left(\dfrac{1}{2^2} - \dfrac{1}{n^2}\right)$, where $n = 3, 4, 5, \ldots$
  Paschen series $\dfrac{1}{\lambda} = R\left(\dfrac{1}{3^2} - \dfrac{1}{n^2}\right)$, where $n = 4, 5, 6, \ldots$
  Brackett series $\dfrac{1}{\lambda} = R\left(\dfrac{1}{4^2} - \dfrac{1}{n^2}\right)$ where $n = 5, 6, 7, \ldots$
  Pfund series $\dfrac{1}{\lambda} = R\left(\dfrac{1}{5^2} - \dfrac{1}{n^2}\right)$, where $n = 6, 7, 8, \ldots$
- **de-Broglie's Comment on Bohr's second Postulate** According to de-Broglie, a stationary orbit is that which contains an integral number of de-Broglie standing waves associated with the revolving electron.

[redacted]

CHAPTER PRACTICE (UNSOLVED)

OBJECTIVE Type Questions

1. Atoms consist of a positively charged nucleus is obvious from the following observation of Geiger-Marsden experiment
   (a) most of α-particles do not pass straight through the gold foil
   (b) many of α-particles are scattered through the acute angles
   (c) very large number of α-particles are deflected by large angles
   (d) None of the above

2. The simple Bohr model cannot be directly applied to calculate the energy levels of an atom with many electrons. This is because **NCERT Exemplar**
   (a) of the electrons not being subject to a central force
   (b) of the electrons colliding with each other
   (c) of screening effects
   (d) the force between the nucleus and an electron will no longer be given by Coulomb's law

3. Taking the Bohr radius as $a_0 = 53$ pm, the radius of Li^{++} ion in its ground state, on the basis of Bohr's model, will be about **NCERT Exemplar**
   (a) 53 pm (b) 27 pm
   (c) 18 pm (d) 13 pm

4. In Bohr's atomic model, in going to a higher level (PE = potential energy, TE = total energy)
   (a) PE decreases, TE increases
   (b) PE increases, TE increases
   (c) PE decreases, TE decreases
   (d) PE increases, TE decreases

5. The kinetic energy in ground state of hydrogen atom is – 13.6 eV. What will be the potential energy of electron in this state?
   (a) – 27.2 eV (b) + 27.2 eV
   (c) – 13.6 eV (d) 0 eV

6. Balmer formula is valid for
   (a) hydrogen
   (b) singly ionised helium
   (c) doubly ionised lithium
   (d) All of the above

7. In hydrogen spectrum, H_α lines lies in
   (a) Lyman series
   (b) Balmer series
   (c) Paschen series
   (d) Brackett or Pfund in one of them

8. The number of spectral lines produced due to transition among three energy levels will be
   (a) 10 (b) 8 (c) 6 (d) 3

9. The de-Broglie wavelength of an electron in first Bohr's orbit is
   (a) equal to $\frac{1}{4}$ of circumference of orbit
   (b) equal to $\frac{1}{2}$ of circumference of orbit
   (c) equal to twice of circumference of orbit
   (d) equal to the circumference of orbit

VERY SHORT ANSWER Type Questions

10. What is the impact parameter for scattering of α-particle by 180°?

11. What is the ratio of mass of an α-particle to that of an electron?

12. Calculate the radius of the first orbit of H-atom. Show that the velocity of electron in the first orbit is 1/137 times the velocity of light.

13. Name the spectral series of H-atom lying in the infrared region.

SHORT ANSWER Type Questions

14. Explain, why scattering of α-particles in Rutherford's experiment is not affected by the mass of the nucleons?

[redacted]

15. Derive the expression for the wavelength of the H-atom for different spectral series.

16. Derive the Bohr's quantisation condition for angular momentum of the orbiting of electron in hydrogen atom, using de-Broglie's hypothesis.
All India 2017

LONG ANSWER Type I Questions

17. Explain the significance of negative energy of an electron in an orbit. What is the energy possessed by an electron for $n = \infty$?

18. Draw energy levels for the hydrogen atom.

19. What does an emperical formula mean? Hence, explain that how Balmer proposed this formula?

20. Using Bohr's postulates, derive the expression for the total energy of the electron revolving in n th orbit of hydrogen atom. Find the wavelength of H_α line, given the value of Rydberg constant, $R = 1.1 \times 10^7 \text{ m}^{-1}$.

21. What is the difference between emission spectra and absorption spectra?

22. Explain the origin of spectral lines of hydrogen using Bohr's theory.

23. How did de-Broglie's equation lead to the quantisation condition laid down by Bohr?

LONG ANSWER Type II Questions

24. (i) State the basic assumption of the Rutherford model of an atom. Explain in brief, why this model cannot account for the stability of an atom?
 (ii) Using Bohr's postulates, derive the expression for radius of electron in nth orbit of electron in hydrogen atom.

25. (i) Using postulates of Bohr's theory of hydrogen atom, show that
 (a) the radii of orbits increase as n^2 and
 (b) the total energy of the electron increases as $1/n^2$, where n is the principal quantum number of the atom.
 (ii) Calculate the wavelength of H_α-line in Balmer series of hydrogen atom.
 Given, Rydberg constant, $R = 1.097 \times 10^7 \text{ m}^{-1}$.

ANSWERS

1. (b) 2. (a) 3. (c) 4. (b) 5. (b)
6. (d) 7. (b) 8. (d) 9. (d)

10. We know that, impact parameter $(b) = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Ze^2 \cot\phi/2}{K}$

Here, $\phi = 180°$

$\therefore \quad b = 0 \qquad [\because \cot 180° = 0]$

So, impact parameter becomes zero when scattering of α-particles occurs at 180°.

11. We have, mass of α-particle

$({}_2\text{He}^4) = 4 \times 1.67 \times 10^{-27}$ kg

Mass of electron $= 9.1 \times 10^{-31}$ kg

$$\therefore \quad \frac{m_\alpha}{m_e} = \frac{4 \times 1.67 \times 10^{-27}}{9.1 \times 10^{-31}} = 7341$$

Hence, α-particle is 7341 times heavier than electron.

12. We have, radius of nth orbit, $r_n = \dfrac{n^2 h^2 \varepsilon_0}{\pi m Z e^2}$ ...(i)

For $n = 1$,

$$r_1 = \frac{h^2\varepsilon_0}{\pi m e^2} = \frac{(6.63 \times 10^{-34})^2 \times (8.85 \times 10^{-12})}{(3.14)(9.1 \times 10^{-31})(1.6 \times 10^{-19})^2} = 0.53\text{Å}$$

By Bohr's postulate, we can also write as

$$v = \frac{nh}{2\pi m r}$$

...(ii)

Putting Eq. (ii) in Eq. (i), we get

$$v = \frac{Ze^2}{2h\varepsilon_0}\cdot\frac{1}{n}$$

For $n = 1$, we get $v_1 = \dfrac{e^2}{2h\varepsilon_0}$ or we can write it as $= \dfrac{c}{137}$

13. Brackett and Pfund series of H-atom lying in the infrared region.

14. The electrostatic force of attraction between α-particles and nucleus is $\approx 10^{36}$ stronger than the gravitational force, i.e.

$$\frac{F_g}{F_e} = \frac{\dfrac{Gm_\alpha M_{\text{nucleus}}}{r^2}}{\dfrac{Kq_\alpha q_{\text{nucleus}}}{r^2}} \approx 10^{-36}$$

Hence, scattering of α-particles is not affected by the mass of nucleus significantly.

15. Refer to text on page 478.

16. Refer to text on page 479.

[redacted]

17. Refer to text on page 477.

18. Refer to text on page 477.

19. An empirical formula is based solely on observation and experiments but not necessarily supported by theory.

The spacing between the spectral lines within certain sets of the hydrogen spectrum decreases in a regular way. As the wavelength decreases, teh lines appear closes together and are weaker in intensity. Balmer found a simple empirical formula for the observed comelengths.

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

20. Refer to text on pages 477 and 478.

21. Refer to text on page 478.

22. Refer to text on page 478.

23. Refer to text on page 479.

24. (i) Refer to the text on pages 473 and 475.
(ii) Refer to the text on page 476.

25. (i) (a) Refer to the text on page 476.
(b) Refer to the text on page 477.

(ii) We know that, $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For H_α-line in Balmer series.

$$n_1 = 2,\ n_2 = 3$$

$$\therefore \quad \frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

$$= 1.097 \times 10^7 \left[\frac{1}{4} - \frac{1}{9}\right]$$

$$= 1.097 \times 10^7 \left[\frac{9-4}{36}\right]$$

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left[\frac{5}{36}\right]$$

or $$\lambda = \frac{36}{5 \times 1.097} \times 10^{-7}$$

$$\lambda = 656 \times 10^{-7}\ \text{m}$$

[redacted]

13

Rutherford established that a central core exists in every atom which contains entire positive charge and more than 99% mass of the atom, this central core was named as Nucleus. In this chapter, we will study the constituents of nucleus and how they are held together. Further, we will proceed with the topics, size, mass, density and stability of Nuclei. And finally, we will have a look at the associated nuclear phenomena such as radioactivity, nuclear fission and nuclear fusion.

NUCLEI

|TOPIC 1|

Nucleus and Its Composition

CHAPTER CHECKLIST
- Nucleus and Its Composition
- Nuclear Energy

In every atom, the positive charge and mass are densely concentrated at the centre of the atom forming its nucleus. The overall dimensions of a nucleus are much smaller than those of an atom. The radius of the nucleus is smaller than the radius of an atom by a factor of 10^4. This means the volume of a nucleus is about 10^{-12} times the volume of the atom.

COMPOSITION OF NUCLEUS

The nucleus was first discovered in 1911 by Lord Rutherford and his associates by experiments on scattering of α-particles by atoms. He found that the scattering results could be explained, if atoms consist of a small, central, massive and positive core surrounded by orbiting electrons.The experimental results indicated that the size of the nucleus is of the order of 10^{-14} m and is thus 10000 times smaller than the size of atom. The study of radioactivity revealed that nucleus is not a composite body, but it is made of nucleons.The positive charge in the nucleus is that of the protons. A proton carries one unit of fundamental charge. A free proton is stable.

Atomic Mass Unit

The mass of an atom is very small. Kilogram cannot be used to measure such small quantity of mass. It is measured by a unit called **atomic mass unit** (amu, i.e. u).

It is defined as

$$1\text{u} = \frac{\text{mass of one } ^{12}\text{C atom}}{12} = \frac{1992647 \times 10^{26}\text{ kg}}{12} = 1660539 \times 10^{27}\text{ kg}.$$

Atomic masses are measured by an instrument called **mass spectrometer**.

[redacted]

Discovery of Neutron

The study of isotopes of hydrogen led to the fact that in addition to protons, the nuclei of atoms contain neutral matter in multiples of basic unit. This hypothesis was verified in 1932 by James Chadwick. Chadwick observed that when beryllium was bombarded with α-particles, some neutral radiations were emitted, which could knock out protons from lighter nuclei such as those of helium, carbon and nitrogen. Application of the principles of conservation of energy and momentum showed that these neutral radiations could not be photons. Chadwick satisfactorily solved this puzzle by assuming that the neutral radiation consists of a new type of neutral particles called **neutrons**.

He estimated the mass of a neutron being roughly equal to mass of a proton. However, unlike a free proton, a free neutron is unstable. The composition of a nucleus can be described by using the following terms and symbols:

Atomic Number (Z)

Atomic number of an element is the number of protons present inside the nucleus of an atom of the element. It is also equal to the number of electrons revolving in various orbits around the nucleus of the neutral atom.

$$\boxed{\begin{aligned}\text{Atomic number, } Z &= \text{Number of protons}\\ &= \text{Number of electrons (in a neutral atom)}\end{aligned}}$$

Mass Number (A)

Mass number of an element is the total number of protons and neutrons inside the atomic nucleus of the element.

$$\boxed{\text{Mass number, } A = \text{Number of protons} + \text{Number of neutrons}}$$

$$= \text{Number of electrons (in a neutral atom)} + \text{Number of neutrons}$$

$$= \text{Atomic number} + \text{Number of neutrons} = Z + N$$

The term **nucleon** is also used for neutron and proton. Thus, the number of nucleons in an atom is its mass number A.

Nuclear species or nuclides are shown by the notation ${}^{A}_{Z}X$, where X is the chemical symbol of the species.

EXAMPLE |1| In a nucleus of ${}_{92}U^{238}$, find the number of protons and the number of neutrons.

Sol. Number of protons, $Z = 92$

$\therefore$ Number of neutrons, $N = A - Z = 238 - 92 = 146$

Size of Nucleus

The size of the nucleus has been measured with the help of a variety of experiments involving the scattering of particles such as neutrons, protons, electrons, etc. From all these experiments, it is found that the volume of the nucleus is directly proportional to the number of nucleons (mass number) constituting nucleus.

If R is the radius of the nucleus having mass number A, then

$$\frac{4}{3}\pi R^3 \propto A \Rightarrow R \propto A^{1/3}$$

$$\Rightarrow \boxed{R = R_0 A^{1/3}}$$

where, $R_0 = 1.2 \times 10^{-15}$ m is the range of nuclear size.

It is also known as **nuclear unit radius**.

Owing to the small size of the nucleus, fermi (fm) is found to be a convenient unit of length in nuclear physics.

It is given as,

$$1 \text{ fermi (fm)} = 10^{-15} \text{ m}$$

EXAMPLE |2| Obtain the approximate value of the radius of a nucleus ${}_{92}U^{238}$. Take, R_0 is 1.2×10^{-15} m.

Sol. Given, $A = 238$, $R_0 = 1.2 \times 10^{-15}$ m

As, $R = R_0 A^{1/3} = 1.2 \times 10^{-15} (238)^{1/3}$

$\therefore$ $R = 7.437 \times 10^{-15}$ m

Nuclear Density

Density of nuclear matter is the ratio of mass of nucleus and its volume.

If m is the average mass of a nucleon and A is the mass number of element, then the mass of nucleus $= mA$. If R is the nuclear radius, then

$$\text{volume of nucleus} = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 A$$

$$\text{As, density of nuclear matter} = \frac{\text{Mass of nucleus}}{\text{Volume of nucleus}}$$

$$\therefore \quad \boxed{\rho = \frac{mA}{\frac{4}{3}\pi R_0^3 A}} \quad \text{or} \quad \boxed{\rho = \frac{3m}{4\pi R_0^3}}$$

Thus, the density of nucleus is a constant, **independent of** A, for all nuclei. Different nuclei are like drop of liquid of constant density. The density of nuclear matter is approximately 2.3×10^{17} kg/m^3. This density is very large as compared to an ordinary matter.

[redacted]

EXAMPLE |3| Given the mass of iron nucleus as 55.85u and $A = 56$. Find the nuclear density. NCERT

Sol. Given, mass, $m = 55.85\text{ u} = 55.85 \times 1.67 \times 10^{-27}$ kg

Volume, $V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (R_0 A^{1/3})^3 = \frac{4}{3}\pi R_0^3 \times A$

$\therefore$ Nuclear density, $\rho = \frac{m}{V}$

$$= \frac{3 \times 55.85 \times 1.67 \times 10^{-27}}{4 \times \frac{22}{7} \times (1.2 \times 10^{-15})^3 \times 56}$$

$$= 2.29 \times 10^{17} \text{ kg/m}^3$$

EXAMPLE |4| Supposing that protons and neutrons have equal masses. Calculate how many times nuclear matter is denser than water? Take, mass of a nucleon $= 1.67 \times 10^{-27}$ kg and $R_0 = 1.2 \times 10^{-15}$ m.

Sol. Density of nucleus (of water),

$$\rho = \frac{3m}{4\pi R_0^3} = \frac{3 \times 1.67 \times 10^{-27}}{4 \times \frac{22}{7} \times (1.2 \times 10^{-15})^3}$$

$$= \frac{7 \times 3 \times 1.67 \times 10^{18}}{88 \times 1.2 \times 1.2 \times 1.2}$$

$$= 2.307 \times 10^{17} \text{ kg/m}^3$$

Density of water, $\rho' = 10^3 \text{ kg/m}^3$

$$\therefore \quad \frac{\rho}{\rho'} = \frac{2.307 \times 10^{17}}{10^3} = 2.307 \times 10^{14}$$

MASS-ENERGY AND NUCLEAR BINDING ENERGY

Mass-Energy

Einstein showed that mass is another form of energy and one can convert mass-energy into other forms of energy.

Einstein's mass-energy equivalence equation is $E = mc^2$

where, E is the energy, m is the mass and c is the velocity of light in vacuum (approximately equal to 3×10^8 m/s).

The mass of a particle measured in a frame of reference in which the particle is at rest is called its rest mass, usually denoted by m_0. The rest mass-energy of a particle would be m_0c^2, which is enormously large on account of large value of c.

If I is kinetic energy of the particle, then its total energy,

$$\boxed{E = mc^2}$$

$$= \text{rest mass-energy} + \text{KE} = m_0c^2 + I$$

where, m is called effective mass of the particle, when it is moving. Clearly, $m > m_0$.

The conservation law of energy states that the initial energy and final energy are equal, provided the energy associated with mass is also included.

Nuclear Binding Energy

The sum of the masses of neutrons and protons forming a nucleus is more than the actual mass of the nucleus. This difference of masses is known as mass defect.

If a certain number of neutrons and protons are brought together to form a nucleus of a certain charge and mass, an energy E_b will be released in the process.The energy E_b is called the binding energy of the nucleus.

Thus, the binding energy of a nucleus may be defined as the energy equivalent to the mass defect of the nucleus.

If we separate a nucleus into its nucleons, we would have to supply a total energy equal to E_b, to those particles from Einstein equation,

$$E = \Delta mc^2 \qquad [\Delta m = \text{mass defect}]$$

$$E_b = [Zm_p + (A - Z)m_n - M]c^2$$

where, M is mass of nucleus, m_p is the mass of proton and m_n is the mass of neutron.

The mass defect reappears as equivalent energy $(\Delta m)c^2$, which is liberated during the formation of nucleus. Conversely, an amount Δmc^2 of external energy is required to break the nucleus into protons and neutrons. This energy is called binding energy.

"The binding energy of a nucleus is defined as the minimum energy required to separate its nucleons and place them at rest and infinite distance apart".

Average Binding Energy Per Nucleon of a Nucleus

It is the average energy spend to remove a nucleon from the nucleus to infinite distance. It is given by total binding energy divided by the mass number of the nucleus.

Binding energy per nucleon

$$= \frac{\text{Total binding energy}}{\text{Number of nucleons } (A)}$$

or $$E_{bn} = \frac{E_b}{A}$$

EXAMPLE |5| A given coin has a mass of 3.0 g. Calculate the nuclear energy that would be required to separate all the neutrons and protons from each other. For simplicity assume that the coin is entirely made of $^{63}_{29}$Cu atoms (of mass 62.92960 u). NCERT

[redacted]

Sol. Given,mass of coin = 3g

Atomic mass of Cu = 63

Mass of $^{63}_{29}Cu$, m = 62.92960 u

Avogadro's number = 6.023×10^{23}

Mass of proton, m_p = 1.007825 u

Mass of neutron, m_n = 1.008665 u

Nuclear energy required to separate neutrons and protons, E_b = ?

Since, each atom of copper contains 29 protons and 34 neutrons. Therefore, mass defect of each atom using the relation,

$\Delta m = [Z m_p + (A - Z) m_n] - M$

$\Delta m = [29\times 1.007825 + 34\times 1.008665] - 62.92960$

$= 0.591935$ u

Number of atoms in 3 g coin $= \dfrac{6.023\times 10^{23}\times 3}{63}$

$= 2.868\times 10^{22}$

Total mass defect of all atoms,

$(\Delta m)_{total} = 0.591935\times 2.868\times 10^{22} = 1.6977\times 10^{22}$

The nuclear energy required (E_b) to separate all the neutrons and protons from each other and can be calculated by using the relation,

$E_b = (\Delta m)\times c^2 - (\Delta m) c^2 \times 931 \text{MeV}/c^2$

[$\because$ 1 u = 931 MeV]

$= 1.6977\times 10^{22}\times 931$ MeV $- 1.58\times 10^{25}$ MeV

EXAMPLE |6| **Find the binding energy per nucleon of $_{20}Ca^{40}$ nucleus. Given, $m\,(_{20}Ca^{40}) = 39.962589$ u, $m_n = 1.008665$ u and $m_p = 1.007825$ u. Take, 1 amu = 931 MeV/c^2.**

Sol. In a nucleus of $_{20}Ca^{40}$,

Number of protons = 20

Number of neutrons = 40 − 20 = 20

Total mass of 20 protons and 20 neutrons

$= 20 m_p + 20 m_n = 20(m_p + m_n)$

$= 20\,(1.007825 + 1.008665)$

$= 40.3298$ u

Mass defect, $\Delta m = 40.3298 - 39.962589 = 0.367211$ u

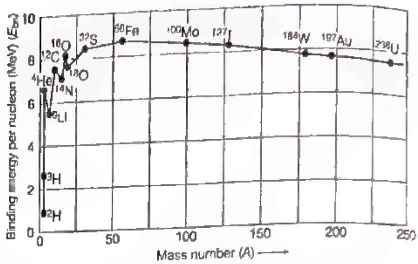
Total binding energy $= 0.367211\times 931 = 341.873441$ MeV

E_b per nucleon, $E_{bn} = \dfrac{341.873441}{40}$

$= 8.547$ MeV/nucleon

Binding Energy Curve

It is a plot of the binding energy per nucleon E_{bn} *versus* the mass number A for a large number of nuclei.



Binding energy per nucleon as a function of mass number

The following are the features of the plot

(i) Average BE/nucleon for lighter nuclei; like $_1H^1$, $_1H^2$, $_1H^3$ is small.

(ii) For mass numbers ranging from 2 to 20, there are sharply defined peaks corresponding to $_2He^4$, $_6C^{12}$, $_8O^{16}$, etc. The peaks indicate that these nuclei are relatively more stable than the other nuclei in their neighbourhood.

(iii) The BE curve has a broad maximum peak in the range $A = 30$ to $A - 120$, which is practically constant corresponding to average binding energy per nucleon is 8.8 MeV per nucleon for $_{26}Fe^{56}$.

(iv) As, the mass number increases, the BE/nucleon decreases gradually falling to about 7.6 MeV per nucleon for $_{92}U^{238}$. The decrease may be due to Coulomb repulsion between the protons. The heavy nuclei are therefore, relatively less stable.

Conclusions

Following conclusions are obtained from the graph

(i) A very heavy nucleus $A = 240$ has lower E_{bn} compared to that of a nucleus with $A = 120$. Thus, if a nucleus $A = 240$ breaks into two $A = 120$ nuclei, nucleons get more tightly bound. Energy would be released in this process (nuclear fission).

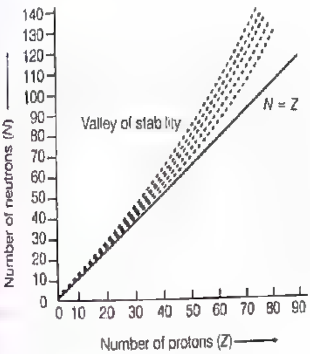
(ii) When two light nuclei ($A \leq 10$) join to form a heavier nucleus, E_{bn} of fused heavier nuclei is more than the E_{bn} of lighter nuclei. Energy would be released in this process (nuclear fusion).

Note This topic has been frequently asked in previous years 2014, 2013,2012, 2011, 2010.

[redacted]

NUCLEAR STABILITY

The stability of a nucleus is determined by the value of its binding energy per nucleon. Higher the binding energy of nucleon, more stable is the nucleus. The stability of nucleus is also determined by its neutron to proton ratio. A plot of number of neutrons and number of protons is shown in the figure below. The solid line shows the nuclei with equal number of protons and neutrons. Only light nuclei are on this line, i.e. they are stable, if they contain approximately same number of protons and neutrons.



Graphical representation of number of neutrons versus number of protons

Heavy nuclei are stable only when they have more neutrons than protons.

The long narrow region shown in the figure, which contains the cluster of short lines representing stable nuclei is referred to as the valley of stability.

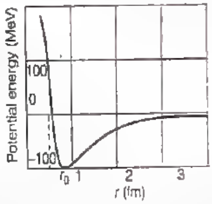
Experimental study shows that the more stable nuclei contain even number of protons or neutrons or both.

NUCLEAR FORCE

From binding energy curve, we have seen that for average mass nuclei, the binding energy per nucleon is approximately 8 MeV, which is much larger than the binding energy in atoms. Thus, for binding a nucleus together, there must be a strong attractive force of a totally different kind.

The force must be strong enough to overcome the repulsion between protons and to bind both protons and neutrons into tiny nuclear volume. The constancy of binding energy per nucleon is a consequence of the fact that nuclear force is short-ranged.

From the plot, it is concluded that potential energy is minimum at a distance $r_0 (\approx 0.8$ fm) which means, the force is attractive for distances larger than 0.8 fm and repulsive for the distances less than 0.8 fm between nucleons.



Graphical representation of potential energy versus distance for a pair of nucleon. For a distance greater than r_0, the force is attractive and for distances less than r_0 the force is strongly repulsive

Some of the important characteristics of these forces are as given below:

(i) Nuclear forces among a pair of neutrons, a pair of protons and also between a neutron-proton pair, is approximately the same. This shows that nuclear forces are independent of charge.

(ii) The nuclear forces are very short range forces. They are operative upto distances of the order of a few fermi.

(iii) The nuclear force is much stronger than the coulomb force acting between charges or gravitational forces between masses.

(iv) Nuclear force between two nucleons falls rapidly to zero as their distance is more than a few femtometres (fm). This leads to saturation of forces in a medium or large sized nucleus, i.e. each nucleon interacts with its immediate neighbours only, rather than with all the other nucleons in the nucleus.

(v) The nuclear forces are dependent on spin or angular momentum of nuclei.

Note Nuclear forces are the strongest attractive forces between nucleons. It is non-conservative force and does not obey inverse square law. It is non-central force also

[redacted]

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. Atomic mass unit (1 u) is
(a) 1/12 of mass of ^{12}C atom
(b) 1/14 of mass of ^{14}C atom
(c) 1/12 of mass of ^{14}C atom
(d) 1/6 of mass of ^{12}C atom

2. Ratio of radius of an atom to the radius of its nucleus is around
(a) 10^{-2} (b) 10^{4}
(c) 10^{12} (d) 10^{15}

3. The number of neutrons in a $_{84}Po^{218}$ nucleus is
(a) 84 (b) 218 (c) 222 (d) 134

4. As compared to ^{12}C atom, ^{14}C atom has
(a) two extra protons and two extra electrons
(b) two extra protons but no extra electrons
(c) two extra neutrons and no extra electrons
(d) two extra neutrons and two extra electrons

5. Density of a nucleus is
(a) more for lighter elements and less for heavier elements
(b) more for heavier elements and less for lighter elements
(c) very less compared to ordinary matter
(d) a constant

6. Energy equivalent of 2 g of a substance is
(a) 18×10^{13} mJ (b) 18×10^{13} J
(c) 9×10^{13} mJ (d) 9×10^{13} J

7. Given, $m\,(^{56}_{26}Fe) = 55.934939$ u and $m\,(^{209}_{83}Bi) = 208.980388$ u
$m_{proton} = 1.007825$ u, $m_{neutron} = 1.008665$ u.
Then, BE per nucleon of Fe
(a) 8.790 MeV (b) 7.75 MeV
(c) 7.5 MeV (d) Data insufficient

8. Nature of nuclear force is
(a) electrical
(b) magnetic
(c) gravitational
(d) None of the above

9. The gravitational force between a H-atom and another particle of mass m will be given by Newton's law $F = G\frac{Mm}{r^2}$, where r is in km. NCERT Exemplar
(a) Gravitational mass of H-atom
(b) Effective mass of H-atom
(c) Nuclear mass of H-atom
(d) Mass of electrons in H-atom

10. Heavy stable nuclei have more neutrons than protons. This is because of the fact that NCERT Exemplar
(a) neutrons are heavier than protons
(b) electrostatic force between protons are repulsive
(c) neutrons decay into protons through beta decay
(d) nuclear forces between neutrons are weaker than that between protons

VERY SHORT ANSWER Type Questions

11. Select the pairs of isotopes and isotones from the following nuclei.
$^{22}_{11}Na$, $^{24}_{12}Mg$, $^{24}_{11}Na$, $^{23}_{10}Na$.

12. Two nuclei have different number of protons and different number of neutrons. Can they have the same radii and same nuclear density?

13. The isotope $^{16}_{8}O$ has 8 protons, 8 neutrons and 8 electrons, while $^{8}_{4}Be$ has 4 protons, 4 neutrons and 4 electrons. Yet the ratio of their atomic masses is not exactly 2. Why?

14. $^{3}_{2}He$ and $^{3}_{1}He$ nuclei have the same mass number. Do they have same binding energy? NCERT Exemplar

15. Why do stable nuclei never have more protons than neutrons? NCERT Exemplar

16. Which property of nuclear forces is responsible for constancy of E_b per nucleon? Comment.

SHORT ANSWER Type Questions

17. From the relation $R = R_0A^{1/3}$, where R_0 is a constant and A is the mass number of a nucleus, show that the nuclear matter density is nearly constant (i.e. independent of A). NCERT

18. Check whether the given statement is correct or incorrect, if incorrect then correct it with proper explaination.

The order of magnitude of density of nuclear matter is 10^4 kg/m^3.

19. The mass of a nucleus is less than the sum of the masses of constituent neutrons and protons. Comment.

[redacted]

20. If both the numbers of protons and neutrons are conserved in a nuclear reaction like

$${}_6C^{12} + {}_6C^{12} \longrightarrow {}_{10}Ne^{20} + {}_2He^4$$

In what way is the mass converted into energy? Explain. **Delhi 2010**

21. Draw a plot of potential energy between a pair of nucleons as a function of their separation. Mark the regions where potential energy is (i) positive and (ii) negative. **Delhi 2013**

22. Explain the processes of nuclear fission and nuclear fusion by using the plot of binding energy per nucleon (BE/A) *versus* the mass number A. **CBSE 2018**

23. Proton and neutron exist together in an extremely small space within the nucleus. How is this possible, when protons repel each other?

24. Why heavy stable nucleus must contain more neutrons than protons?

LONG ANSWER Type I Questions

25. (i) Write three characteristic properties of nuclear force.
    (ii) Draw a plot of potential energy of a pair of nucleons as a function of their separation. Write two important conclusions that can be drawn from the graph. **Delhi 2015**

26. Answer the following.
    (i) Why is the binding energy per nucleon found to be constant for nuclei in the range of mass number (A) lying between 30 and 170?
    (ii) When a heavy nucleus with mass number $A = 240$ breaks into two nuclei, $A = 120$, energy is released in the process. **Foreign 2012**

27. In the study of Geiger-Marsdon experiment on scattering of α-particles by a thin foil of gold, draw the trajectory of α-particles in the Coulomb field of target nucleus. Explain briefly how one gets the information on the size of the nucleus from this study. From the relation $R = R_0 A^{1/3}$, where R_0 is constant and A is the mass number of the nucleus, show that nuclear matter density is independent of A. **All India 2015**

28. Nuclei with magic number of protons $Z = 2, 8, 20, 28, 50, 52$ and magic number of neutrons $N = 2, 8, 20, 28, 50, 82$ and 126 are found to be very stable.
    (i) Verify this by calculating the proton separation energy S_p for ${}^{120}Sn$ ($Z = 50$) and ${}^{121}Sb$ ($Z = 51$). The proton separation energy for a nuclide is the minimum energy required to separate the least tightly bound proton from a nucleus of that nuclide. It is given by

$$S_p = (M_{Z-1,N} + M_H - M_{Z,N})c^2$$

Given, ${}^{119}In = 118.9058$ u, ${}^{120}Sn = 119.902199$ u

${}^{121}Sb = 120.903824$ u,

${}^{1}H - 1.0078252$ u

    (ii) What does the existence of magic number indicate? **NCERT Exemplar**

29. Deuteron is a bound state of a neutron and a proton with a binding energy $B = 2.2$ MeV. A γ-ray of energy E is aimed at a deuteron nucleus to try to break it into a (neutron + proton) such that the n and p move in the direction of the incident γ-ray. If $E - B$, show that this cannot happen. Hence, calculate how much bigger than B must be E be for such a process to happen? **NCERT Exemplar**

NUMERICAL PROBLEMS

30. Obtain approximately the ratio of the nuclear radii of the gold isotope ${}^{197}_{79}Au$ and the silver isotope ${}^{107}_{47}Ag$. **NCERT**

31. Calculate the energy equivalent of 2 g of substance.

32. Calculate the energy in fusion reaction ${}^2_1H + {}^2_1H \longrightarrow {}^3_2He + n$, where BE of ${}^2_1H = 2.23$ MeV and of ${}^3_2He = 7.73$ MeV. **Delhi 2016**

33. Determine the nuclear radii of
    (i) ${}^{60}_{27}Co$
    (ii) ${}^{197}_{79}Au$.

34. A heavy nucleus X of mass number 240 and binding energy per nucleon 7.6 MeV is splitted, into two fragments Y and Z of mass numbers 110 and 130. The binding energy of nucleons in Y and Z is 8.5 MeV per nucleon. Calculate the energy released per fission in MeV. **Delhi 2010**

35. Obtain the binding energy (in MeV) of a nitrogen nucleus (${}^{14}_7N$), given $m({}^{14}_7N) = 14.00307$u. **NCERT**

[redacted]

36. The neutron separation energy is defined as the energy required to remove a neutron from the nucleus. Obtain the neutron separation energies of the nuclei ${}^{41}_{20}Ca$ and ${}^{27}_{13}Al$ from the following data :

$m({}^{40}_{20}Ca) = 39.962591$ u

$m({}^{41}_{20}Ca) = 40.962278$ u

$m({}^{26}_{13}Al) = 25.986895$ u

$m({}^{27}_{13}Al) = 26.981541$ u NCERT

37. A nuclide 1 is said to be the mirror isobar of nuclide 2, if $Z_1 = N_2$ and $Z_2 = N_1$.

(i) What nuclide is a mirror isobar of ${}^{23}_{11}Na$?

(ii) Which nuclide out of the two mirror isobars have greater binding energy and why? NCERT

38. (i) Two stable isotopes of lithium ${}^{6}_{3}Li$ and ${}^{7}_{3}Li$ have respective abundances of 7.5% and 92.5%. These isotopes have masses 6.01512 u and 7.01600 u respectively. Find the atomic mass of lithium.

(ii) Boron has two stable isotopes ${}^{10}_{5}B$ and ${}^{11}_{5}B$. Their respective masses are 10.01294 u and 11.00931 u, and the atomic mass of boron is 10.811 u. Find the abundances of ${}^{10}_{5}B$ and ${}^{11}_{5}B$. NCERT

39. In a periodic table, the average atomic mass of magnesium is given as 24.312 u. The average value is based on their relative natural abundance on the earth.

The three isotopes and their masses are ${}^{24}_{12}Mg$ (23.98504 u), ${}^{25}_{12}Mg$ (24.98584 u) and ${}^{26}_{12}Mg$ (25.98259 u). The natural abundance of ${}^{24}_{12}Mg$ is 78.99% by mass. Calculate the abundances of other two isotopes. NCERT

40. The three stable isotopes of neon ${}^{20}_{10}Ne$, ${}^{21}_{10}Ne$ and ${}^{22}_{10}Ne$ have respective abundances of 90.51%, 0.27% and 9.22%. The atomic masses of three isotopes are 19.99 u, 20.99 u and 21.99 u, respectively. Obtain the average atomic mass of neon. NCERT

41. (i) What is the nuclear density of ${}^{228}_{90}Th$?

(ii) Is the nuclear density of an α-particle (${}^{4}_{2}He$) to be greater than, less than or equal to ${}^{228}_{90}Th$? Explain.

(iii) Determine the nuclear density of an α-particle. NCERT

42. Obtain the binding energy of the nuclei ${}^{56}_{26}Fe$ and ${}^{209}_{83}Bi$ in units of MeV from the following data:

$m({}^{56}_{26}Fe) = 55.934939$ u

$m({}^{209}_{83}Bi) = 208.980388$ u NCERT

HINTS AND SOLUTIONS

1. (a) Atomic mass unit (1 u) is defined as

$$1\text{ u} = \frac{\text{mass of } {}^{12}C \text{ atom}}{12}$$

2. (b) Radius of atom $\simeq 10^{-10}$

Radius of nucleus $\simeq 10^{-14}$

$$\therefore \frac{\text{Radius of atom}}{\text{Radius of nucleus}} = \frac{10^{-10}}{10^{-14}} = 10^4$$

3. (d) Given, ${}_{84}Po^{218}$

Here, $Z = 84$, $A = 218$, $A = Z + N$

$N = A - Z = 218 - 84 = 134$

4. (c) For ${}^{12}_{6}C$, $A = 12 = N + Z$, $Z = 6 \Rightarrow N = 6$

For ${}^{14}_{6}C$, $A = 14 = N + Z$, $Z = 6 \Rightarrow N = 8$

Also, number of electrons in both atoms = number of protons = $Z = 6$

5. (d) Density $= \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{mA}{\frac{4}{3}\pi R_0^3 A}$

$$= \frac{3m}{4\pi R_0^3}, m = m_p = M_N$$

$= 23 \times 10^{17}$ kgm^{-3}, which is a constant

6. (b) Energy, $E = 2\times10^{-3} \times (3\times10^8)^2$ J

$E = 2\times10^{-3} \times 9\times10^{16}$

$= 18\times10^{13}$ J

Thus, if one gram of matter is converted to energy, there is a release of enormous amount of energy

7. (a) ${}^{56}_{26}Fe$ nucleus has 26 protons and 30 neutrons.

$\therefore$ Mass defect $= (26m_p + 30\,m_n) - m({}^{56}_{26}Fe)$

$= 56.46340 - 55.934939 = 0.528461$ amu

Total BE $= 0.528461 \times 931.5 = 492.26$ MeV

$\therefore$ Binding energy per nucleon

$$= \frac{492.26}{56} = 8.790 \text{ MeV}$$

8. (d) Nuclear force is an exchange force, it does not come under electrical, gravitational or magnetic force category.

[redacted]

9. (b) Given, $F = \frac{GMm}{r^2}$

M = effective mass of hydrogen atom

G = gravitational constant

and r = distance between H-atom and particle of mass m

10. (b) Stable heavy nuclei have more neutrons than protons. This is because electrostatic force between protons is repulsive, which may reduce stability.

11. Isotopes $^{22}_{11}Na$, $^{24}_{11}Na$

(both have same atomic number, i.e. 11)

Isotones = $^{24}_{11}Na$, $^{23}_{10}Na$

(both have same number of neutrons, i.e. 13)

12. Since, radius of nucleus (i.e. $R = R_0A^{1/3}$) is proportional to the cube root of its mass number, hence nuclei will have same radii, if their mass numbers are same. But nuclear density is independent on mass number. It remains constant for all nuclei, i.e. 2.3×10^{17} kg/m^3.

13. It is because of the fact that the mass of a nucleus is slightly less than the mass of it's constituent nucleons. This decrease in mass is called mass defect. Since, the mass defect in case of $^{16}_{8}O$ is not exactly twice the mass defect in case of $^{8}_{4}Be$, so the ratio of the atomic masses is not exactly

14. Since, the repulsive force between protons is missing in ^3_1He, so the binding energy of ^3_1He is greater than that of ^3_2He.

15. Because the protons are positively charged, so they repel each other. Since, this repulsion force is more, so that an excess of neutrons are required to reduce this repulsion.

16. Nuclear forces are saturated in character. This property makes E_b per nucleon constant for most of the nuclei.

17. Refer to text on page 502.

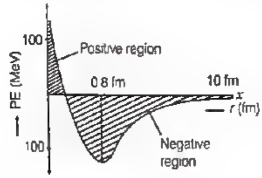
18. Given statement is incorrect because the order of magnitude of density of nuclear matter is the order of 10^{17} kg/m^3.

19. When nucleons approach each other to form a nucleus, they strongly attract each other. Their potential energy decreases and becomes negative. It is the potential energy which holds the nucleons together in the nucleus. The decrease in PE results in the mass of nucleons inside the nucleons.

20. The sum of masses of nuclei of product element is less than sum of masses of reactants and hence, loss of mass takes place during the reaction. This difference of mass of product element and reactant converts into energy and liberated in the form of heat.

Here, sum of masses of $_{10}Ne^{20}$ and $_2He^4$ is less than the sum of two $_6C^{12}$ and conversion of this mass defect is used to produce energy.

21. The graph between the potential energy of a pair of nucleons as a function of their separation is given below

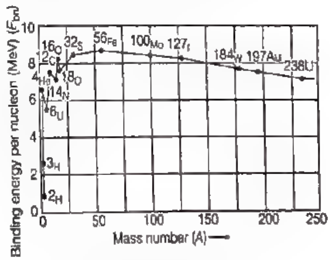


(i) For distance less than 0.8 fm, negative PE decreases to zero and then becomes positive.

(ii) For distances larger than 0.8 fm, negative PE goes on decreasing.

22. From the given plot, we can conclude that,

a very heavy nucleus $A = 240$ has lower E_{bn} compared to that of a nucleus with $A = 120$. Thus, if a nucleus $A = 240$ breaks into two $A = 120$ nuclei, nucleons get more tightly bound. Energy would be released in this process which is known as nuclear fission.



Binding energy per nucleon as a function of mass number

Also, when two light nuclei ($A \leq 10$) join to form a heavier nucleus, E_{bn} of fused heavier nuclei is more than the E_{bn} of lighter nuclei. Energy would be released in this process, which is known as nuclear fusion.

23. Refer to the text on page 505.

24. Refer to the text on page 505.

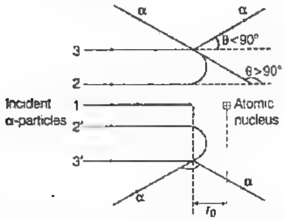
25. Refer to text on page 505.

26. (i) The binding energy per nucleon for nucleus of range, $30 < A < 170$ is close to its maximum value. So, the nucleus belongs to this region is highly stable and does not show radioactivity.

(ii) Binding energy per nucleon is smaller for heavier nuclei than the middle ones, i.e. heavier nuclei are less stable. When a heavier nucleus such as nucleus of mass number 240 splits into lighter nuclei (mass number 120), the BE/nucleon changes from about 7.6 MeV to 8.4 MeV. Greater BE of the product nuclei result in the liberation of energy.

[redacted]

27. Trajectory of α-particles in the coulomb field of target nucleus is shown below:



From this experiment, the following points are observed.

(i) Most of the α-particles pass straight through the gold foil. It means that they do not suffer any collision with gold atoms.

(ii) About one α-particle in every 8000 α-particles deflects by more than 90°. As most of the α-particles go undeflected and only a few get deflected, this shows that most of the space in an atom is empty and at the centre of the atom, there exists a nucleus By the number of α-particles get deflected, the information regarding size of the nucleus can be known.

Refer to text on page 502.

28. (i) The proton separation energy is given by

$$S_{p\,(Sn)} = (M_{119,\,70} + M_H - M_{120,\,70})c^2$$
$$= (118.9058 + 1.0078252 - 119.902199)\,c^2$$
$$= 0.0114262\,c^2$$

Similarly, $S_{p\,(Sb)} = (M_{120,\,70} + M_H - M_{121,\,70})\,c^2$

$$= (119.902199 + 1.0078252 - 120.903822)\,c^2$$
$$= 0.006202\,c^2$$

Since, $S_{p\,(Sn)} > S_{p\,(Sb)}$, Sn nucleus is more stable than Sb nucleus.

(ii) The existence of magic numbers indicates that the shell structure of nucleus is similar to the shell structure of an atom. This also explains the peaks in binding energy/nucleon curve.

29. Apply conservation of energy as well as conservation of momentum.

Given, binding energy, $B = 2.2$ MeV

From the energy conservation law,

$$E - B = K_n + K_p = \frac{p_n^2}{2m} + \frac{p_p^2}{2m} \quad \text{...(i)}$$

From conservation of momentum,

$$p_n + p_p = \frac{E}{c} \quad \text{...(ii)}$$

As, $E = B$

Eq. (i) becomes, $p_n^2 + p_p^2 = 0$

It only happen, if $p_n = p_p = 0$

So, the Eq.(ii) cannot be satisfied and the process cannot take place.

Let $E = B + X$, where $X << B$ for the process to take place.

Put the value of p_n from Eq. (ii) in Eq. (i), we get

$$\Rightarrow \quad X = \frac{\left(\frac{E}{c} - p_p\right)^2}{2m} + \frac{p_p^2}{2m}$$

$$\Rightarrow \quad 2p_p^2 - \frac{2Ep_p}{c} + \frac{E^2}{c^2} - 2mX = 0$$

Using the formula of quadratic equation, we get

$$p_p = \frac{\frac{2E}{c} \pm \sqrt{\frac{4E^2}{c^2} - 8\left(\frac{E^2}{c^2} - 2mX\right)}}{4}$$

For the real value p_p, the determinant is positive

$$\frac{4E^2}{c^2} = 8\left(\frac{E^2}{c^2} - 2mX\right) \Rightarrow 16mX = \frac{4E^2}{c^2}$$

$$\therefore \quad X = \frac{E^2}{4mc^2} = \frac{B^2}{4mc^2} \qquad [\because X << B \Rightarrow E = B]$$

30. Radius of nuclei, $R = R_0 A^{1/3}$

where, A is the mass number of nucleus and R_0 is an empirical constant.

$$\therefore \quad R \propto A^{1/3}$$

$$\Rightarrow \quad \frac{R_{gold}}{R_{silver}} = \left(\frac{A_{gold}}{A_{silver}}\right)^{1/3} = \left(\frac{197}{107}\right)^{1/3} = 1.225 = 1.23$$

31. Given, $m = 2\text{ g} = 2 \times 10^{-3}$ kg

According to mass-energy equivalence equation,

$$E = mc^2 = 2 \times 10^{-3} \times (3 \times 10^8)^2$$
$$= 18 \times 10^{13} = 1.8 \times 10^{14}\text{ J}$$

32. According to question,

$${}_1^2\text{H} + {}_1^2\text{H} \rightarrow {}_2^3\text{He} + {}_0n^1$$

$\therefore$ Energy of fusion = Binding energy of ${}_2^3$He

$- 2 \times$ Binding energy of ${}_1^2$H

$= 7.73 - 2 \times 2.23 = 3.27$ MeV

33. Refer to the Example 2 on page 502.

34. In these type of questions, we have to keep in mind about the difference of mass involved between output and input. Energy will be involved accordingly.

Energy released per fission

$$= (110 + 130) \times 8.5\text{ MeV} - 240 \times 7.6\text{ MeV}$$
$$= 240 \times (8.5 - 7.6)\text{ MeV}$$
$$= 240 \times 0.9 = 216\text{ MeV}$$

[redacted]

35. Mass of proton, $m_p = 1.00783$ u

Mass of neutron, $m_n = 1.00867$ u

In $^{14}_{7}N$, there are 7 protons and 7 neutrons.

$\therefore$ Mass defect, $\Delta m = (7m_p + 7m_n) - m$

$= 7 \times 1.00783 + 7 \times 1.00867 - 14.00307 = 0.11243$ u

Binding energy of nitrogen nucleus $= \Delta m \times 931$MeV

$= 0.11243 \times 931$MeV $= 104.67$ MeV

36. (i) When a neutron is separated from $^{41}_{20}Ca$, we are left with $^{40}_{20}Ca$ and the reaction becomes

$$^{41}_{20}Ca \longrightarrow {}^{40}_{20}Ca + {}_0n^1$$

Mass defect,

$$\Delta m = m(^{40}_{20}Ca) + m(_0n^1) - m(^{41}_{20}Ca)$$
$$= 39.962591 + 1.008665 - 40.962278$$
$$= 0.008978 \text{ u}$$

Energy for separation of neutron $= \Delta m \times 931$

$= 0.008978 \times 931$

$= 8.358$ MeV

(ii) When a neutron is separated from $^{27}_{13}Al$, we are left with $^{26}_{13}Al$. Thus, the reaction becomes

$$^{27}_{13}Al \longrightarrow {}^{26}_{13}Al + {}^1_0n$$

Mass defect, $\Delta m = m(^{26}_{13}Al) + m(_0n^1) - m(^{27}_{13}Al)$

$= 25.986895 + 1.008665 - 26.981541$

$= 0.014019$

$\therefore$ Energy for separation of neutron

$= \Delta m \times 931 = 0.014019 \times 931$

$= 13.06$ MeV

37. (i) According to the question, a nuclide 1 is said to be mirror isobar of nuclide 2, if $Z_1 = N_2$ and $Z_2 = N_1$. Now, in $^{23}_{11}Na$, $Z_1 = 11$, $N_1 = 23 - 11 = 12$

$\therefore$ Mirror isobar of $^{23}_{11}Na$ is $^{23}_{12}Mg$, for which $Z_2 = 12 = N_1$ and $N_2 = 23 - 12 = 11 = Z_1$

(ii) As, $^{23}_{12}Mg$ contains even number of protons (12) against $^{23}_{11}Na$ which has odd number of protons (11), therefore $^{23}_{11}Mg$ has greater binding energy than $^{23}_{11}Na$.

38. (i) Atomic mass of Li = Weighted average of the isotopes

$$= \frac{7.5 \times 6.01512 + 92.5 \times 7.01600}{7.5 + 92.5}$$
$$= \frac{45.1134 + 648.98}{100} = 6.94 \text{ u}$$

(ii) Suppose x and y are the abundances of $^{10}_{5}B$ and $^{11}_{5}B$, respectively.

Atomic mass of boron

= Weighted average of the isotopes

$$= \frac{x \times 10.01294 + y \times 11.00931}{100}$$

$$\Rightarrow \quad 10.811 = \frac{x \times 10.01294 + (100 - x) \times 11.00931}{100}$$
$$[\because y = (100 - x)]$$

$\Rightarrow \quad 1081.1 - 10.01294x + 1100.931 \quad 11.00931x$

$\Rightarrow 0.99637x = 19.831 \Rightarrow x = 19.90$

$y = (100 - x) = 80.1$

So, abundance percent $^{10}_{5}B = 19.90\%$

Abundance percent of $^{11}_{5}B = 80.1\%$

39. Given, atomic mass of Mg = 24.312 u

Mass of $^{24}_{12}Mg = 23.98504$ u

Mass of $^{25}_{12}Mg = 24.98584$ u

Mass of $^{26}_{12}Mg = 25.98259$ u

Abundance of $^{24}_{12}Mg = 78.99\%$

Let the abundance of $^{25}_{12}Mg$ be x %.

The abundance of $^{26}_{12}Mg = 100 - 78.99 - x = (21.01 - x)\%$

Atomic mass = Weight average of masses

$$= \frac{\text{Abundance of the isotopes}}{\text{Total abundance}}$$

$$\Rightarrow \quad 24.312 = \frac{78.99 \times 23.98504 + x \times 24.98548 + (21.01 - x) \times 25.98259}{100}$$

$\Rightarrow \quad x = 9.303\%$

So, the abundance of $_{12}Mg^{25}$ is 9.303% and the abundance of $_{12}Mg^{26}$ is 11.71%.

40. Given, abundance per cent of $Ne^{20} = 90.51\%$

Abundance per cent of $Ne^{21} = 0.27\%$

Abundance per cent of $Ne^{22} = 9.22\%$

Mass of $Ne^{20} = 19.99$ u

Mass of $Ne^{21} = 20.99$ u

Mass of $Ne^{22} = 21.99$ u

Average atomic mass, m = Weighted average of all isotopes

$$= \frac{90.51 \times 19.99 + 0.27 \times 20.99 + 9.22 \times 21.99}{90.51 + 0.27 + 9.22}$$
$$= \frac{1809.29 + 5.67 + 202.75}{100} = \frac{2017.71}{100} = 20.18$$

41. (i) We know that,

$$\rho = \frac{3m}{4\pi R_0^3} = \frac{3 \times 1.6 \times 10^{-17} \text{ kg}}{4 \times 3.14 \times (1.2 \times 10^{-15})^3}$$
$$= 2.3 \times 10^{17} \text{ kg/m}^3$$

(ii) Nuclear density (ρ) is independent on mass number, hence nuclear density of α-particle (^4_2He) and thorium ($^{228}_{90}Th$) is equal to each other.

(iii) For α-particle, also nuclear density is equal to 2.3×10^{17} kg/m^3, as explained earlier.

[redacted]

42. Given, $m_p = 1.00783$ u, $m_n = 1.00867$ u

(i) For $^{56}_{26}$Fe, there are 26 protons and $(56-26)=30$ neutrons.

Δm = mass of nucleons − mass of nucleus

$= 26m_p + 30m_n - m$

$= 26 \times 1.00783 + 30 \times 1.00867 - 55.934939$

$= 0.528741$ u

Total binding energy $= \Delta m \times 931$ MeV

$= 0.528741 \times 931 = 492.26$ MeV

Binding energy per nucleon

$$= \frac{\text{Binding energy}}{\text{Number of nucleons}}$$

$$= \frac{492.26}{56} = 8.790 \text{ MeV}$$

(ii) For $^{209}_{83}$Bi, there are 83 protons and $(209-83)=126$ neutrons.

Δm = mass of nucleons − mass of nucleus

$= 83\,m_p + 126\,m_n - m$

$\Delta m = 83 \times 1.00783 + 126 \times 1.00867 - 208.980388$

$= 1.761922$ u

Binding energy $= \Delta m \times 931$ MeV

$= 1.761922 \times 931$

$= 1640.35$ MeV

Binding energy per nucleon

$$= \frac{\text{Binding energy}}{\text{Number of nucleons}}$$

$$= \frac{1640.35}{209} = 7.848 \text{ MeV}$$

Thus, binding energy per nucleon of Fe is more than Bi.

|TOPIC 2|
Nuclear Energy

NUCLEAR ENERGY

Nuclear energy is the energy released during the transformation of nuclei with less total binding energy to nuclei with greater binding energy.

Two distinct ways of obtaining energy from nucleus are as follows:

1. Nuclear fission
2. Nuclear fusion

Nuclear Fission

Nuclear fission is the phenomenon of splitting of a heavy nucleus (usually $A > 230$) into two or more lighter nuclei by the bombardment of proton, neutron, α-particle, etc. Energies associated with nuclear processes are about a million times larger than chemical process.

In fission, a heavy nucleus like $^{235}_{92}$U breaks into two smaller fragments by the bombardment of thermal neutron (low energy or slow moving).

e.g. $$^{235}_{92}\text{U} + ^1_0n \longrightarrow ^{141}_{56}\text{Ba} + ^{92}_{36}\text{Kr} + 3\,_0n^1 + Q$$

(Energy released)

Q-value here refer to the energy released in the nuclear process, which can be determined using Einstein's mass - energy relation, $E = mc^2$. The Q-value is equal to the difference of mass of products and reactants multiplied by square of velocity of light. Energy released per fission of $^{235}_{92}$U is 200.4 MeV.

The fragment nuclei produced in fission are highly unstable. They are highly radioactive and emit β-particles in succession until each reaches to a stable end product.

Nuclear Chain Reaction

In the nuclear fission reaction, there is a release of extra neutrons. The extra neutrons in turn initiate fission process, producing still more neutrons and so on. Thus, a chain of nuclear fission is set up called **nuclear chain reaction.** The chain reactions may be of two types:

Uncontrolled Chain Reaction During fission reaction, neutrons released are again absorbed by the fissile isotopes, the cycle repeats to give a chain reaction, i.e. self-sustaining and gives off energy at a rate that increases rapidly with time leading to large amount of radiation. This is called **uncontrolled chain reaction.**

Controlled Chain Reaction If by some means, the reaction is controlled in such a way that only one of the neutrons emitted in a fission causes another fission, then the fission rate remains constant and the energy is released steadily. Such a chain reaction is called a controlled chain reaction. It is used in a nuclear reactor.

The sustained fissibility of nuclear chain reaction depends on the multiplication factor or reproduction factor K.

$$K = \frac{\text{Rate of production of neutrons}}{\text{Rate of loss of neutrons}}$$

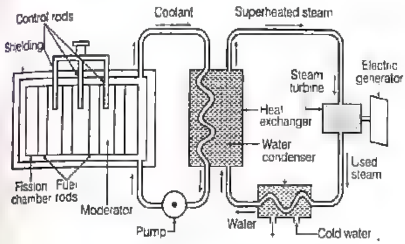
If $K = 1$, the operation of reactor is said to be **critical.** It is what we wish to be for steady power operation.

[redacted]

If $K > 1$, the reaction rate and reactor power increases exponentially. In this case, reaction is super-critical and can even explode. If $K < 1$, the reaction gradually stops. And the condition is called sub-critical.

Nuclear Reactor

It is a device that can initiate a self-sustaining controlled chain reaction of a fissionable material. They are used at nuclear power plants for generating electricity and in propulsion of ships.



Nuclear reactor

Construction

The key components of nuclear reactor are as follows:

(i) **Nuclear fuel** It is a material that can be burned by nuclear fission or fusion to derive nuclear energy. The common fuels used in nuclear reactor are ^{233}U, ^{235}U, ^{239}Pu, etc.

(ii) **Nuclear reactor core** It is the portion of a nuclear reactor containing the nuclear fuel components where the nuclear reaction takes place.

(iii) **Moderator** It is a medium to slow down the fast moving secondary neutrons produced during the fission. Heavy water, graphite, deuterium, paraffins, etc., acts as moderator.

(iv) **Control rods** It is used in nuclear reactors to control the rate of fission of uranium and plutonium. These are made of chemical elements capable of absorbing many neutrons without fissioning themselves such as silver, indium, boron and cadmium.

(v) **Coolant** It is a liquid used to remove heat from nuclear reactor core and transfer it to electrical generator and environment. Ordinary water under high pressure is used as coolant.

(vi) **Shielding** It is the protective covering made of concrete wall to protect from harmful radiations.

India's Atomic Energy Programme

The Atomic Energy Programme of our country was launched around 1950 under the leadership of Homi J Bhabha (1909-1966). The major milestones achieved so far are as below:

(i) First nuclear reactor named Apsara went critical on 4 August, 1956. It used enriched uranium as fuel and water as moderator.

(ii) Another reactor named Canada India Research US (CIRUS) became operative in 1960. It used natural uranium as fuel and heavy water as moderator.

(iii) Indigenous design and construction of plutonium plant at Trombay. It ushered in the technology of fuel reprocessing.

(iv) Research reactors like Zerlina, Purnima, Dhruva and Kamini were commissioned. The last one uses U-233 as fuel.

(v) The fast breeder reactors which use plutonium-239 as fuel do not need moderators. They can be used to produce fissile uranium-233 from thorium-232 and to build power reactors based on them. Considerable work has been done by our scientists in this direction at Kalpakkam nuclear plant.

(vi) We have mastered the complex technologies of mineral exploration, mining, fuel fabrication, heavy water production, fuel reprocessing, etc.

Nuclear Fusion

Nuclear fusion is the phenomenon of fusing two or more lighter nuclei forming a single heavy nucleus.For fusion to take place, the two nuclei must come close enough so that, attractive short range nuclear force is able to affect them. Since both the nuclei are positively charged particles, so they experience Coulomb's repulsion. Therefore, they must have enough energy to overcome this Coulomb barrier.

e.g. $${}_1H^1 + {}_1H^1 \longrightarrow {}_1H^2 + {}_1e^+ + \nu + 0.42\text{ MeV}$$

$${}_1H^2 + {}_1H^2 \longrightarrow {}_2He^3 + {}_0n^1 + 3.27\text{ MeV}$$

$${}_1H^2 + {}_1H^2 \longrightarrow {}_1H^3 + {}_1H^1 + 4.03\text{ MeV}$$

Fusion of hydrogen nuclei into helium nuclei is the source of energy of most of the stars including the sun.

Energy Generation in Stars

Thermonuclear fusion is the source of energy output in the interior of stars. The interior of the sun has a temperature of 1.5×10^7 K, which is considerably less than the estimated

[redacted]

temperature required for fusion of particles of average energy. Fusion in the sun involves protons whose energies are much above the average energy, i.e. protons which are in the high velocity fall of Maxwell-Boltzmann distribution.

The fusion reaction in the sun is a multi-step process in which the hydrogen is fused into helium. The **proton-proton** (p, p) cycle by which this occurs is represented by the following sets of reactions:

$$^1_1H + ^1_1H \rightarrow ^2_1H + e^+ + \nu + 0.42 \text{ MeV} \quad ...(i)$$

$$e^+ + e^- \rightarrow \gamma + \gamma + 1.02 \text{ MeV} \quad ...(ii)$$

$$^2_1H + ^1_1H \rightarrow ^3_2He + \gamma + 5.49 \text{ MeV} \quad ...(iii)$$

$$^3_2He + ^3_2He \rightarrow ^4_2He + ^1_1H + ^1_1H + 12.86 \text{ MeV} \quad ...(iv)$$

For the fourth reaction to occur, the first three reactions must occur twice, in case two light helium nuclei unite to form ordinary helium nucleus. If we consider the combination 2(i) + 2(ii) + 2(iii) + (iv), the net effect is

$$4^1_1H + 2e^- \rightarrow ^4_2He + 2\nu + 6\gamma + 26.7 \text{ MeV}$$

or $(4^1_1H + 4e^-) \rightarrow (^4_2He + 2e^-) + 2\nu + 6\gamma + 26.7 \text{ MeV}$

Thus, four hydrogen atoms combine to form ^4_2He atom with a release of 26.7 MeV of energy.

As the hydrogen in the core gets depleted and becomes helium, the core starts to cool. The star begins to collapse under its own gravity, which increases the temperature of the core.

The age of the sun is about 5×10^9 yr and it is estimated that there is enough hydrogen in the sun to keep it going for another 5 billion years.

Nuclear Holocaust

It is the name given to large scale destruction and devastation that would be caused by the use of nuclear weapons.

During fission of a single nucleus of $_{92}U^{235}$, about 0.9×235 MeV (≈ 200 MeV) energy is released in 10^{-9} s. If each nucleus of about 50 kg of ^{235}U undergoes fission, then the total energy released is 4×10^{15} J. This energy is equivalent to about 20000 tonnes of TNT.

The first explosion occured on 6th August, 1945, when USA dropped an atom bomb on Hiroshima in Japan. This resulted in killing of 66000 persons, injured 69000 persons and 67% of the city structures smashed.

The radioactive waste will hang like a cloud in the earth's atmosphere. It will absorb the sun's radiation and there may be a long nuclear winter.

Controlled Thermonuclear Fusion

The essential condition for carrying out nuclear fusion is to raise the temperature of the material so that particles have enough energy due to their thermal motions alone and they can overcome the Coulomb barrier. This process is called thermonuclear fusion.

The natural thermonuclear fusion in a star is replicated in a thermonuclear fusion device. The aim of controlled thermonuclear fusion is to generate the steady power by heating the nuclear fuel to a temperature in the range of 10^8 K. At these temperature, the fuel is a mixture of positive ions and electrons (plasma).

The challenge is to confine this plasma, since no container can stand such a high temperature. Several countries around the world including india are developing techniques in this connection. If successful, fusion reactors will hopefully supply almost unlimited power to humanity.

Distinction between Nuclear Fission and Nuclear Fusion

(i) Fission is the splitting of large nucleus into two or more smaller ones, on the other hand, fusion is the combining of two or more lighter nuclei to form larger one.

(ii) Fission does not normally occur in nature but fusion occurs in stars such as the sun.

(iii) Fission requires critical mass of the substance and high speed neutrons but in fusion, high density and high temperature environment are required.

(iv) In fission, energy released is million times greater than in chemical reactions, but lower than energy released by nuclear fusion.

(v) Uranium is the primary fuel for fission reaction and hydrogen isotopes are the primary fuel in nuclear fusion reaction.

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. In a nuclear reaction $^{238}_{92}U \rightarrow ^A_ZTh + ^4_2He$, the value of A and Z are
   (a) $A = 234, Z = 94$
   (b) $A = 238, Z = 94$
   (c) $A = 234, Z = 90$
   (d) $A = 238, Z = 90$

[redacted]

2. For sustaining the chain reaction in a sample (of small size) of $^{235}_{92}U$, it is desirable to slow down fast neutrons by
   (a) friction
   (b) elastic damping/scattering
   (c) absorption
   (d) None of the above

3. In a nuclear reactor, moderators slow down the neutrons which come out in a fission process. The moderator used have light nuclei. Heavy nuclei will not serve the purpose, because
   (a) they will break up **NCERT Exemplar**
   (b) elastic collision of neutrons with heavy nuclei will not slow them down
   (c) the net weight of the reactor would be unbearably high
   (d) substances with heavy nuclei do not occur in liquid or gaseous state at room temperature

VERY SHORT ANSWER Type Questions

4. What is nuclear holocaust?

5. Four nuclei of an element undergo fusion to form a heavier nucleus, with release of energy. Which of the two–the parent or the daughter nucleus–would have higher binding energy per nucleon? **CBSE 2018**

SHORT ANSWER Type Question

6. An atomic power nuclear reactor can deliver 300 MW. The energy released due to fission of each nucleus of uranium atoms U^{238} is 170 MeV. What will be the number of uranium atoms fissioned per hour?

LONG ANSWER Type II Question

7. Suppose India had a target of producing by 2020 AD, 200000 MW of electric power, 10% of which was to be obtained from nuclear power plants. Suppose we are given that, on an average, the efficiency of utilisation (i.e. conversion to electric energy) of thermal energy produced in a reactor was 25%. How much amount of fissionable uranium would our country need per year by 2020? Take the heat energy per fission of ^{235}U to be about 200 MeV. **NCERT**

NUMERICAL PROBLEMS

8. Complete the following fission reaction and calculate the amount of energy it releases.

$$^1_0n + ^{235}_{92}U \longrightarrow ^{88}_{38}Sr + ^{136}_{54}Xe + (?)$$

9. Determine the energy released in the following fusion reaction.

$$^1_1H + ^2_1H \longrightarrow ^3_2He + \gamma$$

10. Suppose we think of fission of a $^{56}_{26}Fe$ nucleus into two equal fragments, $^{28}_{13}Al$. Is the fission energetically possible? Argue by working out Q of the process. Given, $m(^{56}_{26}Fe) = 55.93494$ u and $m(^{28}_{13}Al) = 27.98191$ u. **NCERT**

11. The sun is believed to be getting its energy from the fusion of four protons to form a helium nucleus and a pair of positrons. Calculate the release of energy per fusion in MeV.
   Mass of proton = 1.007825 amu, mass of positron = 0.000549 amu, mass of helium nucleus = 4 002603 amu.
   Take, 1 amu = 931.5 MeV.

12. The fission properties of $^{239}_{94}Pu$ are very similar to those of $^{235}_{92}U$. The average energy released per fission is 180 MeV. How much energy in MeV is released, if all the atoms in 1 kg of pure $^{239}_{94}Pu$ undergo fission? **NCERT**

13. The Q-value of a nuclear reaction $A + b \longrightarrow C + d$ is defined by $Q = [m_A + m_b - m_C - m_d]c^2$, where the masses refer to the respective nuclei. Determine from the given data, the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.
   (i) $^1_1H + ^3_1H \longrightarrow ^2_1H + ^2_1H$
   (ii) $^{12}_6C + ^{12}_6C \longrightarrow ^{20}_{10}Ne + ^4_2He$

   Atomic masses are given to be
   $m(_1H^1) = 1.007825$ u, $m(^2_1H) = 2.014102$ u,
   $m(^3_1H) = 3.016049$ u, $m(^{12}_6C) = 12.000000$ u
   $m(^{20}_{10}Ne) = 19.992439$ u **NCERT**

14. Find the Q-value and the kinetic energy of the emitted α-particle in the α-decay of
   (i) $^{226}_{88}Ra$ and (ii) $^{220}_{86}Rn$.

   Given, $m(^{226}_{88}Ra) = 226.02540$ u,
   $m(^{222}_{86}Rn) = 222.01750$ u, $m_\alpha = 4.00260$ u
   $m(^{220}_{86}Rn) = 220.01137$u,
   $m(^{216}_{84}Po) = 216.00189$ u **NCERT**

[redacted]

15. How long can an electric lamp of 100 W be kept glowing by fusion of 2 kg of deuterium? Take the fusion reaction as CBSE SQP (Term-II)

$${}^2_1H + {}^2_1H \longrightarrow {}^3_2He + n + 3.27 \text{ MeV}$$

16. Calculate and compare the energy released by
(i) fusion of 1 kg of hydrogen deep within sun and
(ii) the fission of 1 kg of ^{235}U in a fission reactor. NCERT

17. Distinguish between nuclear fission and fusion. Show how in both these processes energy is released?
Calculate the energy release in MeV in the deuterium-tritium fusion reaction.

$${}^2_1H + {}^3_1H \longrightarrow {}^4_2He + n$$

Using the data,

$$m({}^2_1H) = 2.014102\,u, \quad m({}^3_1H) = 3.016049\,u$$
$$m({}^4_2He) = 4.002603\,u, \quad m_n = 1.008665\,u$$
$$1\,u = 931.5\,\frac{\text{MeV}}{c^2}$$

All India 2015

18. A 1000 MW fission reactor consumes half of its fuel in 5 yr. How much $^{235}_{92}U$ did it contain initially? Assume that the reactor operates 80% of the time that all the energy generated arises from the fission of $^{235}_{92}U$ and that this nuclide is consumed only by the fission process. NCERT

HINTS AND SOLUTIONS

1. (c) $^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^4_2He$
When a α particle is emitted mass number decreases by 4 and atomic number by 2

2. (b) Fast neutrons are slowed down by elastic scattering with light nuclei. Each collision takes away nearly 50% of energy

3. (b) According to the question, the moderator used have light nuclei (like proton). When protons undergo perfectly elastic collision with the neutron emitted, their velocities are exchanged, i.e., neutrons come to rest and protons move with the velocity of neutrons.
Heavy nuclei will not serve the purpose because elastic collisions of neutrons with heavy nuclei will not slow them down.

4. It is the name given to large scale destruction and devastation that would be caused by the uncontrolled release of large energy from the nuclear weapons.

5. According to question,

$$4\ {}_ZX^A \longrightarrow {}_{Z'}Y^{A'} + Q$$
(Parent nuclei) (Daughter nucleus)

As the daughter nucleus is a heavier nucleus as compared to parent nuclei, which are more stable than lighter nuclei, hence daughter nucleus has more binding energy per nucleon than parent nuclei.

6. As, we know that,

$$\text{power} = \frac{\text{energy}}{\text{time}} = 300 \times 10^6\text{ W} = 3 \times 10^8 \text{ J/s}$$

$$170 \text{ MeV} = 170 \times 10^6 \times 1.6 \times 10^{-19} = 27.2 \times 10^{-12}\text{ J}$$

Number of atoms fissioned per second

$$= \frac{3 \times 10^8}{27.2 \times 10^{-12}} = \frac{3 \times 10^{20}}{27.2}$$

$\therefore$ Number of atoms fissioned per hour

$$= \frac{3 \times 10^{20} \times 3600}{27.2}$$
$$= \frac{3 \times 36}{27.2} \times 10^{22} = 4 \times 10^{22}\text{m}$$

7. Total target power $= 200000 = 2 \times 10^5$ MW
Total nuclear power $= 10\%$ of total target power

$$= \frac{10}{100} \times 2 \times 10^5 = 2 \times 10^4 \text{ MW}$$

Energy produced/fission = 200 MeV
Efficiency of power plant = 25%
Energy converted into electrical energy per fission

$$= \frac{25}{100} \times 200 = 50 \text{ MeV}$$
$$= 50 \times 1.6 \times 10^{-13}\text{ J}$$

Total electrical energy to be produced per year

$$= 2 \times 10^4 \text{ MW}$$
$$= 2 \times 10^4 \times 10^6 = 2 \times 10^{10}\text{ W}$$
$$= 2 \times 10^{10}\text{ J/s}$$
$$= 2 \times 10^{10} \times 60 \times 60 \times 24 \times 365 \text{ J/yr}$$

Number of fission in one year,

$$n = \frac{2 \times 10^{10} \times 60 \times 60 \times 24 \times 365}{50 \times 1.6 \times 10^{-13}}$$
$$n = \frac{2 \times 36 \times 24 \times 365}{8} \times 10^{24}$$

Mass of 6.023×10^{23} atoms of $^{235}U = 235$ g

$$= 235 \times 10^{-3}\text{ kg}$$

Mass of $^{235}_{92}U$ required to produce

$$= \frac{2 \times 36 \times 24 \times 365}{8} \times 10^{24} \text{ atoms}$$
$$= \frac{235 \times 10^{-3} \times 2 \times 36 \times 24 \times 365 \times 10^{24}}{6.023 \times 10^{23} \times 8}$$
$$= 3.08 \times 10^4\text{ kg}$$

Thus, the mass of uranium needed per year is 3.08×10^4 kg.

[redacted]

8. By conservation of charge and mass, given equation can be written as

$${}^1_0n + {}^{235}_{92}U \longrightarrow {}^{88}_{38}Sr + {}^{136}_{54}Xe + 12\,{}^1_0n + Q \text{ (Energy)}$$

For amount of energy released, use

$$Q = \Delta m \times 931 \text{ MeV}$$

9. Use $Q = \Delta m \times 931$ MeV

Ans. 5.94 MeV

10. The given reaction for decay process,

$${}^{56}_{26}Fe \longrightarrow 2\,{}^{28}_{13}Al$$

Mass defect, $\Delta m = m({}^{56}_{26}Fe) - 2m({}^{28}_{13}Al)$

$$= 55.93494 - 2(27.98191)$$
$$= -0.02888 \text{ u}$$

$\Rightarrow$ Q $\Delta m \times 931 = -0.02888 \times 931$

$$= -26.88728 \text{ MeV}$$

Because the energy is negative, so the fission is not possible energetically.

11. ${}^1_1H + {}^1_1H + {}^1_1H + {}^1_1H \longrightarrow {}^4_2He + 2\,{}^0_1e + Q$

Initial mass = Mass of 4 hydrogen atoms

$$= 4 \times 1.007825 \text{ amu} = 4.031300 \text{ amu}$$

Final mass $= m({}^4_2He) + 2m({}^0_1e)$

$$= 4.002604 + 2 \times 0.000549$$
$$= 4.002604 + 0.001098 = 4.003702 \text{ amu}$$

Mass defect,

$$\Delta m = 4.031300 - 4.003702 = 0.027598 \text{ amu}$$

$\therefore$ Energy released,

$$Q = 0.027598 \times 931 \text{ MeV} = 25.7 \text{ MeV}$$

12. According to the concept of Avogadro number,
The number of atoms in 239 g of ${}^{239}_{94}Pu = 6.023 \times 10^{23}$

Number of atoms in 1 kg of ${}^{239}_{94}Pu$

$$= \frac{6.023 \times 10^{23} \times 1000}{239} = 2.52 \times 10^{24}$$

The average energy released in one fission

$$= 180 \text{ MeV}$$

So, total energy released in fission of 1 kg of

$${}^{239}_{94}Pu = 180 \times 2.52 \times 10^{24}$$
$$= 4.53 \times 10^{26} \text{ MeV}$$

13. (i) The given reaction, ${}^1_1H + {}^3_1H \longrightarrow {}^2_1H + {}^2_1H$

Mass defect, $\Delta m = m\,({}^1_1H) + m\,({}^3_1H) - 2m\,({}^2_1H)$

$$= 1.007825 + 3.016049 - 2(2.014102)$$
$$= -0.00433 \text{ u}$$

Q-value of the reaction,

$$Q = \Delta m \times 931 = -0.00433 \times 931$$
$$Q = -4.031 \text{ MeV}$$

As, the energy is negative, so the reaction is endothermic.

(ii) The given reaction,

$${}^{12}_6C + {}^{12}_6C \longrightarrow {}^{20}_{10}Ne + {}^4_2He$$

Mass defect, $\Delta m = 2\,m({}^{12}_6C) - m({}^{20}_{10}Ne) - m({}^4_2He)$

$$= 2 \times 12 - 19.992439 - 4.002603$$
$$\Delta m = 0.00495 \text{ u}$$

Q-value of the reaction,

$$Q = \Delta m \times 931 = 0.00495 \times 931$$
$$= 4.62 \text{ MeV}$$

Since, the energy is positive, thus the reaction is exothermic.

14. (i) The process of α-decay of ${}^{226}_{88}Ra$ can be expressed as,

$${}^{226}_{88}Ra \longrightarrow {}^{222}_{86}Rn + {}^4_2He + Q$$

Q-value of the reaction is given by

$$= [m({}^{226}_{88}Ra) - m({}^{222}_{86}Rn) - m_\alpha] \times 931 \text{ MeV}$$
$$(226.02540 - 222.01750 - 4.00260) \times 931$$
$$= 0.0053 \times 931 = 4.93 \text{ MeV}$$

Kinetic energy of emitted α-particle computed using conservation of momentum

$$= \left(\frac{A-4}{A}\right) Q = \left(\frac{226-4}{226}\right) \times 4.93$$
$$= 4.84 \text{ MeV}$$

(ii) The process of α-decay of ${}^{220}_{86}Rn$ can be expressed as,

$${}^{220}_{86}Rn \longrightarrow {}^{216}_{84}Po + {}^4_2He$$

Q-value of the reaction,

$$Q = [m({}^{220}_{86}Rn) - m({}^{216}_{84}Po) - m_\alpha] \times 931 \text{ MeV}$$
$$= [220.01137 - 216.00189 - 4.00260] \times 931$$
$$Q = 6.41 \text{ MeV}$$

$\therefore$ Kinetic energy of emitted α-particle

$$\frac{(A-4)Q}{A} = \left(\frac{220-4}{220}\right) \times 6.41$$
$$= 6.29 \text{ MeV}$$

15. Let t be the time.

According to the Avogadro number concept,

Number of atoms in 2 g of deuterium $= 6.023 \times 10^{23}$

Number of atoms in 2 kg of deuterium

$$= \frac{6.023 \times 10^{23} \times 2 \times 10^3}{2}$$
$$= 6.023 \times 10^{+26} \text{ nuclei}$$

Energy released during fusion of two deuterium

$$= 3.27 \text{ MeV}$$

$\therefore$ Energy released per deuterium = 1.635 MeV

Energy released in 6.023×10^{26} deuterium atoms

$$= 1.635 \times 6.023 \times 10^{26}$$
$$= 9.848 \times 10^{26} \text{ MeV}$$
$$= 9.848 \times 10^{26} \times 1.6 \times 10^{-13}$$
$$= 15.75 \times 10^{13} \text{ J}$$

Energy used by bulb in 1s = 100 J

100 J energy used in time = 1 s

[redacted]

15.75×10^{13} J energy used in time $=\dfrac{1\times15.75\times10^{13}}{100}$

$= 15.75\times10^{11}$ s [$\because$ 1 yr $= 60\times24\times60\times365$ s]

$=\dfrac{15.75\times10^{11}}{60\times24\times60\times365}$ yr $=4.99\times10^{4}$ yr

Thus, the bulb glows for 4.99×10^{4} yr.

16. (i) In sun, four hydrogen nuclei fuse to form a helium nucleus with release of 26 MeV energy.

$\because$ 1 g of hydrogen contains $= 6.023\times10^{23}$ nuclei

$\therefore$ Energy released by fusion of 1 kg ($=1000$ g) of hydrogen, $E_1 = \dfrac{6.023\times10^{23}\times26\times10^{3}}{4}$

$=39\times10^{26}$ MeV

(ii) Energy released in one fission of $^{235}_{92}$U nucleus

$= 200$ MeV

Mass of uranium $=1$ kg $=1000$ g

We know that, 235 g of ^{235}U has 6.023×10^{23} atoms or nuclei.

$\therefore$ Energy released in fission of 1 kg of U^{235},

$$E_2 = \frac{6.023\times10^{23}\times1000\times200}{235} = 5.1\times10^{26}\text{ MeV}$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{39\times10^{26}}{5.1\times10^{26}} = 7.65 \approx 8$$

Thus, the energy released in fusion is 8 times the energy released in fission.

17. Nuclear fission is the phenomenon of splitting of a heavy nucleus (usually $A > 230$) into two or more lighter nuclei.

$$U^{235}_{92} + n^1_0 \longrightarrow Ba^{141}_{56} + Kr^{92}_{36} + 3n^1_0 + Q$$

In this case, the energy released per fission of U^{235}_{92} is 200.4 MeV.

Nuclear fusion is the phenomenon of fusion of two or more lighter nuclei to form a single heavy nucleus.

The mass of the product nucleus is slightly less than the sum of the masses of the lighter nuclei fusing together.

This difference in the masses results the release of tremendous amount of energy.

e.g. $^1_1H + ^1_1H \longrightarrow ^2_1He + e^+ + \nu + 0.42$ MeV

$^2_1H + ^2_1H \longrightarrow ^3_1He + {}_0n^1 + 3.27$ MeV

$^2_1H + ^2_1H \longrightarrow {}_1H^3 + ^1_1H + 4.03$ MeV

$^2_1H + ^3_1H \longrightarrow ^4_2He + n$ MeV

$\Delta m = (2.014102 + 3.016049) - (4.002603 + 1.008665)$

$= 0.018883$ u

Energy released, $Q = 0.018883\times931.5\dfrac{\text{MeV}}{c^2}$

$= 17.589$ MeV

18. Given, power of reactor, $P = 1000$ MW

We will use concept that the energy generated in one fission of $^{235}_{92}$U is 200 MeV.

Number of $^{235}_{92}$U atoms in 1 g $=\dfrac{1}{235}\times6.023\times10^{23}$

$\therefore$ Energy generated per gram of $^{235}_{92}$U

$$=\left(\frac{1}{235}\times6.023\times10^{23}\times200\times1.6\times10^{-13}\right)$$

Total energy generated in 5 yr with 80% of the time

$$=1000\times10^{6}\times5\times365\times24\times60\times60\times\frac{80}{100}$$

[as $E = Pt$]

$\therefore$ Mass of $^{235}_{92}$U consumed in 5 yr,

$$m = \frac{\text{Total energy}}{\text{Energy consumed per gram}}$$

$$=\frac{1000\times10^{6}\times5\times365\times24\times60\times60\times0.8}{\left(\frac{1}{235}\right)\times6.023\times10^{23}\times200\times1.6\times10^{-13}}$$

$=1.538\times10^{6}$ g

≈ 1538 kg

$\therefore$ Initial amount of $^{235}_{92}$U $= (1544\times2) = 3076$ kg

[redacted]

SUMMARY

- Volume of a nucleus is about 10^{-12} times the volume of the atom. But the nucleus contains more than 99% of the mass of an atom.
- The unit used to express atomic masses is **atomic mass unit** (u).

$$1\,u = 1.660539 \times 10^{-27}\,kg$$

- **Isotopes** The atomic species of the same element differing in mass but having the same chemical properties are called isotopes.
- **Nucleus** It consists of protons and neutrons. The positive charge in the nucleus is that of **protons**. A proton is stable.
- **Neutron** was discovered by **James Chadwick**. A free neutron is unstable.
- **Atomic Number** It is the number of protons present inside the nucleus.
- **Mass Number** It is the total number of protons and neutrons inside the nucleus.
- **Nuclear Density** The ratio of the mass of nucleus and its volume. So, it can be given by $\rho = \frac{3m}{4\pi R_0^3}$.
- **Size of Nucleus** The radius R of the nucleus having mass number A can be given by

$$R \propto A^{1/3} \Rightarrow R = R_0 A^{1/3}$$

where, $R_0 = 1.2 \times 10^{-15}\,m$

- **Mass Energy** Einstein showed that mass is another form of energy. Einstein's mass-energy equivalence equation is $E = mc^2$
- **Binding Energy** Minimum energy required to separate the nucleons (present inside the nucleus) and place them at rest and infinite distance apart.
- **Average Binding Energy per Nucleon of Nucleus**

$$= \frac{\text{Total binding energy}}{\text{Number of nucleons } (A)}$$

- **Nuclear Stability** The stability of nucleus is determined by the value of its binding energy per nucleon and its neutron to the proton ratio.
- **Nuclear Force** is the strong attractive forces between nucleons. It is a non-conservative force and does not obey inverse-square law.
- **Nuclear Energy** It is the energy released during the transformation of a nuclei.
- **Nuclear Fission** It is phenomenon of splitting of a heavy nucleus into two or more lighter nuclei by the bombardment of proton, neutron, α-particles, etc.
- **Nuclear Fusion** It is phenomenon of fusing of two or more lighter nuclei forming a single heavy nucleus.

[redacted]

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. If the nuclear radius of ^{27}Al is 3.6 Fermi, the approximate nuclear radius of ^{64}Cu in Fermi is

(a) 2.4 (b) 1.2
(c) 4.8 (d) 3.6

2. How much mass has to converted into energy to produce electric power of 200 MW for one hour?

(a) 2×10^{-6} kg (b) 8×10^{-6} kg
(c) 1×10^{-6} kg (d) 3×10^{-6} kg

3. The mass defect of helium nucleus is 0.0303 amu. The binding energy per nucleon for helium nucleus will be

(a) 28 MeV (b) 7 MeV (c) 14 MeV (d) 1 MeV

4. Binding energy of hydrogen nucleus is

(a) – 13.6 eV (b) 0
(c) 13.6 eV (d) 6.8 eV

5. Two protons are attracting each other, then separation between them is

(a) 10^{-10} m (b) 10^{-2} m
(c) 10^{-8} m (d) 10^{-15} m

6. In fusion reaction occurring in the sun, NCERT Exemplar

(a) hydrogen is converted into carbon
(b) hydrogen and helium are converted into carbon and other heavier metals/elements
(c) helium is converted into hydrogen
(d) hydrogen is converted into helium

ASSERTION AND REASON

Directions (Q. Nos. 7-8) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

7. **Assertion** Nuclear force between neutron-neutron, proton-neutron and proton-proton is approximately the same.

Reason The nuclear force does not depend on the electric charge.

8. **Assertion** Naturally, thermonuclear fusion reaction is not possible on earth.

Reason For thermonuclear fusion to take place, extreme condition of temperature and pressure are required.

CASE BASED QUESTIONS

Directions (Q.Nos. 9-10) *These questions are case study based questions. Attempt any 4 sub-parts from each question. Each question carries 1 mark.*

9. **Discovery of Nucleus**

The nucleus was first discovered in 1911 by Lord Rutherford and his associates by experiments on scattering of α-particles by atoms He found that the scattering results could be explained, if atoms consist of a small, central, massive and positive core surrounded by orbiting electrons.The experimental results indicated that the size of the nucleus is of the order of 10^{-14} m and is thus 10000 times smaller than the size of atom.

(i) Ratio of mass of nucleus with mass of atom is approximately

(a) 1 (b) 10
(c) 10^3 (d) 10^{10}

(ii) Masses of nuclei of hydrogen, deuterium and tritium are in ratio

(a) 1 : 2 : 3 (b) 1 : 1 : 1
(c) 1 : 1 : 2 (d) 1 : 2 : 4

[redacted]

(iii) Density of a nucleus is
(a) more for lighter elements and less for heavier elements
(b) more for heavier elements and less for lighter elements
(c) very less compared to ordinary matter
(d) a constant

(iv) If R is the radius and A is the mass number, then $\log R$ *versus* $\log A$ graph will be
(a) a straight line
(b) a parabola
(c) an ellipse
(d) None of the above

(v) The ratio of the nuclear radii of the gold isotope $^{197}_{79}$Au and silver isotope $^{107}_{47}$Au is
(a) 1.23 (b) 0.216
(c) 2.13 (d) 3.46

10. **Disappeared Mass**

In the year 1939, German scientist Otto Hahn and Strassmann discovered that when an uranium isotope was bombarded with a neutron, it breaks into two intermediate mass fragment. It was observed that, the sum of the masses of new fragments formed were less than the mass of the original nuclei. This difference in the mass appeared as the energy released in the process.

Thus, the phenomenon of splitting of a heavy nucleus (usually $A > 230$) into two or more lighter nuclei by the bombardment of proton, neutron, α-particle, etc with liberation of energy is called nuclear fission.

Fission reaction resulting from the absorption of neutron is known as induced fission.

$$_{92}U^{235} + {_0}n^1 \rightarrow \underset{\text{Unstable nucleus}}{_{92}U^{236}} \rightarrow {_{56}}Ba^{144} + {_{36}}Kr^{89} + 3{_0}n^1 + Q$$

(i) Fission of nuclei is possible because the binding energy per nucleon in them
(a) increases with mass number at high mass numbers
(b) decreases with mass number at high mass numbers
(c) increases with mass number at low mass number
(d) decreases with mass number at low mass number

(ii) For sustaining the nuclear fission chain reaction in a sample (of small size) of $^{235}_{92}$U, it is desirable to slow down fast neutrons by
(a) friction
(b) elastic damping/scattering
(c) absorption
(d) None of the above

(iii) Which of the following is/are fission reaction(s)?

I. $^1_0n + {^{235}_{92}}U \rightarrow {^{236}_{92}}U \rightarrow {^{133}_{51}}Sb + {^{99}_{41}}Nb + 4{^1_0}n$

II. $^1_0n + {^{235}_{92}}U \rightarrow {^{140}_{54}}Xe + {^{94}_{38}}Sr + 2{^1_0}n$

III. $^2_1H + {^2_1}H \rightarrow {^3_2}He + {^1_0}n$

(a) Both II and III (b) Both I and III
(c) Only II (d) Both I and II

(iv) If a nucleus with mass number $A = 240$ with $E_{bn} = 7.6$ MeV breaks into two fragments of $A = 120$ and $E_{bn} = 8.5$ MeV, then released energy is around
(a) 216 MeV
(b) 200 MeV
(c) 100 MeV
(d) Cannot be estimated from given data

(v) In any fission process, ratio of mass of daughter nucleus to mass of parent nucleus is
(a) less than 1
(b) greater than 1
(c) equal to 1
(d) depends on the mass of parent nucleus

11. The neutron is bombarded on a $^{10}_5$B nucleus and an α-particle is emitted. The nuclear reaction involved is **CBSE 2020**

$^1_0n + {^{10}_5}B \longrightarrow {^4_2}He + \dots$

12. Name the particle emitted spontaneously in the following nuclear reaction: **CBSE 2020**

$^{32}_{15}P \longrightarrow {^{32}_{16}}S + \bar{\nu} + \dots\dots$

VERY SHORT ANSWER Type Questions

13. Two nuclei have mass numbers in the ratio of 27 : 512. What is the ratio of their nuclear radii?

14. In the following nuclear reaction, identify unknown labelled X. **CBSE SQP (Term-I)**

$^{22}_{11}Na + X \rightarrow {^{22}_{10}}Ne + \nu_e$

15. Why is it necessary to slow down the neutrons, produced through the fission of $^{235}_{92}$U nuclei (by neutrons) to sustain a chain reaction? What type of nuclei are (preferably) needed for slowing down fast neutrons?

16. Name the materials used as moderators in nuclear reactors and write the reasons for their use as moderator.

[redacted]

SHORT ANSWER Type Questions

17. (i) Write two characteristic features of nuclear force.

(ii) Draw a plot of potential energy of a pair of nucleons as a function of their separation.

All India 2017 C

18. (a) State two distinguishing features of nuclear force. **CBSE 2019**

(b) Draw a plot showing the variation of potential energy of a pair of nucleons as a function of their separation. Mark the regions on the graph where the force is (i) attractive and (ii) repulsive.

19. If both the numbers of protons and neutrons are conserved in a nuclear reaction like

$${}_6C^{12} + {}_6C^{12} \longrightarrow {}_{10}N^{20} + {}_2He^4$$

In what way is the mass converted into the energy? Explain.

20. Calculate for how many years will the fusion of 2.0 kg deuterium keep 800 W electric lamp glowing. Take the fusion reaction as **CBSE 2020**

$${}^2_1H + {}^2_1H \longrightarrow {}^3_2He + {}^1_0n + 3.27 \text{ MeV}$$

LONG ANSWER Type I Questions

21. (a) Give one point of difference between nuclear fission and nuclear fusion.

(b) Suppose we consider fission of a ${}^{56}_{26}Fe$ into two equal fragments of ${}^{28}_{13}Al$ nucleus. Is the fission energetically possible? Justify your answer by working out Q-value of the process.

Given $(m)\ {}^{56}_{26}Fe = 55.93494$ u

and $(m)\ {}^{28}_{13}Al = 27.98191$. **CBSE SQP (Term-I)**

22. Draw the curve showing the variation of binding energy per nucleon with the mass number of nuclei. Using it explain the fusion of nuclei lying on ascending part and fission of nuclei lying on descending part of this curve. **CBSE 2020**

23. A heavy nucleus P of mass number 240 and binding energy 7.6 MeV per nucleon splits in to two nuclei Q and R of mass numbers 110, 130 and binding energy per nucleon 8.5 MeV and 8.4 MeV, respectively. Calculate the energy released in the fission. **CBSE 2020**

LONG ANSWER Type II Questions

24. Define Q-value of a nuclear process. When can a nuclear process not proceed simultaneously?

If both the number of protons and the number of neutrons are conserved in a nuclear reaction, in what way is mass converted into energy (or *vice-versa*) in nuclear reaction?

25. (i) In a typical nuclear reaction, e.g.

$${}^2_1H + {}^2_1H \longrightarrow {}^3_2He + 3.27 \text{MeV}$$

Although number of nucleons is conserved, yet energy is released. How? Explain.

(ii) Show that nuclear density in a given nucleus is independent of mass number A.

ANSWERS

1. (c)	2. (b)	3. (b)	4. (c)	5. (d)
6. (d)	7. (a)	8. (a)		

9.(i) (a) As nearly 99.9% mass of atom is in nucleus.

$$\therefore \frac{\text{Mass of nucleus}}{\text{Mass of atom}} = \frac{99.9}{100} = 0.99 \approx 1$$

(ii) (a) Since, the nuclei of deuterium and tritium are isotopes of hydrogen, they must contain only one proton each. But the masses of the nuclei of hydrogen, deuterium and tritium are in the ratio of 1 : 2 : 3, because of presence of neutral matter in deuterium and tritium nuclei.

(iii) (d) Density $= \dfrac{\text{Mass}}{\text{Volume}}$

$$= \frac{mA}{\frac{4}{3}\pi R_0^3 A} = \frac{3m}{4\pi R_0^3}$$

As, $m = m_p = m_N$

$= 2.3 \times 10^{17}$ kgm^{-3}, which is a constant.

(iv) (a) $R = R_0 A^{1/3}$

$$\log R = \log R_0 + \frac{1}{3}\log A$$

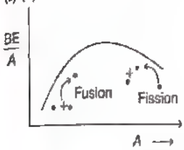
On comparing the above equation of straight line; $y = mx + c$, so the graph between $\log A$ and $\log R$ is a straight line also.

(v) (a) Here $A_1 = 197$ and $A = 107$

$$\therefore \quad \frac{R_1}{R_2} = \left(\frac{A_1}{A_2}\right)^{1/3} = \left(\frac{197}{107}\right)^{1/3} = 1.225 \simeq 1.23$$

[redacted]

10. (i) (*b*)



So, from graph, it is clear that binding energy per nucleon decreases with mass number at high mass numbers.

(ii) (*b*) Fast neutrons are slowed down by elastic scattering with light nuclei as each collision takes away nearly 50% of energy

(iii) (*d*) Reactions I and II represent fission of uranium isotope $^{235}_{92}U$, when bombarded with neutrons that breaks it into two intermediate mass nuclear fragments. However, reaction III represents two deuterons fuses together to form the light isotope of helium.

(iv) (*a*) The energy released (i.e. Q-value) in the fission reaction of nuclei like uranium is of the order of 200 MeV per fissioning nucleus. This is estimated as follows

Let us take a nucleus with $A = 240$ breaking into two fragments each of $A = 120$, then

E_{bn} for $A = 240$ nucleus is about 7.6 MeV.

E_{bn} for the two $A = 120$ fragment nuclei is about 8.5 MeV.

So, gain in binding energy for nucleon is about 0.9 MeV.

Hence, the total gain in binding energy is 240×0.9 or 216 MeV.

(v) (*a*) In fission process, when a parent nucleus breaks into daughter products, then some mass is lost in the form of energy. Thus, mass of fission products < mass of parent nucleus.

$$\Rightarrow \frac{\text{Mass of fission products}}{\text{Mass of parent nucleus}} < 1$$

11. $^1_0n + ^{10}_5B \longrightarrow ^4_2He + ^7_3Li$

12. $^{32}_{15}P \longrightarrow ^{32}_{16}S + \bar{\nu} + e^-$

13. We know that, radius of nucleus in terms of mass number (A), given as

$$R = R_0 A^{1/3}$$

$$\therefore \quad \frac{R_1}{R_2} = \frac{R_0(27)^{1/3}}{R_0(512)^{1/3}} \Rightarrow \frac{3}{8} \Rightarrow 3:8$$

14. In the given nuclear reaction, the atomic number of product side is one less than that of reactant side. So, the X must be an electron and the reaction can be written as

$$^{22}_{11}Na + ^{0}_{-1}e \longrightarrow ^{22}_{10}Ne + \nu_e$$

15. In fission each nucleus of ^{235}U, emits on the average more than two neutrons. If one of these neutrons is absorbed by another ^{235}U nucleus, causing it to fission, we can have a sustainable chain reaction. However, only a slow neutron, rather than a fast neutron has a high cross-section (chance) of absorption. i.e. Why neutrons are slowed down by use of moderator. Heavy nuclei are (preferably) needed for slowing down fast neutrons.

16. Heavy water and graphite are used as moderator in nuclear reactors. The main reason why heavy water and graphite used as moderator because they capture less neutrons than other substance.

17. Refer to text on page 505.

18. Refer to text on page 505. (Nuclear force)

19. Here, sum of masses of constituents of product is less than the sum of masses of constituents of reactants, which causes some mass defect. This mass defect gets converted into energy, as per mass-energy equivalence.

20. Let t be the time.

According to the Avogadro number concept, number of atoms in 2 g of deuterium

$$= 6.023 \times 10^{23}$$

and number of atoms in 2 kg of deuterium

$$= \frac{6.023 \times 10^{23} \times 2 \times 10^3}{2}$$

$$= 6.023 \times 10^{+26} \text{ nuclei}$$

Energy released during fusion of two deuteriums

$$= 3.27 \text{ MeV}$$

$\therefore$ Energy released per deuterium = 1.635 MeV

Energy released in fusion of 6.023×10^{26} deuterium atoms

$$= 1.635 \times 6.023 \times 10^{26}$$
$$= 9.848 \times 10^{26} \text{ MeV}$$
$$= 9.848 \times 10^{26} \times 1.6 \times 10^{-13}$$
$$= 15.75 \times 10^{13} \text{ J}$$

Energy used by bulb in 1s = 800 J $\quad (\because W = J/s)$

As, 800 J of energy used in time = 1 s,

So, 15.75×10^{13} J of energy used in time

$$= \frac{1 \times 15.75 \times 10^{13}}{800}$$
$$= 1.969 \times 10^{11} \text{ s}$$

$[\because 1 \text{ yr} = 60 \times 24 \times 60 \times 365 \text{ s}]$

$$= \frac{1.969 \times 10^{11}}{60 \times 24 \times 60 \times 365} \text{ yr}$$
$$= 6.243 \times 10^3 \text{ yr}$$

Thus, the bulb glows for 6.243×10^3 yr.

21. (a) **Distinction between Nuclear Fission and Nuclear Fusion** Fission is the splitting of large nucleus into two or more smaller ones, on the other hand, fusion is the combining of two or more lighter nuclei to form larger one.

(b) The given reaction for decay process,

$$^{56}_{26}\text{Fe} \longrightarrow 2\,^{28}_{13}\text{Al}$$

Mass defect, $\Delta m = m(^{56}_{26}\text{Fe}) - 2m(^{28}_{13}\text{Al})$

$= 55.93494 - 2(27.98191)$

$= -0.02888$ u

$\Rightarrow \quad Q = \Delta m \times 931$

$= -0.02888 \times 931$

$= -26.88728$ MeV

Because the energy is negative, so the fission is not possible energetically.

22. Refer to text on page 504

23. Energy released $(\Delta E) = \Delta m \times c^2$

where, Δm is the mass defect.

$\Rightarrow \quad \Delta E = \mu c^2 = [110 + 130] \times 8.5 - 240 \times 7.6$ MeV

$(\because c^2 = 931.5 \text{ MeV})$

$= 240 \times (8.5 - 7.6) = 240 \times 0.9$

$= 216$ MeV

24. Q-value; refer to the text on pages 512 and 513
In fact the number of protons and number of neutrons are same before and after nuclear reaction, but the binding energies of nuclei present before and after a nuclear reaction are different. This difference is called mass defect. This mass defect appears as energy of reaction. In this sense, a nuclear reaction is an example of mass-energy inter conversion

25. (i) Refer to Q. 20 on page 507.
(ii) Refer to text on page 502.

[redacted]

14

The basic building blocks of any electronic circuit are the devices which have controlled flow of electrons. Before the discovery of **semiconductor devices**, such devices were mostly vacuum tubes. The vacuum tubes which have two electrodes : anode and cathode, are called **diode valves** and the tubes which have three electrodes : cathode, anode and grid, are called **triode valves**. Such devices were bulky, consume high power, generally operate at high voltages and have limited life and low reliability.

SEMICONDUCTOR ELECTRONICS : MATERIALS, DEVICES AND SIMPLE CIRCUITS

CHAPTER CHECKLIST
- Semiconductor, Diode and Its Applications

The seed of growth and development of modern solid state semiconductor electronics goes back to 1930, when it was realised that some semiconductors and their junctions have the ability of controlling the number and the direction of flow of charge carriers through them. Simple excitation with the help of light, heat or small applied voltage can change the number of mobile charge carriers in a semiconductor. The supply and flow of charge carriers in these devices are within the solid itself, no vacuum or external heating is required. So, these devices are small in size, consume low power, operate at low voltages and have long life and high reliability.

CLASSIFICATION OF METALS, CONDUCTORS AND SEMICONDUCTORS ON THE BASIS OF CONDUCTIVITY

On the basis of the relative values of electrical conductivity (σ) or resistivity ($\rho = 1/\sigma$), the solids are broadly classified as,

(i) **Metals** They possess very low resistivity (or high conductivity).

$$\rho \sim 10^{-2}\text{-}10^{-8}\,\Omega\text{m},\ \sigma \sim 10^{2}\text{-}10^{8}\ \text{Sm}^{-1}$$

(ii) **Semiconductors** They have resistivity or conductivity intermediate to metals and insulators.

$$\rho \sim 10^{-5}\text{-}10^{6}\Omega\text{m},\ \sigma \sim 10^{+5}\text{-}10^{-6}\text{Sm}^{-1}$$

[redacted]

(iii) **Insulators** They have high resistivity (or low conductivity).

$$\rho \sim 10^{11} - 10^{19}\,\Omega m, \sigma \sim 10^{-11} - 10^{-19}\,Sm^{-1}$$

The values of ρ and σ given above are indicative of magnitude and could well go outside the ranges as well.

Our interest in this chapter is in the study of semiconductors, which can be of the following types

(i) **Element semiconductors** These semiconductors are available in natural form.

e.g. Silicon and germanium.

(ii) **Compound semiconductors** These semiconductors are made by compounding the metals. e.g.

(a) Inorganic semiconductors are CdS, GaAs, CdSe, InP, etc.

(b) Organic semiconductors are anthracene, doped pthalocyanines, etc.

(c) Organic polymer semiconductors are polypyrrole, polyaniline, polythiophene, etc.

ENERGY BANDS IN SOLIDS (CONDUCTOR, INSULATOR AND SEMICONDUCTOR)

Energy Band

According to Bohr's atomic model and concept of electronic configuration in an isolated atom, the electrons have certain definite discrete amounts of energy corresponding to different shells and subshells, i.e. there are well-defined energy levels of electrons in an isolated atom.

But in a crystal due to interatomic interaction, valence electrons are shared by more than one atom. Due to this, splitting of energy level takes place. The collection of these closely spaced energy levels is called an energy band. These bands are formed due to the continuous energy variation in different energy levels.

These different energy levels in different electrons are formed because inside the crystal, each electron has a unique position and no two electrons is exactly at the same pattern of surrounding charges.

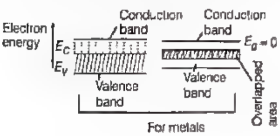
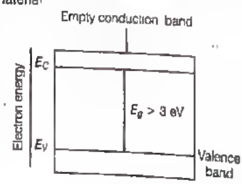
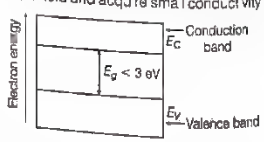
Valence Band

The energy band, which includes the energy levels of the valence electrons is called valence band. This band may be partially or completely filled with electrons but is never empty.

Conduction Band

The energy band above the valence band is called conduction band. At room temperature, this band is either empty or partially filled with electrons. Electrons can gain energy from external electric field, then jump from valence to conduction band and contribute to the electric current.

Difference between Conductor, Insulator and Semiconductor on the basis of Energy Bands

Conductor (Metal)	Insulator	Semiconductor
In conductor, either there is no energy gap between the conduction band which is partially filled with electrons and valence band or the conduction band and valence band overlap each other.	In insulator, the valence band is completely filled, the conduction band is completely empty. In this energy gap is quite large and even energy from any other source cannot help electrons to overcome it.	In semiconductor, the valence band is totally filled and the conduction band is empty but the energy gap between conduction band and valence band, unlike insulators is very small.
Thus, many electrons from below the fermi level can shift to higher energy levels above the fermi level in the conduction band and behave as free electrons by acquiring a little more energy from any other sources.	Thus, electrons are bound to valence band and are not free to move. Hence, electric conduction is not possible in this type of material.	Thus, at room temperature some electrons in the valence band acquire thermal energy greater than energy band gap and jump over to the conduction band where they are free to move under the influence of even a small electric field and acquire small conductivity.
		

[redacted]

Energy Band Gap

The minimum energy required for shifting electrons from valence band to conduction band is called energy band gap (E_g). It is the gap between the top of the valence band and bottom of the conduction band. It can be zero, small or large depending upon the material.

Note If λ is the wavelength of radiation used in shifting the electron from valence band to conduction band, then energy band gap is

$$\boxed{E_g = h\nu = hc/\lambda}$$

where, h is called Planck's constant and c is the velocity of light.

Fermi Energy

It is the maximum possible energy possessed by free electrons of a material at absolute zero temperature (i.e. 0 K). The value of fermi energy is different for different materials.

SEMICONDUCTORS

The materials whose conductivity lie between metals and insulators are known as semiconductors. They are characterised by narrow energy gap (less than 3eV) between the valence band and conduction band. At absolute zero temperature, all states in valence band are filled and all states in conduction band are empty. An applied electric field cannot give so much energy to the valence electrons that they could cross the gap and enter the conduction band. Hence, at low temperatures, pure semiconductors are insulators.

Electrons and Holes in Semiconductors

At room temperature, however some of the valence electrons acquire thermal energy greater than E_g and move into conduction band. A vacancy is created in the valence band at each place where an electron was present before moving into conduction band. This vacancy is called hole. It is a seat of positive charge of magnitude equal to the charge of an electron. Thus, free electrons in the conduction band and the holes are created in the valence band, which can move even under a small applied field. The solid is therefore conducting.

On the basis of purity, semiconductors are of two types

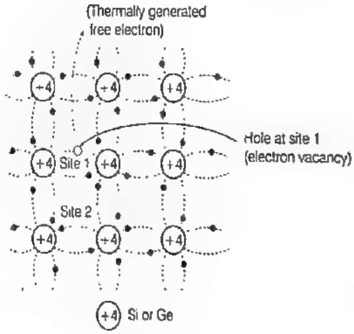
Intrinsic Semiconductors

This type of semiconductor is also called an undoped semiconductor or *i*-type semiconductor. It is a pure semiconductor without any significant presence of dopant species. Pure germanium, silicon in their natural state are intrinsic semiconductors.

The number of charge carriers is determined by the properties of the material itself instead of the amount of impurities. In intrinsic semiconductors, the number of excited electrons is equal to number of holes, i.e. $n_h = n_i$ where n_i is called intrinsic carrier concentration. At temperature 0 K, the valence band is filled. The energy gap is 0.72 eV and the conduction band is totally empty.

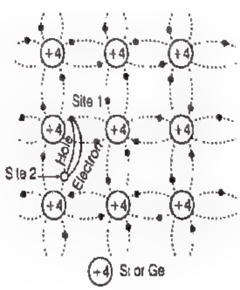
Under the action of an electric field, holes move towards negative potential giving hole current I_h. The total current I is the sum of the electron current I_e and the hole current I_h, i.e. $I = I_e + I_h$.

It may be noted that apart from the process of generation of conduction in electrons and holes, a simultaneous process of recombination occurs in which the electrons recombine with the holes. At equilibrium, the rate of generation is equal to rate of recombination of charge carriers. The recombination occurs due to an electron colliding with a hole.



(a)

Fig. (a) is representing the generation of hole at site 1 and conduction electron due to thermal energy at moderate temperatures



(b)

Fig. (b) is representing possible thermal motion of a hole. The electron from the lower left hand covalent bond (site 2) goes to the earlier hole site 1, leaving a hole at its site indicating an apparent movement of the hole from site 1 to site 2

[redacted]

An intrinsic semiconductor behaves like an insulator at $T = 0\,K$. The thermal energy at higher temperature is the only reason which excites some electrons from the valence band to the conduction band.

In Fig. (b) these thermally excited electrons at $T > 0\,K$, partially occupy the conduction band. They have come from the valence band leaving equal number of holes there.

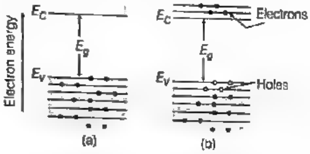


Fig (a) an intrinsic semiconductor at T = 0K behaves like insulator Fig. (b) is representing four thermally generated electron-hole pairs at T > 0K

Extrinsic Semiconductors

The conductivity of intrinsic semiconductors is very low at room temperature. But, it can be significantly increased, if some pentavalent or trivalent impurity is mixed with it. Hence, those semiconductors in which some impurity atoms are embedded are known as extrinsic or impurity semiconductors.

NOTE When some desirable impurity is added to intrinsic semiconductors deliberately then this process is called doping and the impurity are called dopants. The process of adding impurity to an intrinsic semiconductor in a controlled manner is called **doping.**

There are two types of dopants used in doping.

(i) Trivalent (valency 3) atoms: e.g., Indium (In), Boron (B), aluminium (Al), etc.

(ii) Pentavalent (valency 5) atoms: e.g., Arsenic (As), Antimony (Sb), Phosphorous (P), etc.

Extrinsic semiconductors are basically of two types

(i) *n*-type semiconductors (ii) *p*-type semiconductors

n-Type Semiconductors

This type of semiconductor is obtained when pentavalent impurity is added to Si or Ge. During doping, four electrons of pentavalent element bond with the four silicon neighbours while fifth remains very weakly bound to its parent atom. Also the ionisation energy required to set this electron free is very small.

Hence, these electrons are almost free to move. In other words, we can say that these electrons are donated by the impurity atoms. So, these are also known as **donor atoms** and the conduction inside the semiconductor will take place with the help of the negatively charged electrons. Due to this negative charge, these semiconductors are known as **n-type semiconductors.** When the semiconductors are placed at room temperature, then the covalent bond breakage takes place. So, more free electrons are generated. As a result, same number of holes generation takes place. But as compared to the free electrons, the number of holes are comparatively less due to the presence of donated electrons, i.e. $n_e >> n_h$.

Therefore, major conduction in *n*-type semiconductors is due to electrons. So, electrons are known as majority carriers and the holes are known as the minority carriers.

This means, $n_e >> n_h$; $I_e >> I_h$

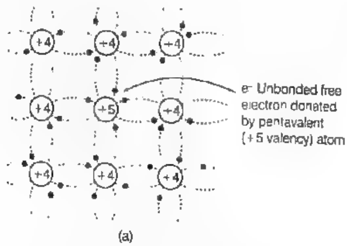


Fig. (a) Pentavalent donor atom (As, Sb, P, etc.) doped for tetravalent Si or Ge giving *n*-type semiconductor

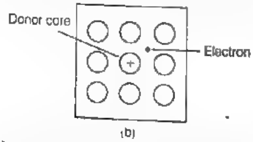


Fig. (b) Commonly used schematic representation for *n*-type material which shows only the fixed cores of the substituent donors with one additional effective positive charge and its associated extra electron.

p-Type Semiconductors

This type of semiconductor is obtained when a trivalent impurity is added to Si or Ge.

So, the three valence electrons of the doped impure atoms will form the covalent bonds with silicon atoms but silicon atoms have four electrons in its valence shell. Hence, one covalent bond will be improper.

This means, one more electron is needed for the proper covalent bonding. This need of one electron is fulfilled from any of the bond between two silicon atoms. So, the bond between the silicon and impurity atoms will be completed. After bond formation, the doped impurity will get ionised. As we know that, ions are negatively charged. So, the impurity will also get negative charge.

[redacted]

As, hole was created when the electron come from silicon-silicon bond moved to complete the bond between the doped impurity and silicon. Due to this, an electron will now move from any one of the covalent bond to fill the empty hole. This will further result in a new hole formation. So, in p-type semiconductor, the holes movement results in the formation of the current. This means, in this type of semiconductor majority charge carriere are holes, i.e. positively charged and minority charge carriers are electrons, i.e. $n_h >> n_e$; $I_h >> I_e$. Hence, these conductors are known as p-type semiconductors or acceptor type semiconductors.

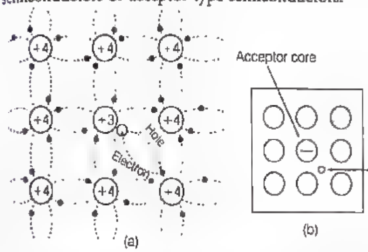


Fig. (a) Trivalent acceptor atom (In, Al, B, etc.) doped in tetravalent Si or Ge lattice giving p-type semiconductor. Fig (b) Commonly used schematic representation of p-type material which shows only the fixed core of the substituent acceptor with one effective additional negative charge and its associated hole.

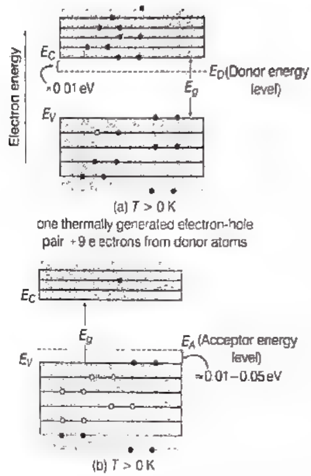
The electron and hole concentration in a semiconductor in thermal equilibrium is given by $n_e n_h = n_i^2$.

Note The energy gaps of C,Si and Ge are 5.4 eV 1.1 eV and 0.7 eV, respectively.
Sn is a group IV element as its energy gaps is zero.

Energy Band in Extrinsic Semiconductors

In extrinsic semiconductors, additional energy states due to donor impurities (E_D) and acceptor impurities (E_A) also exist. In the energy band diagram of n-type semiconductor, the donor energy level E_D is slightly below the bottom E_C of conduction band and the electrons from this level move into conduction band with very small supply of energy.

In p-type semiconductors, the acceptor energy level E_A is slightly above the top energy level E_V of the valence band. With very small supply of energy an electron from the valence band can jump to the level E_A and ionise the acceptor negatively.



Energy bands of (a) n-type semiconductor at $T > 0$ K, (b) p-type semiconductor at $T > 0$ K

EXAMPLE |1| The number of silicon atoms per m^3 is 5×10^{28}. This is doped simultaneously with 5×10^{22} atoms per m^3 of arsenic and 5×10^{20} atoms per m^3 of indium. Calculate the number of electrons and holes. Given that, $n_i = 1.5 \times 10^{16} \text{m}^{-3}$. Is the material n-type or p-type?

NCERT

Sol. For each atom doped with arsenic, one free electron is received. Similarly, for each atom doped of indium, a vacancy is created. So, number of free electrons introduced by pentavalent impurity is

$$N_{As} = 5 \times 10^{22} \text{ m}^{-3}$$

The number of holes introduced by trivalent impurity added is $N_{In} = 5 \times 10^{20} \text{ m}^{-3}$

So, net number of electrons added is

$$n_e = N_{As} - N_{In} = 5 \times 10^{22} - 5 \times 10^{20}$$
$$= 4.95 \times 10^{22} \text{ m}^{-3}$$

We know that,

$$n_e n_h = n_i^2$$

So, $$n_h = \frac{n_i^2}{n_e} = \frac{(1.5 \times 10^{16})^2}{4.95 \times 10^{22}}$$
$$= 4.54 \times 10^9 \text{ m}^{-3}$$

As, $n_e > n_h$ (number of holes). So, the material is n-type semiconductor.

[redacted]

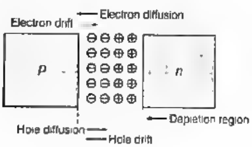
p-n JUNCTION

It is an arrangement made by a close contact of n-type semiconductor and p-type semiconductor. There are various methods of forming p-n junction. In one method, an n-type germanium crystal is cut into thin slices called wafers. An aluminium film is laid on an n-type wafer, which is then heated in an oven at a temperature of about 600° C. Aluminium then diffuses into the surface of wafer. In this way, a p-n junction is formed.

Formation of Depletion Region in p-n Junction

In an n-type semiconductor, the concentration of electrons is more than that of holes. Similarly, in a p-type semiconductor, the concentration of holes is more than that of electrons. During the formation of p-n junction and due to the concentration gradient across p and n-sides, holes diffuse from p-side to n-side ($p \to n$) and electrons diffuse from n-side to p-side ($n \to p$). The diffused charge carriers combine with their counterparts in the immediate vicinity of the junction and neutralise each other.

Thus, near the junction positive charge is built on n-side and negative charge on p-side.



p-n junction formation process

This sets up potential difference across the junction and an internal electric field E_i directed from n-side to p-side. The equilibrium is established when the field E_i becomes strong enough to stop further diffusion of the majority charge carriers (however, it helps the minority charge carriers to diffuse across the junction).

The region on either side of the junction which becomes depleted (free) from the mobile charge carriers is called **depletion region or depletion layer.** The width of depletion region is of the order of 10^{-6} m.

The potential difference developed across the depletion region is called the **potential barrier**. It depends on dopant concentration in the semiconductor and temperature of the junction.

Note

- Due to the diffusion of holes from p-side to n-side and electrons from n-side to p-side at the junction, a current rises from p-side to n-side, which is called **diffusion current.**
- If an electron-hole pair is created on the depletion region due to thermal collision, the electrons are pushed by the electric field towards the n-side and the holes towards the p-side, which gives rise to a current from n-side to p-side known as **drift current.**
- In steady state, diffusion current = drift current

SEMICONDUCTOR DIODE OR p-n JUNCTION DIODE

It is basically a p-n junction with metallic contacts provided at the ends for the application of an external voltage. It is a two terminal device.

It is represented by the symbol $\overset{p}{-}\!\!\triangleright\!\!|\!\overset{n}{-}$.

The direction of arrow indicates the conventional direction of current.

Forward Biasing and Reverse Biasing of Junction Diode

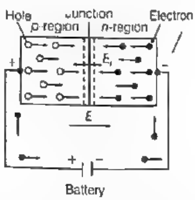
Biasing is the method of connecting external battery or emf source to a p-n function diode. The junction diode can be connected to an external battery in two ways, called **forward biasing** and **reverse biasing** of the junction.

Forward Biasing

A junction diode is said to be forward biased when the positive terminal of the external battery is connected to the p-side and negative terminal to the n-side of the diode.

Flow of Current in Forward Biasing

In this situation, the forward voltage opposes the potential barrier, due to which both the potential barrier and width of the depletion layer decreases. Under the effect of external electric field, holes in the p-region and electrons in the n-region, both move towards the junction. These holes and electrons mutually combine just near the junction and cease to exist. For each electron-hole combination, a covalent bond breaks up in the p-region near the positive terminal of the battery. Out of the hole and electron so produced, the hole moves towards the junction, while the electron enters the positive terminal of the battery through the connecting wire.



Forward biasing of junction diode

Just at this moment, an electron is released from the negative terminal of the battery which enters the n-region to replace the electron lost by combining with a hole at the junction. Thus, a current called forward current, is

[redacted]

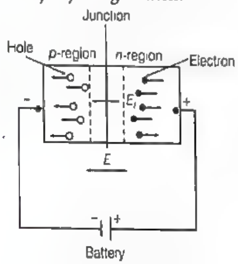
constituted by the motion of majority charge carriers across the junction. In forward bias, the junction diode offers low resistance.

Reverse Biasing

A junction diode is said to be reverse biased when the positive terminal of the external battery is connected to the *n*-side and negative terminal to the *p*-side of the diode.

Flow of Current in Reverse Biasing

In this situation, the reverse voltage supports the potential barrier, due to which both the potential barrier and with of the depletion layer increases. Under the effect of external electric field, holes in the *p*-region and electrons in the *n*-region are pushed away from the junction i.e. they cannot be combined at the junction. So, there is almost no flow of current due to majority charge carriers.



Reverse biasing of junction diode

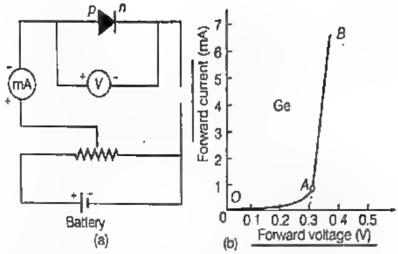
However, a very small current due to minority charge carriers, flows across the junction. This current is called reverse current.

I-V (CURRENT-VOLTAGE) CHARACTERISTICS OF *p-n* JUNCTION DIODE

The graphical relations between voltage applied across *p-n* junction and current flowing through the junction are called *I-V* characteristics of junction diode.

Forward Biased Characteristics

The circuit diagram for studying forward biased characteristics is shown in the figure (a). Starting from a low value, forward bias voltage is increased step by step (measured by voltmeter) and forward current is noted (by ammeter). A graph is plotted between voltage and current is shown in figure (b).



Forward biased characteristic of a diode

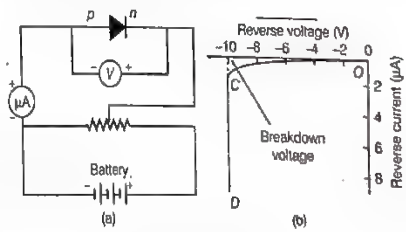
At the start when applied voltage is low, the current through the diode is almost zero. It is because of the potential barrier, which opposes the applied voltage.

Till the applied voltage exceeds the potential barrier, the current increases very slowly with increase in applied voltage (*OA* portion of the graph).

With further increase in applied voltage, the current increases very rapidly (*AB* portion of the graph), in this situation the diode behaves like a conductor. The forward voltage beyond which the current through the junction starts increasing rapidly with voltage is called **knee voltage** or **threshold voltage**. If line *AB* is extended back, it cuts the voltage axis at potential barrier voltage.

Reverse Biased Characteristics

The circuit diagram for studying reverse biased characteristics is shown in the figure (a).



Reverse biased characteristic of a diode

In reverse biased, the applied voltage supports the flow of minority charge carriers across the junction. So, a very small current flows across the junction due to minority charge carriers. Motion of minority charge carriers is also supported by internal potential barrier, so all the minority carriers cross over the junction.

[redacted]

Therefore, the small reverse current remains almost constant over a sufficiently long range of reverse bias, increasing very little with increasing voltage (*OC* portion of the graph). This reverse current is voltage independent upto certain voltage known as **breakdown voltage** and this voltage independent current is called **reverse saturation current**.

Note If the reverse bias is equal to the breakdown voltage, then the reverse current through the junction increases very rapidly (*CD* portion of the graph). This situation is called **avalanche breakdown** and the junction may get damaged due to excessive heating if this current exceeds the rated value of *p-n* junction.

In diodes, a resistance is offered by the function which depends on the applied voltage, which is called **dynamic resistance**. It is the ratio of small change in voltage to the small change in current produced.

Dynamic resistance, $r_d = \dfrac{\Delta V}{\Delta I}$.

DIODE AS A RECTIFIER

The process of converting alternating voltage/current into direct voltage/current is called rectification. Diode is used as a rectifier for converting alternating current/voltage into direct current/voltage.

Principle

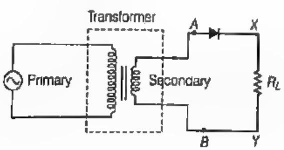
From the *V-I* characteristic of a junction diode, we see that it allows current to pass only when it is forward biased. So, if an alternating voltage is applied across a diode, the current flows only in that part of the cycle when the diode is forward biased. This property is used to rectify the current/voltage.

There are two ways of using a diode as a rectifier, i.e.

(i) Diode as a half-wave rectifier
(ii) Diode as a full wave rectifier

Diode as a Half-Wave Rectifier

In this, the AC voltage to be rectified is connected to the primary coil of a step-down transformer and secondary coil is connected to the diode through resistor R_L across which, output is obtained.

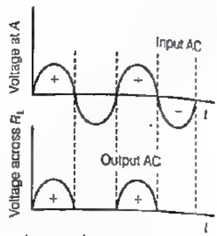


Circuit diagram of half-wave rectifier

Working

During positive half cycle of the input AC, the *p-n* junction is forward biased. Thus, the resistance in *p-n* junction becomes low and current flows. Hence, we get output in the load.

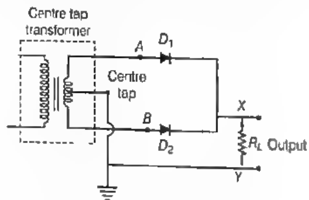
During negative half cycle of the input AC, the *p-n* junction is reverse biased. Thus, the resistance of *p-n* junction is high and current does not flow. Hence, no output is in the load.



Input and output waveforms

Diode as a Full Wave Rectifier

In the full wave rectifier, two *p-n* junction diodes, D_1 and D_2 are used. This arrangement is shown in the diagram below.

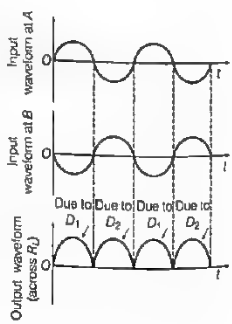


Circuit diagram of full wave rectifier

Working

During the positive half cycle of the input AC, the diode D_1 is forward biased and the diode D_2 is reverse biased. The forward current flows through diode D_1.

During the negative half cycle of the input AC, the diode D_1 is reverse biased and diode D_2 is forward biased. Hence, current flows through diode D_2. Hence, we find that during both the halves, current flows in the same direction.



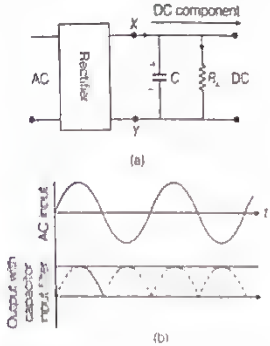
Input and output waveforms

[redacted]

Role of Filters

In order to get the steady DC output from the pulsating voltage normally, a capacitor is connected across the output terminals (parallel to load R_L). An inductor can also be used in series for the same purpose.

As these additional circuits appear to filter out the AC ripple and provide a pure DC voltage, so they are called filters.



A full wave rectifier with capacitor filter Fig. (a) and input and output voltage of rectifier in Fig. (b)

Let us discuss the role of capacitor in filtering. When the voltage across the capacitor is rising, it gets charged. If there is no external load, it remains charged to the peak voltage of the rectified output. When there is a load, it gets discharged through the load and the voltage across it begins to fall. In the next half cycle of the rectified output, it again gets charged to the peak value (see the above figure). The rate of fall of voltage across the capacitor depends upon the inverse product of capacitor C and the effective resistance R_L used in the circuit and is known as time constant. To make the time constant large value of C should be large. So, capacitor input filters use large capacitors. The output voltage obtained by using capacitor input filter is nearer to the peak voltage of the rectified voltage.

CHAPTER PRACTICE

(SOLVED)

OBJECTIVE Type Questions

1. The conductivity of a semiconductor increases with increase in temperature, because **NCERT Examplar**

(a) number density of free current carriers increases

(b) relaxation time increases

(c) both number density of carriers and relaxation time increase

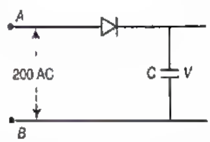
(d) number density of carriers increases, relaxation time decreases but effect of decrease in relaxation time is much less than increase in number density

2. The substance which is doped in an intrinsic semiconductor to make p-type semiconductor is

(a) phosphorus (b) antimony

(c) aluminium (d) arsenic

3. A 220 V AC supply is connected between points A and B (figure). What will be the potential difference V across the capacitor? **NCERT Examplar**



(a) 220V (b) 110 V (c) 0 V (d) $220\sqrt{2}$V

4. The ratio of output frequencies of half wave rectifier and a full-wave rectifier, when an input of frequency 200 Hz is fed at input?

(a) 1 : 2 (b) 2 : 1

(c) 4 : 1 (d) 1 : 4

Directions (Q. Nos. 5-7) *In the following questions, two statements are given- one labeled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

5. **Assertion** The conductivity of an intrinsic semiconductor depends on its temperature.

Reason The conductivity of an intrinsic semiconductor is slightly higher than that of a lightly doped p-type semiconductor.

6. The ability of a junction diode to .... an alternating voltage, is based on the fact that it allows current to pass only when it is forward biased. **Delhi 2020**

7. The ...., a property of materials C, Si and Ge depends upon the energy gap between their conduction and valence bands.

[redacted]

VERY SHORT ANSWER Type Questions

8. Sn, C and Si, Ge are all group XIV elements. Yet, Sn is a conductor, C is an insulator while Si and Ge are semiconductors. Why? **NCERT Exemplar**

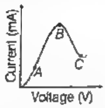
9. Show variation of resistivity of Si with temperature in a graph. **Delhi 2014**

10. Is the ratio of number of holes and the number of conduction electrons in an n-type extrinsic semiconductor more than, less than or equal to 1?

11. How does the width of a depletion region of a p-n junction vary, if doping concentration is increased? **CBSE SQP (Term-I)**

12. What do you mean by reverse current in p-n junction diode?

13. The graph shown in the figure represents a plot of current *versus* voltage for a given semiconductor Identify the region, if any, over which the semiconductor has a negative resistance. **All India 2013**



14. Can the potential barrier across a p-n junction be measured by simply connecting a voltmeter across the junction? **NCERT Exemplar**

15. When a voltage drop across a p-n junction diode is increased from 0.70 V to 0.71V, the change in the diode current is 10 mA .What is the dynamic resistance of diode?

16. In half-wave rectification, what is the output frequency if input frequency is 25 Hz?

17. Why are elemental dopants for silicon or germanium usually chosen from group XIII or group XV? **NCERT Exemplar**

SHORT ANSWER Type Questions

18. Write two characteristic features to distinguish between n-type and p-type semiconductors. **All India 2012**

19. Can a slab of p-type semiconductor be physically joined to another n-type semiconductor slab to form p-n junction? Justify your answer.

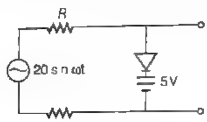
20. In a p-n junction diode the forward bias resistance is low as compared to the reverse bias resistance. Give reason.

21. Briefly explain how a potential barrier is set up across a p-n junction as a result of diffusion and drift of the charge carriers. **CBSE 2020 All India**

22. Explain with the help of a circuit diagram, the working of a p-n junction diode as a half-wave rectifier. **All India 2014**

23. Write any two distinguishing features between conductors, semiconductors and insulators on the basis of energy band diagrams. **All India 2014**

24. Assuming an ideal diode, draw the output waveform for the circuit given in the figure, explain the waveform. **NCERT Exemplar**

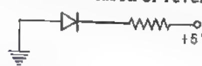


25. The ionisation energy of isolated pentavalent phosphorous atom is very large. How is it possible that when it goes into silicon lattice position, it releases its 5th electron at room temperature, so that n-type semiconductor is obtained?

26. Define the following terms used in electronic devices.
   (i) Reverse breakdown voltage
   (ii) V-I characteristic of forward biased diode

27. Write the two processes that take place in the formation of a p-n junction. Explain with the help of a diagram, the formation of depletion region and barrier potential in a p-n junction. **Delhi 2017**

28. (i) In the following diagram, is the junction diode forward biased or reverse biased?



(ii) Draw the circuit diagram of a full wave rectifier and state how it works? **All India 2017 C**

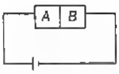
29. Draw a labelled diagram of a full wave rectifier circuit. State its working principle. Show the input-output waveforms. **All India 2019, 20**

[redacted]

Or Draw the circuit diagram of a full wave rectifier. Explain its working principle. Draw the input and output waveform. **All India 2017C**

30. A student wants to use two p-n junction diodes to convert alternating current into direct current. Draw the labelled circuit diagram she would use and explain how it works. **CBSE 2018**

31. There are two semiconductor materials A and B which are made by doping germanium crystal with indium and arsenic, respectively. As shown in the figure, the junction of two is biased with a battery. Will the junction be forward bias and reverse bias?



LONG ANSWER Type I Questions

32. Draw the energy band diagram when intrinsic semiconductor (Ge) is doped with impurity atoms of antimony (Sb). Name the extrinsic semiconductor so obtained and majority charge carriers in it. **CBSE SQP (Term-II)**

33. (a) Explain the formation of energy bands in crystalline solids.
(b) Draw the energy band diagrams of (i) a metal and (ii) a semiconductor.

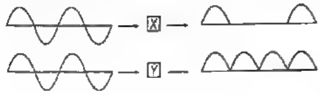
34. Explain the formation of potential barrier and depletion region in a p-n junction diode. What is effect of applying forward bias on the width of depletion region? **Delhi 2020**

35. As we know that an n-type semiconductor has large number of electrons but it is still electrically neutral. Why?

LONG ANSWER Type II Questions

36. (i) State briefly the processes involved in the formation of p-n junction explaining clearly how the depletion region is formed?
(ii) Using the necessary circuit diagrams, show how the V-I characteristics of a p-n junction are obtained in
(a) forward biasing (b) reverse biasing
How are these characteristics made use of in rectification? **Delhi 2014**

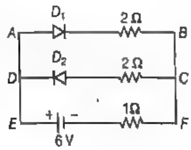
37. An AC signal is fed into two circuits X and Y and the corresponding output in the two cases have the waveforms as shown in below.



(i) Identify the circuits X and Y. Draw their labelled circuit diagrams.
(ii) Briefly explain the working of circuit Y.
(iii) How does the output waveform circuit Y get modified when a capacitor is connected across the output terminals parallel to the load resistor?

NUMERICAL PROBLEMS

38. Assuming that the two diodes D_1 and D_2 used in the electric circuit as shown in the figure are ideal, find out the value of the current flowing through $1\,\Omega$ resistor. **Delhi 2013, (2 M)**

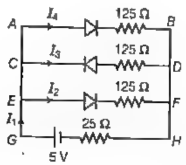


39. The impurity levels of dopped semiconductor are 30 eV below the conduction band. Determine whether the semiconductor is n-type or p-type. At the room temperature, thermal collisions occur as a result of which, the extra electron loosely bound to the impurity ion gets an amount of energy kT and hence this electron can jump into conduction band. What is the value of T? Take, k is Boltzmann constant $= 8.62 \times 10^{-5}$ eV/K.

40. A potential barrier of 0.4 V exists across p-n junction.
(i) If the depletion region is 4.0×10^{-7} m wide, what is the intensity of the electric field in this region?
(ii) If an electron with speed 4×10^5 m/s approaches the p-n junction from the n-side, find the speed with which it will be p-side.

[redacted]

41. If each diode in figure has a forward bias resistance of 25 Ω and infinite resistance in reverse bias, what will be the values of the currents I_1, I_2, I_3 and I_4?



42. In half-wave rectification, what is the output frequency, if the input frequency is 50 Hz? What is the output frequency of a full wave rectifier for the same input frequency? NCERT

43. Predict the effect on the electrical properties of a silicon crystal at room temperature, if every millionth silicon atom is replaced by an atom of indium. Given, concentration of silicon atoms $= 5\times10^{28}\,m^{-3}$, intrinsic carrier concentration $= 1.5\times10^{16}\,m^{-3}$, $H_e - 0.135\,m^3/V\text{-}s$ and $H_h = 0.048 m^3/V\text{-}s$.

HINTS AND SOLUTIONS

1. (d) The conductivity of a semiconductor increases with increase in temperature, because the number density of current carriers increases, relaxation time decreases but effect of decrease in relaxation is much less than increase in number density.

2. (c) In an intrinsic semiconductor, when an impurity of trivalent group such as aluminium, boron, etc., mixed in very small quantity, then the resultant crystal will be p-type semiconductor.

3. (d) As p-n junction conducts during positive half cycle only, the diode connected here will work is positive half cycle. Potential difference across C = peak voltage of the given AC voltage $- V_0 = V_{rms}\sqrt{2} = 220\sqrt{2}$ V.

4. (a) Output frequency of full wave rectifier is twice the output frequency of half wave rectifier.

$$\therefore \frac{f_{\text{half wave}}}{f_{\text{full wave}}} = \frac{1}{2}$$

5. (c) The conductivity of an intrinsic semiconductor is less than that of a lightly doped p-type semiconductor.

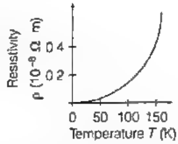
6. rectify

7. conductivity

8. A material is a conductor, if in its energy band diagram, there is no energy gap between conduction band and valence band. For insulator, the energy gap is large and for semiconductor, the energy gap is moderate.

The energy gap for Sn is 0 eV, for C is 5.4 eV, for Si is 1.1 eV and for Ge is 0.7 eV, related to their atomic size. Therefore, Sn is a conductor, C is an insulator and Ge and Si are semiconductors.

9. Graph of resistivity of Si as a function of temperature is given alongside (resistivity of metals increases with increase in temperature).



10. The ratio of number of holes and the number of conduction electrons in an n-type extrinsic semiconductor is less than 1.

11. The width of a depletion region of a p-n junction of inversely proportional to the concentration of dopants. So, if the doping concentration is increased, then the width of depletion region decreases.

12. When a diode is reversed biased, then very small current due to minority charge carries flows across the junction. This current is called reverse current.

13. Resistance of a material can be found out by the slope of the curve V *versus* I. Part BC of the curve shows the negative resistance as with the increase in current, there is a decrease in voltage.

14. We cannot measure the potential barrier across a p-n junction by a voltmeter because the resistance of voltmeter is very high as compared to the junction resistance.

15. The dynamic resistance of a diode is

$$r_d = \frac{\text{Change in diode voltage } (\Delta V)}{\text{Change in diode current } (\Delta I)}$$

Here, $\Delta V = 0.71 - 0.70 = 0.01$ V and $\Delta I = 10\,\text{mA} = 10\times10^{-3}$ A

$$\therefore \quad r_d = \frac{0.01}{10\times10^{-3}} = 1\,\Omega$$

16. The output frequency of a half-wave rectifier is same as that as input frequency, i.e. 25 Hz.

17. The size of the dopant atom should be such that their presence in the pure semiconductor does not distort the semiconductor but easily contribute the charge carriers on forming covalent bonds with Si or Ge atoms, which are provided by group XIII or group XV elements

18. Refer to text on page 528.

19. No, two different slabs of p-type and n-type semiconductor cannot be physically joined to form p-n junction. It is because, two different slabs have different extent of doping of impurity atom in them. So, the characteristics of a p-n junction diode are not met by this process.

[redacted]

20. It is because in forward biased condition, the potential barrier is low in p-n a junction as compared to that in reverse biased condition. So, the resistance is low in forward biasing as compared to that in reverse biasing.

21. Refer to text on page 530.

22. Refer to text on page 532.

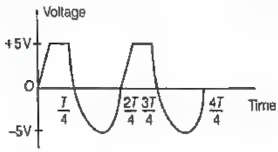
23. Refer to text on page 526.

24. When the input voltage is equal to or less than 5 V, diode will be reverse biased. It will offer high resistance in comparison to resistance (R) in series. Now, diode appears in open circuit. The input waveform is then passed to the output terminals. The result with sine wave input is to dip off all positive going portion above 5 V.

If input voltage is more than + 5 V, diode will be conducting as if forward biased offering low resistance in comparison to R. But there will be no voltage in output beyond 5 V as the voltage beyond + 5 V will appear across R.

When input voltage is negative, there will be opposition to 5 V battery. In p-n junction, input voltage becomes more than − 5 V, the diode will be reverse biased. It will offer high resistance in comparison to resistance R in series. Now, junction diode appears in open circuit. The input waveform is then passed on to the output terminals.

The output waveform is shown here in the figure



25. Refer to text on page 529.

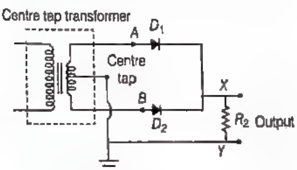
26. (i) Refer to text on page 532.
(ii) Refer to text on page 531.

27. Refer to text on page 530.

28. Refer to text on pages 531 and 532.

29. Refer to text on page 532.

30. A rectifier is used to convert alternating current into direct current, whose labelled circuit is given below.

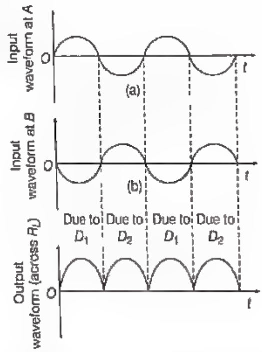


Circuit diagram of full wave rectifier

Working

During the positive half cycle of the input AC, the diode D_1 is forward biased and the diode D_2 is reverse biased. The forward current flows through diode D_1.

During the negative half cycle of the input AC, the diode D_1 is reverse biased and diode D_2 is forward biased. Thus, current flows through diode D_2. Thus, we find that during both the halves, current flows in the same direction.

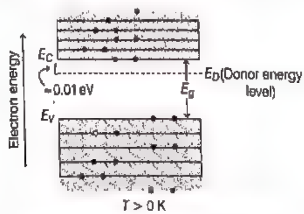


31. As, semiconductor A is doped with indium, so it behaves as p-type semiconductor and B is doped with arsenic, so it behaves as n-type semiconductor. Thus, the figure shows that it is forward bias condition.

32. When intrinsic semiconductor (Ge) is doped with impurity atoms of antimony (Sb), which is a pentavalent atom, the extrinsic semiconductor so formed is of n-type.

The energy band diagram for n-type semiconductor

In extrinsic semiconductors, additional energy states due to donor impurities (E_D) also exist. In the energy band diagram of n-type semiconductor, the donor energy level E_D is slightly below the bottom E_C of conduction band and the electrons from this level move into conduction band with very small supply of energy.



In n-type semiconductor, the majority charge carriers are electrons.

[redacted]

33. (a) Refer to text on page 526.

(b) Refer to text on page 526.

34. Refer to text on page 530.

On applying forward bias, the width of the depletion region decreases.

35. n-type semiconductor is formed by doping it with pentavalent impurities. These impurities or dopant takes the atoms in the crystal and its four electrons take part in chemical bonding with four electrons of intrinsic semiconductor or pure semiconductor. Whereas the last electrons are left free. Since, as whole atom is electrically neutral, so n-type semiconductor is also neutral.

36. (i) Refer to text on page 530.

(ii) (a) Refer to text on page 531.

(b) Refer to text on pages 531 and 532.

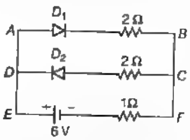
37. (i) X-Half wave rectifier

Y Full wave rectifier.

(ii) Refer to text on pages 532 and 533.

(iii) Refer to text on pages 532 and 533.

38. According to the question,



Equivalent resistance, $R_{AB} = 2 + 1 = 3\,\Omega$

$$\frac{1}{R'} = \frac{1}{2} + \frac{1}{3} = \frac{3+2}{6} = \frac{5}{6}\,\Omega$$

or $R' = \frac{6}{5}\,\Omega$

$$\Rightarrow \quad I_{EF} = \frac{V}{R'} = \frac{6}{6/5} = 5\text{ A}$$

39. The separation of impurity energy level from conduction band is less in case of n-type semiconductor and more in case of p-type semiconductor. As, energy separation of impurity is 30×10^{-3} eV is much smaller than energy gap of pure semiconductor, i.e. $E \approx 1$ eV. Therefore, the doped semiconductor is n-type.

$$E_g = 30 \times 10^{-3}\text{ eV} = kT$$

$$\Rightarrow \quad T = \frac{E_g}{k} = \frac{30 \times 10^{-3}}{8.62 \times 10^{-5}} = 348.02\text{ K}$$

40. Given, $V = 0.4$ V

(i) $d = 4 \times 10^{-7}$ m, $E = ?$

$$\text{Electric field, } E = \frac{V}{d} = \frac{0.4}{4 \times 10^{-7}} = 1 \times 10^{6}\text{ V/m}$$

(ii) $v_1 = 4 \times 10^{5}$ m/s, $v_2 = ?$

Suppose v_1 be the speed of electron when it enters the depletion layer and v_2 be the speed when it comes out of the depletion layer.

According to principle of conservation of energy,

KE before entering the depletion layer = Gain in PE + KE after crossing the depletion layer

$$\Rightarrow \quad \frac{1}{2}mv_1^2 = e \times V + \frac{1}{2}mv_2^2$$

$$\Rightarrow \quad \frac{1}{2} \times 9.1 \times 10^{-31} \times (4 \times 10^{5})^2 = 1.6 \times 10^{-19} \times 0.4 + \frac{1}{2} \times 9.1 \times 10^{-31} \times v_2^2$$

$$\therefore \quad v_2 = 1.39 \times 10^{5}\text{ m/s}$$

41. Given, forward biased resistance = 25 Ω

Reverse biased resistance = ∞

As the diode in branch CD is in reverse biased which having resistance infinite,

So, $I_3 = 0$

Resistance in branch $AB = 25 + 125 = 150\,\Omega$ (say R_1)

Resistance in branch $EF = 25 + 125 = 150\,\Omega$ (say R_2)

AB is parallel to EF

So, resultant resistance, $\frac{1}{R'} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{150} + \frac{1}{150} = \frac{2}{150}$

$\Rightarrow \quad R' = 75\,\Omega$

Total resistance, $R = R' + 25 = 75 + 25 = 100\,\Omega$

Current, $I_1 = \frac{V}{R} = \frac{5}{100} = 0.05$ A

$I_1 = I_4 + I_2 + I_3$ [here, $I_3 = 0$]

So, $I_1 = I_4 + I_2$

Here, the resistances R_1 and R_2 are same.

i.e. $I_4 = I_2$

$\therefore \quad I_1 = 2I_2$

$\Rightarrow \quad I_2 = \frac{I_1}{2} = \frac{0.05}{2} = 0.025$ A and $I_4 = 0.025$ A

Thus, $I_1 = 0.05$ A, $I_2 = 0.025$ A, $I_3 = 0$

and $I_4 = 0.025$ A

42. Given, input frequency = 50 Hz

For a half-wave rectifier, the output frequency is equal to the input frequency.

∴ Output frequency = 50 Hz

For a full wave rectifier, the output frequency is twice the input frequency.

∴ Output frequency = $2 \times 50 = 100$ Hz.

43. As, concentration of Si atom $= 5 \times 10^{28}\,/\text{m}^3$

The doping of indium is 1 atom in 10^{6} atoms of Si. But indium has three valence electrons and each doped indium atom creates one hole in Si crystal. Hence, it acts as an acceptor atom.

$\therefore$ Concentration of acceptor atoms,

$$n_h = 5\times10^{28}\times10^{-6} = 5\times10^{22}/\text{m}^3$$

Intrinsic carrier concentration, $n_i = 1.5\times10^{16}/\text{m}^3$

$\therefore$ Hole concentration is increased,

$$= \frac{n_h}{n_i} = \frac{5\times10^{22}}{1.5\times10^{16}} = 3.33\times10^6$$

New electron concentration,

$$n_e = \frac{n_i^2}{n_h} = \frac{(1.5\times10^{16})^2}{5\times10^{22}} = 0.45\times10^{10}/\text{m}^3$$

Electron concentration has been reduced

$$= \frac{n_i}{n_e} = \frac{1.5\times10^{16}}{0.45\times10^{10}} = 3.33\times10^6/\text{m}^3$$

This means that the hole concentration has been increased over its intrinsic concentration by the same amount with which the electron concentration has been decreased.

The conductivity of doped silicon is given by

$$\sigma = e(n_e H_e + n_h H_h)$$
$$= 1.6\times10^{-19}(0.45\times10^{10}\times0.135 + 5\times10^{22}\times0.048)$$
$$= 384 \text{ S/m}$$

Resistivity, $\rho = \frac{1}{\sigma} = \frac{1}{384} = 0.0026\ \Omega\text{-m}$

Conductivity of pure Si crystal,

$$\sigma = en_i(H_e + H_h) = 1.6\times10^{-19}\times1.5\times10^{16}(0.135+0.048)$$
$$= 0.4392\times10^{-3}\text{ S/m}$$

Resistivity, $\rho = \frac{1}{\sigma} = \frac{1}{0.4392\times10^{-3}} = 2276.8\ \Omega\text{-m}$

Thus, we see that the conductivity of Si doped within become much greater than its intrinsic conductivity and the resistivity has become much smaller than the intrinsic resistivity.

[redacted]

SUMMARY

- **Semiconductors** are the basic material used in the present solid state electronic devices like diode, transistor, ICs etc
- Metal have low resistivity (10^{-2} to 10^{-8} Ω-m), insulators have very high resistivity (10^{8} Ω-m) while semiconductors have intermediately values of resistivity
- **Valence Band** is the energy band which includes the energy levels of the valence electrons. This band may be partially or complete filled with electrons.
- **Conduction Band** is the energy band above the valence band. At room temperature, this band is either empty or partially filled with electrons.
- The minimum energy required for shifting electrons from valence band to conduction band is called energy **band gap.**
- **Fermi Energy** is the maximum possible energy possessed by free electrons of a material at absolute zero temperature
- **An intrinsic semiconductor** is also called an undoped semiconductor or *i*-type semiconductor
- **Extrinsic Semiconductor** Those semiconductors in which some impurity atoms are embedded are known as extrinsic semiconductor
- In *n*-type semiconductors $n_e \geq n_h$ while in *p*-type semiconductors $n_h >> n_e$.
- *n*-type semiconductor Si or Ge is obtained by doping with pentavalent atoms (donors) like As, Sb, P, etc., while *p*-type Si or Ge can be obtained by doping with trivalent atom like B, Al, In, etc
- In all cases, $n_e n_h = n_i^2$ further the material possesses an overall charge neutrality.
- A ***p-n* junction** is an arrangement made by a close contact of *n*-type semiconductor and *p*-type semiconductor
- The region on either side of the junction which becomes depleted (free) from the mobile charge carriers is called **depletion region.**
- The potential difference developed across the depletion region is called the **potential barrier**
- A **semiconductor diode** is basically a *p-n* junction with metallic contacts provided at the ends for the application of an external voltage
- In forward bias (*n*-side is connected to negative terminal of the battery and *p*-side is connected to positive), the barrier is decreased while the barrier increases in reverse bias. Hence, forward bias current is more (mA) while it is very small (μA) in a *p-n* junction diode
- Diodes can be used for rectifying an AC voltage. With the help of a capacitor or a suitable filter, a DC voltage can be obtained

[redacted]

CHAPTER PRACTICE (UNSOLVED)

OBJECTIVE Type Questions

1. In an n-type silicon, which of the following statements is correct?

(a) Electrons are majority charge carriers and trivalent atoms are the dopants

(b) Electrons are minority charge carriers and pentavalent atoms are the dopants

(c) Holes are minority charge carriers and pentavalent atoms are the dopants

(d) Holes are majority charge carriers and trivalent atoms are the dopants

2. In an unbiased p-n junction, holes diffuse from the p-region to n-region because

(a) free electrons in the $n$region attract them

(b) they moves across the junction by the potential difference

(c) hole concentration in p-region is more as compared to hole concentration in n-region

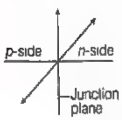
(d) All of the above

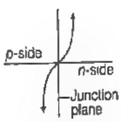
3. The potential barrier of germanium diode is

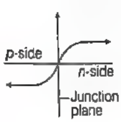
(a) 0.1 V (b) 0.3 V

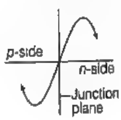
(c) 0.5 V (d) 0.7 V

4. Which of these graphs shows potential difference between p-side and n-side of a p-n junction in equilibrium?

(a) 

(b) 

(c) 

(d) 

5. If reverse biasing potential is increased beyond a certain critical (breakdown) value, then

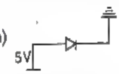
(a) diode gets destroyed due to overheating

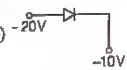
(b) no current flows through the diode

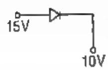
(c) after breakdown a heavy current flows from p to n-side

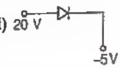
(d) potential barrier becomes zero

6. Which is reverse biased diode?

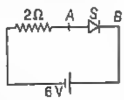
(a) 

(b) 

(c) 

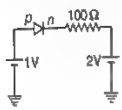
(d) 

7. The diode shown in the circuit is a silicon diode. The potential difference between the points A and B will be



(a) 6 V (b) 0.6 V (c) 0.7 V (d) 0 V

8. The current through an ideal p-n junction shown in the following circuit diagram will be



(a) zero (b) 1 mA

(c) 10 mA (d) 30 mA

VERY SHORT ANSWER Type Questions

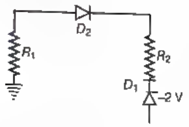
9. At what temperature would an intrinsic semiconductor behave like a perfect insulator?

[redacted]

10. What type of charge carriers are there in an n-type semiconductor?

11. What do you mean by dynamic resistance of a p-n junction diode?

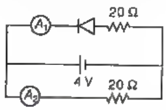
12. Which one of the two diodes D_1 and D_2 in the given figure is



(i) forward biased? (ii) reverse biased

SHORT ANSWER Type Question

13. Assuming that the resistances of the meters are negligible, what will be the readings of the ammeters A_1 and A_2 in the circuit shown in figure?



LONG ANSWER Type I Question

14. Distinguish between an intrinsic semiconductor and p-type semiconductor. Give reason, why a p-type semiconductor crystal is electrically neutral, although $n_h >> n_e$?

LONG ANSWER Type II Question

15. (i) Draw a typical shape of the V-I characteristics of a p-n junction diode both in (i) forward (b) reverse bias configuration. How do we infer from these characteristics that a diode can be used to rectify alternating voltages.
(ii) Draw the circuit diagram of a full wave rectifier using a centre tap transformer and two p-n junction diodes. Give a brief description of the marking of this circuit.

ANSWERS

1. (c)	2. (c)	3. (b)	4. (c)	5. (c)
6. (b)	7. (a)	8. (a)		

9. At 0K, intrinsic semiconductor behaves like a perfect insulator.

10. Majority charge carriers are electrons and minority charge carriers are holes.

11. It is the ratio of small change in voltage to the small change in current produced, $r_d = \frac{\Delta V}{\Delta I}$

12. (i) D_2 (ii) D_1

13. In the given circuit, the diode is reverse biased. In the upper port of the circuit, no current flows through the upper resistance

Reading of ammeter, $A_1 = 0$

Reading of ammeter, $A_2 = \frac{4}{20} = 0.2\,A$

14. Refer to text on pages 527 and 528.

15. (i) Refer to text on pages 531 and 532.
(ii) Refer to text on page 532.

[redacted]

SAMPLE QUESTION PAPER 1

A HIGHLY SIMULATED SAMPLE QUESTION PAPER FOR CBSE CLASS XII EXAMINATIONS

PHYSICS (FULLY SOLVED)

GENERAL INSTRUCTIONS

1. All questions are compulsory. There are 33 questions in all.
2. This question paper has five sections: Section A, Section B, Section C, Section D and Section E.
3. **Section A** contains ten very short answer questions and four assertion reasoning MCQs of 1 mark each, **Section B** has two case based questions of 4 marks each, **Section C** contains nine short answer questions of 2 marks each, **Section D** contains five short answer questions of 3 marks each and **Section E** contains three long answer questions of 5 marks each.
4. There is no overall choice. However internal choice is provided. You have to attempt only one of the choices in such questions.

TIME : 3 HOURS **MAX. MARKS : 70**

SECTION-A

All questions are compulsory. In case of internal choices, attempt anyone of them.

1. A photon and an electron have the same de-Broglie wavelength, which one has higher total energy?

2. Name any one method by which eddy currents can be minimised in the metal core of transformer on which coils are wound.

Or

Find the self-inductance of a coil, in which magnetic flux of 40 mWb is produced when 2A current flows through it.

3. When a ray is refracted from one medium to another, the wavelength changes from 6000 Å to 4000 Å. Find the critical angle for the interface.

4. Write any two distinguishing features between conductors, semiconductors and insulators on the basis of energy band diagrams.

5. A magnet of magnetic moment 2.5 A-m^2, weighs 66g. If the density of the material of the magnet is 7500 kg m^{-3}, then find the intensity of magnetisation.

Or

An electron moves in a circular path of radius 15 cm in a magnetic field of 4 G. Find the velocity of the electron in this field.

6. Four nuclei of an element undergo fusion to form a heavier nucleus, with release of energy. Which of the two—the parent or the daughter nucleus—would have higher binding energy per nucleon?

Or

If the orbital radius of the electron in a hydrogen atom is 4.7×10^{-11} m. Compute the kinetic energy of the electron in hydrogen atom.

7. In photoelectric effect, if the intensity of light is doubled, then what will be the change in maximum kinetic energy of photoelectrons?

8. When a charged capacitor is disconnected from a battery and if its plates are separated further, then find the change in its potential energy.

FULLY SOLVED

[redacted]

9. What is the frequency range of visible rays?

10. A p-n junction diode is forward biased. Write the effect on its potential barrier.

Or

What do you mean by reverse current in p-n junction diode?

For question numbers 11, 12, 13 and 14, two statements are given-one labelled Assertion (A) and the other labelled **Reason** (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

11. Assertion If a proton and an α-particle enter a uniform magnetic field perpendicularly with the same speed, the time period of revolution of α-particle is double than that of proton.

Reason In a magnetic field, the period of revolution at a charged particle is directly proportional to the mass of the particle and inversely proportional to the charge of particle.

12. Assertion If the inner solenoid was much shorter than (and placed well inside) the outer solenoid, then we could still have calculated the flux linkage $N_1\phi_1$.

Reason The inner solenoid is effectively immersed in a uniform magnetic field due to the outer solenoid.

13. Assertion Propagation of light through an optical fibre is due to total internal reflection taking place at the core-clade interface.

Reason Refractive index of the material of the core of the optical fibre is greater than that of air.

14. Assertion Photocell is also called electric eye.

Reason Photocell can see the things placed infront of it.

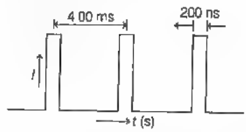
SECTION-B

Questions 15 and 16 are case study based questions and are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

Electric Pulse

15. We define average current as $I_{av} = \dfrac{\Delta Q}{\Delta t}$ and instantaneous current as $I = \dfrac{dQ}{dt}$. Obviously, $Q = \int I dt$. Further, electric energy delivered per unit time by a source, i.e. power, $P = \dfrac{\Delta E}{\Delta t}$, where ΔE is the energy delivered by the source in time Δt.

In a certain accelerator, electrons emerge with energies of 40.0 MeV (1MeV $= 1.60 \times 10^{-13}$ J). The electrons do not emerge in steady stream, but in pulses that repeat 250 times per second.



This corresponds to a time between each pulse of 4.00 ms in figure. Each pulse lasts for 200 ns and the electrons in the pulse constitute a current of 250 mA. The current is zero between the pulses. While the pulse is ON, the current is constant.

(i) Which of the following relation is correct for conductivity σ of solid conductor?

(a) $\sigma = \dfrac{ne^2}{m}\tau$

(b) $\sigma = \dfrac{2ne^2}{m}\tau$

(c) $\sigma = \dfrac{ne^2}{2m}\tau$

(d) $\sigma = \dfrac{ne^2}{4m}\tau$

(ii) The charge delivered by the acceleration per pulse is

(a) 5.00×10^{-6} C (b) 5.00×10^{-8} C

(c) 10.00×10^{-8} C (d) 10.00×10^{-6} C

[redacted]

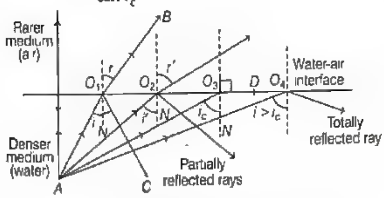
(iii) The number of electrons delivered per pulse is
(a) 6×10^{11} (b) 6×10^{9}
(c) 3.13×10^{11} (d) 3.13×10^{18}

(iv) The average current delivered by the acceleration is
(a) 12.5 A (b) 12.5 mA
(c) 12.5 μA (d) 1.25 A

(v) The maximum power delivered by the electron beam is
(a) 100 W (b) 10 kW
(c) 1MW (d) 10 MW

Total Internal Reflection

16. Total internal reflection is the phenomenon of reflection of light into denser medium at the interface of denser medium with a rarer medium. Light must travel from denser to rarer and angle of incidence in denser medium must be greater than critical angle (i_c) for the pair of media in contact, we can show $\mu=\frac{1}{\sin i_c}$.



(i) In total internal reflection,
(a) light ray travelling through a denser medium is completely reflected back to denser medium
(b) light ray travelling through a denser medium is completely refracted to rarer medium
(c) light ray is partially reflected back to denser medium and partially refracted to rarer medium
(d) light ray is absorbed completely by denser medium

(ii) Total internal reflection of a light ray travelling from denser medium to rarer medium occurs only when angle of incidence is
(a) 45°
(b) 90°
(c) acute
(d) more than a certain value

(iii) Critical angle for water-air interface is 48.6°. What is the refractive index of water?
(a) 1 (b) 3/2
(c) 4/3 (d) 3/4

(iv) Light is travelling from air to water at $\angle i=50°$, which is greater than critical angle for air-water interface. What fraction of light will be totally reflected?
(a) 100% (b) 50%
(c) 25% (d) None of these

(v) Critical angle for glass-air interface where refractive index μ of glass is 3/2 is
(a) 41.8° (b) 60°
(c) 30° (d) 44.3°

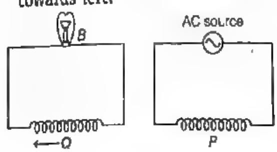
SECTION-C

All questions are compulsory. In case of internal choices, attempt anyone.

17. A germanium *p-n* junction diode is connected to a battery with milliammeter in series. What should be the minimum voltage of battery, so that current may flow in the circuit? What happens, if the diode is now made of silicon? Give reason for your answer.

18. In a region of uniform magnetic induction $B=10^{-2}$ T, a circular coil of radius 30 cm and resistance $\pi^2\Omega$ is rotated about an axis which is perpendicular to the direction of B and which forms a diameter of the coil. If the coil rotates at 200 rpm, find the amplitude of the alternating current induced in the coil.

Or A coil Q is connected to low voltage bulb B and placed near another coil P as shown in the figure. Give reasons to explain the following observations.
(i) The bulb B lights.
(ii) Bulb gets dimmer, if the coil Q is moved towards left.



FULLY SOLVED

[redacted]

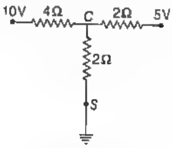
19. Suppose that, the electric field part of an electromagnetic wave in vacuum is

$$E = (3.1\,N/C)\cos[(1.8\,\text{rad/m})y + 5.4\times 10^{8}\,\text{rad/s})t]\hat{i}$$

(i) What is the direction of propagation?

(ii) What is the wavelength λ?

20. As the switch S is closed in the circuit shown in figure below, then find current passed through it



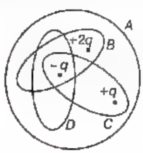
Or

Three materials A, B and C have electrical conductivities σ, 2σ and 2σ, respectively. Their number densities of free electrons are n, $2n$ and n, respectively. For which material, is the average collision time of free electrons maximum ?

21. Write some important properties of electric field lines.

Or

Rank the Gaussian surfaces as shown in the figure. In order of increasing electric flux, starting with the most negative.



22. (i) Which lens is used as a magnifying lens?

(ii) Raghav's grandfather was using spectacles of power $-1D$ for distant vision. Now, he also needs to use reading glass of $+0.2D$. Explain it.

23. Two electric bulbs P and Q have their resistances in the ratio of 1 : 2. They are connected in series across a battery. Find the ratio of the power dissipation in these bulbs.

24. Two wavelengths of sodium light 590 nm, 596 nm are used in turn to study the diffraction taking place at a single slit aperture 2×10^{-4} cm. The distance between the slit and the screen is 1.5 m. Calculate the separation between the positions of first maximum of the diffraction pattern obtained in the two cases.

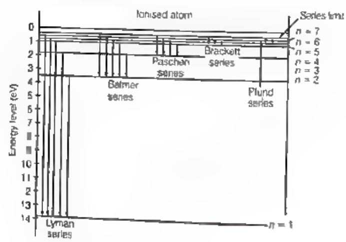
25. The mass of a nucleus is less than the sum of the masses of constituent neutrons and protons. Comment.

SECTION-D

All questions are compulsory. In case of internal choices, attempt anyone.

26. Hydrogen spectrum consists of discrete bright lines in a dark background and it is specifically known as hydrogen emission spectrum.There is one more type of hydrogen spectrum that exists where we get dark lines on the bright background, which is known as absorption spectrum.

Line spectra of the hydrogen atom is given below, whose series limit corresponds to the wavelength for $n\to\infty$.



By using above spectra, write the expression for the series limit for all the series obtained.

Or (i) Using Bohr's second postulate of quantisation of orbital angular momentum, show that the circumference of the electron in the nth orbital state in H-atom is n-times the de-Broglie wavelength associated with it.

(ii) The electron in H-atom is initially in the third excited state. What is the maximum number of spectral lines which can be emitted when it finally moves to the ground state?

FULLY SOLVED

[redacted]

27. A circular coil of 200 turns and radius 10 cm is placed in a uniform magnetic field of 0.5 T, parallel to the plane of the coil. If the current in the coil is 3 A, then calculate the

(i) total torque on the coil,

(ii) total force on the coil and

(iii) average force on each electron in the coil, due to the magnetic field.

Assume the area of cross-section of the wire to be 10^{-5} m^2 and the free electron density is 10^{29} m^{-3}.

28. What are extrinsic semiconductors? On the basis of valence band model, explain how can a pure semiconductor of Ge or Si be converted into n-type semiconductor?

29. (i) How does the choke coil help in the working of tubelight?

(ii) What is the effective resistance of a choke coil?

(iii) How can one reduces high frequency alternating current? What should be changed to reduce low frequency AC ?

30. Describe a photo-cell and mention few of its applications.

Or

What are the observations made from the expression of de-Broglie wavelength?

SECTION-E

All questions are compulsory. In case of internal choices, attempt anyone.

31. (i) A point charge is placed at the centre of spherical gaussian surface. How will the electric flux ϕ change, if

(a) the sphere is replaced by a cube of same or different volume?

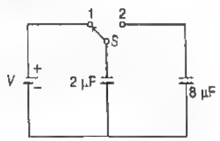
(b) a second charge is placed near and outside the original sphere?

(c) the original charge is replaced by an electric dipole?

(ii) Two point charges $q_A = 3\mu C$ and $q_B = -3\mu C$ are located 20 cm apart in vacuum. What is the electric field and its direction at the mid-point O of the line AB joining the two charges?

Or (i) A molecule of a substance has a permanent electric dipole moment of magnitude 10^{-29} C-m. A mole of this substance is polarised (at low temperature) by applying a strong electrostatic field of magnitude 10^6 Vm^{-1}. The direction of the field is suddenly changed by an angle of 60°. Calculate the heat released by the substance in aligning its dipoles along the new direction of the field.

(ii) A capacitor of $2\mu F$ is charged as shown in the figure. When the switch S is turned to position 2, then find the percentage of its stored energy dissipated.



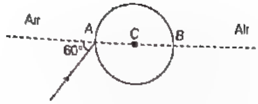
32. (i) Unpolarised light is passed through a polaroid P_1. When this polarised beam passes through another polaroid P_2 and if the pass axis of P_2 makes an angle θ with the pass axis of P_1, then write the expression for the polarised beam passing through P_2. Draw a plot showing the variation of intensity, when θ varies from 0 to 2π.

(ii) What do you understand by sign convention in measuring distances for lenses? Write it in your own words.

Or (i) A ray of light falls on a transparent sphere with centre C as shown in the figure. The ray emerges from the sphere parallel to the line AB. Find the angle of refraction of A, if the refractive index of material of sphere is $\sqrt{3}$. Also draw the refracted ray in the given figure.

FULLY SOLVED

[redacted]



(ii) The image obtained with a convex lens is erect and its length is four times the length of the object.

If the focal length of the lens is 20 cm, calculate the object and image distances.

33. (i) A solenoid of length 0.5 m has a radius of 1 cm and is made up of 500 turns. It carries a current of 5A. What is the magnetic field inside the solenoid?

(ii) A solenoid of length 1.0 m and 3.0 cm diameter has 5 layers of windings of 850 turns each and carries a current of 5 A. What is the magnetic field at the centre of solenoid? Also, calculate the magnetic flux from a cross-section of the magnetic flux solenoid at the centre of solenoid.

(iii) Two long parallel straight wires A and B carrying currents of 4.0 A and 5.0 A in same direction separated by a distance of 4×10^{-2} m. Calculate the force on a 0.20 m section of wire A and its direction.

Or

(i) The earth's magnetic induction at a certain point is 7×10^{-5} Wbm^{-2}. This is to be annulled by the magnetic induction at the centre of a circular conducting loop of radius 5 cm. Find the required current in the loop.

(ii) Is it possible to have a magnetic field configuration with three poles?

Also, if magnetic monopoles existed, how would Gauss's law of magnetism be modified?

(iii) A solenoid has core of a material with relative permeability 400. The windings of the solenoid are insulated from the core and carry a current of 2A.

If the number of turns is 1000 per metre, calculate (a) H and (b) M.

SOLUTIONS

1. Total energy of an electron, $E_e = mc^2$

Total energy of a photon, $E_p = \frac{hc}{\lambda}$

de-Broglie wavelength of electron of mass m moving with velocity v,

$$\lambda = \frac{h}{mv}$$

$$\Rightarrow \quad m = \frac{h}{\lambda v}$$

$\therefore$ Energy of an electron, $E_e = mc^2 = \frac{hc^2}{\lambda v}$

$$\therefore \quad \frac{E_e}{E_p} = \frac{\frac{hc^2}{\lambda v}}{\frac{hc}{\lambda}} = \frac{c}{v}$$

As $c >> v$, therefore the total energy of electron is more than the total energy of photon. [1]

2. Eddy currents are minimised by using laminations of metal to make a metal core. [1/2]

This laminations reduces the strength of the eddy current. So, the dissipation of the strength of electric current or heat loss is substantially reduced. [1/2]

Or

Here, $\phi = 40$ mWb $= 40 \times 10^{-3}$ Wb

and $I = 2$A

Self-inductance, $L = \frac{\phi}{I}$ [1/2]

$$= \frac{40 \times 10^{-3}}{2} = 2 \times 10^{-2} \text{ Wb}$$ [1/2]

3. As, $\mu_2 = \frac{1}{\sin C}$

$$\Rightarrow \quad \frac{\mu_2}{\mu_1} = \frac{\lambda_1}{\lambda_2} = \frac{1}{\sin C}$$ [1/2]

$$\frac{6000}{4000} = \frac{1}{\sin C}$$

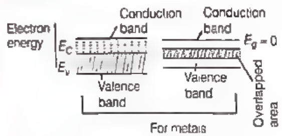
or $\quad C = \sin^{-1}\left(\frac{2}{3}\right)$

[1/2]

[redacted]

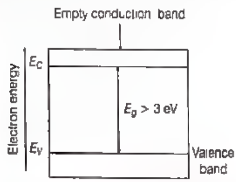
4. Conductor (Metal) In conductor, either there is no energy gap between the conduction band which is partially filled with electrons and valence band or the conduction band and valence band overlap each other.

Thus, many electrons from below the fermi level can shift to higher energy levels above the fermi level in the conduction band and behave as free electrons by acquiring a little more energy from any other sources.



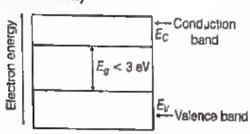
Insulator In insulator, the valence band is completely filled, the conduction band is completely empty. In this, energy gap is quite large and even energy from any other source cannot help electrons to overcome it.

Thus, electrons are bound to valence band and are not free to move. Hence, electric conduction is not possible in this type of material.



Semiconductor In semiconductor, the valence band is totally filled and the conduction band is empty but the energy gap between conduction band and valence band, unlike insulators is very small.

Thus, at room temperature, some electrons in the valence band acquire thermal energy greater than energy band gap and jump over to the conduction band where they are free to move under the influence of even a small electric field and acquire small conductivity.



5. Volume $= \dfrac{\text{Mass}}{\text{Density}} = \dfrac{66 \times 10^{-3}\ \text{kg}}{7500\ \text{kgm}^{-3}}$ **[1/2]**

$$= \frac{66 \times 10^{-5}}{75}\ \text{m}^3$$

Magnetisation, $M = \dfrac{m}{V} = \dfrac{2.5}{\dfrac{66 \times 10^{-5}}{75}}$

$$= 2.84 \times 10^5\ \text{Am}^{-1}$$ **[1/2]**

Or

As, $v = \dfrac{reB}{m} = \dfrac{0.15 \times 1.6 \times 10^{-19} \times 4 \times 10^{-4}}{9 \times 10^{-31}}$ **[1/2]**

$$= 1.07 \times 10^7\ \text{m/s}$$ **[1/2]**

6. According to question,

$$4\ {}_ZX^A \longrightarrow {}_{Z'}Y^{A'} + Q$$

(Parent nuclei) (Daughter nucleus)

As the daughter nucleus is a heavier nucleus as compared to parent nuclei, which are more stable than lighter nuclei, hence daughter nucleus has more binding energy per nucleon than parent nuclei.

Or

Given, orbital radius, $r = 4.7 \times 10^{-11}$ m

Kinetic energy,

$$K = \frac{e^2}{8\pi\varepsilon_0 r} = \frac{(9 \times 10^9\ \text{Nm}^2/\text{C}^2)(1.6 \times 10^{-19}\text{C})^2}{(2)(4.7 \times 10^{-11}\text{m})}$$ **[1/2]**

$$= 2.45 \times 10^{-18}\ \text{J}$$

$$= 2.45 \times 10^{-18}/1.6 \times 10^{-19}\ \text{eV}$$

$$= 15.3\ \text{eV}$$ **[1/2]**

7. The maximum kinetic energy of emitted photoelectrons depends on the frequency of radiation source and nature of the material of plate, but is independent of the intensity of light.

So, it will remain unchanged. **[1]**

8. When a charged capacitor is disconnected from a battery and if its plates are separated further, then its potential energy will rise.

Since, charge, Q = constant

and potential energy, $E = \dfrac{Q^2}{2C} = \dfrac{Q^2 d}{2\varepsilon_0 A}$ **[1/2]**

where, d is the separation between the plates

So $E \propto d$

Thus, with the increase in d, E also increases/rises. **[1/2]**

9. The frequency range of visible rays is 4×10^{14} Hz to 8×10^{14} Hz. **[1]**

[redacted]

10. As p-n junction diode is forward biased, so the applied voltage opposes the barrier voltage. Due to this, the potential barrier across the junction is lowered. **[1]**

Or

When a diode is reversed biased, then very small current due to minority charge carrier flows across the junction. This current is called reverse current. **[1]**

11. (a) Time period, $T = \frac{2\pi m}{Bq} \Rightarrow T \propto \frac{m}{q}$

As, $\left(\frac{m}{q}\right)_\alpha = 2\left(\frac{m}{q}\right)_p$

$\Rightarrow \quad T_\alpha = 2T_p$

Therefore, both A and R are true and R is the correct explanation of A. **[1]**

12. (a) If the inner solenoid was much shorter than (and placed well inside) the outer solenoid, then we could still have calculated the flux linkage $N_1\phi_1$ because the inner solenoid is effectively immersed in a uniform magnetic field due to the outer solenoid.

Therefore, both A and R are true and R is the correct explanation of A. **[1]**

13. (b) Optical fibre communication is based on the phenomenon of total internal reflection at core-clad interface.

The refractive index of the material of core is higher than that of the cladding, hence light striking at core-cladding interface gets totally internally reflected. The light undergoes repeated total internal reflection and reaches the other end of the fibre.

Therefore, both A and R are true but R is not the correct explanation of A. **[1]**

14. (c) Photocell is technical application of the photoelectric effect. It is a device which converts light energy into electric energy. It is also called an electric eye. Photocell are used in the reproduction of sound in motion picture and in the television camera.

Therefore, A is true but R is false. **[1]**

15. (i) (a) The correct relation of conductivity of a solid conductor is $\sigma = \frac{ne^2}{m}\tau$ **[1]**

(ii) (b) Here, current, $I = 250 \times 10^{-3}$A

Time, $t = 200$ ns $= 200 \times 10^{-9}$s

So, charge delivered by the acceleration per pulse

i.e. $\quad I = \frac{Q_{pulse}}{t}$

$\Rightarrow Q_{pulse} = It = (250 \times 10^{-3}\text{A})(200 \times 10^{-9}\text{s})$

$= 5.00 \times 10^{-8}$ C **[1]**

(iii) (c) Number of electrons delivered per pulse,

$$n = \frac{Q_{pulse}}{e} = \frac{5.00 \times 10^{-8}\text{ C}}{1.6 \times 10^{-19}\text{C}}$$

$= 3.13 \times 10^{11}$ electron / pulse **[1]**

(iv) (c) Average current delivered by the acceleration,

$$I_{av} = \frac{Q_{pulse}}{\Delta t} = \frac{5.00 \times 10^{-8}\text{ C}}{4.00 \times 10^{-3}\text{ s}} = 12.5\mu\text{A}$$ **[1]**

(v) (d) Maximum power delivered by the electron beam,

$$P = \frac{\Delta E}{\Delta t}$$

$$= \frac{(3.13 \times 10^{11}\text{ electrons/pulse})(40.0\text{ MeV/electron})}{2.00 \times 10^{-7}\text{ s/pulse}}$$

$(6.26 \times 10^{19}\text{ MeV/s})(1.6 \times 10^{-13}\text{ J/MeV})$

$= 1.00 \times 10^7$ W $= 10.0$ MW **[1]**

16. (i) (c) In total internal reflection, light travels from an optically denser medium to a rarer medium at the interface, it is partly reflected back into the same medium and partly refracted back to the second medium. **[1]**

(ii) (d) When the angle of incidence is more than a certain value, the angle of refraction becomes more than 90°. It result into total internal reflection (critical angle.) **[1]**

(iii) (c) Critical angle, $\mu = \frac{1}{\sin i_c} = \frac{1}{\sin 48.6} = \frac{1}{0.75} = \frac{4}{3}$ **[1]**

(iv) (d) Light cannot undergoes total internal reflection when it is travelling from air to water, i.e. from rarer to denser medium. **[1]**

(v) (a) From total internal reflection of light,

As we know that, $\mu = \frac{1}{\sin i_c} \Rightarrow \sin i_c = \frac{1}{\mu}$

As, $\sin i_c = 1/(3/2) = \frac{2}{3} = 0.6667$

$i_c = \sin^{-1}(0.6667) = 41.8°$ **[1]**

17. The internal potential barrier of germanium is 0.3 V therefore to overcome this barrier the potential of battery should be equal to or more than 0.3 V. **[1]**

Thus, the minimum voltage of battery = 0.3 V. If the diode is made of silicon, then the value of minimum voltage of battery is 0.7 V, as for silicon the potential barrier is 0.7 V. **[1]**

18. When a coil of N number of turns and area A is rotated in external magnetic field B, magnetic flux linked with the coil changes and hence an emf is induced in the coil. At an instant t, if e is the emf

FULLY SOLVED

[redacted]

induced in the coil, then alternating emf induced is given by

$$e = e_0 \sin \omega t \quad [1]$$

Maximum current, $i_0 = \frac{e_0}{R} = \frac{NBA\omega}{R}$

Given, $N = 1, B = 10^{-2}$ T

$r = 30 \text{ cm} = 0.3 \text{ m}$

$A = \pi(0.3)^2 \text{ m}^2$

$R = \pi^2 \, \Omega$

$f = \frac{200}{60} \text{ s}^{-1}$

and $\omega = 2\pi f = 2\pi\left(\frac{200}{60}\right)$

$\therefore \quad i_0 = \frac{1 \times 10^{-2} \times \pi (0.3)^2 \times 2\pi \times 200}{60 \times \pi^2}$

$= 6 \times 10^{-3} \text{ A} = 6 \text{ mA}$ [1]

Or

(i) Due to varying current in coil P, the flux linked with P changes. Hence, flux linked with coil Q changes which in turn induces an emf in Q. Thus, bulb B lights up. [1]

(ii) When Q is moved towards left or away from P, less amount of flux change takes place in Q. This leads to decrease in the value of rate of change of magnetic flux and hence, lesser emf and bulb B gets dimmer. [1]

19. (i) From the given electric field expression, we can say that wave is propagating along negative y-direction or its direction is $-\hat{j}$. [1]

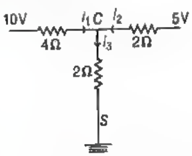
(ii) Comparing the given equation with the standard equation,

$$E = E_0 \cos\left[2\pi\left(\frac{y}{\lambda} + vt\right)\right]$$

we get $\frac{2\pi}{\lambda} = 1.8$ rad/m

or wavelength, $\lambda = \frac{2\pi}{1.8} = \frac{2 \times 3.14}{1.8} = 3.5$ m [1]

20. Let V be the potential at C.



Using Kirchhoff's first law, $i_1 + i_2 = i_3$ [1]

$$\frac{10 - V}{4} + \frac{5 - V}{2} = \frac{V - 0}{2}$$

$\Rightarrow \quad 10 - V + 10 - 2V = 2V$

$\Rightarrow \quad 5V = 20$

$\Rightarrow \quad V = 4 \text{ V}$

$\Rightarrow \quad i_3 = V/2 = 4/2 = 2\text{A}$ [1]

Or

Conductivity, $\sigma = \frac{ne^2\tau}{m}$

$\Rightarrow$ Relaxation time, $\tau = \frac{\sigma m}{ne^2}$

For material A, $\tau_A = \frac{\sigma m}{ne^2}$ [1]

For material B, $\tau_B = \frac{2\sigma m}{2ne^2} = \frac{\sigma m}{ne^2}$

For material C, $\tau_C = \frac{2\sigma m}{ne^2}$

$\therefore$ From the above relations, we can say that,

$$\tau_C > \tau_B = \tau_A \quad [1]$$

21. Electric field lines follow some important properties which are discussed below

(i) Electric field lines start from positive charges and end at negative charges. In the case of a single charge, they may start or end at infinity.

(ii) Tangent to any point on electric field lines shows the direction of electric field at that point. [1]

(iii) Two field lines can never intersect each other because if they intersect, then two tangents drawn at that point will represent two directions of field at that point, which is not possible.

(iv) In a charge free region, electric field lines can be taken to be continuous curves without any breaks. [1]

Or

Since, surface D enclosed negative charge, hence it has least flux negative. [1]

In parts C and A, there is zero net charge, hence flux is zero, surface B has most flux, which is positive in nature, since it consist positive charge, i.e. $+2q$. [1]

22. (i) Converging lens, i.e. convex lens. [1/2]

(ii) The power of spectacles was 1 D, i.e. for an object at infinity, image is formed at 100 cm by the lens

Power of accommodation gets reduced in old age. Generally, the ability of the lens to become thick and reduce its focal length (to see nearly objects) is lost. So, near vision is affected, this

[redacted]

can be corrected by using a convex (converging) lens of suitable focal length

Here, the reading glass he need is of power + 0.2 D. This implies his near point has receded to 26.3 cm from 25 cm. This can be calculated using thin lens formula.

Here, $P = +0.2\text{ D}$

$\Rightarrow \quad f = 5\text{ m} = 500\text{ cm}$

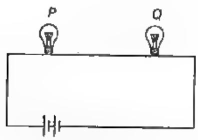
also, $u = 25\text{ cm}$

$\Rightarrow \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{25} = \frac{1}{500} - \frac{1}{25} = \frac{-19}{500}$

$\Rightarrow \quad v = -26.3\text{ cm}$ [1½]

23. Given, $\frac{R_P}{R_Q} = \frac{1}{2}$

$\therefore \quad R_Q = 2R_P$ ... (i)



In series, power dissipated is given by the relation

$$P = I^2R$$

or $\quad P \propto R$ (1)

$\therefore \quad \frac{P_P}{P_Q} = \frac{R_P}{R_Q}$ ... (ii)

Using Eqs. (i) and (ii), we get

$\therefore \quad \frac{P_P}{P_Q} = \frac{R_P}{2R_P} = \frac{1}{2}$ (1)

24. Here, $\lambda_1 = 590\text{ nm} = 590 \times 10^{-9}\text{ m}$,

$\lambda_2 = 596\text{ nm}$
$= 596 \times 10^{-9}\text{m}$

$d = 2 \times 10^{-4}\text{ cm}$
$= 2 \times 10^{-6}\text{ m}$

and $\quad D = 1.5\text{ m}$

Distance of first secondary maximum from the centre of the screen,

$$x = \frac{3}{2}\frac{D\lambda}{d}$$

For the two wavelengths, we have,

$$x_1 = \frac{3}{2}\frac{D\lambda_1}{d}$$

and $\quad x_2 = \frac{3}{2}\frac{D\lambda_2}{d}$ [1]

Spacing between the first two maximum of sodium lines

$$\therefore x_2 - x_1 = \frac{3D}{2d}(\lambda_2 - \lambda_1)$$

$$= \frac{3 \times 1.5}{2 \times 2 \times 10^{-6}}(596 \times 10^{-9} - 590 \times 10^{-9})$$

$$= 6.75 \times 10^{-3}\text{ m}$$

$$= 6.75\text{ mm}$$ [1]

25. When nucleons approach each other to form a nucleus, they strongly attract each other. Their potential energy decreases and becomes negative. It is the potential energy which holds the nucleons together in the nucleus. The decrease in PE results from the decrease in the mass of nucleons inside the nucleons. [2]

Hence, the mass of the nucleus is less than the sum of the masses of constituent neutrons and protons.

26. The wavelengths of spectral line in these series can be expressed by the following formulae

(i) **For Lyman series**

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \text{ where } n = 2, 3, 4, \ldots$$

For, $\quad n = \infty, \lambda = \frac{1}{R}$

(ii) **For Balmer series**

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \text{ where } n = 3, 4, 5, \ldots$$

For, $\quad n = \infty, \lambda = \frac{4}{R}$

(iii) **For Paschen series**

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), \text{ where } n = 4, 5, 6, \ldots$$ [1½]

For, $\quad n = \infty, \lambda = \frac{9}{R}$

(iv) **For Brackett series**

$$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right), \text{ where } n = 5, 6, 7, \ldots$$

For, $\quad n = \infty, \lambda = \frac{16}{R}$

(v) **For Pfund series**

$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right), \text{ where } n = 6, 7, 8, \ldots$$

For, $\quad n = \infty, \lambda = \frac{25}{R}$ [1½]

Or

(i) Bohr's second postulate states that, the electron revolves around the nucleus in certain privileged

[redacted]

orbit which satisfy certain quantum condition that angular momentum of an electron is an integral multiple of $\frac{h}{2\pi}$, where h is Planck's constant

i.e. $L = mvr = \frac{nh}{2\pi}$ [1]

where, m = mass of electron, v = velocity of electron and r = radius of orbit of electron.

$\Rightarrow \quad 2\pi r = n\left(\frac{h}{mv}\right)$

$\therefore$ Circumference of electron in n th orbit

$= n \times$ de-Broglie wavelength associated with electron $\left[\because \lambda = \frac{h}{mv}\right]$ [1]

(ii) Given, the electron in H-atom is initially in third excited state

$\therefore \quad n = 4$

and the total number of spectral lines of an atom that can exist is given by the relation

$$= \frac{n(n-1)}{2}$$ [1/2]

Here, $n = 4$

So, number of spectral lines

$$= \frac{4(4-1)}{2} = \frac{4 \times 3}{2} = 6$$

Hence, when a H-atom moves from third excited state to ground state, it emits six spectral lines. [1/2]

27. Given, $N = 200$ turns, $r = 10\,\text{cm} = 0.1\,\text{m}$, $B = 0.5\,\text{T}$

$\theta = 90^\circ$ and $I = 3\text{A}$

(i) As, $\tau = NIAB$

$= 200 \times 3 \times [\pi(0.1)^2] \times 0.5 \quad [\because A = \pi r^2]$

$\Rightarrow \tau = 9.42$ N-m [1]

(ii) The net magnetic force on circular loop is zero. [1/2]

(iii) Average force on electron,

$F = (e)(v_d) B \sin 90^\circ$

But, $I = neAv_d$

[where, A = cross-section of the wire]

$v_d = \frac{I}{neA}$

$\therefore \quad F = (e)\left(\frac{I}{neA}\right)B$

$F = \frac{IB}{nA} = \frac{3 \times 0.5}{10^{29} \times 10^{-5}}$

$\Rightarrow \quad F = \frac{1.5}{10^{24}}$

$\Rightarrow \quad F = 1.5 \times 10^{-24}$ N [1½]

28. Semiconductors in which some impurity atoms are embedded are known as extrinsic or impure semiconductors.

Extrinsic semiconductors are basically of two types

(i) n-type semiconductors

(ii) p-type semiconductors

n-type Semiconductors

This type of semiconductor is obtained when pentavalent impurity such as phosphorus (P), arsenic (As), etc is added to Si or Ge

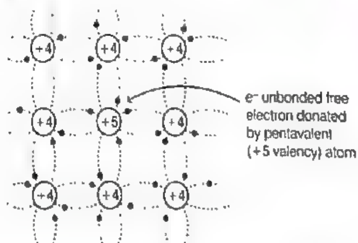
During doping four electrons of pentavalent atom bond with the four silicon neighbours while fifth remains very weakly bound to its parent atom. Also the ionisation energy required to set this electron free is very small

Hence these electrons are almost free to move. In other words, we can say that these electrons are donated by the impurity atoms.

So these are also known as **donor atoms** and the conduction inside the semiconductor will take place with the help of the negatively charged electrons. Due to this negative charge, these semiconductors are known as **n-type semiconductors.** [2]

Therefore major conduction in n-type semiconductors is due to electrons. So, electrons are known as **majority carriers** and the holes are known as the **minority carriers.**

This means, $n_e >> n_h$; $I_e >> I_h$



[1]

29. (i) A choke coil reduces the current in the AC circuit without any heat loss. [1]

(ii) The effective resistance of a choke coil

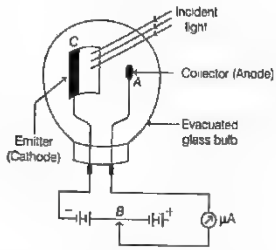
$Z = \sqrt{R^2 + (\omega L)^2}$

$\because$ Choke coil is a L-R device. [1]

(iii) One can control high frequency alternating current by using choke coil with air cores, these are called rf choke coils. While in case of low frequency AC, the core is of laminated soft iron. [1]

[redacted]

30. It is a device which converts light energy into electrical energy. It is also called an **electric eye.** As the photoelectric current is set up in the photoelectric cell corresponding to incident light it provides the information about the objects as done by our eye in the presence of light.



[1½]

A photocell consists of a semi-cylindrical photosensitive metal plate C (emitter) and a wire loop A (collector) supported in an evacuated glass or quartz bulb. When light of suitable wavelength falls on the emitter C, photoelectrons are emitted.

Some applications of photocell are given below

(i) Used in television camera for telecasting scenes and in photo telegraphy.

(ii) Reproduction of sound in cinema film.

(iii) Used in burglar alarm and fire alarm. [1½]

Or

According to de-Broglie hypothesis, the wavelength of wave associated with moving material particle is given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

which is the expression for de-Broglie wavelength. From the above expression the following observations are made [1]

(i) The de-Broglie wavelength $\lambda \propto \frac{1}{v}$. So if the particle moves faster, then the wavelength will be smaller and *vice-versa*.

(ii) If the particle is at rest ($v = 0$), then the de-Broglie wavelength is infinite ($\lambda = \infty$). Such a wave cannot be visualised.

(iii) The de-Broglie waves cannot be electromagnetic in nature because electromagnetic waves are produced by motion of accelerated charged particles.

(iv) The wavelength of a wave associated with moving particle defines a region of uncertainty, within which the whereabouts of the particle are unknown. [2]

31. (i) By Gauss's law, $\phi = \oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$

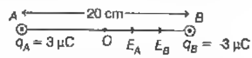
where, q is the net charge enclosed in the gaussian surface.

(a) ϕ does not change because it depends only on the total charge enclosed by the gaussian surface and not on its shape or size. [1/2]

(b) ϕ will not change because the total flux is determined by the charge inside the surface, not on the charge outside. [1/2]

(c) ϕ becomes zero because a dipole consists of two equal and opposite charges and so the net charge inside the surface is zero. Thus, the flux is also zero. [1/2]

(ii) Given, $AB = 20$ cm



$$AO = OB = 10\text{ cm} = 0.1\text{m}$$
$$q_A = 3\,\mu C = 3 \times 10^{-6}\text{ C}$$
$$q_B = -3\,\mu C = -3 \times 10^{-6}\text{ C}$$

The electric field at a point due to a charge q is

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$ [1/2]

where r is the distance between charge and the point.

Electric field due to q_A at O,

$$E_A = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|q_A|}{(AO)^2}$$

$$E_A = \frac{9 \times 10^9 \times 3 \times 10^{-6}}{(0.1)^2} = \frac{27 \times 10^3}{0.1 \times 0.1}$$

$$= 2.7 \times 10^{-6}\text{ N/C}$$

The direction of E_A is A to O, i.e. towards O or towards OB as the electric field is always directed away from positive charge. [1]

Electric field due to q_B at O,

$$E_B = \frac{1}{4\pi\varepsilon_0} \cdot \frac{|q_B|}{(OB)^2}$$

$$E_B = \frac{9 \times 10^9 \times 3 \times 10^{-6}}{(0.1)^2}$$

FULLY SOLVED

[redacted]

$$= \frac{27 \times 10^3}{0.1 \times 0.1} = 2.7 \times 10^6 \text{ N/C}$$

The direction of E_B is O to B, i.e. towards B or towards OB as the electric field is always directed towards the negative charge.

Now, we see that both E_A and E_B are in same direction. So, the resultant electric field at O is E.

Hence, $E = E_A + E_B = 2.7 \times 10^6 + 2.7 \times 10^6$

$$= 5.4 \times 10^6 \text{ N/C}$$

The direction of E (resultant electric field) will be from O to B or towards B. **[2]**

Or

(i) Here, the dipole moment of each molecule $= 10^{-29}$ C-m.

1 mole of the substance contains 6×10^{23} molecules.

Therefore, total dipole moment of all molecules,

$$p = 6 \times 10^{23} \times 10^{-29} \text{C-m} = 6 \times 10^{-6} \text{ C-m}$$

Initial potential energy,

$$U_i = -pE\cos\theta = -6 \times 10^{-6} \times 10^6 \cos 60° = -3 \text{ J}$$

Final potential energy (when $\theta = 0°$),

[finally p and E are aligned in the same direction]

$$U_f = -6 \times 10^{-6} \times 10^6 \cos 0° = -6 \text{ J}$$

Change in potential energy

$$= -6\text{J} - (-3\text{J}) = -3 \text{ J}$$

So, there is a loss in potential energy. This must be the energy released by the substances in the form of heat in aligning its dipole. **[3]**

(ii) Initially, energy stored in the capacitor can be given as $E = \frac{1}{2}C_1V^2$

When the switch S is connected to point 2, then energy dissipated on connecting across 8 μF will be

$$E' = \frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)V^2$$

$$= \frac{1}{2}\left(\frac{2 \times 8}{10}\right)V^2$$

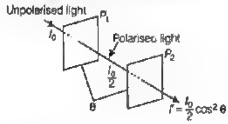
$$= \frac{1}{2} \times \frac{16}{10} \times V^2$$

Therefore, % loss of energy

$$= \left(\frac{E'}{E}\right) \times 100\% = \left(\frac{\frac{1}{2} \times \frac{16}{10}V^2}{\frac{1}{2} \times 2 \times V^2}\right) \times 100\% = 80\%$$

[2]

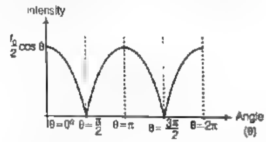
32. (i) The figure when unpolarised light beam is passed through polaroid light is shown below.



By law of Malus,

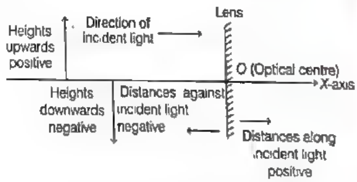
Intensity received after $P_2 = I' = \frac{I_0}{2}\cos^2\theta$. **[1]**

Variation of intensity with angle θ is shown below.



[1]

(ii) To derive the relevant formulae for refraction by spherical lenses, we must first adopt a sign convention for measuring distances as shown below



According to the cartesian sign convention,

(a) All the distances are measured from the optical centre(O) of the lens.

(b) The principal axis of lens is taken as X-axis and optical centre as origin.

(c) Distances measured in the direction of the incident light are taken as positive and opposite to the direction of incident light as negative.

(d) The heights measured upwards with respect to X-axis and normal to the principal axis of the lens are taken as positive and the heights measured downwards are taken as negative. **[3]**

Or

(i) Given $i = 60°$ and $\mu = \sqrt{3}$

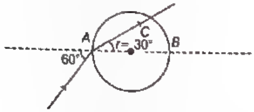
From Snell's law, we have,

$$\frac{\sin i}{\sin r} = \mu \Rightarrow \frac{\sin 60°}{\sin r} = \sqrt{3}$$ **[1]**

FULLY SOLVED

[redacted]

$$\sin r = \frac{\sin 60^\circ}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$$



$$\sin r = 0.5$$

$\Rightarrow \quad r = \sin^{-1}(0.5)$

$\Rightarrow \quad r = 30^\circ$ **[1]**

(ii) **For convex lens** For erect image

$u = -ve, v = +ve$

Magnification, $m = \frac{I}{O} = \frac{-v}{u}$

where, O = length of object

and I = length of image.

Given, $f = +20$ cm, $I = 4 \times$ length of object

$\Rightarrow \quad \frac{I}{O} = 4 \Rightarrow \frac{v}{-u} = 4$

$\Rightarrow \quad v = -4u$ **[1]**

Using lens formula,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{(-4u)} - \frac{1}{(-u)} \Rightarrow \frac{1}{f} = -\frac{1}{4u} + \frac{1}{u}$$

$\Rightarrow \quad \frac{1}{20} = \frac{4-1}{4u} = \frac{3}{4u}$

$\Rightarrow \quad u = \frac{20 \times 3}{4} = 15$ cm **[1]**

$\Rightarrow u = 15$ cm, $v = 4u = 15 \times 4 = 60$ cm

Distance of the object, $u = 15$ cm

Distance of the image, $v = 60$ cm

The image is on the same side of the object. **[1]**

33. (i) The number of turns per unit length is

$n = \frac{N}{l} = \frac{500}{0.5} = 1000$ turns/m **[1/2]**

The length $(l) = 0.5$ m and radius $(r) = 1$cm $= 0.01$ m.

Thus, $l/r = 50$, i.e. $l >> r$.

Hence, we can use the long solenoid formula i.e.

$$B = \mu_0 nI = 4\pi \times 10^{-7} \times 10^3 \times 5$$

$$= 6.28 \times 10^{-3} \text{ T}$$ **[1]**

(ii) Number of turns, $N = 850 \times 5$, $l = 1$ m, $I = 5$ A

Area of cross-section,

$$A = \pi r^2 = \frac{22}{7}\left(\frac{3}{2} \times 10^{-2}\right)^2 \text{m}^2$$

Magnetic field at the centre of solenoid,

$B = \mu_0 NI/l$

$= 4\pi \times 10^{-7} \times (850 \times 5) \times 5/1$

$= 2.671 \times 10^{-2}$ T **(1)**

$\therefore$ Magnetic flux $= BA$

$$= 2.671 \times 10^{-2} \times \frac{22}{7} \times \left(\frac{3}{2} \times 10^{-2}\right)^2$$

$= 1.89 \times 10^{-5}$ Wb **[1]**

(iii) Given, $I_1 = 4$A, $I_2 = 5$A, $d = 4 \times 10^{-2}$ m

and $l = 0.20$ m

Force on a current carrying wire,

$$F = \frac{\mu_0 I_1 I_2 l}{2\pi d} = \frac{4\pi \times 10^{-7} \times 4 \times 5}{2 \times \pi \times 4 \times 10^{-2}} \times 0.20$$

$= 2 \times 10^{-5}$ N, towards B **[1½]**

Or

(i) Magnetic field induced at the centre of a circular loop, i.e.

$$B = \frac{\mu_0 I}{2r}$$

$$\Rightarrow 7 \times 10^{-5} = \frac{4\pi \times 10^{-7} \times I}{2 \times 5 \times 10^{-2}}$$

$\therefore$ Current in the circular loop,

$$I = \frac{7 \times 10^{-5}}{4\pi \times 10^{-6}} = 5.6 \text{ A}$$ **[1½]**

(ii) Yes, it is possible to have magnetic field configuration with three poles. It can be done by putting north poles or south poles of two magnets together.

Gauss's law of magnetism can be modified as

$$\Sigma \mathbf{B} \cdot d\mathbf{S} = \mu_0 m$$

where, m is the strength of the monopole. **[1½]**

(iii) (a) The field H is dependent of the material core and is expressed as

$$H = nI = 1000 \times 2.0 = 2 \times 10^3 \text{ Am}^{-1}$$

(b) Magnetisation,

$M = (B - \mu_0 H)/\mu_0$

As, $B = \mu_r \mu_0 H$

$\therefore \quad M = (B - \mu_0 H)/\mu_0$

$= (\mu_r \mu_0 H - \mu_0 H)/\mu_0$

$= (\mu_r - 1) H = (400 - 1) \times 2 \times 10^3$

$= 8 \times 10^5$ Am^{-1} **[2]**

FULLY SOLVED

[redacted]

SAMPLE QUESTION PAPER

A HIGHLY SIMULATED SAMPLE QUESTION PAPER FOR CBSE CLASS XII EXAMINATIONS

PHYSICS (UNSOLVED)

GENERAL INSTRUCTIONS

1. All questions are compulsory. There are 33 questions in all.
2. This question paper has five sections: Section A, Section B, Section C, Section D and Section E.
3. **Section A** contains ten very short answer questions and four assertion reasoning MCQs of 1 mark each, **Section B** has two case based questions of 4 marks each, **Section C** contains nine short answer questions of 2 marks each, **Section D** contains five short answer questions of 3 marks each and **Section E** contains three long answer questions of 5 marks each.
4. There is no overall choice. However internal choice is provided. You have to attempt only one of the choices in such questions.

TIME : 3 HOURS **MAX. MARKS : 70**

SECTION-A

All questions are compulsory. In case of internal choices, attempt anyone of them.

1. A telescope consists of two lenses of focal lengths 20 cm and 5 cm. Obtain its magnifying power when the final image is at 25 cm from the eye. [Ans. – 4.8]

Or

A concave lens of focal length 5 cm produces an image $\frac{1}{4}$ times than that of the size of the object. Calculate the distance of the object from the lens. [Ans. – 15 cm]

2. Electromotive force of primary cell is 2.4 V. When cell is short-circuited, then current becomes 4 A. What is the internal resistance of cell? [Ans. 0.6 Ω]

3. Two identical induction coils, each of inductance L joined in series are placed very close to each other such that the winding direction of one is exactly opposite to that of other. Find the net inductance.

Or A copper loop and a silver loop are removed from a magnetic field in the same time-interval. In which loop will the induced emf and induced current be greater?

4. What is the electric field intensity at a point between two parallel plates with like charges of same surface charge densities (σ)? [Ans. zero]

Or

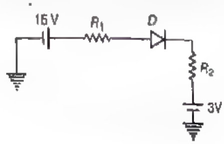
A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is C, then find the resultant capacitance. [Ans. $(n-1)C$]

5. What changes occur in electrical conductivity of a pure semiconductor, on heating?

6. What is the number of neutrons in a ${}_{84}Po^{218}$ nucleus? [Ans. 134]

7. A platinum wire has resistance of 10Ω at 0°C and of 20Ω at 273°C. Find the temperature coefficient of resistance of platinum wire. [Ans. 1/213 $°C^{-1}$]

8. In the following diagram, is the junction diode forward biased or reverse biased?



[redacted]

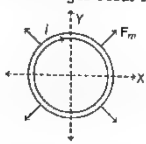
UNSOLVED

9. What is the electrostatic potential at the surface of a silver nucleus of diameter 12.4 fermi? Atomic number (Z) for silver is 47. [Ans. 1.09×10^7 V]

10. A circular coil of 30 turns and radius 8 cm carrying a current of 6 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1 T. The field lines make an angle 60° with the normal of the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning. [Ans. 3.13 N-m]

Or

A conducting loop carrying a current I is placed in a uniform magnetic field pointing into the plane of the paper as shown in figure. What changes occur in the loop?



For question numbers 11, 12, 13 and 14, two statements are given-one labelled Assertion (A) and the other labelled **Reason** (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

11. **Assertion** Magnetism is relativistic in nature, i.e. stationary charges do not produce magnetic field.

Reason When we move along the charge i.e. there is no relative motion, then we find no magnetic field associated with the charge.

12. **Assertion** Inductance coils are made of copper.

Reason Induced current is more in wire having less resistance.

13. **Assertion** Atoms of each element are stable and emit characteristic spectrum.

Reason The spectrum provides useful information about the atomic structure.

14. **Assertion** The applied voltage (in forward bias of a p-n junction) mostly drops across the depletion region and the voltage drop across the p-side and n-side of the junction is negligible.

Reason Resistance of depletion region is large compared to resistance of n or p-side.

Answers

11. (a)	12. (a)	13. (b)	14. (a)

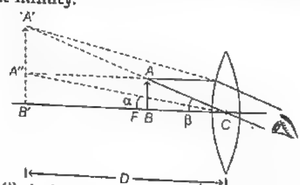
SECTION-B

Questions 15 and 16 are case study based questions and are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

Simple Microscope

15. Microscope is an optical instrument which forms large image of close and minute objects. A simple microscope is a converging lens of small focal length. When an object is at a distance less than the focal length of the lens, the image obtained is virtual, erect and magnified.

When the object is at a distance equal to the focal length of the lens, the image is formed at infinity.



(i) A simple microscope has a limited maximum magnification
(a) greater than 9 (b) lesser than 9
(c) equal to 9 (d) Both (b) and (c)

(ii) In order to increase the angular magnification of a simple microscope, one should increase
(a) the object size
(b) the aperture of the lens
(c) the focal length of the lens
(d) the power of the lens

[redacted]

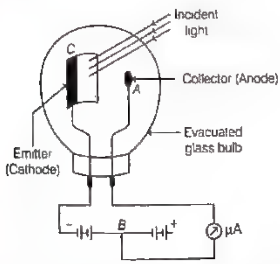
(iii) The image formed by an objective of a compound microscope is
(a) virtual and diminished
(b) real and diminished
(c) real and enlarged
(d) virtual and enlarged

(iv) The distance between the second focal point of the objective f_o and first focal point of the eyepiece, i.e. f_e is called
(a) tube length
(b) focal length
(c) image distance
(d) radius of curvature

(v) For compound microscope, $f_o = 1$ cm, $f_e = 2.5$ cm. An object is placed at distance 1.2 cm from object lens. What should be the length of microscope for normal adjustment?
(a) 8.5 cm (b) 8.3 cm
(c) 6.5 cm (d) 6.3 cm

Photocell

16. Photocell is a device which converts light energy into electrical energy. It is also called an electric eye. As, the photoelectric current sets up in the photoelectric cell corresponding to incident light, it provides the information about the objects as has been seen by our eye in the presence of light.



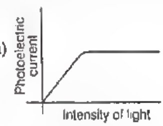
A photocell consists of a semi-cylindrical photosensitive metal plate C (emitter) and a wire loop A (collector) supported in an evacuated glass or quartz bulb. When light of suitable wavelength falls on the emitter C, photoelectrons are emitted.

(i) A photocell cannot be used
(a) for reproduction of sound in motion pictures
(b) in burglar alarms
(c) as a fire alarm
(d) to illuminate a room

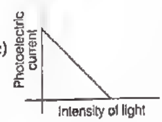
(ii) It is observed that no electrons are emitted when frequency of light is less than a certain minimum frequency.
This minimum frequency depends on
(a) potential difference of emitter and collector plates
(b) distance between collector and the emitter plate
(c) size (area) of the emitter plate
(d) material of the emitter plate

(iii) The work function of a metal used in photocell is hc/λ_0. If light of wavelength λ is incident on its surface, then the essential condition for the electron to come out from the metal surface is
(a) $\lambda \geq \lambda_0$
(b) $\lambda \geq 2\lambda_0$
(c) $\lambda \leq \lambda_0$
(d) $\lambda \leq \lambda_0/2$

(iv) Variation of photoelectric current with intensity of light for a photocell is

(a)


(b)


(c)


(d)


(v) A photon of energy 3.4 eV is incident on a metal surface of a photocell whose work function is 2 eV. Maximum kinetic

[redacted]

energy of the photoelectron emitted by the metal surface will be

(a) 1.4 eV
(b) 1.7 eV
(c) 5.4 eV
(d) 6.8 eV

Answers

15. (i) d (ii) d (iii) c (iv) a (v) b
16. (i) b (ii) d (iii) c (iv) d (v) a

SECTION-C

All questions are compulsory. In case of internal choices, attempt anyone.

17. For spherically symmetrical charge distribution with charge density varying as

$\rho(x)=\rho_0\left(\frac{5}{4}-\frac{x}{R}\right)$

where, $x<r$, upto $r=R$

The electric field at a distance $(r<R)$ comes out to be $E=\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$

Justify the relation obtained.

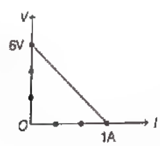
18. The bombardment of lithium with protons gives rise to the following reaction

${}^{7}_{3}\text{Li}+{}^{1}_{1}\text{H}\longrightarrow{}^{4}_{2}\text{He}+{}^{4}_{2}\text{He}+\text{Energy}$

The atomic masses of lithium, hydrogen and helium are 7.016 amu, 1.008 amu and 4.004 amu, respectively. Find the initial energy of each helium atom.

(Take, 1 amu $=931\ \text{MeV/c}^2$) [Ans. 7.488 MeV]

19. The plot of the variation of potential difference across a combination of three identical cells in series *versus* current is as shown in the figure. What is the emf of each cell?



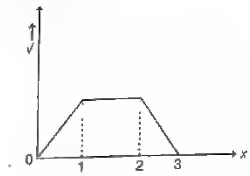
Or

(i) Two wires of equal lengths, one of copper and other of manganin have the same resistance. Which wire will be thicker?

(ii) How does the drift velocity of electrons in a metallic conductor vary with the increase in temperature?

20. What are extrinsic semiconductors? Write the names and types of dopants.

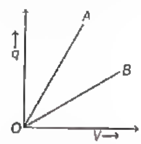
21. The electric potential V as a function of distance x is shown in the figure. Construct a graph of the electric field strength E *versus* distance x.



Or

The graph shows the variation of charge q *versus* potential difference V for two capacitors, C_1 and C_2. The two capacitors have the same plate separation but the plate area of C_2 is double than that of C_1.

Which of the lines in the graph corresponds to C_1 & C_2 and why?



22. A small telescope has an objective lens of focal length 150 cm and an eyepiece of focal length 5 cm. If this telescope is used to view a 100 m high tower 3 km away, find the height of the final image, when it is formed 25 cm away from the eyepiece and magnification of telescope is 36.

[Ans. – 30 cm]

[redacted]

23. (i) Name the electromagnetic waves which
(a) maintain the earth's warmth and
(b) are used in aircraft navigation.
(ii) To which regions of the electromagnetic spectrum do the following wavelength belong
(a) 250 nm (b) 1500 nm

Or

Show that the radiation pressure exerted by an EM wave of intensity I on a surface kept in vacuum is $\frac{I}{c}$.

24. (i) Write two characteristic features of nuclear force.
(ii) Draw a plot of potential energy of a pair of nucleons as a function of their separation.

25. If light of wavelength, $\lambda = 4000\ \text{Å}$ and intensity $100\ \text{W/m}^2$ incident on a metal plate of threshold frequency 5.5×10^{14} Hz, what will be the maximum kinetic energy, and work function of photoelectron? (Take, $h = 6.6 \times 10^{-34}$ J-s). [Ans. 1.32×10^{-19} J]

SECTION-D

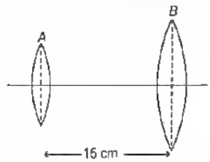
All questions are compulsory. In case of internal choices, attempt anyone.

26. A metallic rod of length l and resistance R is rotated with a frequency ν, with one end hinged at the centre and the other end at the circumference of a circular metallic ring of radius l, about an axis passing through the centre and perpendicular to the plane of the ring. A constant and uniform magnetic field B parallel to the axis is present everywhere.
(i) Derive the expression for the induced emf and the current in the rod.
(ii) Due to the presence of the current in the rod and of the magnetic field, find the expression for the magnitude and direction of the force acting on this rod.

Or

(i) The reactance of a capacitor of capacitance C is X. If both the frequency and capacitance be doubled, then give the expression of the new reactance of circuit.
(ii) A wire of resistance R is connected in series with an inductor of reactance ωL, then derive the mathematical expression of quality factor of RL circuit.

27. Two convex lenses A and B of an astronomical telescope having focal lengths 5 cm and 20 cm, respectively are arranged as shown below



(i) Which one of the two lenses you will select as the objective lens and why?
(ii) What should be the change in the distance between the lenses to have the telescope in its normal adjustment position? [Ans. 10]
(iii) Calculate the magnitude of magnifying power of the telescope in the normal adjustment position. [Ans. 4]

28. The force experienced by a unit charge when placed at a distance of 0.10 m from the middle of an electric dipole on its axial line is 0.025 N and when it is placed at a distance of 0.2 m, the force is reduced to 0.002 N. Calculate the dipole length. [Ans. 0.10 m]

Or

A point charge causes an electric flux $-3 \times 10^{-14}\ \text{N-m}^2/\text{C}$ to pass through a spherical Gaussian surface.
(i) Calculate the value of the point charge. [Ans. -2.655×10^{-25} C]
(ii) If the radius of the Gaussian surface is doubled, how much flux would pass through the surface?

29. A p-n junction diode is basically a p-n junction with metallic contacts provided at the ends for the application of an external voltage. It can be biased in two

[redacted]

ways, i.e. an external battery can be connected to it two ways. One is forward biasing in which positive terminal of the battery is connected to p-side and negative terminal to n-side of the diode. In this large amount of current passes through the diode. However, the second is reverse biasing in which positive terminal of the battery is connected to n-side and negative terminal to p-side of the diode. In this negligible amount of current passes through the diode.

Now, on the basis of above mentioned information, draw an observation in your words, when an input waveform as shown in Fig. (a) is applied to the circuit as shown in Fig. (b).

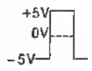


Fig. (a)

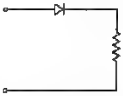


Fig. (b)

30. With the help of a suitable diagram, explain in brief about the sensitivity of Wheatstone bridge.

SECTION-E

All questions are compulsory. In case of internal choices, attempt anyone.

31. Show that the refractive index of the material of a prism is given by

$$\mu = \frac{\sin \frac{(A+\delta_m)}{2}}{\sin\left(\frac{A}{2}\right)}$$

where, symbols have their usual meanings.

Or

(i) When the width of the slit is made double, how would this effect the size and intensity of the central diffraction band? Justify your answer with the help of diagram.

(ii) Write three characteristic features to differentiate between diffraction and interference.

32. (i) The coil area of a galvanometer is $25 \times 10^{-4}\, m^2$. It consists of 150 turns of a wire and is in a magnetic field of 0.15 T. The restoring torque constant of the suspension fibre is 10^{-6} N-m per degree.

Assuming the magnetic field to be radial, calculate the maximum current that can be measured by the galvanometer, if the scale can accommodate 30° deflection.

[Ans. 5.3×10^{-4} A]

(ii) An electron in H-atom circles around the proton with a speed $3 \times 10^6\ ms^{-1}$ in an orbit of radius 6×10^{-11}m.

Calculate

(a) the equivalent current and

(b) magnetic field produced at the proton.

Given, charge on electron is 1.6×10^{-19} C and $\mu_0 - 4\pi \times 10^{-7}\ TmA^{-1}$.

[Ans. (a) 1.28×10^{-3} A, (b) 13.4 T]

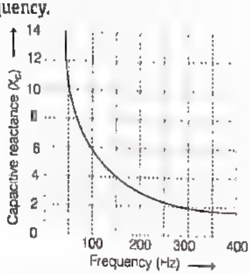
Or

(i) Write the four important properties of the magnetic field lines due to a bar magnet.

(ii) A wire of length L is bent in the form of a circle of radius R and carries current I. What is its magnetic moment?

(iii) A bar magnet of length 0.1 m and a pole strength 10^{-4} A-m is placed in a magnetic field of 30 Wb/m at an angle 30°. Determine the couple acting on it.

33. Figure shown below, shows how the reactance of a capacitor varies with frequency.



(i) Use the information of the graph to calculate the value of capacitance of capacitor.

[Ans. 2.65×10^{-4} F]

[redacted]

(ii) An inductor of inductance L has the same reactance as the capacitor at 100 Hz. Find the value of L.
[Ans. 9.55×10^{-3} H]

(iii) Using the same axes, draw a graph of reactance against the frequency for the inductor given in part (ii).

(iv) If this capacitor and inductor were connected in series to a resistor of 10 Ω, what would be the impedance of the combination at 300 Hz?

(v) A charged 30 μF capacitor is connected to a 27 mH inductor. What is the angular frequency of free oscillations of the circuit? [Ans. 1.1×10^{-3} rad s^{-1}]

Or

Draw a labelled diagram of AC generator, explain its theory and working. An armature coil consists of 20 turns of wire, each of area, $A = 0.09\,\text{m}^2$ and total resistance 15 Ω rotates in a magnetic field of 0.5T at a constant frequency $(150/\pi)$ Hz.

Calculate the value of maximum emf produced in the coil. [Ans. 270 V]

[redacted]

SAMPLE QUESTION PAPER 3

A HIGHLY SIMULATED SAMPLE QUESTION PAPER FOR CBSE CLASS XII EXAMINATIONS

PHYSICS (UNSOLVED)

GENERAL INSTRUCTIONS

1. All questions are compulsory. There are 33 questions in all.
2. This question paper has five sections: Section A, Section B, Section C, Section D and Section E.
3. **Section A** contains ten very short answer questions and four assertion reasoning MCQs of 1 mark each, **Section B** has two case based questions of 4 marks each, **Section C** contains nine short answer questions of 2 marks each, **Section D** contains five short answer questions of 3 marks each and **Section E** contains three long answer questions of 5 marks each.
4. There is no overall choice. However internal choice is provided. You have to attempt only one of the choices in such questions.

TIME : 3 HOURS **MAX. MARKS : 70**

SECTION-A

All questions are compulsory. In case of internal choices, attempt anyone of them.

1. The electric field of an electromagnetic wave is given by

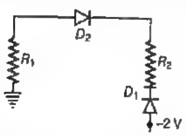
$E_z = 5\times10^{-7}\sin(2\times10^3 x + 8.28\times10^{12} t)$

What is the wavelength of the electromagnetic wave? [Ans. 0.32 cm]

Or

Give the definition of electromagnetic wave and general equation of electric field in electromagnetic wave.

2. Which one of the two diodes D_1 and D_2 in the given figure is



(i) forward biased (ii) and reverse biased?

3. A single slit of width d is illuminated by violet light of wavelength 400 nm and the width of the diffraction pattern is measured as y. When half of the slit width is covered and other half illuminated by yellow light of wavelength 600 nm, then how much the width of diffraction pattern becomes?

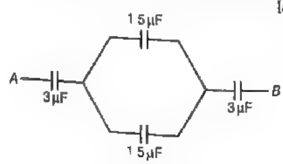
Or

What are the conditions for total internal reflection to take place?

4. On which principle a transformer works?

5. What is the motion of charge carriers in forward bias and reverse bias of a p-n junction?

6. Find the equivalent capacitance between the points A and B in the following circuit. [Ans. 1 μF]



Or

Can a body has charge of 1.5e, where e is the electronic charge?

[redacted]

7. Write the name of spectral series of hydrogen atom, which lies in visible range of electromagnetic wave.

8. The maximum voltage in AC circuit is 141 V. What is the effective voltage in circuit? [Ans. 100 V]

Or

Voltage V and current i in AC circuit are given by, $V = 100\sin(50t)$ V and $i = 200\sin\left(50t + \frac{\pi}{3}\right)$mA. Calculate the power dissipated in circuit. [Ans. 10 W]

9. Why does the conductivity of metals decrease with increase in temperature?

10. Why does the width of depletion layer of p-n junction increase in reverse biasing?

For question numbers 11, 12, 13 and 14, two statements are given-one labelled Assertion (A) and the other labelled **Reason** (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

11. Assertion If Gaussian surface does not enclose any charge, then **E** at any point on the Gaussian surface must be zero.

Reason No net charge is enclosed by Gaussian surface, so net flux passing through the surface is non-zero.

12. Assertion Critical angle of light passing from glass to air is minimum for violet colour.

Reason The wavelength of violet light is greater than the light of other colours.

13. Assertion To observe diffraction of light the size of obstacle aperture should be of the order of 10^{-7}m.

Reason 10^{-7}m is the order of wavelength of visible light.

14. Assertion When two long straight wires are connected to a battery, they may come close to each other.

Reason Force of attraction acts between two wires carrying current.

Answers

11. (d)	12. (c)	13. (a)	14. (c)

SECTION-B

Questions 15 and 16 are case study based questions and are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

Gauss' Law in Magnetism

15. We can write Gauss' law for magnetostatics as $\int \mathbf{B} \cdot dA = \mu_0 m$, where $\int \mathbf{B} \cdot dA$ is the magnetic flux and m is the net magnetic pole strength inside a closed surface.

It was found that, magnetic flux through a closed surface is always zero. Thus, we reach at a conclusion that magnetic monopoles do not exist. A bar magnet always attains north-south poles no matter how many times it is cut into pieces.

(i) The net magnetic flux due to a bar magnet over a closed surface is

(a) zero (b) $\frac{\mu_0}{4\pi}$

(c) $4\pi\mu_0$ (d) $\frac{4\mu_0}{\pi}$

(ii) The presence of magnetic monopoles is ruled out by

(a) Gauss' law of electrostatics

(b) Gauss law of magnetism

(c) Faraday's law

(d) Ampere's circuital law with Maxwell's addition

(iii) A bar magnet is separated four times, the number of monopoles and dipoles formed are

(a) 0, 8 (b) 0, 4

(c) 8, 2 (d) 8, 1

(iv) The number of magnetic field lines passing through a surface area normally is called

(a) magnetic field (b) electric field

(c) magnetic flux (d) electric flux

UNSOLVED

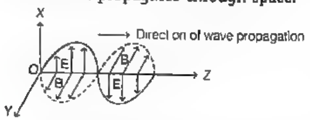
[redacted]

UNSOLVED

(v) The dimensional representation of magnetic flux density is
(a) $[MLT^{-2}]$ (b) $[MLT^{-2}A^{-1}]$
(c) $[MT^{-2}A^{-1}]$ (d) $[MLT^{-2}A^{2}]$

Oscillating Charge

16. An oscillating charge is an example of accelerating charge. It produces an oscillating electric field in space, which produces an oscillating magnetic field, which in turn produces an oscillating electric fields and so on. The oscillating electric and magnetic fields regenerate each other as a wave which propagates through space.



(i) Electromagnetic waves can be deflected by
(a) only electric field
(b) only magnetic field
(c) Both (a) and (b)
(d) None of the above

(ii) Total energy density of electromagnetic waves in vacuum is given by the relation
(a) $\frac{1}{2}\cdot\frac{E^2}{\varepsilon_0}+\frac{B^2}{2\mu_0}$
(b) $\frac{1}{2}\varepsilon_0 E^2+\frac{1}{2}\mu_0 B^2$
(c) $\frac{E^2+B^2}{c}$
(d) $\frac{1}{2}\varepsilon_0 E^2+\frac{B^2}{2\mu_0}$

(iii) The speed of electromagnetic wave in vacuum depends upon the source of radiation
(a) increases as we move from γ-rays to radio waves
(b) decreases as we move from γ-rays to radio waves
(c) is same for all of them
(d) None of the above

(iv) Solar radiation is
(a) transverse electromagnetic wave
(b) longitudinal electromagnetic wave
(c) stationary wave
(d) None of the above

(v) A plane electromagnetic wave of frequency 25 MHz travels in free space along the x-direction. At a particular point in space and time, $E = 6.3$ J-V/m. What is B at this point?
(a) $2.1\times 10^{-8}\hat{k}$ T
(b) $2.1\times 10^{8}\hat{k}$ T
(c) $3.5\times 10^{6}\hat{k}$ T
(d) $3.0\times 10^{5}\hat{k}$ T

Answers

15. (i) a (ii) b (iii) b (iv) c (v) c
16. (i) d (ii) d (iii) c (iv) a (v) a

SECTION-C

All questions are compulsory. In case of internal choices, attempt anyone.

17. A 10 kg satellite circles earth once energy 2h in an orbit having a radius of 8000 km. Assuming that Bohr's angular momentum postulate applies to satellites just as it does to an electron in hydrogen atom, find the quantum number of the orbit of the satellite. [Ans. 5.3×10^{45}]

18. Why is it that while using a moving coil galvanometer as a voltmeter, a high resistance in series is required? Also draw the circuit diagram for a voltmeter.

19. Car batteries are often rated in unit ampere hours. Does this unit designate the amount of current, energy, power or charge that can be drawn from the battery? Explain.

20. Draw energy band diagrams for germanium and wood.

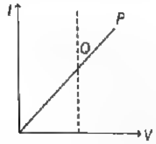
21. Four identical cells, each of emf 8 V and internal resistance 2.5 Ω are connected in series and charged by a 100 V DC supply, using a 24 Ω resistor in series.
Calculate
(i) charging current in the circuit
(ii) and potential difference across the cells during recharging. [Ans. (i) 2 A, (ii) 52 V]

Or

Figure below shows a plot of current *versus* voltage for two different

[redacted]

materials P and Q. Which of the following two materials satisfies Ohm's law?



22. Define the following terms and give their source of origin.
(i) Plane wavefront
(ii) Cylindrical wavefront

23. Give any two applications of eddy currents.

Or

Define the following terms
(i) Power factor
(ii) Wattless current

24. Derive an expression for the force between two long parallel current carrying conductors.

25. Draw a graph to show the variation of stopping potential with frequency of radiation incident on a metal plate. How can the value of Planck's constant be determined from this graph?

Or

Consider figure for photoemission. How would you reconcile with momentum conservation? Note light (photons) have momentum in a different direction than the emitted electrons.

SECTION-D

All questions are compulsory. In case of internal choices, attempt anyone.

26. (i) Why do we need the oil drops in Millikan's experiment to be of microscopic sizes? Why cannot we carry out the experiment with bigger drops?
(ii) What happens to the wavelength of a photon after it collides with an electron?
(iii) Can X-rays cause photoelectric effect?

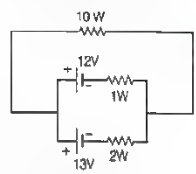
27. (i) Why is the core of a nuclear reactor one of its most important part?
(ii) Why is the number of neutrons in heavier nuclei more than the number of protons?
(iii) Name the element with which control rods in nuclear reactors are made up.

28. For a given lens, the magnification was found to be twice as large when the object was 0.15 m distant from it than when the distance was 0.2 m. What is the focal length of the lens? [Ans. 0.10 m]

Or

An astronomical telescope has objective and eyepiece of focal lengths 40 cm and 4 cm, respectively. Find the distance by which the lenses must be separated, so that image of an object 200 cm away from the objective can be seen at infinity. Also draw the ray diagram. [Ans. 54 cm]

29. Two batteries with emf 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω, respectively. The voltage across the load can be calculated as below



For parallel combination of cells,

$$E_{eq} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{\frac{12}{1} + \frac{13}{2}}{\frac{1}{1} + \frac{1}{2}} = \frac{37}{3} \text{ V}$$

Potential drop across 10 Ω resistance,

$$V = \left(\frac{E}{R_{total}}\right) \times 10 = \frac{37/3}{\left(10 + \frac{2}{3}\right)} \times 10$$

$$= 11.56 \text{ V}$$

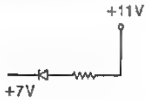
$\therefore \quad V = 11.56$ V

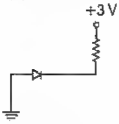
Now, give an alternate solution of above problem.

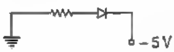
[redacted]

UNSOLVED

30. In the following figures, indicate which of the diodes are forward biased and which are reverse biased?

(i) 

(ii) 

(iii) 

Or

Predict the effect on the electrical properties of a silicon crystal at room temperature, if every millionth silicon atom is replaced by an atom of indium. Given, concentration of silicon atoms $= 5 \times 10^{28}\ m^{-3}$, intrinsic carrier concentration $= 1.5 \times 10^{16}\ m^{-3}$, $H_e = 0.135\ m^3/V\text{-}s$ and $H_h = 0.048 m^3/V\text{-}s$.

SECTION-E

All questions are compulsory. In case of internal choices, attempt anyone.

31. (i) What is the focal length of the combination of a convex lens of focal length 30 cm in contact with a concave lens of focal length 20 cm? Is the system a converging or a diverging lens? Ignore thickness of the lenses. [Ans. 60 cm]

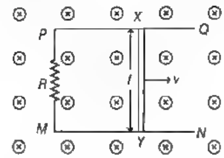
(ii) At what angle should a ray of light be incident on the face of a prism of refracting angle 60°, so that it just suffers total internal reflection at the other face? The refractive index of the material of the prism is 1.524. [Ans. 29°75']

Or

(i) In Young's double slit experiment using monochromatic light L_1 of wavelength 700 nm, 10th bright fringe was obtained at a certain point P on a screen. Which bright fringe will be obtained at the same point P, if monochromatic light of wavelength 500 nm is used in place of L_1? (No other alterations were made in the experimental set-up.) [Ans. 14]

(ii) Monochromatic light of wavelength 650 nm falls normally on a slit of width 1.3×10^{-4} cm and the resulting Fraunhofer diffraction is obtained on a screen. Find the angular width of the central maxima. [Ans. 1 rad]

32. (i) PQ and MN are two parallel conductors at a distance l apart and connected by a resistance R as shown in figure. They are placed in a magnetic field B which is perpendicular to the plane of the conductors.



A wire XY is placed over PQ and MN and, then made to slide over PQ and MN with a velocity v. Neglecting the resistance of PQ, MN and wire XY, calculate the work done per second to slide the wire XY.

(ii) Magnetic flux through a coil of resistance R changes by an amount $\Delta\phi$ during a small time interval Δt. Calculate the total quantity of charge that passes through any cross-section in the coil during this time interval.

Or

(i) State the underlying principle of a transformer. How is the large scale transmission of electric energy over long distances done with the use of transformers?

(ii) A step-down transformer operated on a 2.5 kV line. It supplies a load with 20 A. The ratio of the primary winding to the secondary is 10 : 1. If the transformer is 90% efficient, calculate

[redacted]

(a) the power output,
(b) the voltage
(c) and the current in the secondary coil.
[Ans. (a) 4.5×10^4 W, (b) 250 V and (c) 180 A]

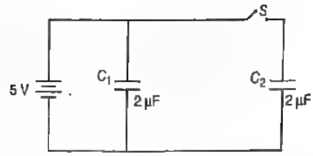
33. (i) Two insulated charged copper spheres A and B have identical sizes and charge 6.5×10^{-7} C on each and their centres are separated by distance of 50 cm.

A third sphere C of same size but uncharged is brought in contact with first, then brought in contact with the second and finally removed from both, find the new force of interaction between spheres A and B.
[Ans. 5.7×10^{-3} N]

(ii) Figure shows two identical capacitors C_1 and C_2, each of $2\,\mu$F capacitance, connected to a battery of 5 V. Initially switch S is closed.

After sometime, S is left open and dielectric slabs of dielectric constant $K = 5$ are used and inserted to fill completely the space between the plates of the two capacitors.

How will the (a) charge and (b) potential difference between the plates of the capacitors be affected after the slabs are inserted?
[Ans. (b) 1 V]

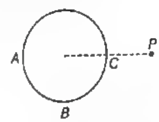


Or

(i) Intensity of electric field at a perpendicular distance of 0.5 m from an infinitely long line charge having linear charge density (λ) is 3.6×10^3 Vm^{-1}. Find the value of λ.
[Ans. 10^{-7} cm^{-1}]

(ii) Three equal charges of 5.0 μC each are placed at the three vertices of an equilateral triangle of side 5.0 cm each. Calculate the electrostatic potential energy of the system of charges.
[Ans. 13.5 J]

(iii) A hollow conducting sphere is placed in an electric field produced by a point placed at P as shown in figure.



Let V_A, V_B and V_C be the potentials at points A, B and C respectively.
Then, find out the relation between V_A, V_B and V_C.

[redacted]

SAMPLE QUESTION PAPER 4

A HIGHLY SIMULATED SAMPLE QUESTION PAPER FOR CBSE CLASS XII EXAMINATIONS

PHYSICS (UNSOLVED)

GENERAL INSTRUCTIONS

1. All questions are compulsory. There are 33 questions in all.
2. This question paper has five sections: Section A, Section B, Section C, Section D and Section E.
3. **Section A** contains ten very short answer questions and four assertion reasoning MCQs of 1 mark each, **Section B** has two case based questions of 4 marks each, **Section C** contains nine short answer questions of 2 marks each, **Section D** contains five short answer questions of 3 marks each and **Section E** contains three long answer questions of 5 marks each.
4. There is no overall choice. However internal choice is provided. You have to attempt only one of the choices in such questions.

UNSOLVED

TIME : 3 HOURS **MAX. MARKS : 70**

SECTION-A

All questions are compulsory. In case of internal choices, attempt anyone of them.

1. The north pole of a long horizontal bar magnet is being brought closer to a vertical conducting plane along the perpendicular direction. What will be the direction of induced current in the conducting plane?

Or

An alternating voltage is connected in series with a resistance R and inductance L. If the potential drop across the resistance is 200 V and across the inductance is 150 V, then find the applied voltage. [Ans. 250 V]

2. If the energy of a photon of sodium light ($\lambda = 580\,\text{nm}$) equals the band gap of semiconductor, then find the minimum energy required to create hole-electron pair. [Ans. 2.1 eV]

3. What is the effect of heating of a conductor on the drift velocity of free electrons?

4. Two charges $5\mu C$ and $10\mu C$ are placed 1 m apart. What amount of work is done to bring these charges at a distance 0.5 m from each other? ($k - 9\times10^9$ SI) [Ans. 9×10^{-1} J]

5. What is the de-Broglie wavelength of a electron accelerated through a potential difference of 100 V? [Ans. 1.227 Å]

Or

In what way has the wave nature of electron beam exploited in electron microscope?

6. A galvanometer having internal resistance $10\,\Omega$ requires 0.01 A for a full scale deflection. To convert this galvanometer to a voltmeter of full scale deflection at 120 V, what value of series resistance is needed? [Ans. $11990\,\Omega$]

7. The electron in hydrogen atom is initially in the third excited state. What is the maximum number of spectral lines which can be emitted when it finally moves to the ground state? [Ans. 6]

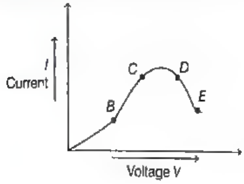
8. Give any two characteristics of semiconductor material.

9. Give the ratio of radii of the orbits corresponding to second excited state and ground state in a H-atom. [Ans. 9 : 1]

Or What is the ratio of nuclear densities of the two nuclei having mass numbers in the ratio 1 : 4? [Ans. 1 : 1]

[redacted]

10. Graph given below shows the variation of current *versus* voltage for a material GaAs.



Identify the region of negative resistance.

Or

How is current kept continuous inside a conductor of finite length?

For question numbers 11, 12, 13 and 14, two statements are given-one labelled Assertion (A) and the other labelled **Reason** (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A

(c) A is true but R is false.

(d) A is false and R is also false.

11. Assertion Ferromagnetic substances are those which gets strongly magnetised when placed in an external magnetic field.

Reason The individual atoms (or ions or molecules) in a ferromagnetic material possess a dipole moment as in a paramagnetic material.

12. Assertion E in outside vicinity of a conductor depends only on the local charge density σ and it is independent of the other charges present anywhere on the conductor.

Reason E in outside vicinity of a conductor is given by $\frac{\sigma}{\varepsilon_0}$.

13. Assertion If a plane glass slab is placed on the letters of different colours all the letters appear to be raised up to different height.

Reason Different colours have different wavelengths.

14. Assertion The drift velocity of electrons in a metallic wire decreases when temperature of the wire is increases.

Reason On increasing temperature, conductivity of metallic wire decreases.

Answers

11. (c)	12. (d)	13. (a)	14. (b)

SECTION-B

Questions 15 and 16 are case study based questions and are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

Electrical Resonance

15. Electrical resonance is said to take place in a series *L-C-R* circuit when the circuit allows maximum current for a given frequency of the source of alternating supply for which capacitive reactance becomes equal to the inductive reactance. Impedance of this *L-C-R* circuit is minimum and hence current is maximum. Resonant circuits are used to respond selectively to signals of a given frequency while discriminating against signals of different frequencies. If the response of the circuit is more narrowly peaked around the chosen frequency, we say that the circuit has higher "selectivity or sharpness". This sharpness is measures with the help of Q-factor.

(i) Bandwidth of the resonant *L-C-R* circuit is

(a) $\frac{R}{L}$ (b) $R/2L$

(c) $\frac{2R}{L}$ (d) $\frac{4R}{L}$

(ii) To reduce the resonant frequency in an *L-C-R* series circuit with a generator

(a) the generator frequency should be reduced

(b) another capacitor should be added in paralle to the first

(c) the iron core of the inductor should be removed

(d) dielectric in the capacitor should be removed

UNSOLVED

[redacted]

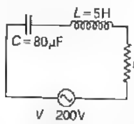
(iii) In a series L-C-R circuit, the capacitance C is changed to $4C$. To keep the resonant frequency same, the inductance must be changed by

(a) $2L$ (b) $L/2$
(c) $4L$ (d) $L/4$

(iv) In non-resonant circuit, what will be the nature of circuit for frequencies higher than the resonant frequency?

(a) Resistive (b) Capacitive
(c) Inductive (d) None of these

(v) Figure shows a series L-C-R circuit, connected to a variable frequency 200 V source. $C = 80\ \mu F$ and $R = 40\ \Omega$. The source frequency which drives the circuit at resonance is



(a) 25 Hz (b) $\frac{25}{\pi}$ Hz
(c) 50 Hz (d) $\frac{50}{\pi}$ Hz

Dual Nature of Matter

16. Matter cannot exist both as a particle and as a wave simultaneously. At a particular instant of time, it is either the one or the other aspect, i.e. the two aspects are complementary to each other.

According to de-Broglie, a wave is associated with moving material particle which controls the particle in every respect. The wave associated with moving material particle is called matter wave or de-Broglie wave whose wavelength is called de-Broglie wavelength.

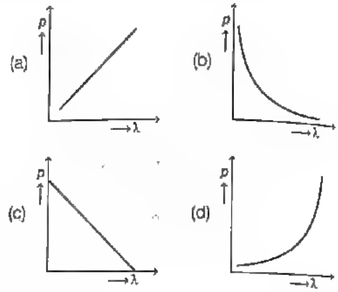
(i) The de-Broglie wave of a moving particle does not depend on

(a) mass (b) charge
(c) velocity (d) momentum

(ii) The de-Broglie wavelength of a particle of KE, K is λ. What will be the wavelength of the particle, if it's kinetic energy is $\frac{K}{9}$?

(a) λ (b) 2λ
(c) 3λ (d) 4λ

(iii) Which of the following figures represent the variation of particle momentum and the associated de-Broglie wavelength?



(iv) de-Broglie wavelength associated with an electron, accelerating through a potential difference of 100 V lies in the region of

(a) Gamma rays (b) X-rays
(c) Ultraviolet rays (d) Visible region

(v) A proton and an α-particle are accelerated through the same potential difference. The ratio of de-Broglie wavelength λ_p to that of λ_α is

(a) $\sqrt{2}:1$ (b) $\sqrt{4}$ 1
(c) $\sqrt{6}$ 1 (d) $\sqrt{8}:1$

Answers

15. (i) b (ii) b (iii) d (iv) c (v) b
16. (i) b (ii) c (iii) b (iv) b (v) d

SECTION-C

All questions are compulsory. In case of internal choices, attempt anyone.

17. Write two characteristic features to distinguish between n-type and p-type semiconductors.

18. A rectangular loop of length $l = 2$m and breadth $b = 0.3$m is placed at distance of $x = 0.6$m from infinitely long wire carrying current, $I = 2$A such that the direction of current is parallel to breadth. If the loop moves away from the current wire in a direction perpendicular to it with a velocity

[redacted]

$v = 3\text{ms}^{-1}$, what will be the magnitude of emf in the loop? [Ans. 4.61×10^{-7} V]

Or

A resistance of 20 Ω is connected to a source of alternating current rated 110 V, 50 Hz. Find the
(i) rms current
(ii) maximum instantaneous current in the resistor [Ans. (i) 5.5 A , (ii) 7.8 A]

19. Name the part of the electromagnetic spectrum which is
(i) suitable for radar systems used in aircraft navigation.
(ii) produced by bombarding a metal target with high speed electrons.

20. How many electrons pass through a lamp in 1 min, if the current is 300 mA? Given, the charge on an electron is 1.6×10^{-19} C. [Ans. 1.125×10^{20}C]

Or

Find the current flow through a copper wire of length 0.2 m, area of cross-section 1 mm^2, when connected to a battery of 4 V. Given that, for electron mobility is $4.5 \times 10^{-6} \text{m}^2\text{s}^{-1}\text{V}^{-1}$ and charge on an electron is 1.6×10^{-19}C. The number density of electrons in copper wire is 8.5×10^{28} m^{-3}. [Ans. 1.22 A]

21. Give any two differences between electric charge and mass.

22. Two lenses of power 10D and –5D are placed in contact.
(i) Calculate the power of lens combination. [Ans. 5D]
(ii) Where should an object be held from the lens, so as to obtain a virtual image of magnification 2? [Ans. – 10 cm]

23. Draw a labelled graph to show, how electrical resistance varies with temperature for
(i) a metallic wire
(ii) a piece of carbon.

24. State Bohr's postulate of hydrogen atom that gives the relationship for the frequency of emitted photon in a transition.

Or

Would the Bohr's formula for the H-atom remains unchanged, if proton had a charge $(+4/3)\,e$, and electron had a charge $(-3/4)e$, where, $e = 1.6 \times 10^{-19}$ C . Give reasons for your answer.

25. Even though an electric field E exerts a force qE on a charged particle yet electric field of an electromagnetic wave does not contribute to the radiation pressure (but transfers energy). Explain.

SECTION-D

All questions are compulsory. In case of internal choices, attempt anyone.

26. (i) Why is the mass of a nucleus always less than the sum of masses of constituents, neutrons and protons?
(ii) What is obtained by fusion of two deuterons?
(iii) $^3_2$He and $^3_1$He nuclei have the same mass number .
Do they have same bindng energy?

27. (i) A diverging lens of focal length f is cut into two identical parts, each forming a plano-convex lens. What is the focal length of each part? [Ans. 2f]
(ii) A ray of light passes through an equilateral glass prism such that the angle of incidence is equal to angle of convergence and each of these angles is equal to $\frac{3}{4}$ of angle of prism. What is the value of angle of deviation? [Ans. 30°]

28. A straight wire carrying a current of 10 A is bent into a semi-circular arc of radius 2.0 cm as shown in the figure. What is the magnetic field at O due to
(i) straight segments
(ii) the semi-circular arc?

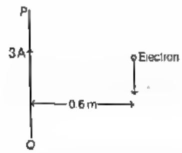
Or

(i) A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries a current of 0.40 A. What is the magnitude of magnetic field at the centre of the coil? [Ans. 3.14×10^4 T]

[redacted]

(ii) PQ is a long straight conductor carrying a current of 3A as shown in figure below. An electron moves with a velocity of 2×10^7 ms^{-1} parallel to it. Find the force acting on the electron.

[Ans. 32×10^{-19} N]



29. (i) Out of blue and red lights, which is more deviated by a prism? Give reason.

(ii) Give one application of prism.

(iii) If a prism of 5° angle gives deviation of 3.2°, then what will be the refractive index of prism? [Ans. 1.64]

30. (i) What is the ratio of the number of holes and the number of conduction electrons in an intrinsic semiconductor?

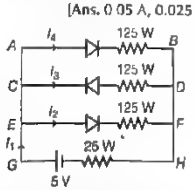
(ii) Draw the energy band diagram of n-type semiconductor.

(iii) Draw I *versus* V graph of a forward biased junction diode.

Or

If each diode in figure has a forward bias resistance of 25 Ω and infinite resistance in reverse bias, what will be the values of the currents I_1, I_2, I_3 and I_4?

[Ans. 0.05 A, 0.025 A, 0 A, 0.05 A]

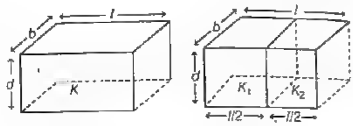


SECTION-E

All questions are compulsory. In case of internal choices, attempt anyone.

31. (i) A aluminium sheet of area 1m^2 is fixed on the top of a 2m insulating slab by a man outside his house one evening. Will he get an electric shock, if he touches the metal sheet next morning? [Ans. Yes]

(ii) Two identical capacitors of plate dimensions $l \times b$ and plate separation d have dielectric slabs filled in between the space of the plates as shown in the figures. $\left[\text{Ans. } \frac{2k}{k_1 + k_2}\right]$



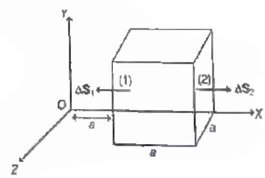
Find the ratio capacitance in each case.

(iii) In SI unit, V at a point in an electric field is given by $V = -\frac{6}{x} + 2$. Find the value of **E** in the field at the point (2, 0, 0). [Ans. 1.5 units]

Or

State Gauss's law in electrostatics.

The electric field components in figure shown are $E_x = \alpha x^{1/2}$, $E_y = E_z = 0$ in which $\alpha = 800\ \text{N/Cm}^{1/2}$.



Calculate

(i) the electric flux through the cube

(ii) the charge within the cube

where, the side of cube = 0.1 m.

[Ans. (i) 1.05 Nm^{2}C^{-1}, (ii) 9.2×10^{-12}C]

32. What is diffraction of light? Draw a graph showing the variation of intensity with angle in a single slit diffraction experiment. Write one feature which distinguishes the observed pattern from the double slit interference pattern.

How would the diffraction pattern of a single slit be affected when

[redacted]

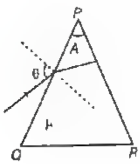
(i) the width of the slit is decreased?

(ii) the monochromatic source of light is replaced by a source of white light?

Or

(i) In Young's double slit experiment, the intensity of central maxima is I. What will be the intensity at the same place, if one slit is closed?

(ii) Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ and ray incident at an angle θ, on the face PQ. Now prove that, the ray only get transmitted through the face PR of prism, if $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$



(iii) Write conditions for sustained interference.

33. An electron beam passes through a region in which a magnetic field of 2×10^{-3} T and an electric field 3.4×10^4 V/m both acting simultaneously at right angles to each other. If the path of the electron remains undeflected, calculate the speed of the electrons. If the electric field is removed, what will be the radius of the circular path of electrons? [Ans. 1.7×10^7 m/s and 5 cm]

Or

(i) The magnetic field B and the magnetic intensity H in a material are found to be 1.6 T and 1000 Am^{-1}, respectively. Determine the relative permeability μ_r and the susceptibility χ_m of the material. [Ans. 1.27×10^3]

(ii) A solenoid of 600 turns per metre is carrying a current of 4 A. Its core is made of iron with relative permeability of 5000. Calculate the magnitudes of magnetic intensity, intensity of magnetisation and magnetic field inside the core. [Ans. 15 T]

[redacted]

SAMPLE QUESTION PAPER 5

A HIGHLY SIMULATED SAMPLE QUESTION PAPER FOR CBSE CLASS XII EXAMINATIONS

YSICS (UNSOLVED)

GENERAL INSTRUCTIONS

1. All questions are compulsory. There are 33 questions in all.
2. This question paper has five sections: Section A, Section B, Section C, Section D and Section E.
3. **Section A** contains ten very short answer questions and four assertion reasoning MCQs of 1 mark each, **Section B** has two case based questions of 4 marks each, **Section C** contains nine short answer questions of 2 marks each, **Section D** contains five short answer questions of 3 marks each and **Section E** contains three long answer quest ons of 5 marks each.
4. There is no overall choice. However internal choice is provided. You have to attempt only one of the choices in such questions.

TIME : 3 HOURS **MAX. MARKS : 70**

UNSOLVED

SECTION-A

All questions are compulsory. In case of internal choices, attempt anyone of them

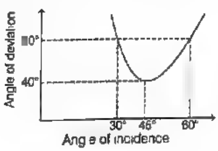
Two long straight wires are set parallel to each other. Each carries a current in the same direction and the separation between them is $2r$. Find out the intensity of the magnetic field mid-way between them. [Ans. 0]

When the voltage drop across a p-n junction diode is increased from 0.65 V to 0.70 V, the change in the diode current is 5 mA. What is the dynamic resistance of diode? [Ans. 10 Ω]

What are the dark lines seen in the solar spectrum called?

Draw the graphs that shows in a pure resistor, the voltage and current are in phase?

A plot of angle of deviation D *versus* angle of incidence, for a triangular prism is shown below



Calculate the angle of incidence for which the light ray travels parallel to the base.

Or

A slit of size 0.15 cm is placed at 2.1 m from a screen. On illuminating it by a light of wavelength 5×10^{-5} cm, calculate the width of central maxima.

6. When an electron jumps from the orbit $n=2$ to $n=4$, then calculate the wavelength of the radiations absorbed. (R is Rydberg's constant) [Ans. 16/3R]

Or

Two nuclei have mass numbers in the ratio 125 : 216, what is the ratio of their radii? [Ans. 5/6]

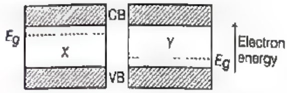
7. 2 m long wire is moved with a velocity 1 ms^{-1} in a magnetic field of intensity 0.5 Wbm^{-2} in a direction perpendicular to the field. What is the value of emf induced? [Ans. 1 V]

Or If an AC main supply is given to be 220 V, then what would be the average emf during a positive half-cycle? [Ans. 198 V]

8. Write the formula for kinetic mass of a moving photon.

9. The energy band diagrams for two semiconductor samples of silicon are as shown below

[redacted]



What will you infer from the above diagrams?

10. What is the angle of refraction made by a ray of light inside a prism, i.e. an equilateral glass prism in the minimum deviation?

[Ans. 30°]

Or

Two monochromatic light waves of same amplitudes of 2 A interferring at a point have a phase difference of 60°. What is the relation between intensity and amplitude?

[Ans. $I \propto 12\,A^2$]

For question numbers 11, 12, 13 and 14, two statements are given-one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

11. **Assertion** Ferromagnetic substances are those which gets strongly magnetised when placed in an external magnetic field.

Reason The individual atoms (or ions or molecules) in a ferromagnetic material possess a dipole moment as in a paramagnetic material.

12. **Assertion** Resonance phenomenon is exhibited by a circuit only, if both L and C are present in the circuit.

Reason Voltage across L and C cancel each other and the current amplitude is V_m/R, the total source voltage appearing across R causes resonance.

13 **Assertion** Heinrich Hertz observed that high voltage spark across detector loop were enhanced when the emitter plate was illuminated by UV-light.

Reason Light shining on the metal surface facilitate the escape of free electrons.

14 **Assertion** The applied voltage (in forward bias of a p-n junction) mostly drops across the depletion region and the voltage drop across the p-side and n-side of the junction is negligible.

Reason Resistance of depletion region is large compared to resistance of n or p-side.

SECTION-B

Questions 15 and 16 are case study based questions and are compulsory. Attempt any 4 sub parts from each question. Each question carries 1 mark.

Microscope

15. The instruments like microscope, telescope, etc., which are used to assist the eye in viewing an object are known as optical instruments.

Microscope is used to see very small object. It forms a large image of closed and minute objects.

There are of two types; simple microscope and compound microscope

Simple microscope consists of a single convex lens (converging lens) of small focal length. In this type of microscope, when an object is at a distance less than the focal length of the lens, the image obtained is virtual, erect and magnified.

On the other hand, compound microscope consists of two convex lenses coaxially separated by some distance. The lens nearer to the object is called the objective. The lens through which the final image is viewed is called the eyepiece.

(i) Which one of the following statements is incorrect?

(a) A simple magnifier or microscope is a converging lens of small focal length.

(b) For microscope the magnification in case of image formed at infinity is one more than the magnification when image is at the near point.

(c) For larger magnifications, one uses two lenses, one compounding the effect of the other. This is known as a compound microscope.

[redacted]

(d) The lens nearest the object is called objective and the lens near eye is called eyepiece.

(ii) In order to increase the angular magnification of a simple microscope, one should increase
(a) the object size
(b) the aperture of the lens
(c) the focal length of the lens
(d) the power of the lens

(iii) The image formed by an objective of a compound microscope is
(a) virtual and diminished
(b) real and diminished
(c) real and enlarged
(d) virtual and enlarged

(iv) For realistic focal length, a simple microscope has a limited maximum magnification
(a) greater than 9 (b) lesser than 9
(c) equal to 9 (d) Both (b) and (c)

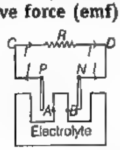
(v) A compound microscope has an objective with magnification 20, an eyepiece with magnification 12.5 and a tube length of 20 cm, then the magnification of this microscope is
(a) 200 (b) 250
(c) 100 (d) 150

UNSOLVED

EMF of a Cell

16. An electric cell is a source of energy that maintains a continuous flow of charge in a circuit. It changes chemical energy into electrical energy. It has two electrodes, positive electrode (PA) and negative electrode (NB) as shown below.

Electric cell has to do some work in maintaining the current through a circuit. The work done by the cell in moving unit positive charge through the whole circuit (including the cell) is called the **electromotive force (emf)** of the cell.



(i) When two electrodes (positive and negative) of a cell are immersed in an electrolytic solution, the charges are exchanged between
(a) positive electrode and electrolyte only
(b) negative electrode and electrolyte only
(c) Both electrodes and electrolytes
(d) directly between two electrodes

(ii) The current flowing in the cell is
(a) $I = \frac{\varepsilon}{R+r}$ (b) $I = \frac{R+r}{\varepsilon}$
(c) $I = \frac{R}{\varepsilon}$ (d) $I = r/\varepsilon$

(iii) The maximum current that can be drawn from a cell is for
(a) R – infinity
(b) R = finite non-zero resistance
(c) $R = 0$ (d) $R = r$

(iv) When R is infinite, then potential difference V between P and N is
(a) ε (b) 2ε
(c) $\varepsilon/2$ (d) $\varepsilon/4$

(v) For the given circuit, if the cell has an emf of 2V and the internal resistance of this cell is $0.1\,\Omega$, it is connected to resistance of $3.9\,\Omega$, the voltage across the cell will be
(a) 1.95 V (b) 1.5 V
(c) 2 V (d) 1.8 V

SECTION-C

All questions are compulsory. In case of internal choices, attempt anyone.

17. What is the difference between the values of potential difference across the two terminals of a cell in an open circuit and closed circuit?

Or

Define mobility of a charge carrier. Write the relation expressing mobility in terms of relaxation time. Give its SI unit.

18. (i) Define one tesla using the expression of the magnetic force acting on a particle of charge q moving with velocity v in a magnetic field B.

(ii) Is it possible to decrease or increase the range of given voltmeter? Explain it.

Or Two identical magnets with a length 100 cm are arranged freely with their like poles facing in a vertical glass tube. The upper

[redacted]

magnet hangs in air above the lower one so that the distance between the nearest poles of the magnet is 3 mm. If the pole strength of the pole of these magnets is 6.64 A-m, then determine the force between the two magnets. [Ans., $F = 0.49$ N]

19. Complete the following fission reaction and how do we calculate the amount of energy it releases?

$$^{1}_{0}n + ^{235}_{92}\text{U} \rightarrow ^{88}_{38}\text{Sr} + ^{136}_{54}\text{Xe} + ?$$

20. A ray of light is normally incident on one face of an equilateral prism. Trace the course of the ray through the prism and emerging from it. ($\mu_g = 3/2$)

21. (i) Identify the electromagnetic waves whose wavelengths vary as

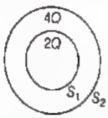
(a) 10^{-12} m $< \lambda < 10^{-8}$ m

(b) 10^{3}m $< \lambda < 10^{1}$ m

[Ans. (a) X-ray, (b) radio wave]

(ii) Describe the displacement current.

22. Consider two hollow concentric spheres S_1 and S_2 enclosing charges $2Q$ and $4Q$ respectively as shown in the figure.



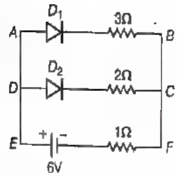
(i) Find out the ratio of the electric flux through them. [Ans. 1 : 8]

(ii) How will the electric flux through the sphere S_1 changes, if a medium of dielectric constant ε_r is introduced in the space inside S_1 in place of air. Deduce the necessary expression. [Ans. $2Q/\varepsilon_0\varepsilon_r$]

23. Suppose a pure Si crystal has 5×10^{28} atoms m^{-3}. It is doped by 1 ppm concentration of pentavalent. Calculate the number of electrons and holes. Given that, $n_i = 1.5\times 10^{16}$ m^{-3}.

[Ans. $n_e = 5\times 10^{22}$ m^{-3}, $n_h = 4.5\times 10^{9}$m^{-3}]

Or Assuming that the two identical diodes D_1 and D_2 are used in the electric circuit as shown in the figure are ideal. Find out the value of current flowing through 1 Ω resistor. [Ans. (30/11) A]



24. Answer the following questions.

(i) In any AC circuit, is the applied instantaneous voltage equal to the algebraic sum of instantaneous voltage across the series elements of the circuit? Is the same true for rms voltage?

(ii) An applied voltage signal consists of superposition of a DC voltage and an AC voltage of high frequency. The circuit consists of an inductor and a capacitor in series. Show that the DC signal will appear across C (capacitor) and the AC signal across L (inductance).

25. An AC input signal of frequency 60 Hz is rectified by

(i)half-wave and (ii) full wave rectifier.

Draw the output waveform and write the output frequency in each case.

[Ans. (i) 60 Hz, (ii) 120 Hz]

SECTION-D

All questions are compulsory. In case of internal choices, attempt anyone.

26. A neutron of mass $(m) = 1.66\times 10^{-27}$kg having energy $(E) = 8.28\times 10^{-21}$J at 127° C is moving in a waveform, then its de-Broglie wavelength can be calculated as.

[given, Boltzmann constant, $k = 1.38\times 10^{-23}$J mol^{-1} K^{-1} and Planck's constant, $h = 6.63\times 10^{-34}$ J-s.]

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.63\times 10^{-34}}{\sqrt{2\times 1.66\times 10^{-27}\times 8.28\times 10^{21}}}$$

$$\lambda = 1.264\times 10^{-10}\text{ m}$$

$$= 1.264\text{ Å}$$

If the energy of neutron will not be given, then suggest an alternative method to find the wavelength. [Ans. 1.264Å]

[redacted]

27. (i) If ε_0 & μ_0 are the electric permittivity & magnetic permeability of free space and ε & μ are the corresponding quantities in the medium. Find the index of refraction of the medium in terms of above parameter. [Ans. $(\mu\varepsilon/\mu_0\varepsilon_0)^{1/2}$]

(ii) An electromagnetic wave is travelling in vacuum with a speed of 3×10^8 m/s. Find the velocity in a medium having relative electric permittivity and magnetic permeability 2 and 1, respectively. [Ans. $\frac{3}{\sqrt{2}} \times 10^8$ m/s]

28. (i) A charged particle is free to move in an electric field. Will it always move along the line of force?

(ii) Two point charges of unknown magnitudes and signs are placed at a distance apart. The electric field is zero at a point, not between the charges but on the line joining them. Write two essential conditions for this to happen.

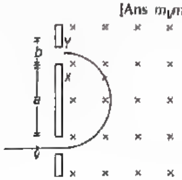
Or A parallel plate capacitor of capacitance C is charged to a potential V by a battery. Without disconnecting the battery, the distance between the plates is tripled and a dielectric medium of $K = 10$ is introduced between the plates of the capacitor. Explain giving reasons, how will the following be affected

(i) capacitance of the capacitor,
(ii) charge on the capacitor and
(iii) energy density of the capacitor.

[Ans. (i) 10/3 C, (ii) 10/3 CV and (iii) 10U]

29. A beam of equally charged particles after being accelerated through a voltage V enters into a magnetic field B as shown in figure. It is found that all the particles hit the plate between X and Y, then what is the ratio between the masses of the heaviest and lightest particles of the beam?

[Ans $m_1/m_2 = (a+b)^2/a^2$]



Or

A current carrying loop consists of 3 identical quarter circles of radius R, lying in the positive quadrants of the x-y, y-z and z-x planes with their centres at the origin, joined together. Find the direction and magnitude of B at the origin.

30. (i) When four hydrogen nuclei combine to form a helium nucleus, estimate the amount of energy in MeV released in this process of fusion. (Neglect the masses of electrons and neutrinos). Given, mass of ${}^1_1H = 1.007825$ u and mass of helium nucleus $= 4.002603$ u.

[Ans. 26.72 MeV]

(ii) A proton and an electron have same kinetic energy. Which one has greater de-Broglie wavelength and why?

SECTION-E

All questions are compulsory. In case of internal choices, attempt anyone.

31. (i) A rod of length l is moved horizontally with a uniform velocity v in a direction perpendicular to its length through a region in which a uniform magnetic field is acting vertically downward. Derive the expression for the emf induced across the ends of the rod. How does one understand this motional emf by involving the Lorentz force acting on the free charge carriers of the conductor? Explain.

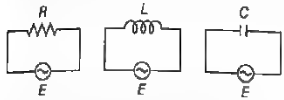
(ii) With the help of a neatly drawn labelled diagram, prove that the magnitude of motional emf e is given by $e = Blv$, where l is the length of a metallic rod and v is the velocity with which it is pulled in a transverse magnetic field B.

Or (i) When an AC source is connected to an ideal inductor, show that the average power supplied by the source over a complete cycle is zero.

(ii) Three electrical circuits having AC sources of variable frequency are shown in the figures. Initially, the current flowing in each of these is same. If the

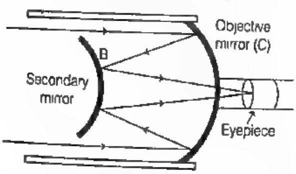
[redacted]

frequency of the applied AC source is increased, then how will the current flowing in these circuits be affected? Give the reason for your answer.



32. (i) Double convex lenses are to be manufactured from a glass of refractive index 1.55 with both faces of the same radius of curvature. What is the radius of curvature required, if the focal length is 30 cm? [Ans. 33 cm]

(ii) A Cassegrain telescope (reflecting telescope) uses two mirrors as shown in figure below. Such a telescope is built with the mirrors 20 mm apart. If the radius of curvature of the large mirror is 220 mm and of the small mirror is 140 mm, then where will be the final image of an object at infinity?



Or

(i) In a single slit diffraction experiment, a slit of width d is illuminated by red light of wavelength 650 nm. For what value of d will

(a) the first minimum fall is at an angle of diffraction of 60° and

(b) the first maximum fall is at an angle of diffraction of 60°?

(ii) In Young's double slit experiment, the two slits 0.15 mm apart are illuminated by monochromatic light of wavelength 450 nm. The screen is 1.0 m away from the slits.

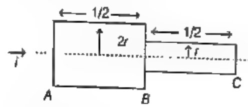
(a) Find the distance of the second

I. bright fringe

II. and dark fringe from the central maxima.

(b) How will the fringe pattern change, if the screen is moved away from the slits?

33. (i) Two bars of radii r and $2r$ are kept in contact as shown in the figure. An electric current I is passed through the bars. Find the ratio of heat produced in bars AB and BC.

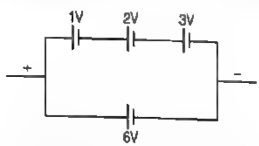


(ii) Two conducting wires A and B of same diameter but different materials are joined in series across a battery. If the number density of electrons in A is that in y, find the ratio of drift velocity of electrons in the two wires.

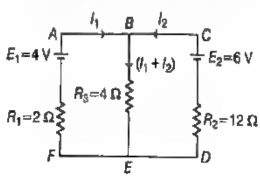
(iii) A secondary cell after a long use has an emf of 1.9 V and a large internal resistance of 380Ω. What maximum current can be drawn from the cell? Could the cell drive the starting motor of a car? [Ans. 0.005 A]

Or

(i) Find the emf of the battery shown in the figure: [Ans. 6 V]



(ii) In the circuit given E_1 and E_2 are two cells of emfs 4 V and 6 V respectively, having negligible internal resistances. Applying Kirchhoff's laws of electrical networks, find the values of I_1 and I_2. [Ans. $I_1 = 0.5$ A and $I_2 = 0.25$ A]



[redacted]

All in one

COMPLETE STUDY | COMPLETE PRACTICE | COMPLETE ASSESSMENT

Physics

CBSE Class 12

All in one Physics for Class 12th has been written specially for students studying in Class 12th with CBSE Curriculum. It is written by an experienced examiner, it provides all the explanation and guidance, you need to study efficiently and succeed in the exam.

The whole syllabus has been divided into chapters as per CBSE Curriculum. To make the students understand the chapter completely, each chapter has been divided into Individual Topics and each such topic has been treated as a separate chapter. Each topic has Detailed Theory, supported by Examples, Tables, Diagrams etc., followed by the questions grouped as Objective Type, Very Short Answer Type Questions, Short Answer Type Questions, Long Answer Type Questions. These questions cover NCERT & NCERT Exemplar Questions, Previous Years' CBSE Examinations' Questions and other Important Questions from examination point of view. To facilitate the easy learning and practice, explanations to all the questions have been given step-to-step.

FEATURES OF THE BOOK

- For the students to check their understanding of the chapter, a Chapter Practice has been given.
- Summary in each chapter is newly added features of the book.
- At the end of book, 5 Sample Question Papers have been given.

arihant

Arihant Prakashan (School Division Series)

Published by ARIHANT PUBLICATIONS (INDIA) LIMITED

Follow us on

9789326196437

Code : F976 ₹575.00

[redacted]

Document Outline